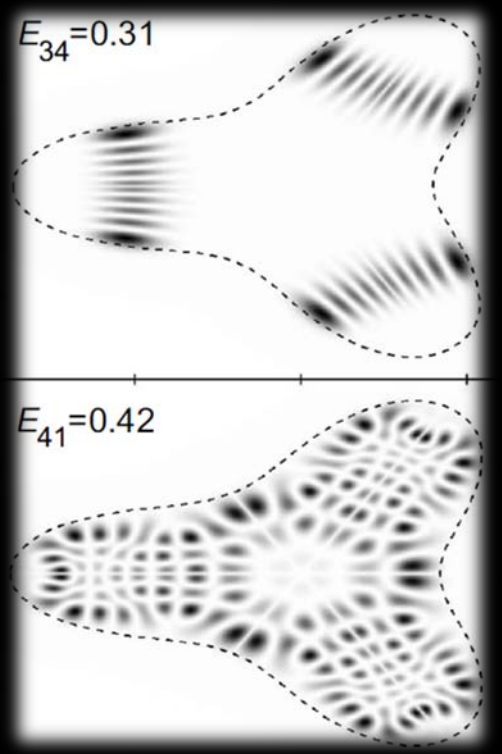
Suppression of chaos and occurrence of rotational bands at high energies



Based on

- ✓ *Occurrence of high-lying rotational bands in the interacting boson model*
  M.Macek, J.Dobeš, P.Cejnar
  Physical Review C 82 (2010) 014308
- ✓ *Regularity-induced separation of intrinsic and collective dynamics*
  M.Macek, J.Dobeš, P.Stránský, P.Cejnar
  Physical Review Letters 105 (2010) 072503

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Program:

- Adiabatic separation of intrinsic and collective motions
- A few words on chaos in nuclei
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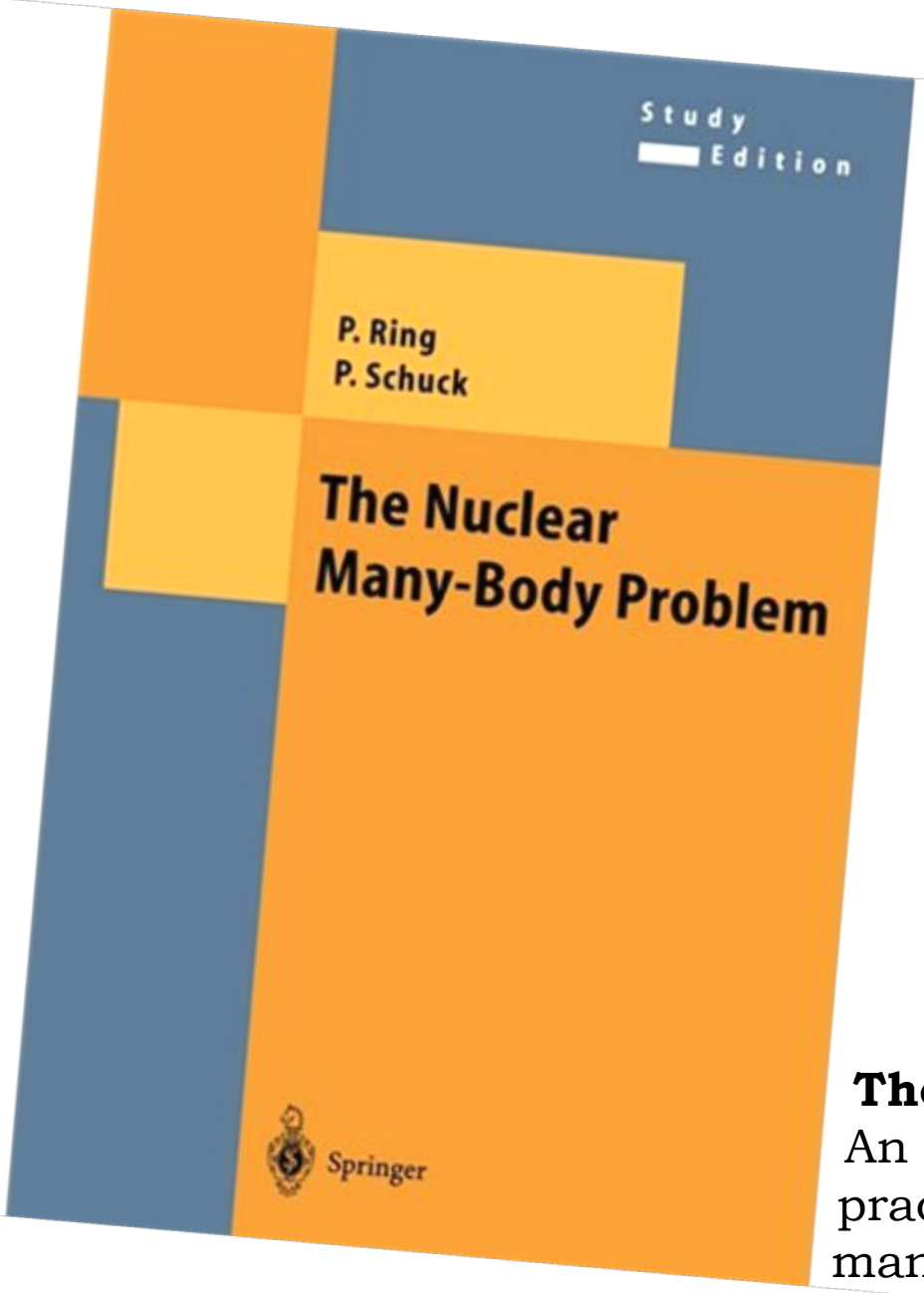
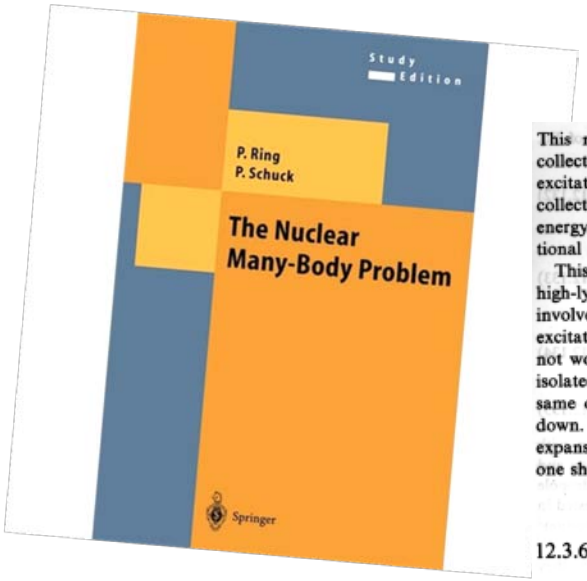


We all know this book!

The first edition in 1980...
An essential reference for all practitioners of almost any many-body technique...

Page 508 and below: **"The adiabatic approximation..."**



12.3.5.2 The Validity of the Adiabatic Approximation. We are now able to give a rough criterion for cases of the adiabatic approximation to be valid [BV 78]. We therefore restrict our considerations to one collective variable q (which is, in fact, no restriction, since q could be the exact solution of the ATDHF equations).

The adiabatic approach is equivalent to the fact that the time-odd component in any single-particle wave function

$$e^{i\hat{\chi}}|i\rangle = |i\rangle + i\hat{\chi}|i\rangle \tag{12.125}$$

should have a small norm:

$$|\hat{\chi}|i\rangle|^2 = \sum_m |\langle m|\hat{\chi}|i\rangle|^2 \ll 1 \qquad \text{for all } i.$$

If, as a rough estimate, in Eq. (12.83) we neglect the residual interaction—that is, if we use the Inglis formula for the collective mass—we find for the kinetic energy involved in this motion

$$\mathcal{K} = \sum_{mi} \chi_{mi}^*(\epsilon_m - \epsilon_i)\chi_{mi} \simeq \Delta\epsilon \sum_{mi} |\chi_{mi}|^2, \tag{12.126}$$

where $\Delta\epsilon$ is a typical *ph* energy. If we furthermore assume that the admixtures of odd components is equally distributed over N occupied states, from (12.126) we get the condition for adiabaticity:

$$\mathcal{K} \simeq \Delta\epsilon \cdot N \sum_m |\chi_{mi}|^2 \ll \Delta\epsilon \cdot N. \tag{12.127}$$

This means that the adiabatic approximation is good as long as the collective kinetic energy is small compared to a typical single-particle excitation energy times the number of single-particle states involved in the collective motion. We should note that it is not the quotient of collective energy to single-particle energy which should be small, but that an additional factor N comes into play which measures the collectivity.

This is the reason why we can also use the adiabatic assumption for high-lying collective states in the RPA. As long as there are many particles involved, the collective energy can be much higher than the single-particle excitations. On the other hand, we see that the adiabatic assumption does not work in situations in which only one particle is involved ($N = 1$). At isolated level crossings, for instance, the kinetic energy can become of the same order of magnitude as $\Delta\epsilon$ and the adiabatic approximation breaks down. This feature is similar to that already encountered in the momentum expansion of the GCM theory. In cases where many level crossings occur, one should therefore use these methods with extreme care [St 77].

12.3.6 Applications of the ATDHF Method

12.3.6.1. Quadrupole Vibrations for a $Q \cdot Q$-Force. An early version of the ATDHF theory was given in the pairing-plus-quadrupole model (see Sec. 7.4) of Baranger and Kumar for the description of transitional nuclei [BK 68, Ku 74a]. In this case, the theory becomes extremely simple because of the separability of the force. Since the model includes pairing correlations, Baranger and Kumar solved the adiabatic time-dependent Hartree–Fock–Bogoliubov problem. In this chapter we will restrict ourselves to the pure HF case and in the following, therefore, present a version which neglects pairing. We also consider only pure axially symmetric deformations ($Y = r^2 Y_{20}$) and neglect exchange terms as discussed in Section 7.4.

The matrix element $\bar{v}$ in this case is given by

$$\bar{v}(1,2) = -kY(1) \cdot Y(2), \tag{12.128}$$

where k is a force constant.

We start according to Section 12.3.3 with a family of static densities $\rho_0(q)$ that are obtained, for instance, from the solution of the CHF problem

$$[h_0(\rho_0) - qY, \rho_0] = 0. \tag{12.129}$$

$|m\rangle$, $|i\rangle$ will be a set of simultaneous eigenvectors of ρ_0 and of the *pp* and *hh* parts of h_0. We furthermore find that Γ_1 of Eq. (12.74) vanishes.

$$\Gamma_1 = -k \cdot Y \cdot \mathrm{Tr}(Y\rho_1) = 0 \tag{12.130}$$

because Y is time-even and ρ_1 is time-odd and the trace is real. This produces a tremendous simplification, because in this case the matrix B of Eq. (12.82) vanishes and A becomes diagonal.

From Eqs. (12.84), (12.100), and (12.101) we can calculate the time-odd part:

$$\chi_{mi} = \hbar^2 \frac{\langle m|\dot{\rho}_0|i\rangle}{\epsilon_m - \epsilon_i} = \dot{q}\hbar^2 \frac{\langle m|\partial/\partial q|i\rangle}{\epsilon_m - \epsilon_i}. \tag{12.131}$$

The matrix elements of $\partial/\partial q$ are obtained from the perturbation theory of Eq. (12.129), which in this case is equivalent to a linear response approach:

$$\langle m|\partial/\partial q|i\rangle = \frac{\langle m|Y|i\rangle}{\epsilon_m - \epsilon_i}. \tag{12.132}$$

This, together with Eq. (12.103), gives the Inglis formula for the mass:

$$M(q) = 2\hbar^2 \sum_{mi} \frac{|\langle m|\partial/\partial q|i\rangle|^2}{\epsilon_m - \epsilon_i} = 2\hbar^2 \sum_{mi} \frac{|\langle m|Y|i\rangle|^2}{(\epsilon_m - \epsilon_i)^3}. \tag{12.133}$$

From the collective Hamiltonian, therefore, we obtain:

$$\mathcal{K} = \tfrac{1}{2}M\dot{q}^2 + V(q), \tag{12.134}$$

where

$$V(q) = \mathrm{Tr}(\epsilon_0\rho_0(q)) - \tfrac{1}{2}k\,\mathrm{Tr}(Y\rho_0(q)) \cdot \mathrm{Tr}(Y\rho_0(q)). \tag{12.135}$$

Baranger and Kumar [BK 68] obtained the same collective Hamiltonian with minor differences. They took the pairing-plus-quadrupole model Hamiltonian and in this way derived a collective Hamiltonian depending on the five quadrupole deformation parameters $\alpha_{2\mu}$ and the pairing gap Δ. Since they are not interested in pairing vibrations, they restricted Δ to be the solution of the static BCS equations for each value of the five other coordinates.

After a transformation to intrinsic coordinates (as discussed in Sec. 1.5) they ended up with a classical Bohr Hamiltonian (1.47)

$$\mathcal{K} = \frac{1}{2}\left(\sum_i \vartheta_i \omega_i^2 + B_{\beta\beta}\dot{\beta}^2 + B_{\beta\gamma}\dot{\beta}\dot{\gamma} + B_{\gamma\gamma}\dot{\gamma}^2\right) + V, \tag{12.136}$$

where the seven functions $\vartheta_1, \vartheta_2, \vartheta_3, B_{\beta\beta}, B_{\beta\gamma}, B_{\gamma\gamma}$, and V depend on the variables β and γ. They are calculated microscopically. The inertial parameters correspond to the Belyaev formula (3.93). This Hamiltonian has been requantized as discussed in Sec. 1.5. Energy levels and wave functions were then calculated numerically [BK 67b, 68].

In this method the coupling between rotations and vibrations and the mixing between different phonon states is fully taken into account. In this way Baranger and Kumar investigated the transition region around the osmium and the samarium isotopes [KB 68, Ku 74a] and found good agreement with experimental data. In particular, they obtained strong deviations from the rotational picture at the low-A end of the W-Os-Pt-transition region and large deviations from the phonon model at the upper end. Their collective wave functions are often smeared out over all possible shapes.

So far, only quadrupole shapes have been used. From Eqs. (12.120) and (12.132) we see that the constraint used in the next step of the procedure, as discussed in Section 12.3.4, would be

$$Q_{mi} = \frac{\hbar^2}{M}\frac{\langle m|Y|i\rangle}{(\epsilon_m - \epsilon_i)^2}. \tag{12.137}$$

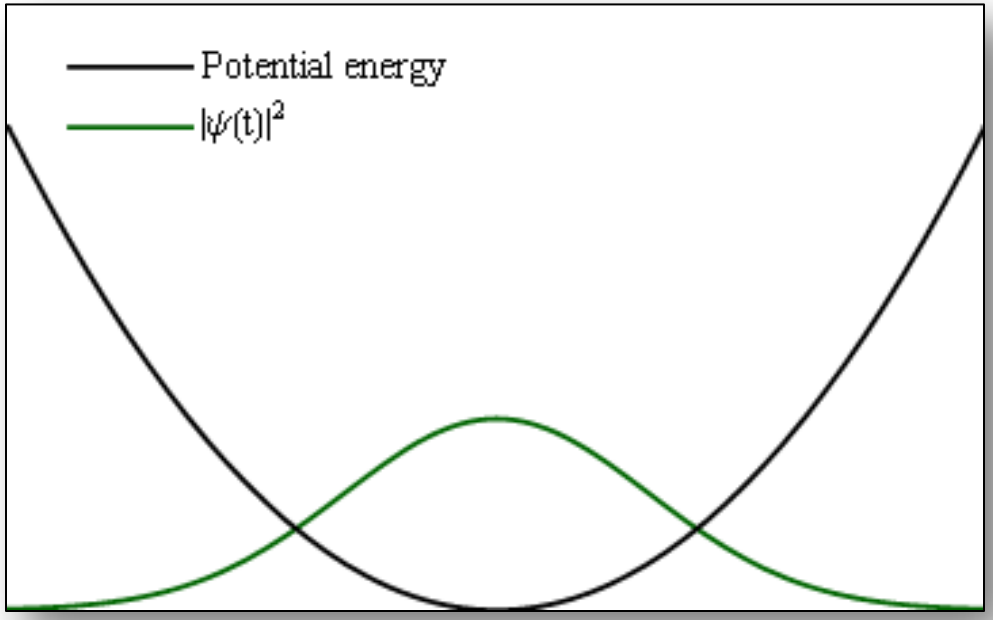
Only in cases where the *ph* energies are almost degenerate, this corresponds to the original constraining operator Y.

12.3.6.2. Further Applications. The ATDHF theory has also been applied to *exactly soluble models* such as the Lipkin model of Sec. 6.2 [KG 74] and a three-level

Adiabatic theorem of quantum mechanics

Hamiltonian $\boldsymbol{H(\lambda)}$ driven by $\boldsymbol{\lambda = t/\tau}$ from λ=0 (at t=0) to λ=1 (at t=τ). System evolves from $\Psi(t=0) \equiv \boldsymbol{\Phi_i(0)}$, the i^{th} eigenstate of $H(0)$, to an unknown final state $\Psi(t=\tau)$. If the change is "*very slow*", the final state is "*almost precisely*" the i^{th} eigenstate of $H(1)$. So $\Psi(t=\tau) \approx \boldsymbol{\Phi_i(1)}$ with $\left|\langle \Phi_i(1) | \Psi(t=\tau) \rangle\right|^2 \approx 1$ for

$$\tau >> \hbar \max_{0 \le \lambda \le 1} \left| \left\langle \Psi_{i\pm1}(\lambda) \left| \frac{d}{d\lambda} H \right| \Psi_i(\lambda) \right\rangle \right| \Big/ \left[\min_{0 \le \lambda \le 1} | E_{i\pm1}(\lambda) - E_i(\lambda) | \right]^2$$



http://en.wikipedia.org/wiki/File:HO_adiabatic_process.gif

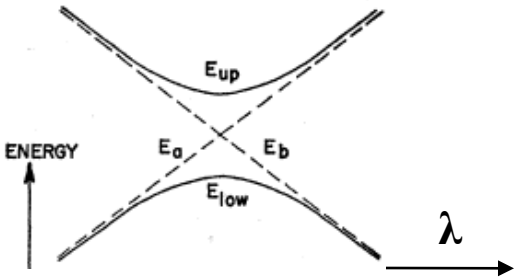
Adiabatic theorem of quantum mechanics

Hamiltonian $\boldsymbol{H(\lambda)}$ driven by $\boldsymbol{\lambda = t/\tau}$ from λ=0 (at t=0) to λ=1 (at $t=\tau$). System evolves from $\Psi(t=0)\equiv\boldsymbol{\Phi_i(0)}$, the i^{th} eigenstate of $H(0)$, to an unknown final state $\Psi(t=\tau)$. If the change is "*very slow*", the final state is "*almost precisely*" the i^{th} eigenstate of $H(1)$. So $\Psi(t=\tau)\approx\boldsymbol{\Phi_i(1)}$ with $\left|\langle\Phi_i(1)|\Psi(t=\tau)\rangle\right|^2 \approx 1$ for

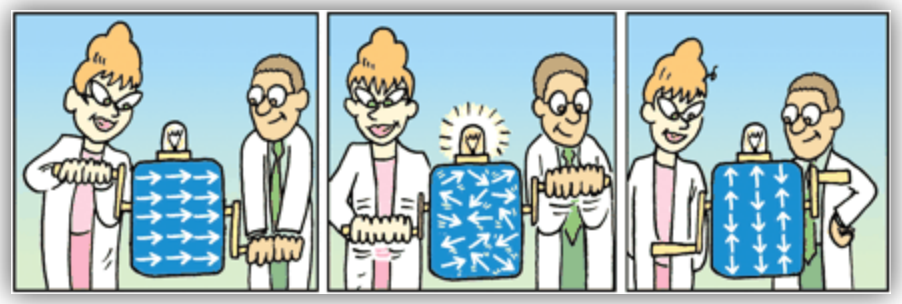
$$\tau >> \hbar \max_{0\le\lambda\le1} |\langle\Psi_{i\pm1}(\lambda)|\tfrac{d}{d\lambda}H|\Psi_i(\lambda)\rangle| \Big/ [\min_{0\le\lambda\le1} |E_{i\pm1}(\lambda)-E_i(\lambda)|]^2$$

Some applications:

- **Landau-Zener mechanism**
  Landau (1932), Zener (1932), Stueckelberg (1932), Majorana (1932)
  Non-adiabatic passage through an avoided level crossing



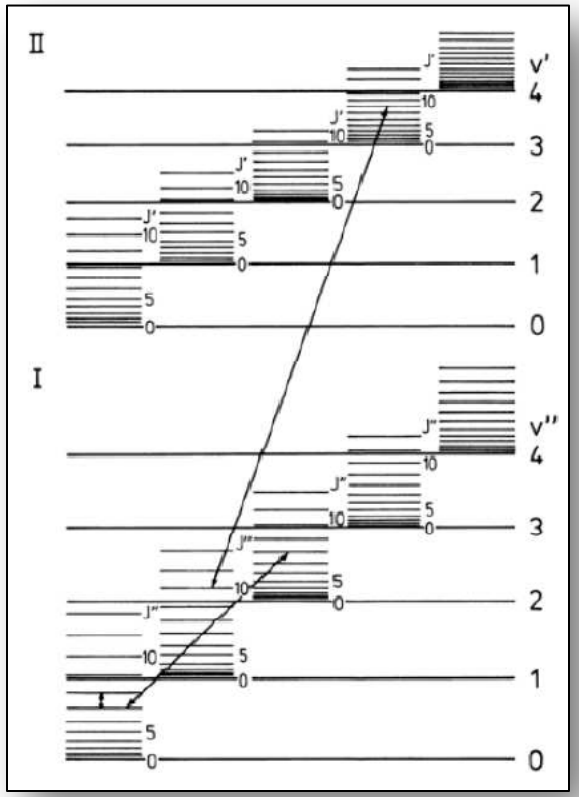
- **Geometric phase**
  Pancharatnam (1956), Berry (1984)
  Phase acquired along a closed path in parameter space around a degeneracy point



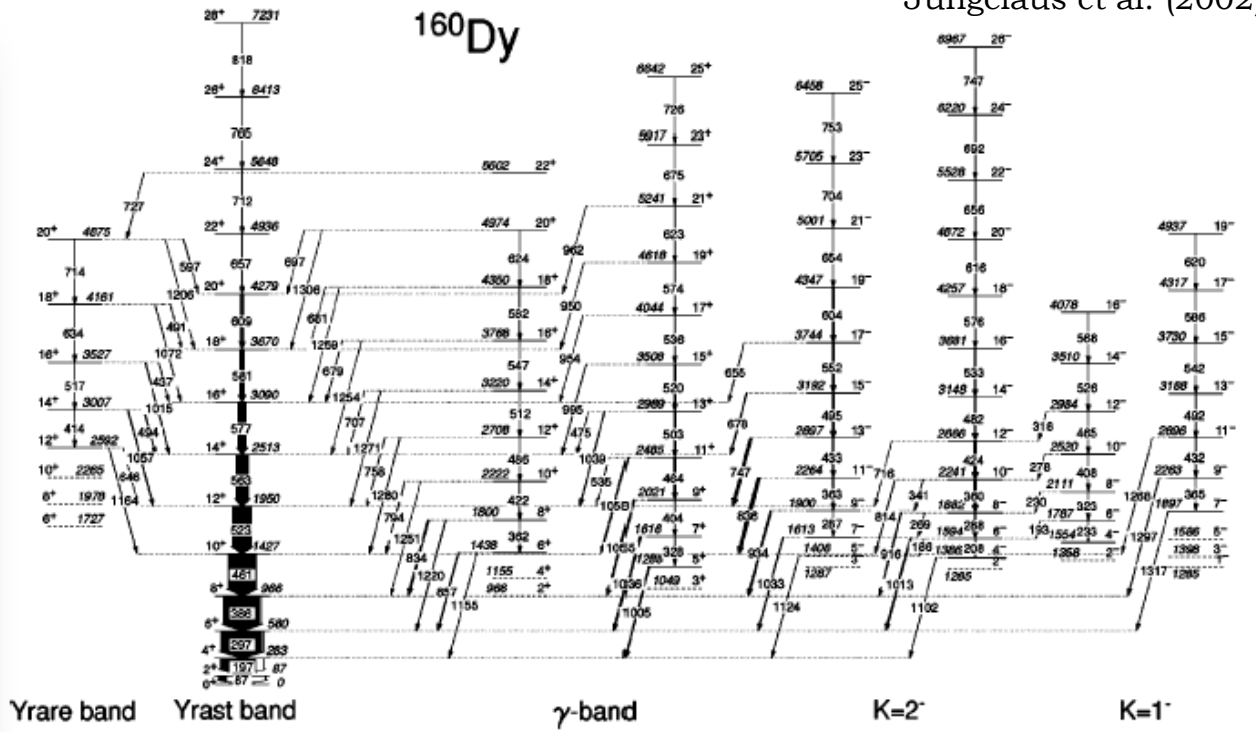
- **Adiabatic quantum computation**
  Fahri et al. (2000)
  System driven to an unknown ground state carrying the result of some computation
- **Born-Oppenheimer approximation**
  Born, Oppenheimer (1927), Eckart (1935)
  Separation of intrinsic and collective wave functions of a molecule or nucleus

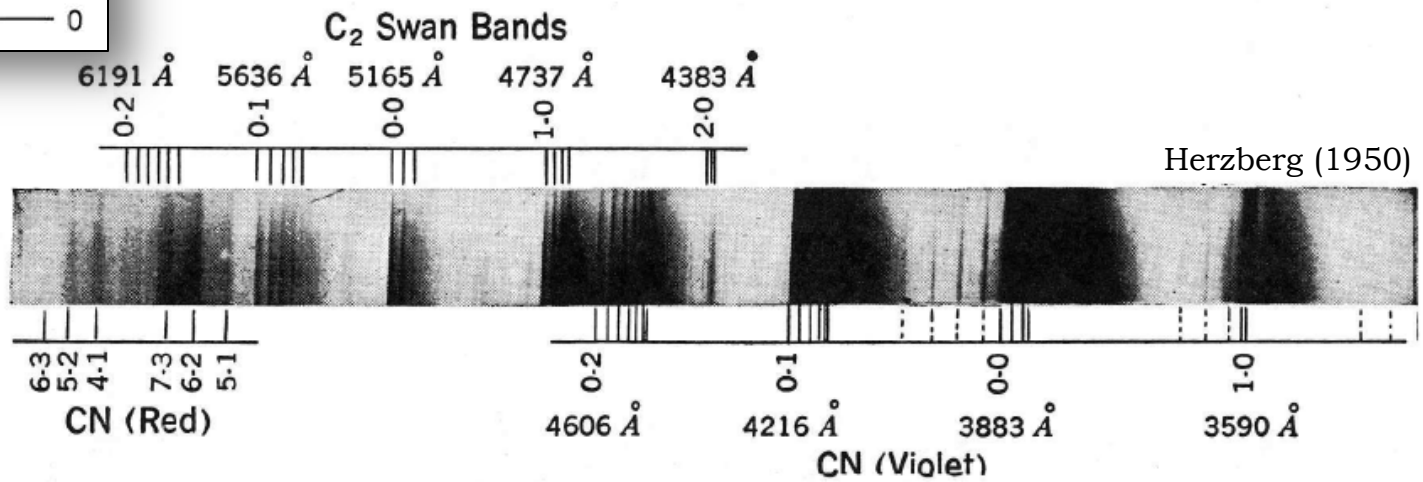
$\Rightarrow$ bands of rotational & vibrational states "built on" individual intrinsic ("electronic" or "nucleonic") states of the molecule (nucleus)

Separation of intrinsic, vibrational, and rotational excitations in molecules & nuclei



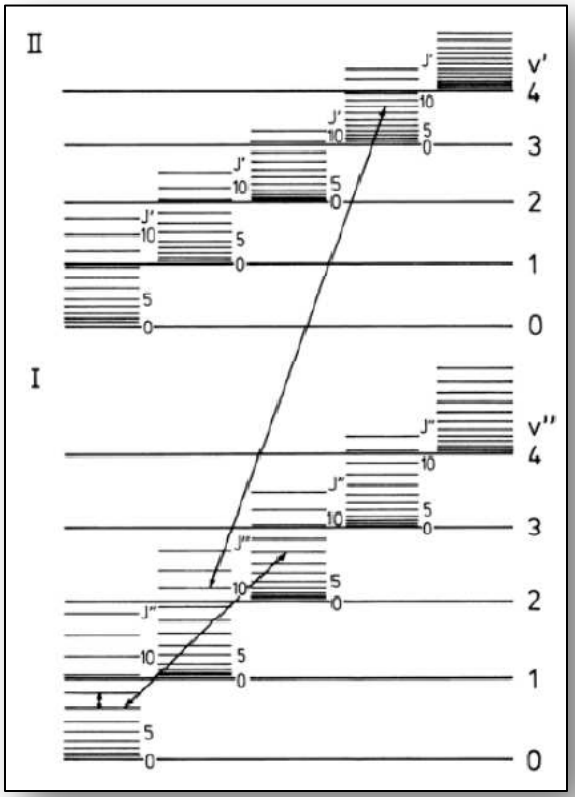
Jungclaus et al. (2002)





Herzberg (1950)

Separation of intrinsic, vibrational, and rotational excitations in molecules & nuclei



Questions:

- How far in excitation energy can such bands be followed?
- How good is the description of collective bands based on the adiabatic separation?
- Are there some requirements on the given intrinsic state to support the existence of well distinguished collective bands?

Hints:

- Various intrinsic states may show different susceptibility to external perturbations (like the Coriolios term).
- In this respect, a very important role may be played by the degree of chaos in the intrinsic dynamics (**chaos => instability**).

Program:

- Adiabatic separation
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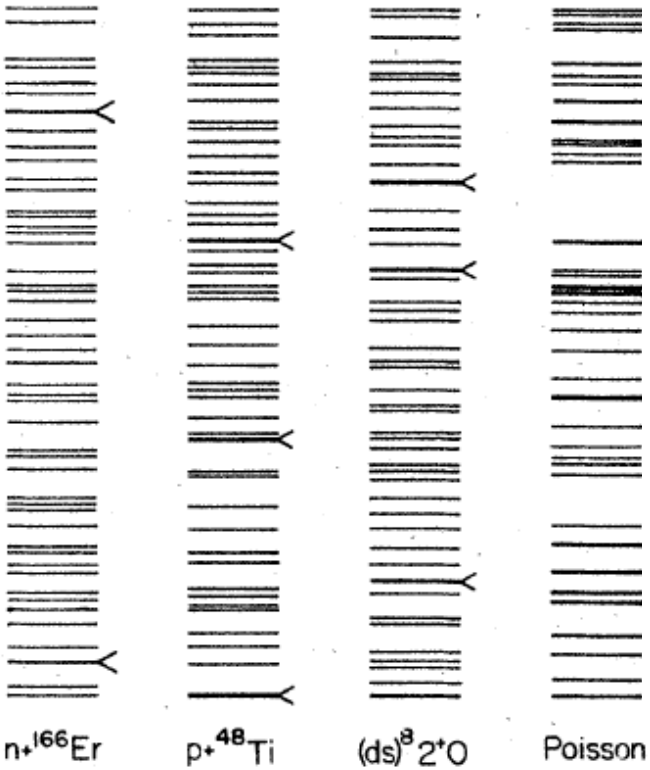
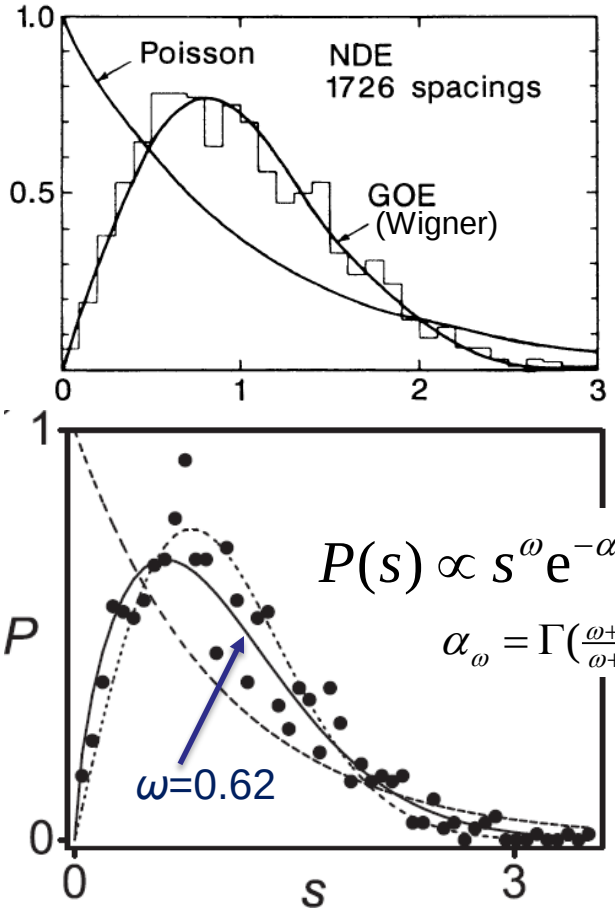
Classical & quantum chaos

Classical
⇒ sensitivity to initial conditions
⇒ practical loss of predictability
⇒ quasi ergodic trajectories in the phase space

Quantum
⇒ no genuinely quantum definition of chaos
⇒ link to quantum-classical correspondence
⇒ Bohigas (1984): Chaos on quantum level affects statistical properties of energy spectra. Chaotic systems yield correlations consistent with the GOE or GUE.





Nuclei show signatures of quantum chaos:
✓ n & p resonances
Bohigas et al. (1983)
✓ low-energy levels
Von Egidy et al. (1987)

Nearest-neighbor spacing

$$s_i = \frac{E_i - E_{i-1}}{\langle E_i - E_{i-1} \rangle}$$

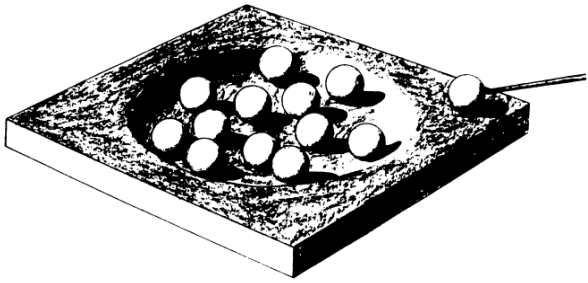
$$P(s) \propto s^{\omega} e^{-\alpha_\omega s^{\omega+1}}$$

$$\alpha_\omega = \Gamma\left(\frac{\omega+2}{\omega+1}\right)^{\omega+1}$$

Brody

Poisson ↔ Wigner
ω=0 order chaos ω=1

Origin of chaos in nuclei



Many-body dynamics

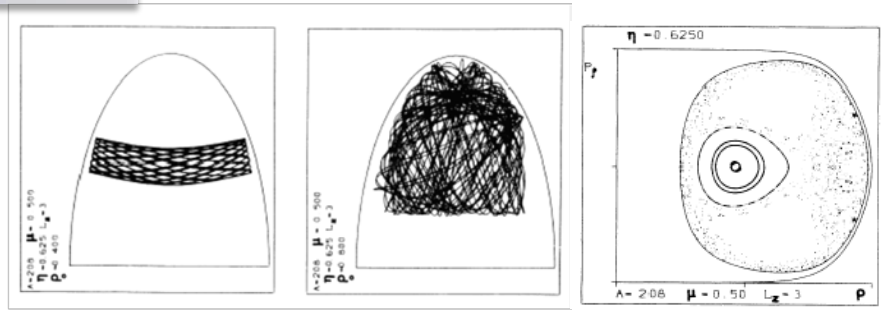
Complex interactions of all particles in the nucleus

Shell-model approach: Zelevinsky et al. 1990s-2000s

Single-particle dynamics

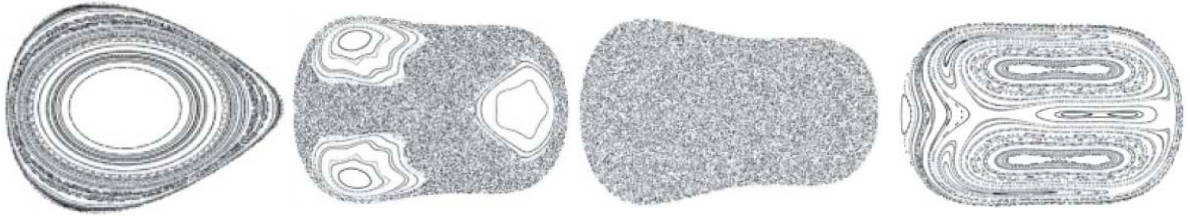
Nucleonic motions in deformed nuclear potentials

Arvieu et al. 1987; Rozmej, Arvieu 1992; Heiss, Nazmitdinov, Radu 1994-95



Collective dynamics

Nuclear vibrations and rotations



Interacting Boson Model Iachello, Arima 1975

Paar, Vorkapic, Dieperink 1988–1992; Alhassid, Whelan, Novoselsky 1990–93; Mizusaki et al. 1991; Canetta, Maino 2000; Cejnar, Jolie, Macek, Heinze, Casten, Dobeš, Stránský 1998–2010

PLB 420,241(1998); PRE 58,387 (1998); PRL 93,132501(2004); PRC 75,064318(2007); PRC 80,014319(2009); PRC 82,014308(2010); PRL 105,072503(2010)

Geometric Model Bohr 1952

Cejnar, Stránský, Kurian, Hruška 2004–10

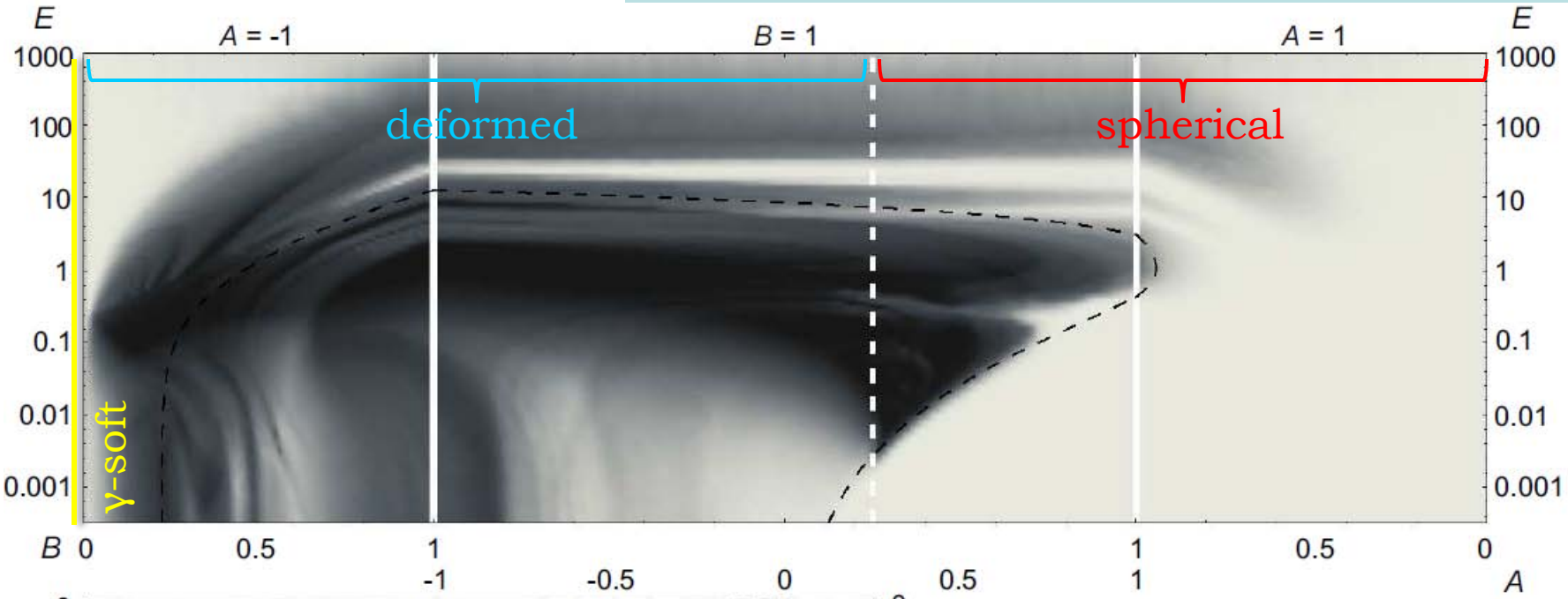
PRL 93,102502(2004); PRC 74,014306(2006); PRE 79, 046202(2009); PRE 79,066201(2009)

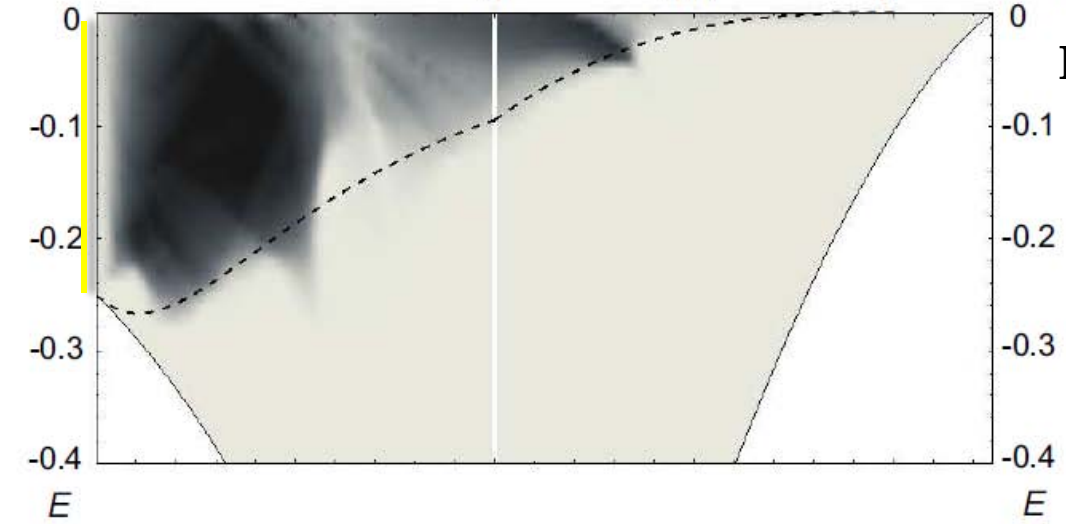
Geometric model

classical chaos map for $J = 0$

Stránský et al. 2010 JPconf.239,012002

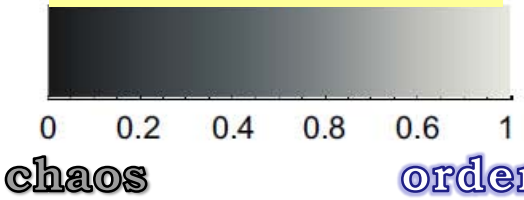
$$H = \underbrace{\sum_{\bullet=x,y,z} \frac{J_\bullet'^2}{2\mathcal{I}_\bullet(\beta,\gamma)}}_{T_{\mathrm{rot}}} + \underbrace{\frac{1}{2K}\left(\pi_\beta^2 + \frac{\pi_\gamma^2}{\beta^2}\right)}_{T_{\mathrm{vib}}} + \underbrace{A\beta^2 + B\beta^3\cos 3\gamma + C\beta^4}_{V}$$





Regular phase space fraction

$$f_{\mathrm{reg}} = \Omega_{\mathrm{reg}} / \Omega_{\mathrm{tot}}$$

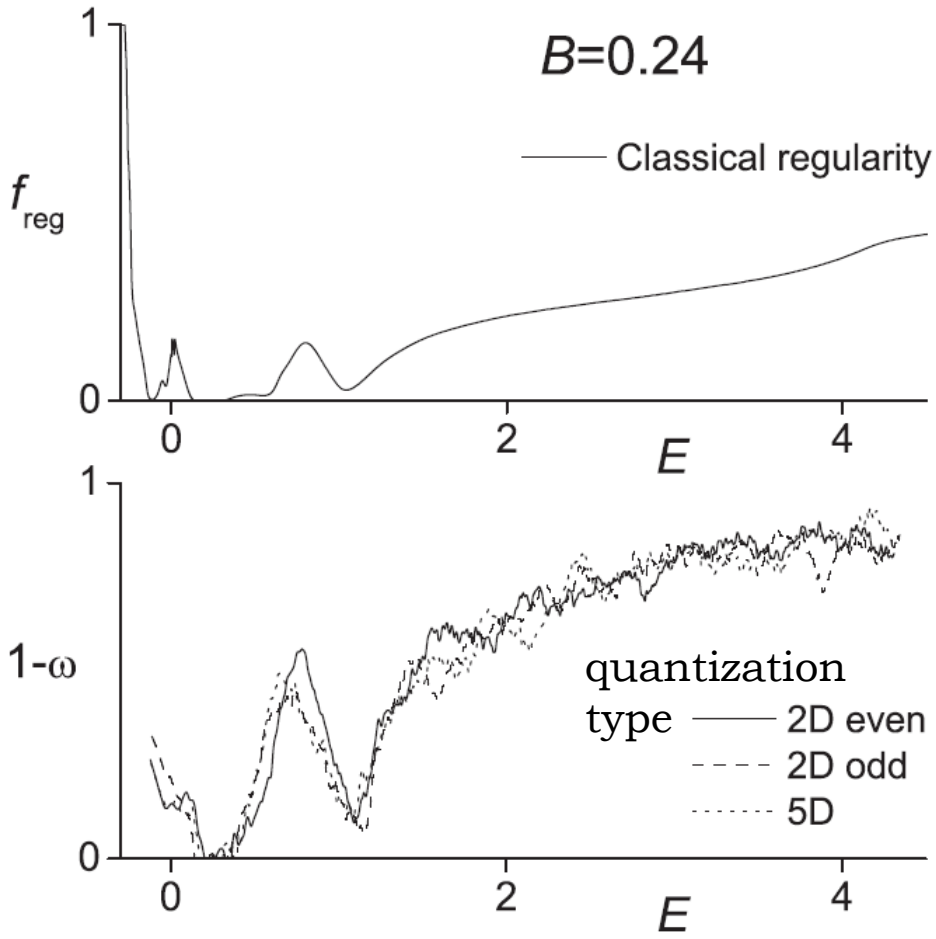


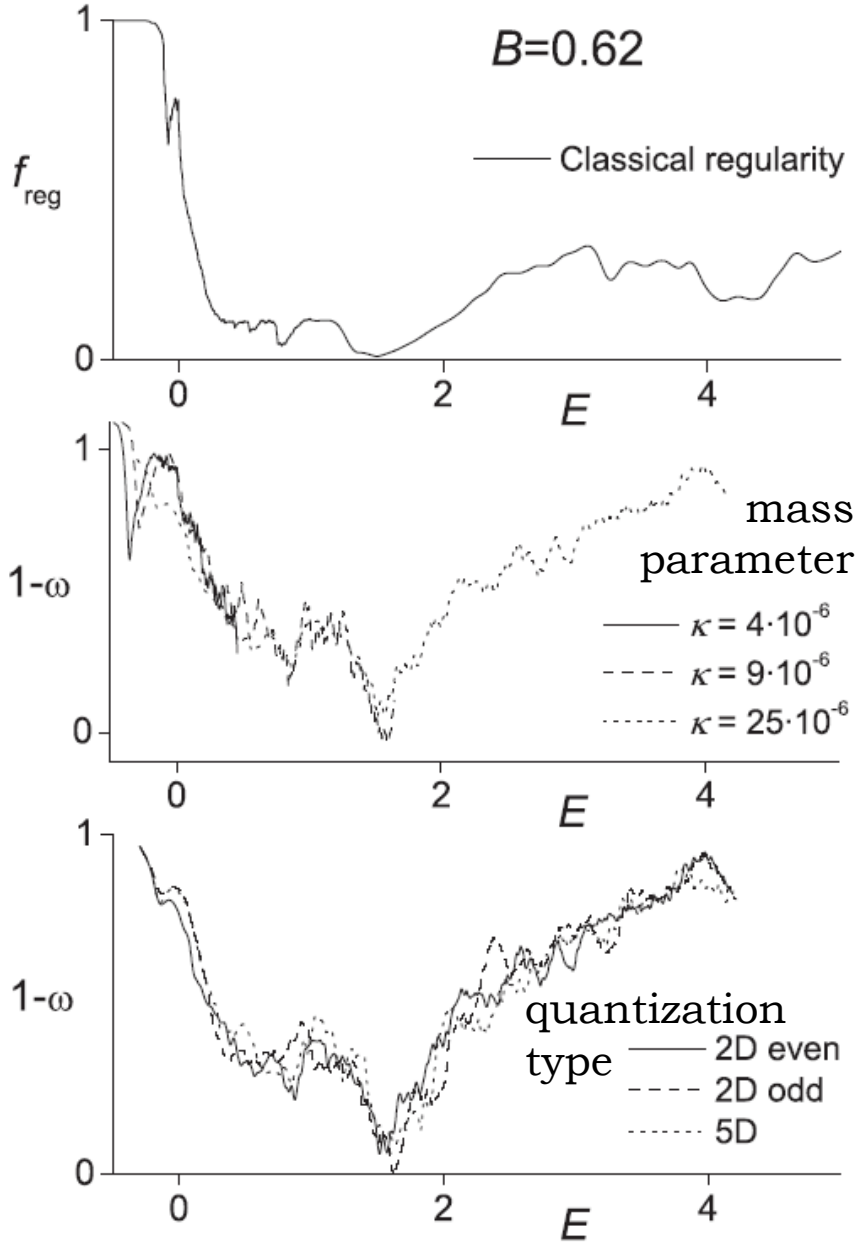
Geometric model

comparison of classical & quantal measures for $J = 0$

Stránský et al. 2009 PRE 79,046202
PRE 79,066201

f_{reg} … classical regular fraction
$1-\omega$ … adjunct of Brody parameter



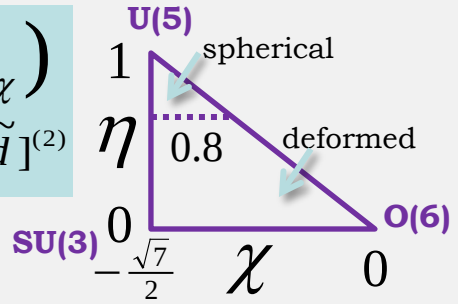


Interacting boson model

comparison with the GM

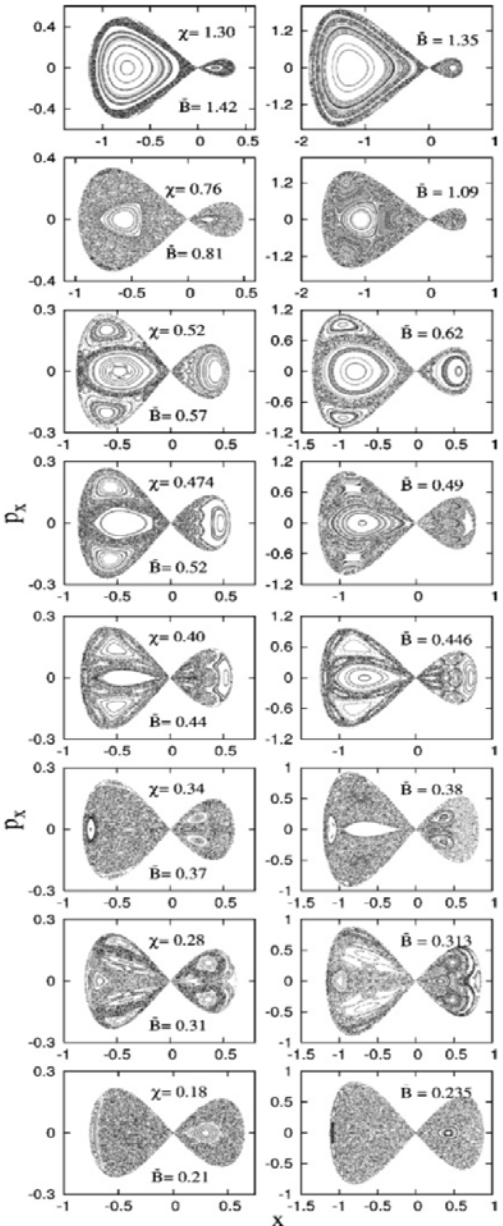
$$H = \eta\, n_d - \frac{1}{N}(1-\eta)\left(Q_\chi \cdot Q_\chi\right)$$

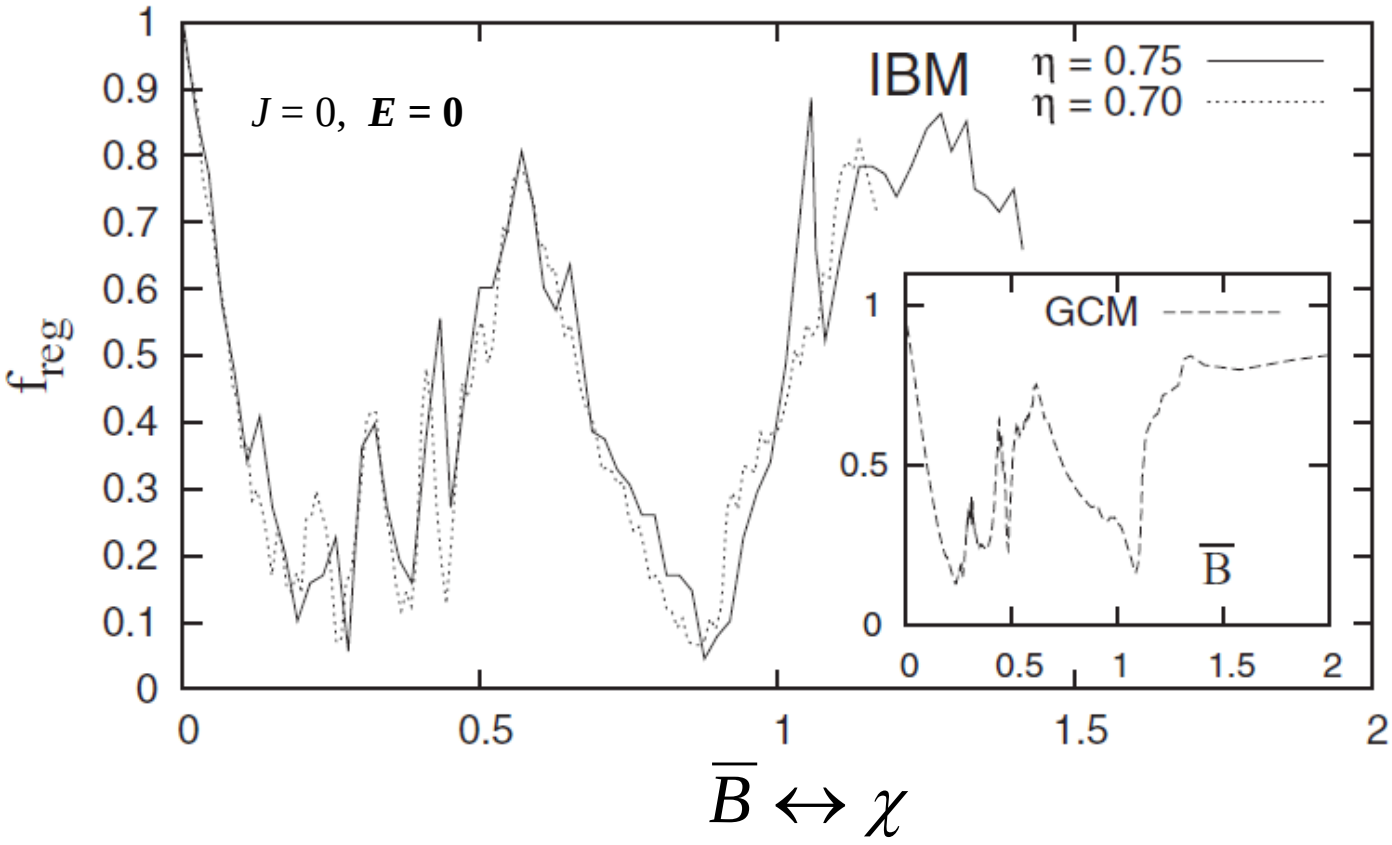
$$n_d = d^+ \cdot \tilde{d} \qquad Q_\chi = d^+ s + s^+ \tilde{d} + \chi [d^+ \tilde{d}]^{(2)}$$



Macek et al. 2007 PRC 75,064318
Macek et al. 2009 PRC 80,014319
Macek 2010 dissertation

>1 control parameter => multi-dimensional chaotic map. Nevertheless, there exist regions of almost full compatibility with the geometric model.



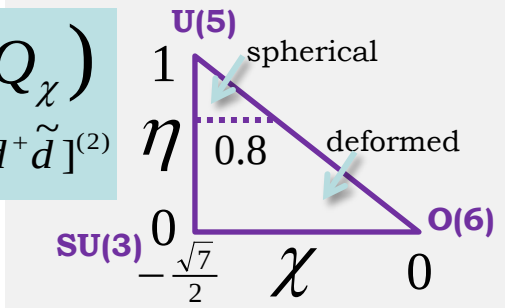


Interacting boson model

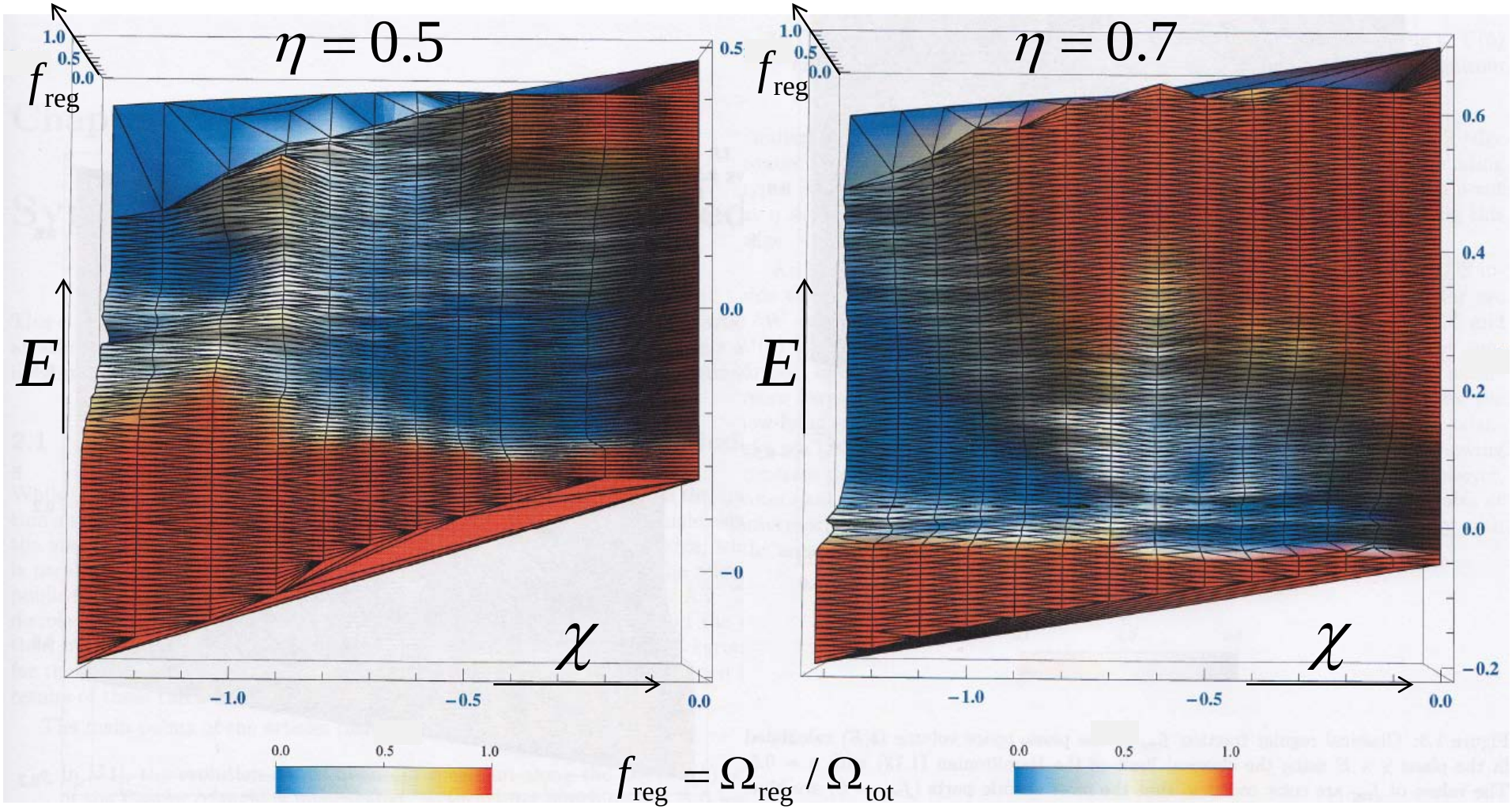
classical chaos map for $J = 0$

$$H = \eta n_d - \frac{1}{N}(1-\eta)(Q_\chi \cdot Q_\chi)$$

$$n_d = d^+ \cdot \tilde{d} \qquad Q_\chi = d^+ s + s^+ \tilde{d} + \chi [d^+ \tilde{d}]^{(2)}$$



Macek et al. 2007 PRC 75,064318
Macek et al. 2009 PRC 80,014319
Macek 2010 dissertation



$$f_{reg} = \Omega_{reg} / \Omega_{tot}$$

Interacting boson model

classical chaos map for $J = 0$

$$H = \eta n_d - \frac{1}{N}(1-\eta)(Q_\chi \cdot Q_\chi)$$

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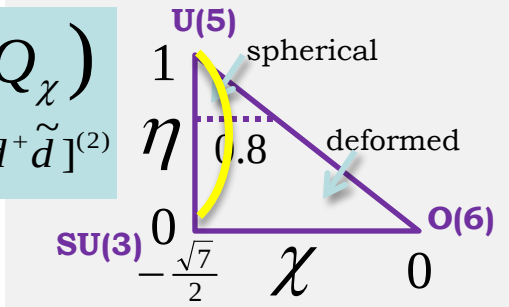
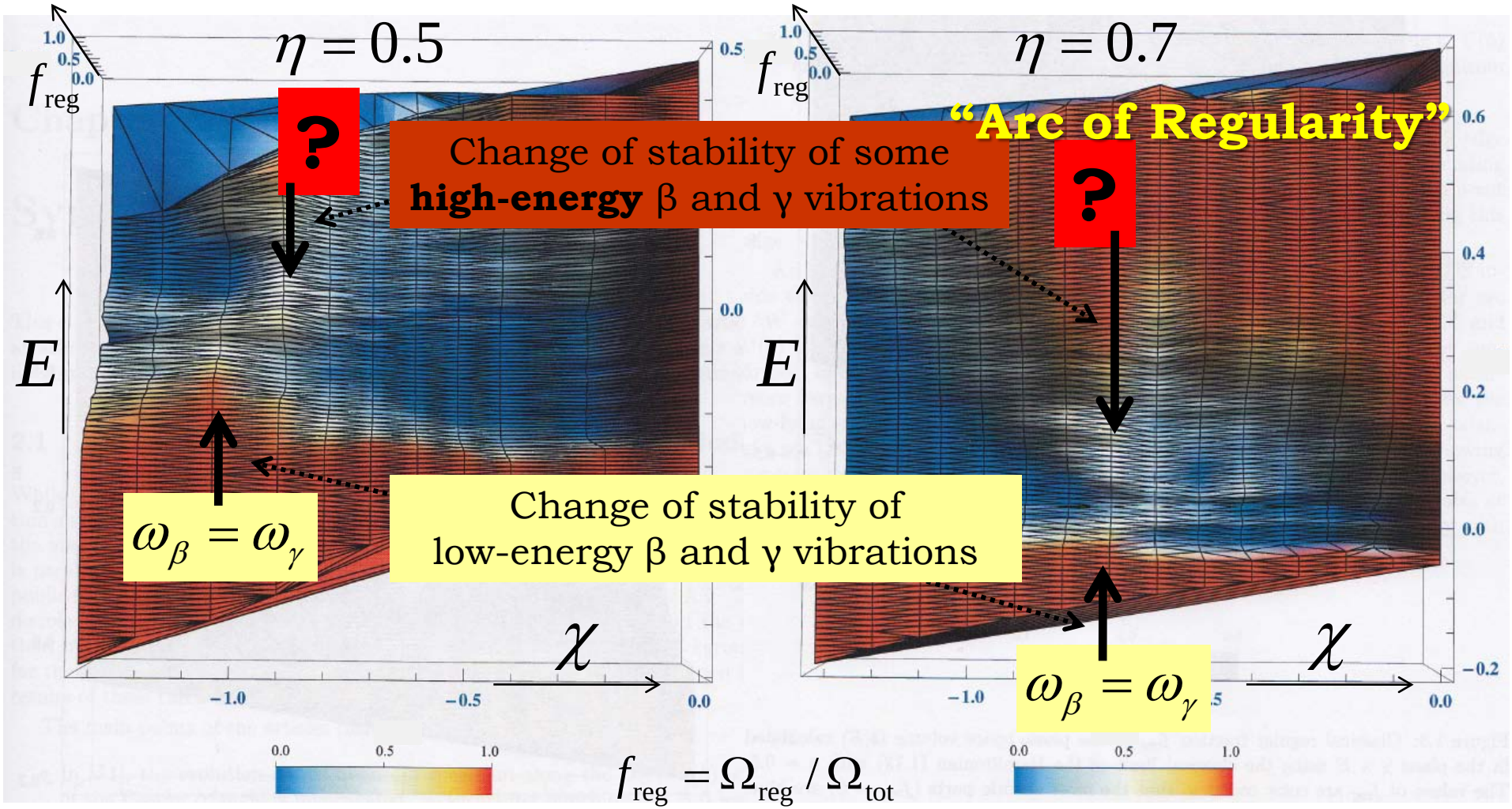


Macek et al. 2007 PRC 75,064318
Macek et al. 2009 PRC 80,014319
Macek 2010 dissertation



Program:

- Adiabatic separation of intrinsic and collective motions
- A few words on chaos in nuclei
- Adiabatic separation and chaos in IBM
- Conclusions and outlook

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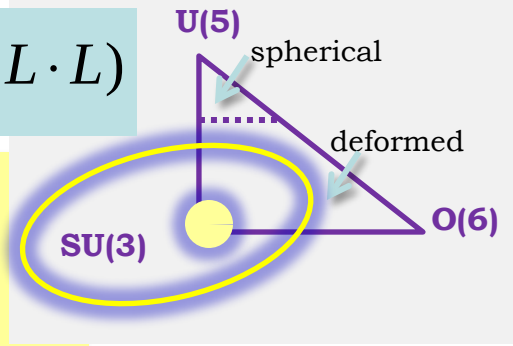
SU(3) limit of IBM

home of rotational bands

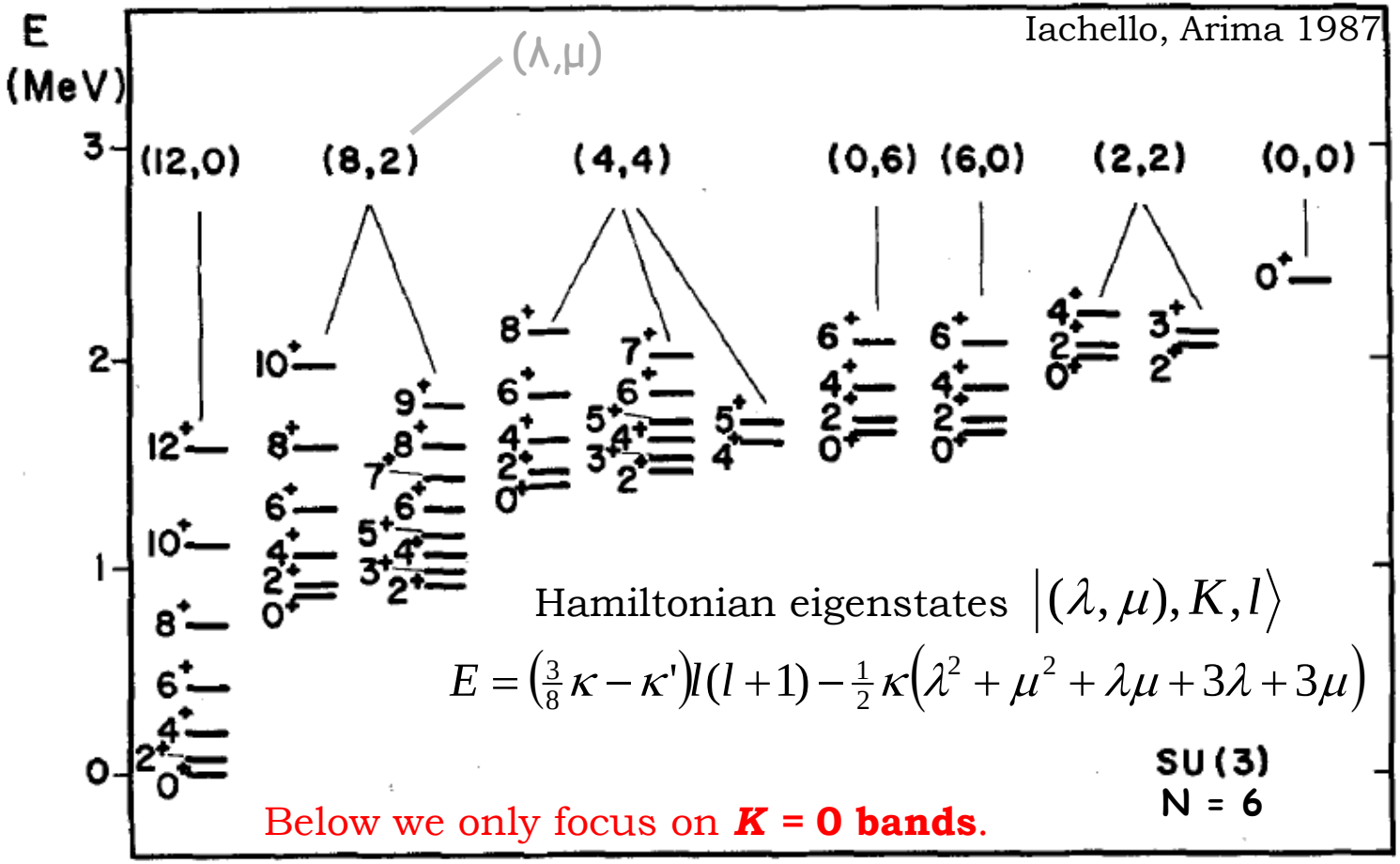
$$H = -\kappa(Q_{\chi=-\frac{\sqrt{7}}{2}} \cdot Q_{\chi=-\frac{\sqrt{7}}{2}}) - \kappa'(L \cdot L)$$



- (λ,μ) = … the SU(3) quantum numbers
- K = … missing label of SU(3)>O(3) reduction

} define the **intrinsic state**

- l =K,K+1,K+2,…K+max{λ,μ} enumerates states in **rotational bands**

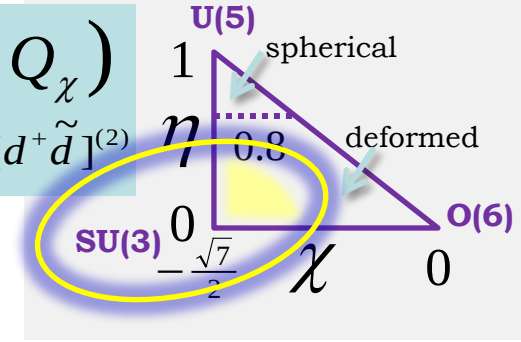


Away from SU(3) limit

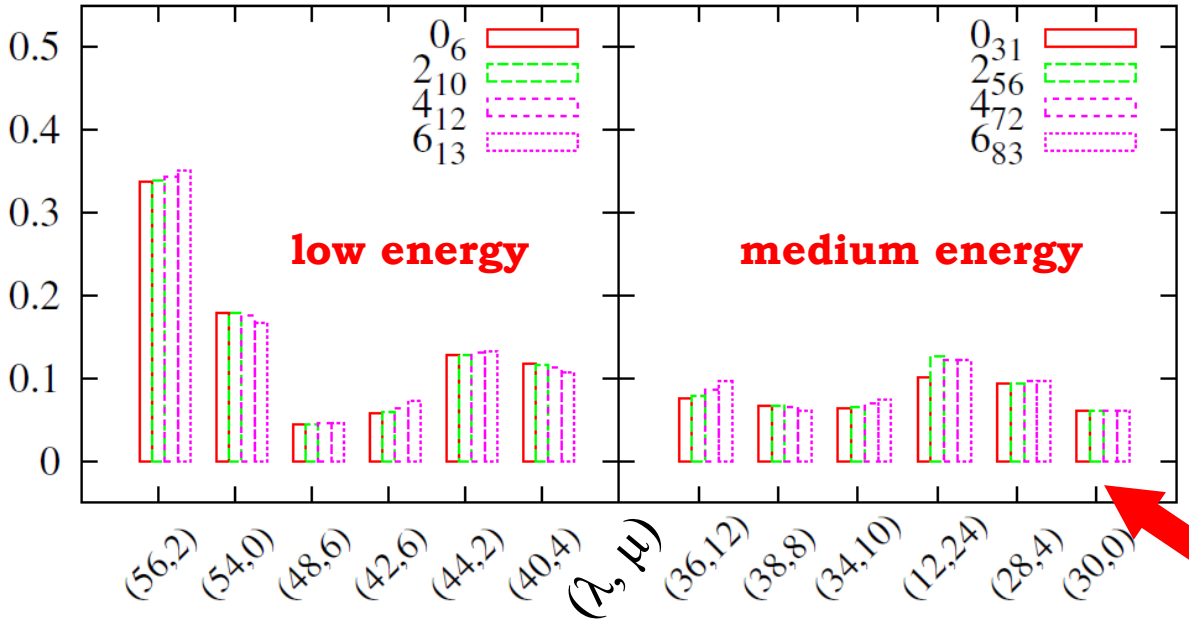
SU(3) quasidynamical symmetry

$$H = \eta\, n_d - \frac{1}{N}(1-\eta)(Q_\chi \cdot Q_\chi)$$

$$n_d = d^+ \cdot \tilde{d} \qquad Q_\chi = d^+ s + s^+ \tilde{d} + \chi [d^+ \tilde{d}]^{(2)}$$



Macek et al. 2010 PRL 105,072503



N=30

$$|l_i\rangle = \sum_{(\lambda,\mu),K} A^{l_i}_{(\lambda,\mu),K} |(\lambda,\mu),K,l\rangle$$

actual eigenstate

amplitude

SU(3) eigenstate

probability

$$P^{l_i}_{(\lambda,\mu)} = \sum_K \left| A^{l_i}_{(\lambda,\mu),K} \right|^2$$

$l_i \equiv i^{\text{th}}$ state with angular momentum l

Quasidynamical symmetry
© Rowe et al. 1988…2004

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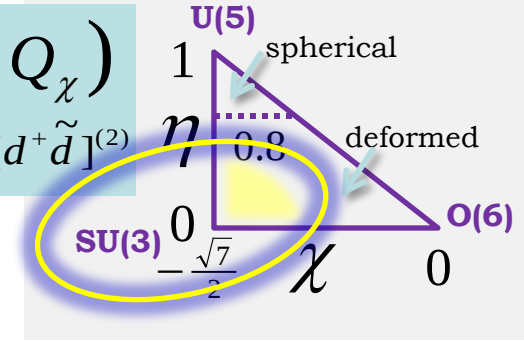

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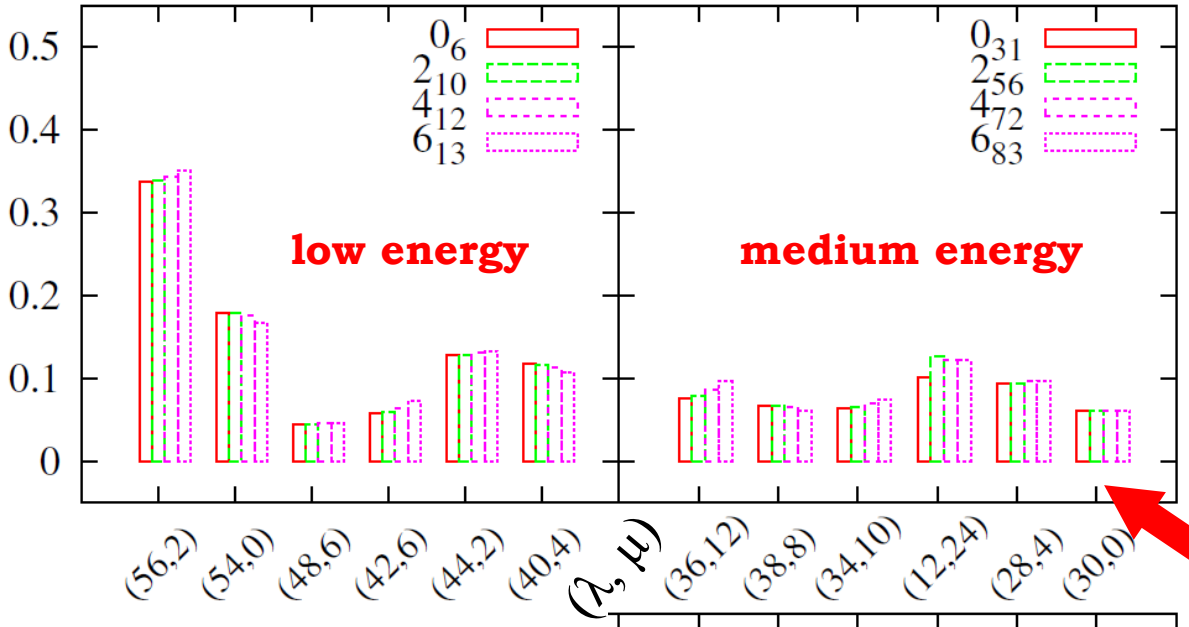

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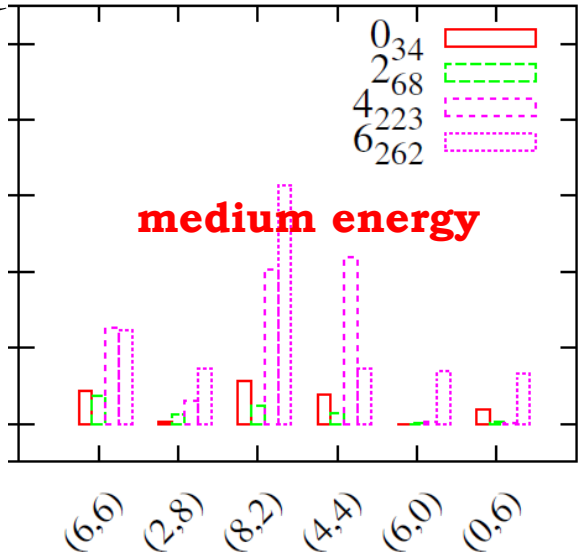

$$\left|l_i\right\rangle = \sum_{(\lambda,\mu),K} A^{l_i}_{(\lambda,\mu),K} \left|(\lambda,\mu),K,l\right\rangle$$

actual eigenstate — amplitude — SU(3) eigenstate

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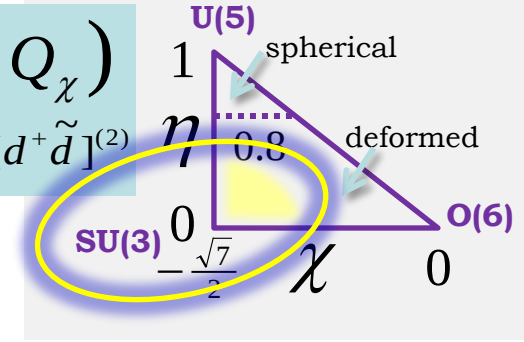

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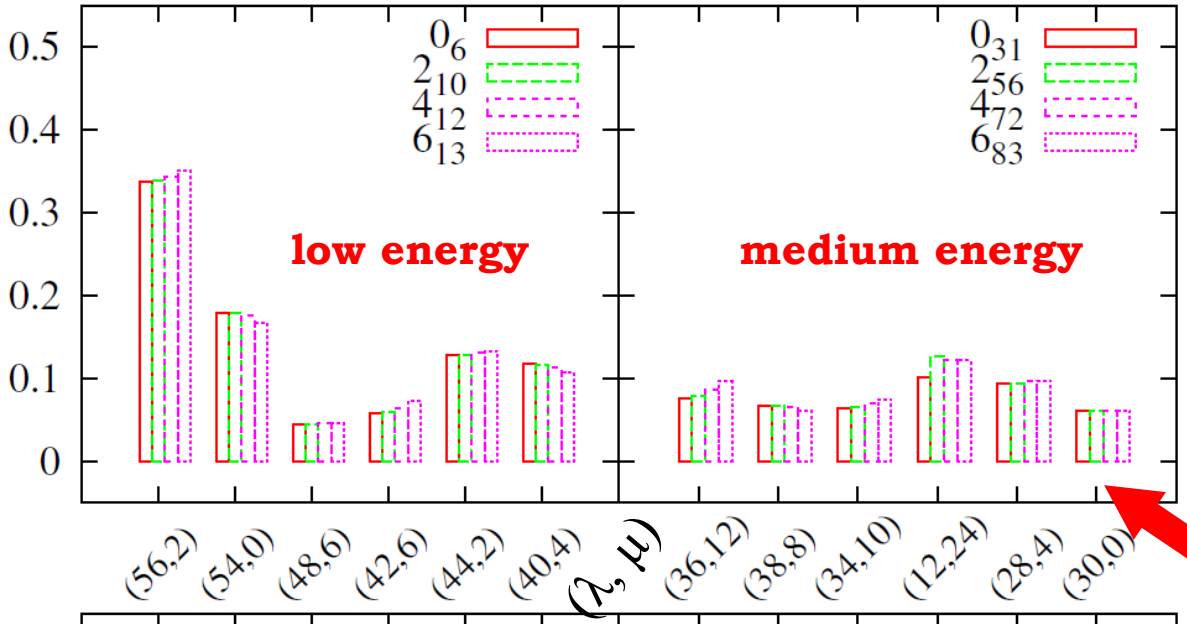

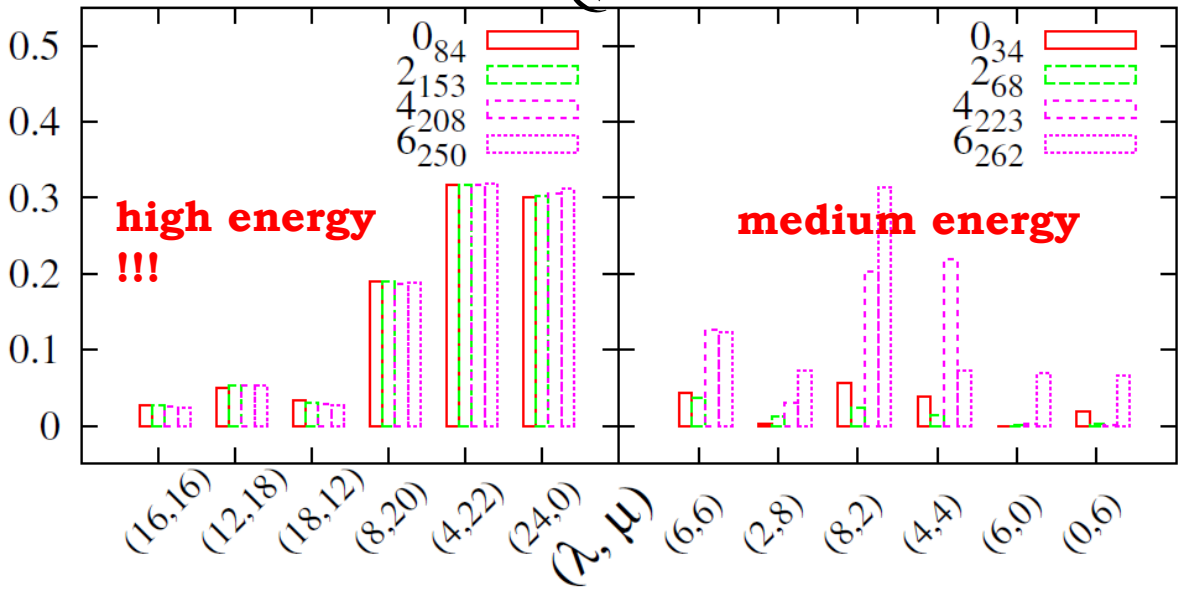

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Away from SU(3) limit

search for rotational bands

Macek et al. 2010 PRL 105,072503

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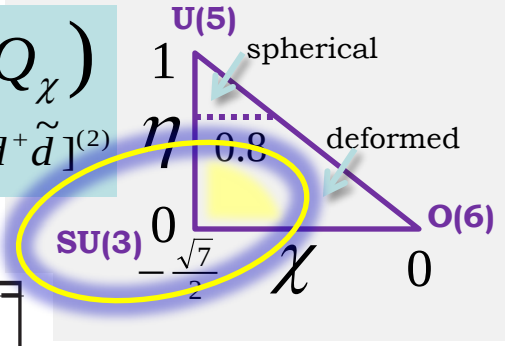
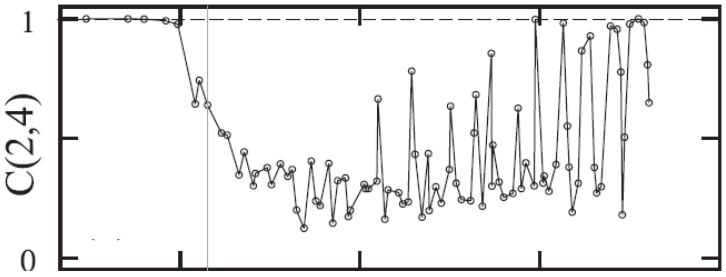


$\eta = 0.5,\ \chi = -0.9$



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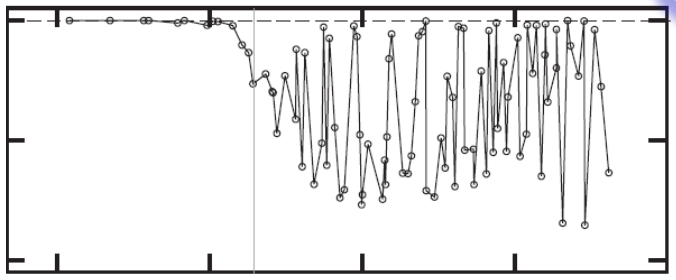

N=30

- Pearson **correlation coefficient** $\pi(\vec{x},\vec{y}) = \frac{1}{n-1}\sum_k \frac{(x_k-\bar{x})}{s_x}\frac{(y_k-\bar{y})}{s_y}$ (for 2 statistical vectors $\vec{x}$ and $\vec{y}$)

  -1 (perfect anticorrelation)

  0 (null correlation)

  $+1$ (perfect correlation)

- As the vectors $\vec{x}$ and $\vec{y}$ take probabilities from the SU(3) decomposition: $\vec{x} \equiv \{P^{0_i}_{(\lambda,\mu)}\}, \vec{y} \equiv \{P^{l_j}_{(\lambda,\mu)}\} \longrightarrow \pi(0_i, l_j)$
- Search for the l_j **states** that **maximize correlation** with a given 0_i **state** with $E(l_>) > E(l_<)$. These set of states form natural candidates for rotational bands.
- Calculate: $C(2,4)_i = \max_j[\pi(0_i,2_j)]\max_k[\pi(0_i,4_k)]$

Away from SU(3) limit

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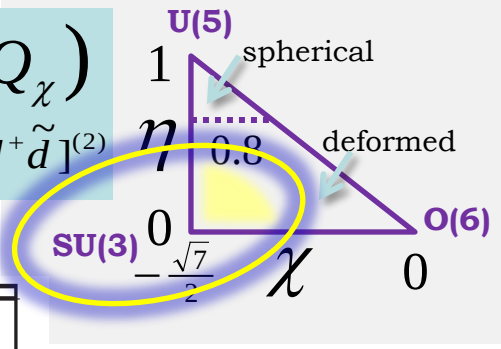

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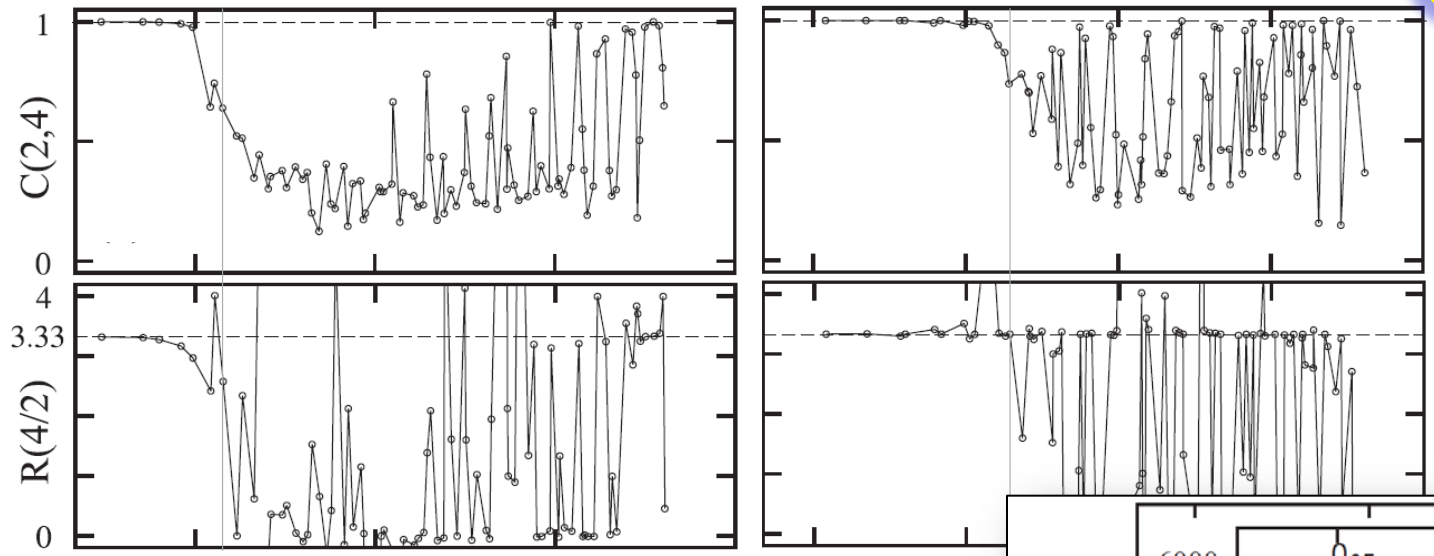
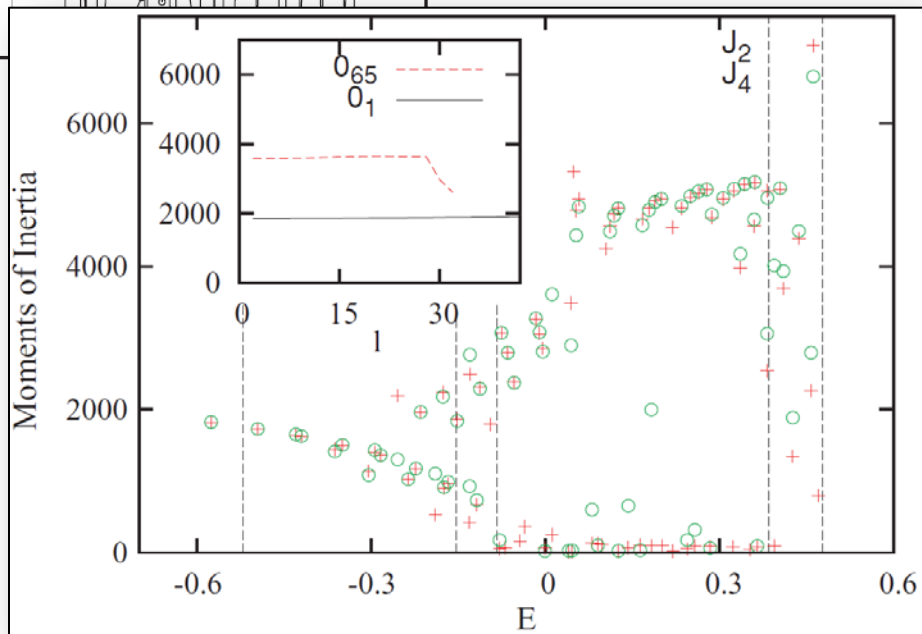


N=30

- For members of the same rotational band we require: $E(l_j) - E(0_i) = \frac{1}{2\Im_l} l(l+1)$ with $\Im_l \approx \Im_{l'}$
- In particular, $\frac{E(4_k) - E(0_i)}{E(2_j) - E(0_i)} \equiv R(4/2)_i \approx 3.33$



Macek et al. 2010 PRC 82,014308

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search for rotational bands

Macek et al. 2010 PRL 105,072503

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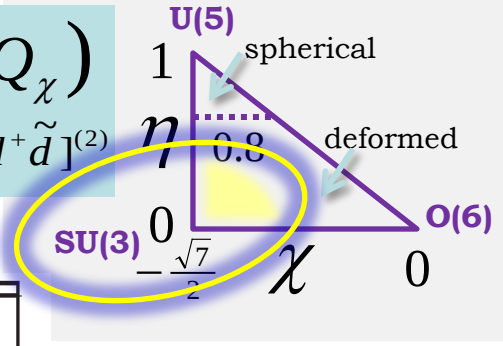
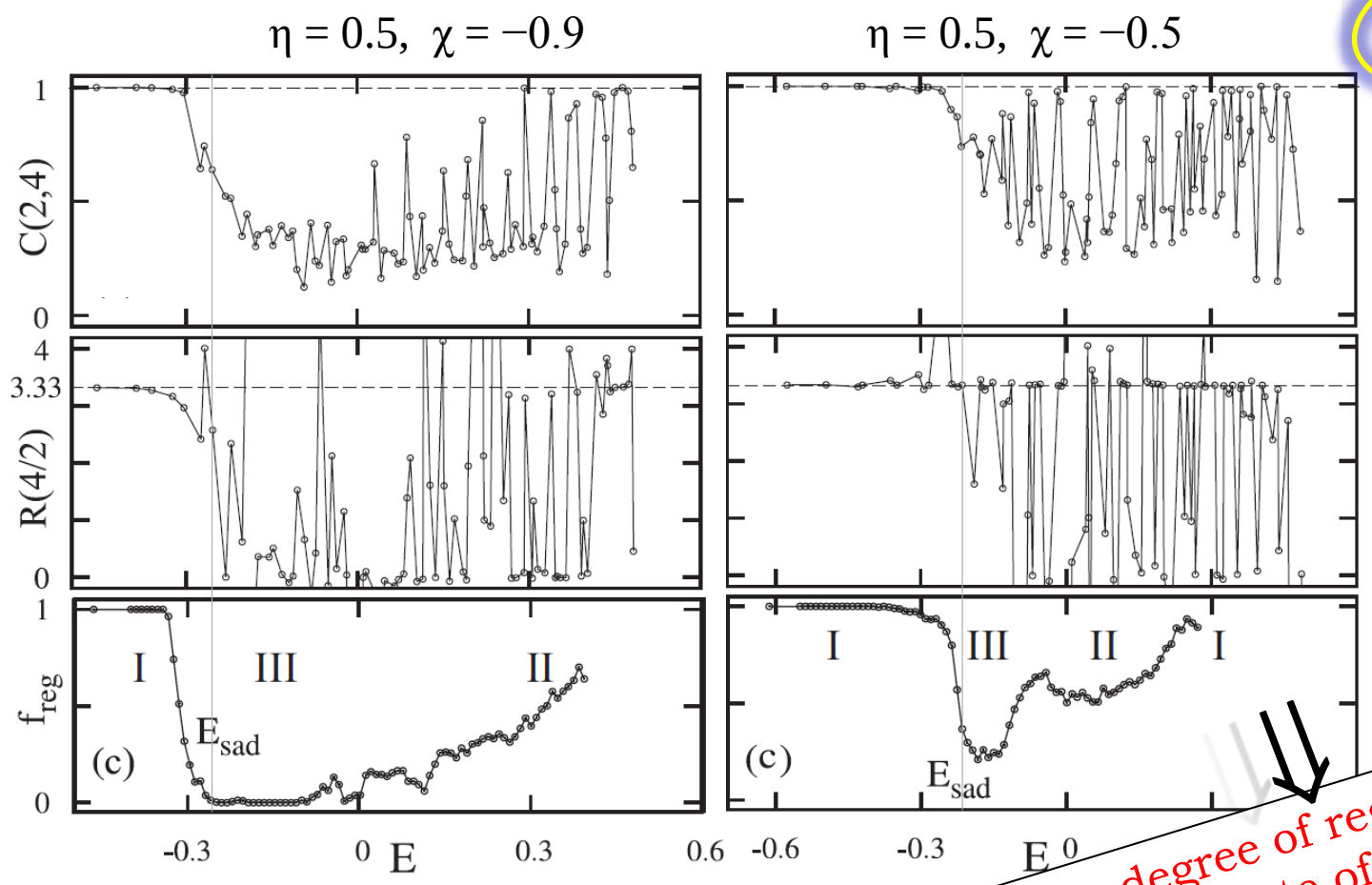
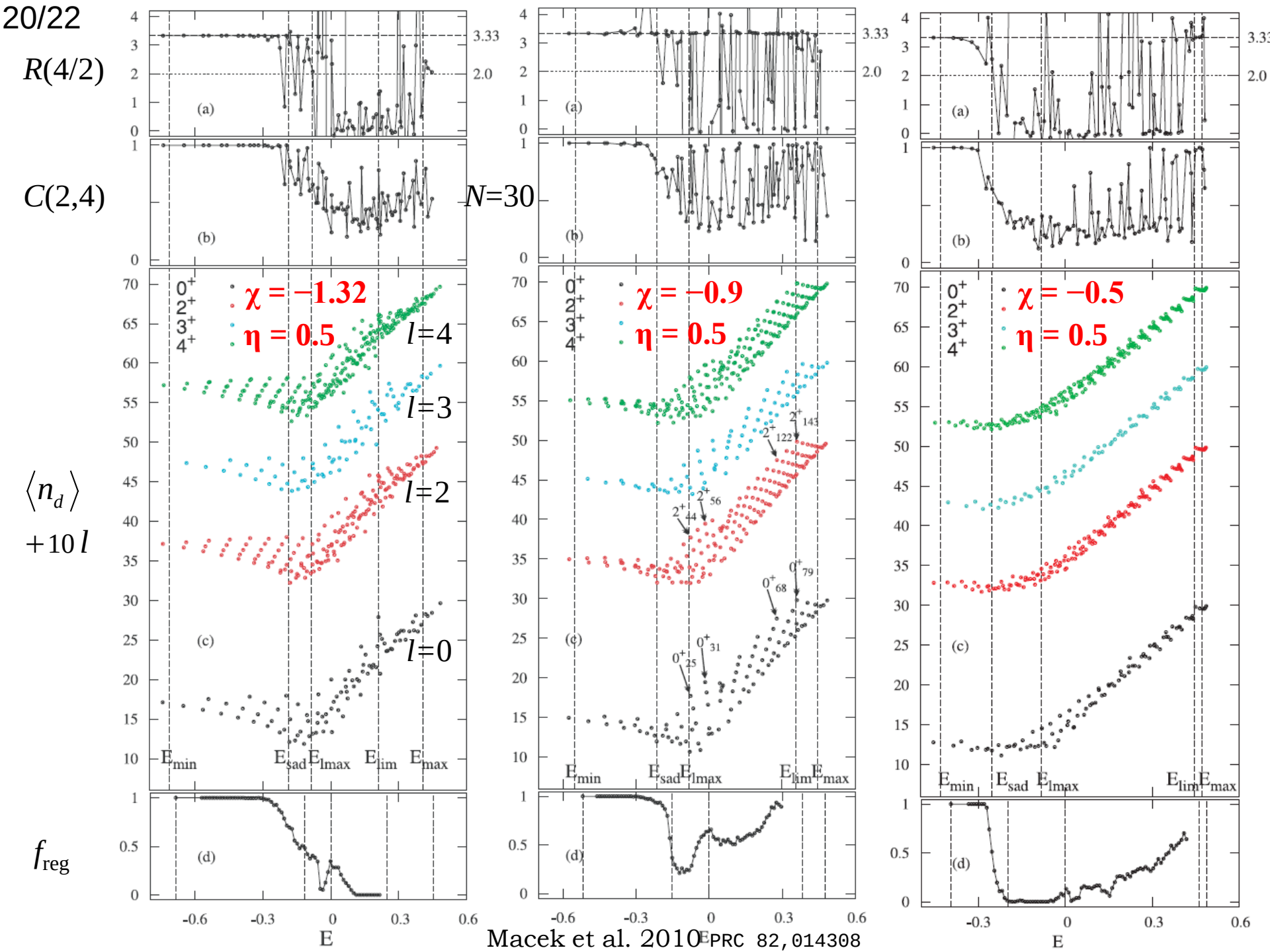




N=30

higher degree of regularity => higher occurrence rate of rotational bands

$$f_{\mathrm{reg}} = \Omega_{\mathrm{reg}} / \Omega_{\mathrm{tot}}$$



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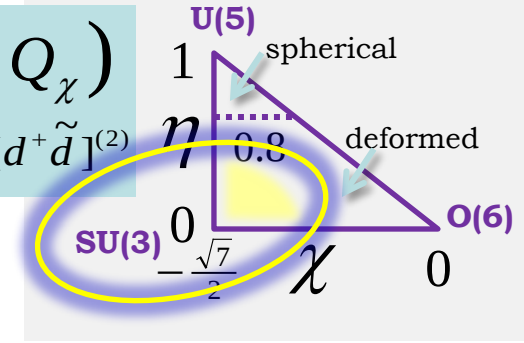


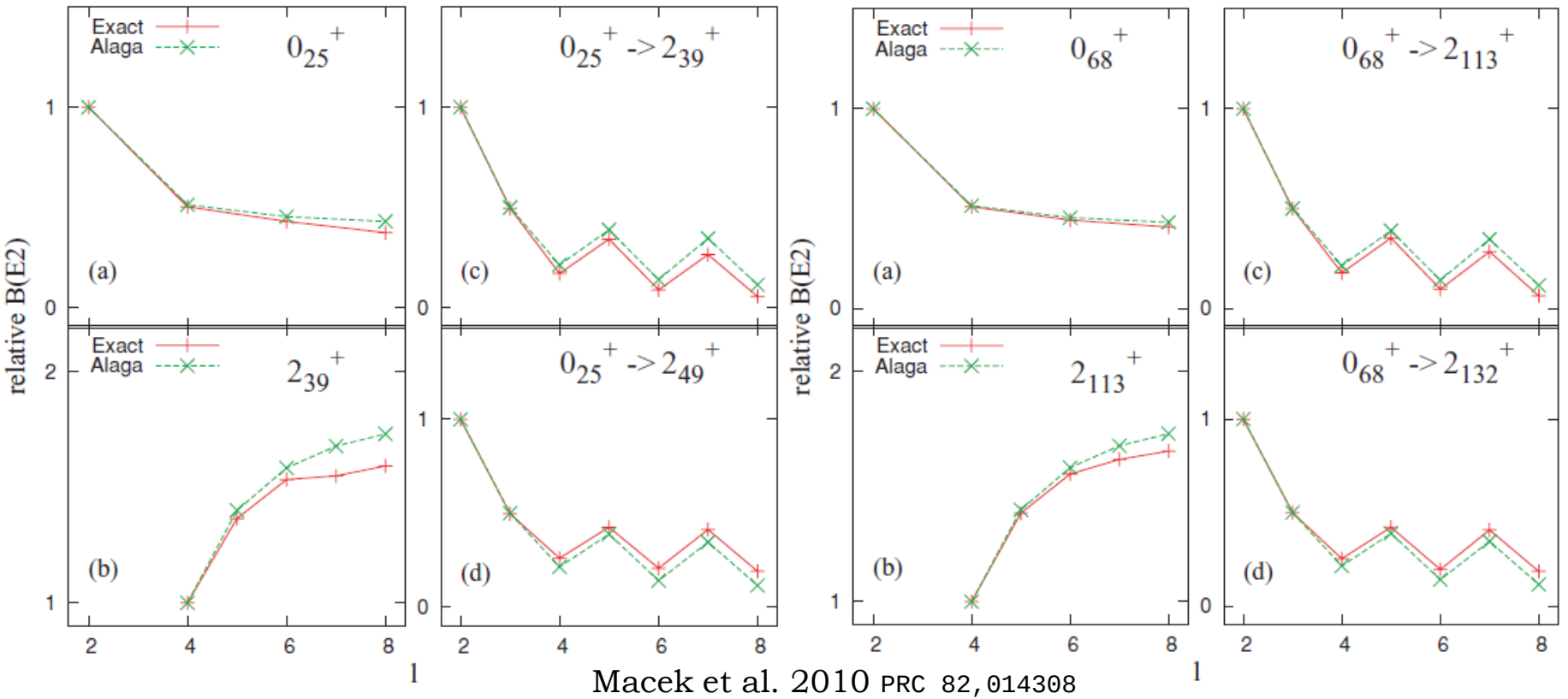
Alaga rule

$$B^{E2}(l, K \to l', K') = (lK2\Delta K \,|\, l'K')^2 \left\langle K' \middle| T^{E2} \middle| K \right\rangle$$

$$\Rightarrow \frac{B^{E2}(l_1, K \to l'_1, K')}{B^{E2}(l_2, K \to l'_2, K')} = \frac{(l_1 K 2\Delta K \,|\, l'_1 K')^2}{(l_2 K 2\Delta K \,|\, l'_2 K')^2}$$

medium energies N=30 **high energies**

intra-band inter-band intra-band inter-band



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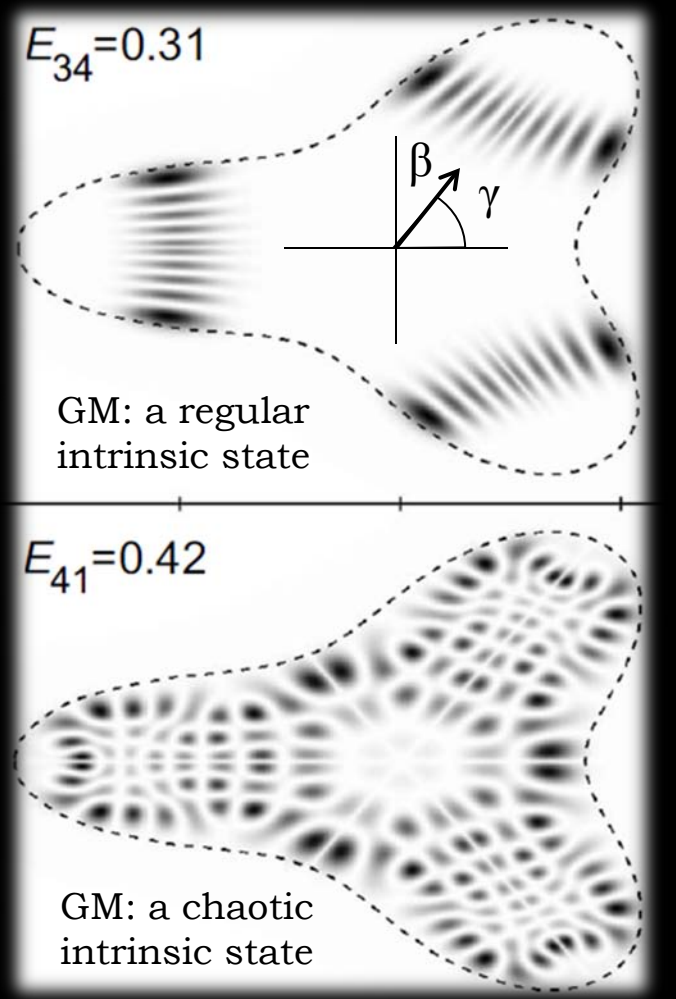
• A tentative criterion for a statistically enhanced occurrence of (adiabatically separated) rotational bands within the IBM: increased regularity of intrinsic (vibrational) motions.

• Why so? The form of wave function makes a typical chaotic state very vulnerable to external perturbations (ergodicity => large overlaps, mixing). In contrast, regular states are more resistant.

• This simple criterion may be applicable in general many-body systems. It should be further tested in various models.



Thanks to collaborators:

Michal Macek (now Hebrew Uni Jerusalem)
Pavel Stránský (now UNAM Mexico)
Jan Dobeš (NPI Řež near Prague)

Thank you for attention!