Experimental Evidence for Magic Numbers

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Outline

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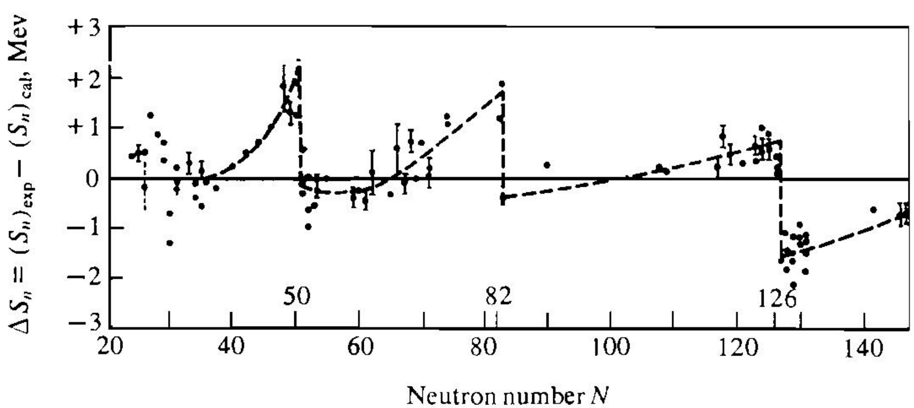
Introduction

- The nuclei exhibit extra stability when either proton number Z or neutron number N has the value $2, 8, 20, 28, 50, 82$, or 126. These numbers are the so-called magic numbers.

- The difference between neutron separation energy observed and calculated with semi-empirical mass formula in liquid drop model (LDM):

$$\Delta S_n = S_n(A, Z)_{\mathrm{exp}} - S_n(A, Z)_{\mathrm{calc}}$$

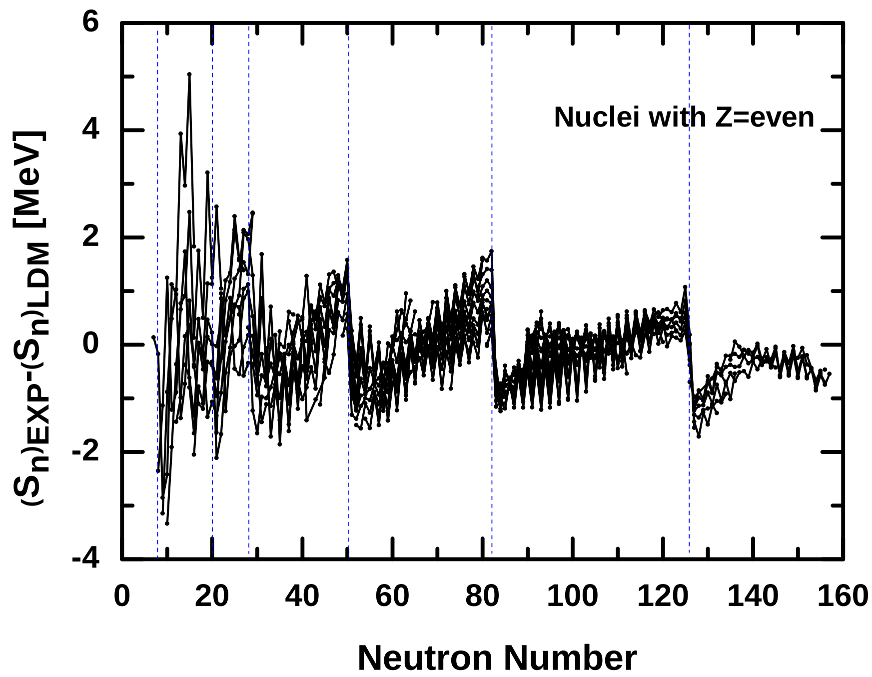
Sharp discontinuities are evident for $N = 50, 82$ and 126 showing a sudden decrease in separation energy of $(N + 1)$th neutron, which reflects the existence of well-known magic number N.



R. D. Evans , The Atomic Nucleus, 1955.

Introduction

- The odd-even effect brings in discontinuities to ΔS_n which are not associated to shell closure. $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ is more appropriate for shell closure study as two related nuclei in S_{2n} have the same neutron number parity.



This work

Appearance of new magic numbers and disappearance of known magic numbers are studied through $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ with new experimental data (G. Audi et al., NPA 729 (2003) 337).

Outline

General formulas

- Difference between two-neutron separation energy observed and calculated with semi-empirical mass formula in LDM:

$$\Delta S_{2n(2p)} = (S_{2n(2p)})_{\mathrm{EXP}} - (S_{2n(2p)})_{\mathrm{LDM}}. \tag{1}$$

a. Two-nucleon separation energy $S_{2n(2p)}$ is defined as

$$S_{2n}(A,Z) = BE(A,Z) - BE(A-2,Z) \tag{2}$$

$$S_{2p}(A,Z) = BE(A,Z) - BE(A-2,Z-2). \tag{3}$$

From the semi-empirical mass formula in LDM:

$$BE(A,Z) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_A(A-2Z)^2 A^{-1}, \tag{4}$$

$S_{2n}(A,Z)$ can be written as

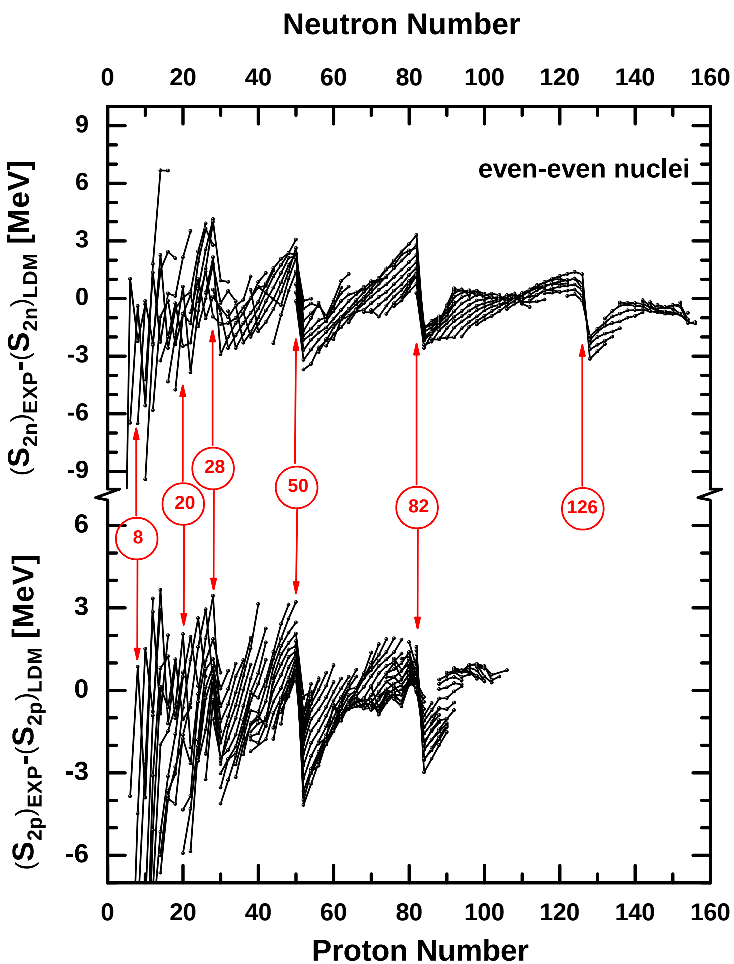
$$S_{2n}(A,Z) \approx 2(a_V - a_A) - \frac{4}{3}a_S A^{-1/3} + \frac{2}{3}a_C Z(Z-1)A^{-4/3} + 8a_A \frac{Z^2}{A(A-2)},$$

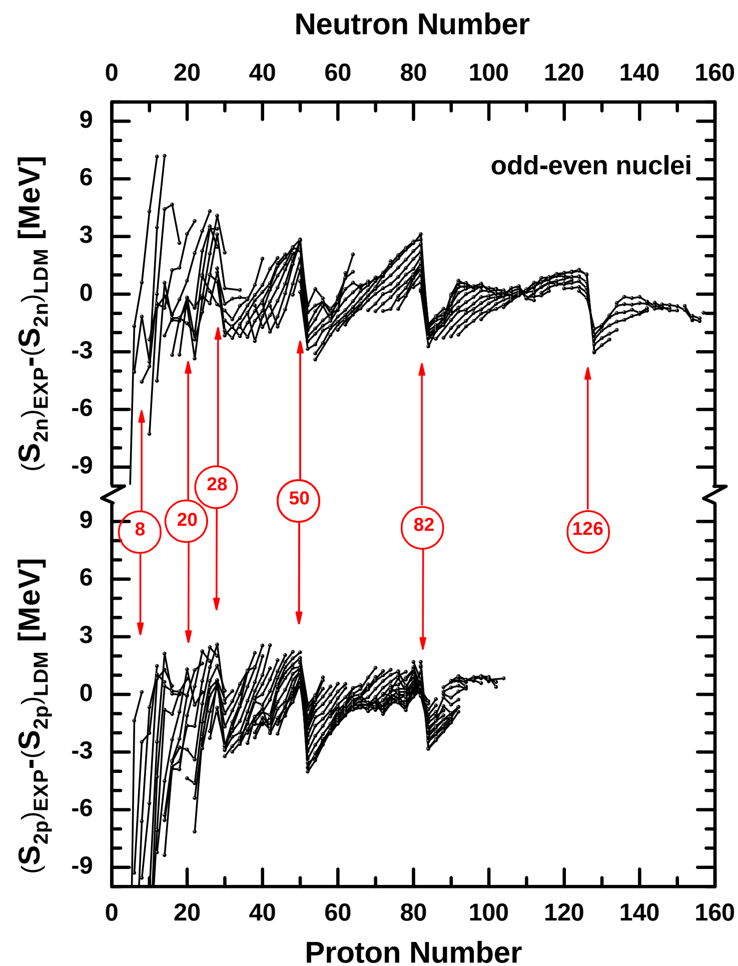
where the values for the LDM parameters: $a_V = 15.85$ MeV, $a_C = 0.71$ MeV, $a_S = 18.34$ MeV and $a_A = 23.21$ MeV. A. H. Wapstra et al., *Nucl. Data Tables* 9 (1971) 267.

b. The experimental data of two-neutron separation energy is taken from G. Audi et al., *Nucl. Phys. A* 729 (2003) 337.

A magic number appears as a sudden decrease in $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ at the neutron number $(N+2)$ following the neutron magic number N.

Neutron separation energy and magic number





- Sudden decreases of the order of 3 MeV are evident for $N = 28, 50, 82, 126$, which reflects the existence of known magic number.
- Two-neutron separation energy differences are complicated in light nuclei, thus further study in this region is needed.

Outline

Odd-even nuclei

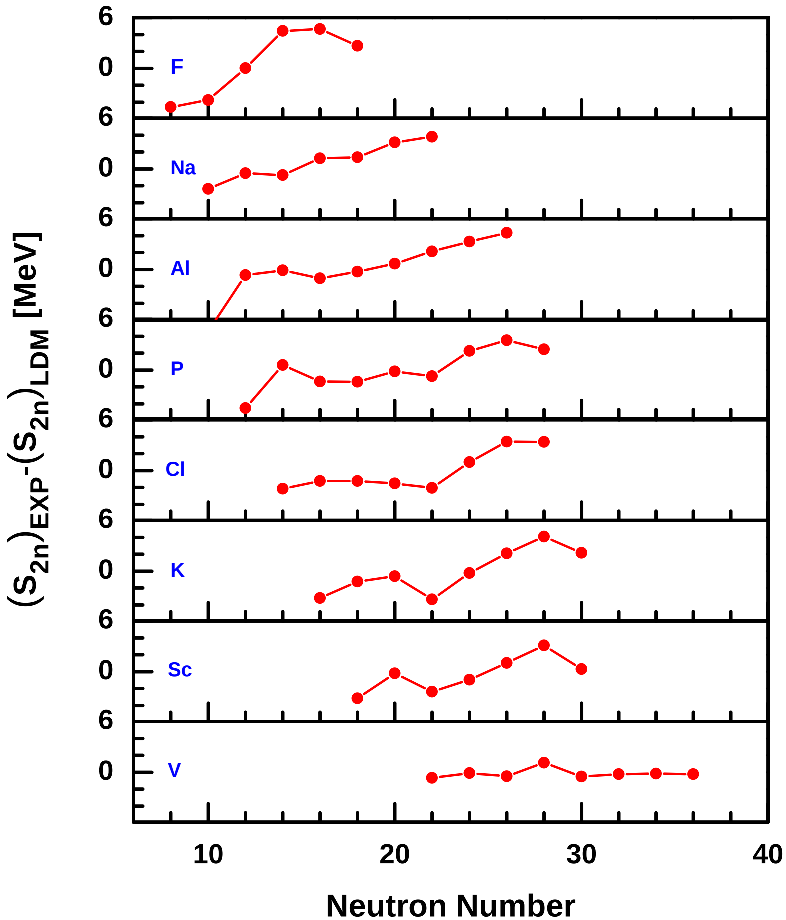


Figure: $(S_{2n})_{\text{EXP}} - (S_{2n})_{\text{LDM}}$ for odd-even nuclei as a function of neutron number.

- Well-known magic number
  a. $N = 8$: F isotope chain, no decrease, indicating disappearance of magic number;
  b. $N = 20$:
     * Na and Al isotope chains, no decreases;
     * P to Sc isotope chains, sudden decreases of 0.55, 0.51, 2.76 and 2.19 MeV respectively;
     * From Na to Sc isotopes, shell closure appears for $Z \geq 15$ and shell effect first increases and then decreases with Z;
  c. $N = 28$: K to V isotope chains, sudden decreases of 1.92, 2.80 and 1.65 MeV respectively. Shell effect first increases and then decreases with Z;
- Possible new magic number
  a. $N = 14$: Al and P isotope chains;
  b. $N = 16$: F isotope chain;
  c. $N = 26$: P isotope chain;

Sudden decreases exist, an indication of new magic number.

Even-even nuclei

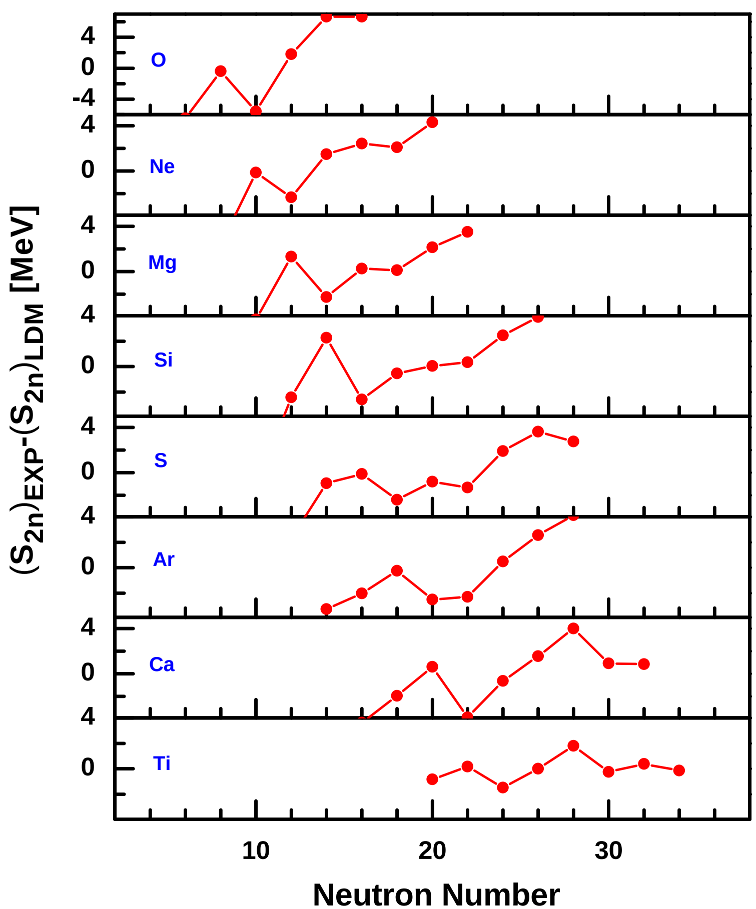


Figure: $(S_{2n})_{\text{EXP}} - (S_{2n})_{\text{LDM}}$ for even-even nuclei as a function of neutron number.

- Well-known magic number
  a. $N = 8$: O isotope chain, sudden decrease of 5.19 MeV ; Ne isotope chain, no decrease;
  b. $N = 20$:
     * Mg, Si, Ar and Ti isotope chains, no decreases;
     * S and Ca isotope chains, sudden decreases of 0.51 and 4.47 MeV respectively;
     * From Mg to Ti isotopes, similar evolution of shell effects as odd-even nuclei is absent;
  c. $N = 28$: Ca and Ti isotope chains, sudden decreases of 3.07 and 2.05 MeV respectively.
- Possible new magic numbers
  a. $N = Z$: Sudden decrease is evident in each isotope chain, which is caused by special properties of nuclei with $N = Z$ or shell effect;
  b. $N \neq Z$ region
     * $N = 16$, Ne and Mg isotope chains;
     * $N = 26$, S isotope chain;
     * $N = 32$, Ti isotope chain;

  Sudden decreases exist, an indication of new magic number.

Outline

Extra binding energy

- Surfaces of experimental masses of even-even and odd-odd nuclei exhibit a sharp discontinuity at $N = Z$. This cusp, reflecting an additional binding in nuclei with neutrons and protons occupying the same shell model orbitals, is knownly attributed to neutron-proton pairing correlations. W. Satula et al., *PLB* 407 (1997) 103, E. P. Wigner, *Phys. Rev.* 51 (1937) 106.

$\Rightarrow$ In order to understand the reason of sudden decrease of $(S_{2n})_{\rm EXP} - (S_{2n})_{\rm LDM}$ at $N = Z$, the extra binding should be removed first.

Assumptions of extra binding energy extraction

- Chasman's method R. R. Chasman, PRL 99 (2007) 082501 :
  a. Extra binding energy is related to $|N - Z|$;
- Present method in even-even nuclei:
  a. Extra binding energy occurs in $N = Z$ nuclei;

$\ast$ The binding energies in these nuclei are decomposed into a part $\tilde{B}$ which varies smoothly with N and Z and extra binding energy Δ.

Extract extra binding energy

- Double-difference formula δV, involving smooth contribution $\delta\tilde{S}_{2n}(Z,N)$ and extra binding Δ, is written as

$$\begin{aligned}\delta V(Z,N) &= [B(Z,N) - B(Z,N-2)] - [B(Z-2,N) - B(Z-2,N-2)] \\ &= \delta\tilde{S}_{2n}(Z,N) + 2\Delta,\end{aligned} \tag{5}$$

where

$$\delta\tilde{S}_{2n}(Z,N) = \left[\tilde{B}(Z,N) - B(Z,N-2)\right] - \left[B(Z-2,N) - \tilde{B}(Z-2,N-2)\right].$$

- Average double-difference formula, $\overline{\delta V}$, is defined as

$$\begin{aligned}\overline{\delta V}(Z,N) &= \frac{1}{4}[\delta V(Z-2,N) + \delta V(Z+2,N) + \delta V(Z,N-2) + \delta V(Z,N+2)] \\ &= \overline{\delta\tilde{S}}_{2n}(Z,N) - \Delta,\end{aligned} \tag{6}$$

where

$$\overline{\delta\tilde{S}}_{2n}(Z,N) = \frac{1}{4}\left[\delta\tilde{S}_{2n}(Z-2,N) + \delta\tilde{S}_{2n}(Z+2,N) + \delta\tilde{S}_{2n}(Z,N-2) + \delta\tilde{S}_{2n}(Z,N+2)\right].$$

If we consider $\delta\tilde{S}_{2n}(Z,N)$ approximately equals to $\overline{\delta\tilde{S}}_{2n}(Z,N)$, from Eq. (5) and Eq. (6) extra binding energy is extracted as

$$\Delta = \frac{1}{3}\left[\delta V(Z,N) - \overline{\delta V}(Z,N)\right]. \tag{7}$$

* Result of Chasman's method: $\Delta = \frac{1}{4}\left[\delta V(Z,N) - \overline{\delta V}(Z,N)\right]$.

Extra binding energy

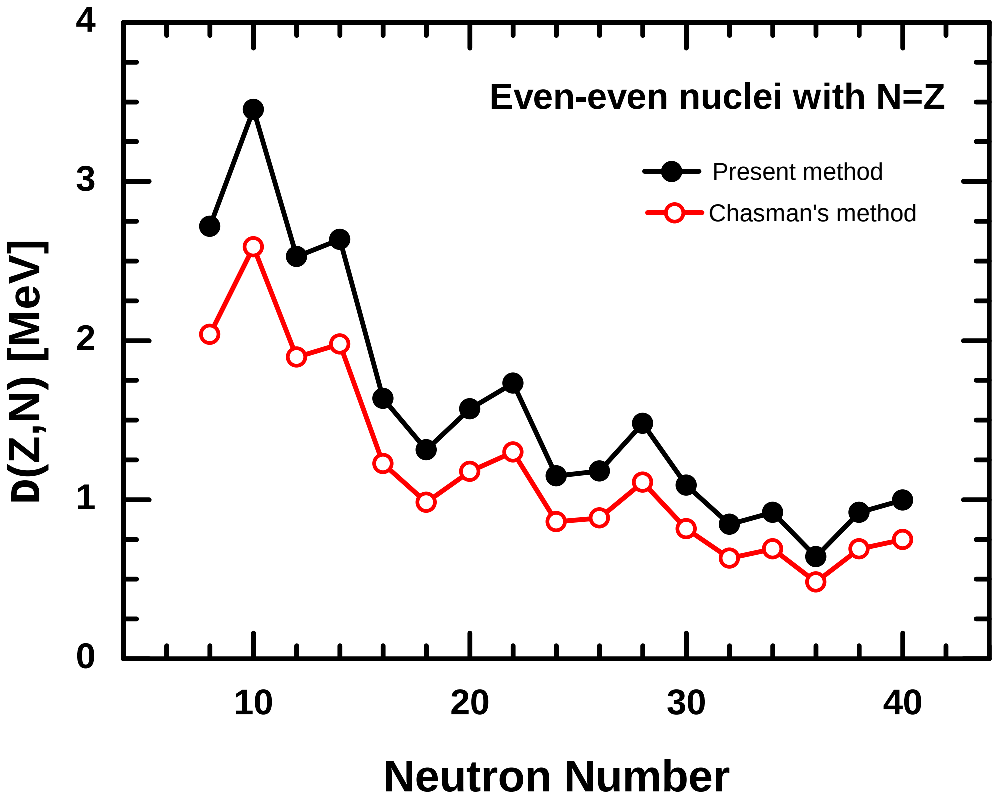
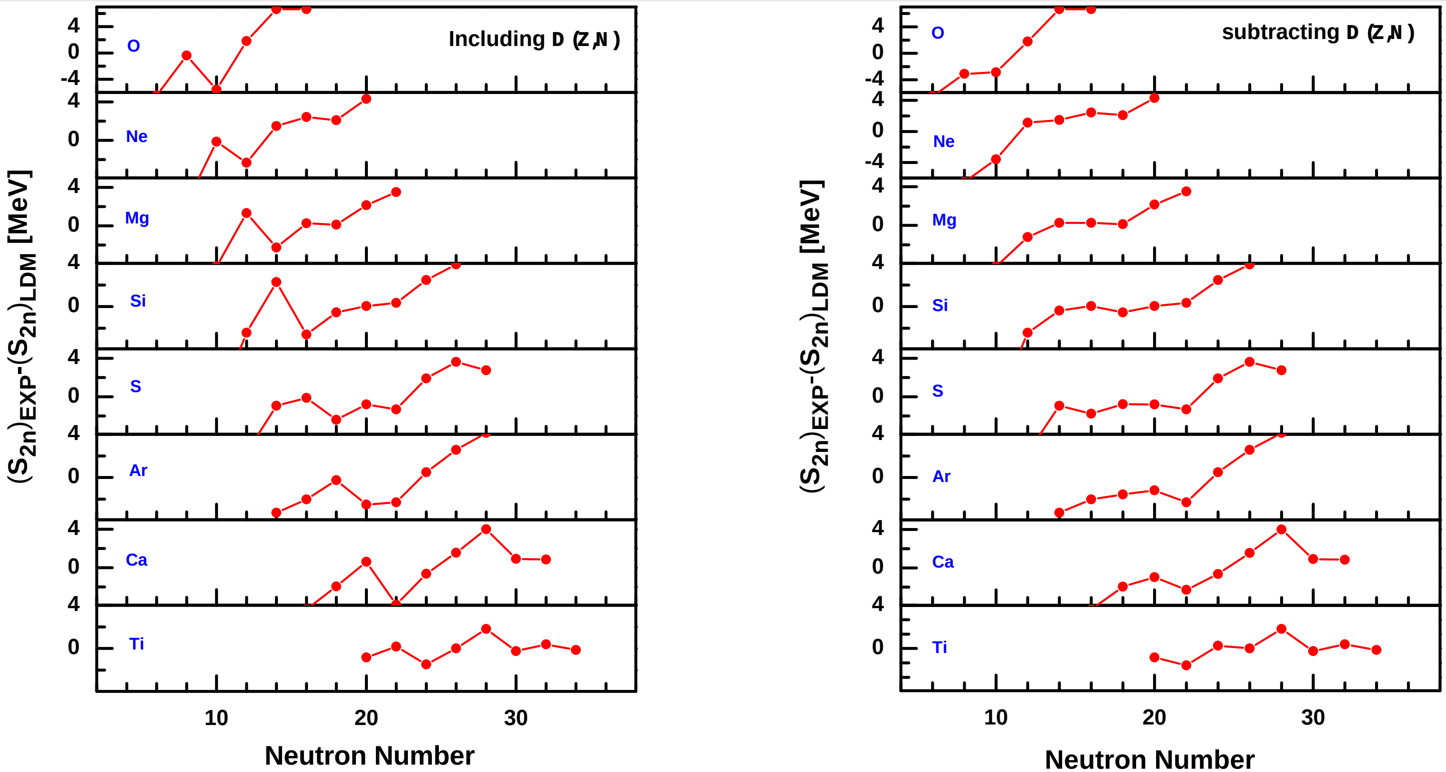


Figure: Extra binding energy in $N = Z$ even-even nuclei as a function of neutron number. The black line and red line are obtained from the present method and Chasman's method respectively.

- The extra binding energies extracted with both methods have the same behavior with increasing neutron number;
- The gap between the results of two methods is attributed to different assumptions.

Neutron separation energy after removing extra binding



- $N = Z$ for non-magic nuclei: no decreases remain, no indication of new magic numbers;
- $N = 20$ for Ar and Ti isotope chains: Recovering of decreases indicates the existence of this magic number. From Mg to Ti isotopes, shell closure appears for $Z \geq 16$; shell effect first increases and then decreases with Z and reaches a maximum at double magic nucleus ^{40}Ca;
- $N = 14$ for S isotope chain and $N = 16$ for Si isotope chain: Occurrence of new decreases indicates appearance of new magic numbers;
- $N = Z$ for magic nuclei: Decrease remains for ^{40}Ca, but disappears for ^{16}O, because the extra binding energy extracted with present method partly mixes the shell effect in very light nuclei.

Outline

Possible new magic number and disappearance of known magic number

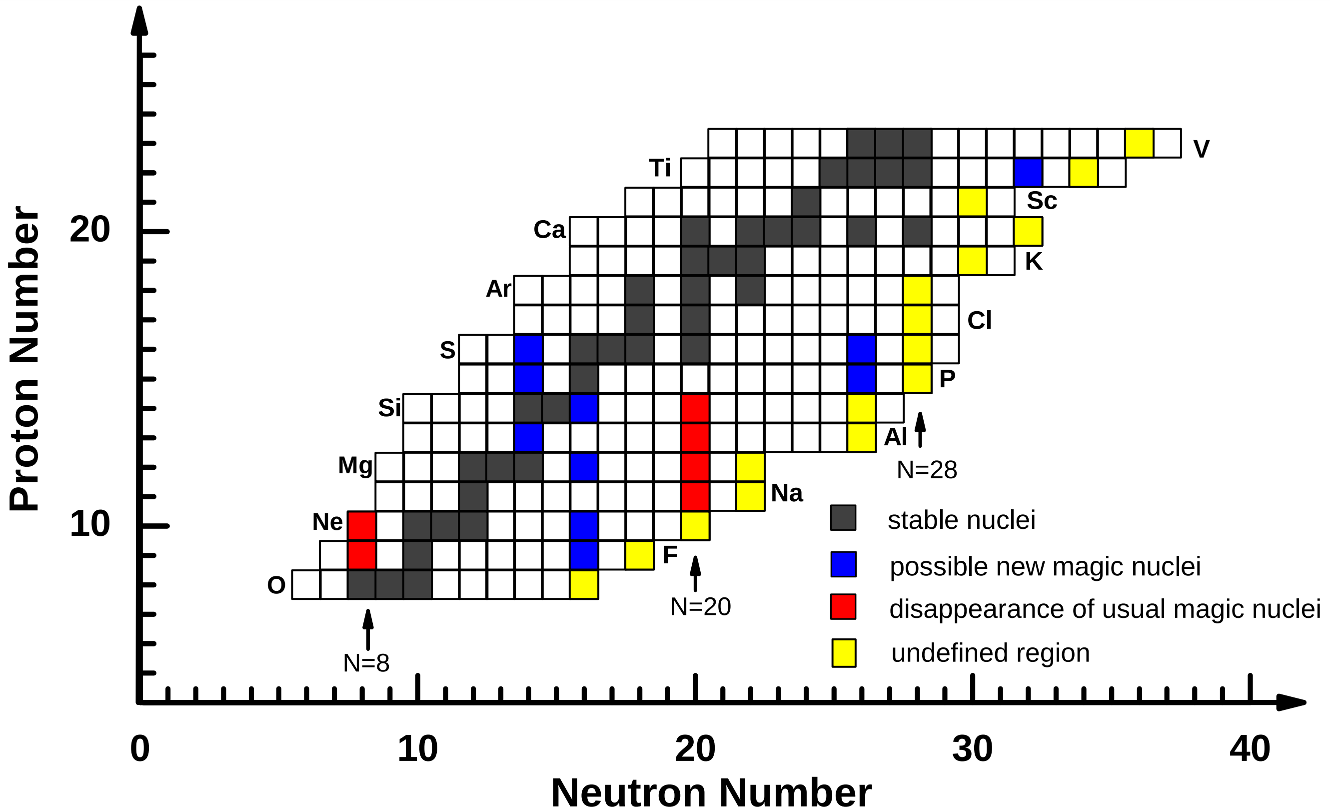


Figure: A partial nuclear chart indicating the disappearance of known magic number and appearance of new magic number.

- Disappearance of known magic number:
  a. $N = 8$, F and Ne isotopes,
  b. $N = 20$, Na to Si isotopes.
- Possible new magic number:
  a. $N = 14$, Al, P and S isotopes,
  b. $N = 16$, F, Ne, Mg and Si isotopes,
  c. $N = 26$, P and S isotopes,
  d. $N = 32$, Ti isotope.

Outline

Summary

- Difference between two-neutron separation energy observed and calculated with semi-empirical mass formula in LDM is calculated for all even-even and odd-even nuclei in Audi 2003 mass table.
- Well-known magic numbers $N = 50, 82, 126$ are observed in medium heavy nuclei.
- Appearance of new magic numbers and disappearance of known magic numbers in light nuclei are studied:
  a. Indicate the disappearance of known magic numbers:
     * $N = 8$, F and Ne isotopes; $N = 20$, Na to Si, Ar and Ti isotopes;

     possible new magic numbers:
     * $N = Z$, all even-even nuclei;
     * $N = 14$, Al and P isotopes; $N = 16$, F, Ne and Mg isotopes; $N = 26$, S isotope; $N = 32$, Ti isotope;
  b. A method to extract extra binding for even-even nuclei with $N = Z$ is proposed;
  c. Indicate disappearance of known magic numbers $N = 8, 20$ after removing extra binding:
     * $N = 8$, F and Ne isotopes;
     * $N = 20$, Na and Si isotopes;

     possible new magic numbers $N = 14, 16, 26, 32$:
     * $N = 14$, Al, P and S isotopes;
     * $N = 16$, F, Ne, Mg and Si isotopes;
     * $N = 26$, P and S isotopes;
     * $N = 32$, Ti isotope.

Thank You!!!

Outline

总结

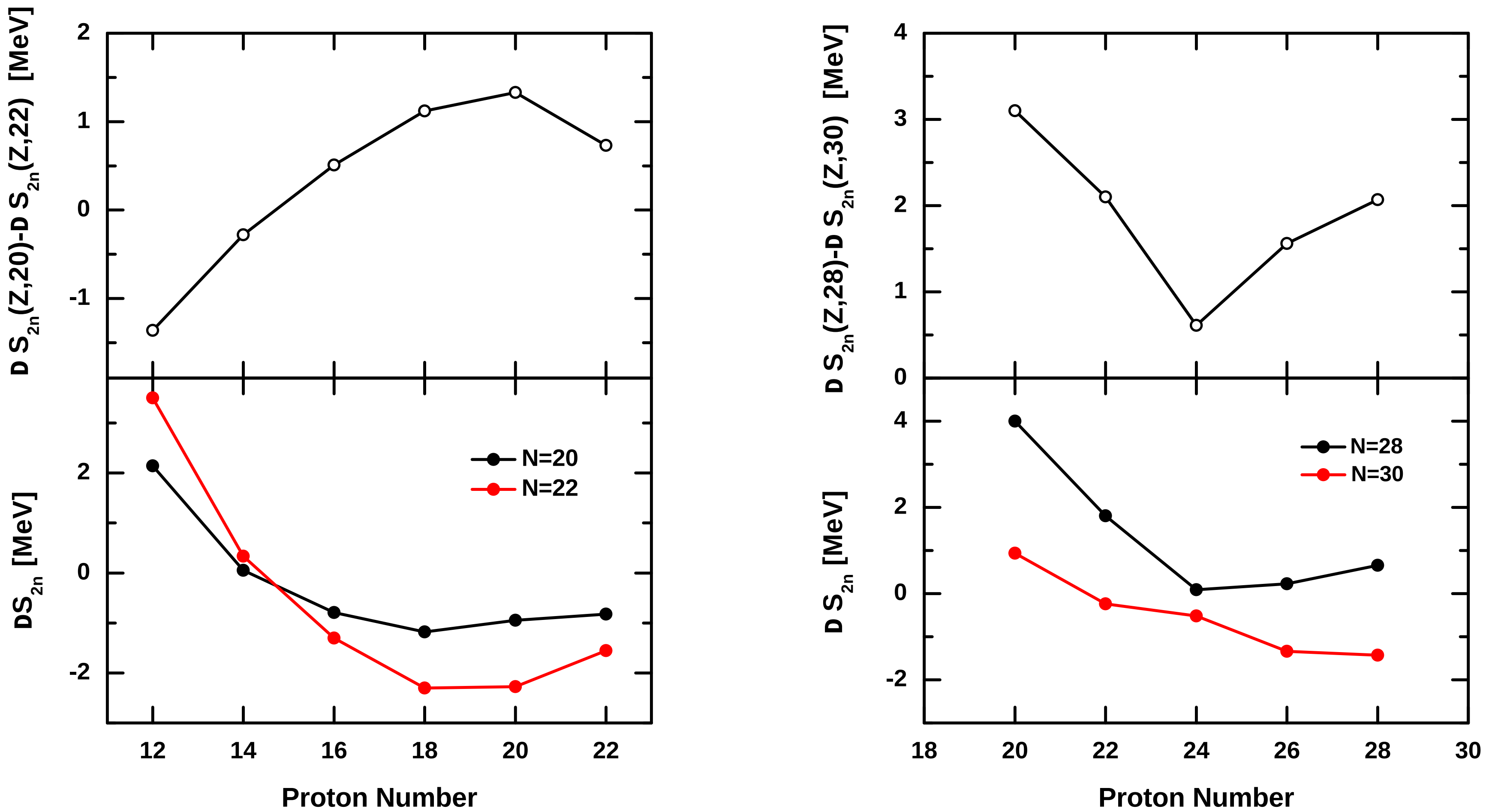
- 对 Audi 2003 质量表中所有的偶偶核与奇偶核，计算了双中子分离能的实验值与液滴模型半经验质量公式的计算值之差.
- 在轻质量区，研究了传统幻数的消失与新幻数的产生：
  a. 观察到传统幻数 $N = 8, 20$ 消失：
     * $N = 8$，F 与 Ne 同位素链；
     * $N = 20$，Na 到 Si, Ar 与 Ti 同位素链；

     以及可能新幻数的产生：
     * $N = Z$，所有 $N = Z$ 的偶偶核；
     * $N = 14$，Al 与 P 同位素；
     * $N = 16$，F, Ne 与 Mg 同位素；
     * $N = 26$，S 同位素；

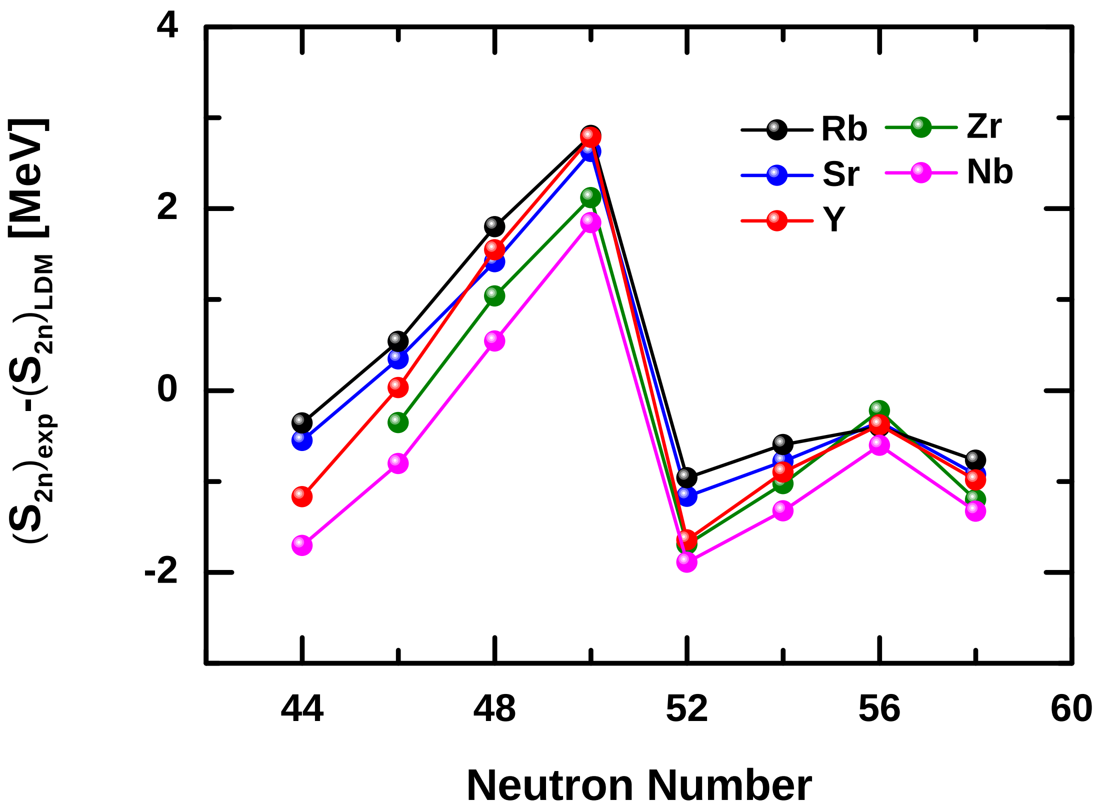
  b. 对 $N = Z$ 的偶偶核，给出了提取额外束缚能的方法；
  c. 扣除额外束缚能后，预示了传统幻数 $N = 8, 20$ 的消失：
     * $N = 8$，F 与 Ne 同位素；
     * $N = 20$，Na 到 Si 同位素；

     以及可能新幻数 $N = 14, 16, 26, 32$ 的产生：
     * $N = 14$，Al, P 及 S 同位素；
     * $N = 16$，F, Ne, Mg 和 Si 同位素；
     * $N = 26$，P 与 S 同位素；
     * $N = 32$，Ti 同位素.

双中子分离能之差随质子数的演化



可能的新幻数



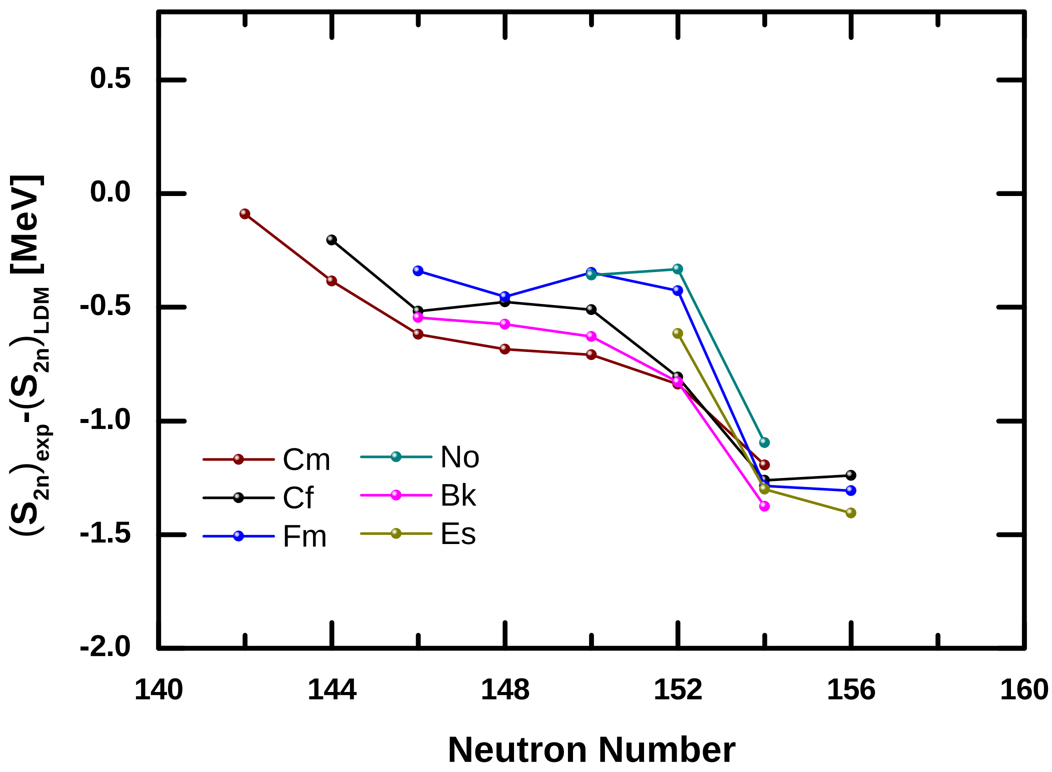
- 在 $N = 50$ 处，对 Sr 到 Nb 同位素链，双中子分离能之差都出现了明显跳变.

$\Rightarrow$ 传统幻数 $N = 50$ 对 Sr 到 Nb 同位素链依然成立.

- 在 $N = 56$ 处，对 Sr 到 Nb 同位素链，双中子分离能之差都出现了明显跳变，大小为 0.37 MeV, 0.58 Mev, 0.61 Mev, 0.98 MeV, 0.72 MeV.

$\Rightarrow$ $N = 56$ 对 Sr 到 Nb 同位素链，可能是一个新的幻数.

可能的新幻数



- 在 $N = 152$ 处，对 Cm 到 No 同位素链，双中子分离能之差都出现了明显跳变.

⇒ 对 Cm 到 No 同位素，$N = 152$ 可能是一个新的幻数.

额外束缚能提取

Surface of experimental masses of even-even and odd-odd nuclei exhibit a sharp slope discontinuity at $N = Z$. This cusp, reflecting an additional binding in nuclei with neutrons and protons occupying the same shell model orbitals, is usually attributed to neutron-proton pairing correlations.
实验上给出的偶偶核与奇奇核的质量曲面在 $N = Z$ 处表现出明显的斜率不连续性. 这个不连续点通常认为是由中子-质子对关联引起的，反映了在质子与中子占据相同壳模型轨道的原子核中存在额外束缚能. E. P. Wigner, *phys. Rev.* 51 (1937) 106, W. Satula, *PLB* 407 (1997) 103.

双中子分离能表达式推导

原子核双核子分离能的半经验质量公式：

$$S_{2n}(A,Z) = BE(A,Z) - BE(A-2,Z) \quad (8)$$
$$S_{2p}(A,Z) = BE(A,Z) - BE(A-2,Z-2). \quad (9)$$

从液滴模型（LDM）半经典质量公式：

$$BE(A,Z) = a_V A - a_S A^{2/3} - a_C Z(Z-1)A^{-1/3} - a_A (A-2Z)^2 A^{-1}, \quad (10)$$

可以得到：

$$S_{2n}(A,Z) \approx 2(a_V - a_A) - \frac{4}{3}a_S A^{-1/3} + \frac{2}{3}a_C Z(Z-1)A^{-4/3} + 8a_A \frac{Z^2}{A(A-2)},$$

证明过程如下：

- 体积项：

$$a_V A - a_V(A-2) = 2a_V, \quad (11)$$

- 对称能项：

$$-a_A \frac{(A-2Z)^2}{A} + a_A \frac{(A-2Z-2)^2}{A-2} = -2a_A + 8a_A \frac{Z^2}{A(A-2)}. \quad (12)$$

以上两项贡献的计算是严格的。

双中子分离能表达式推导

- 表面项的贡献：

$$a_S A^{2/3} - a_S (A-2)^{2/3} \approx (\Delta A)\frac{d(a_S A^{2/3})}{dA} \quad (13)$$

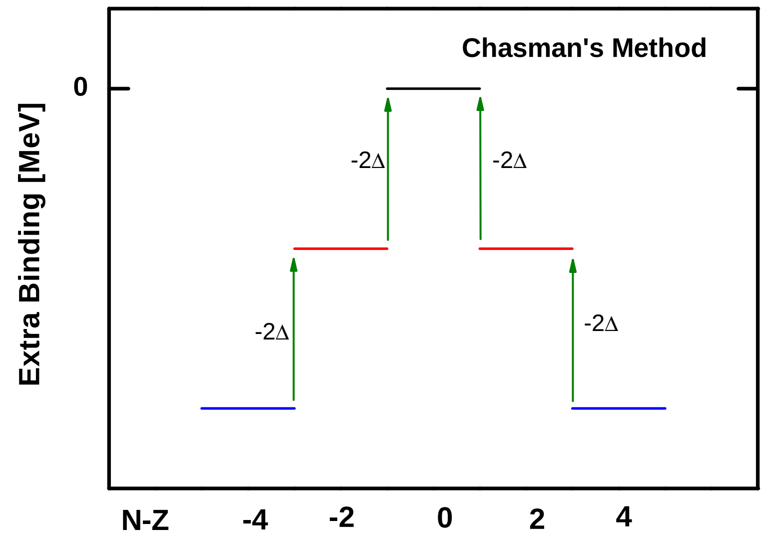
$$= \frac{4}{3} a_S A^{-1/3}, \quad (14)$$

- 库仑项：

$$-a_c Z(Z-1)A^{-1/3} + a_C Z(Z-1) = a_c Z(Z-1)(\frac{2}{3}A^{-4/3}). \quad (15)$$

表面项与库仑项的推导采用了近似：将表面项及库仑项的贡献近似的用其倒数及质量数变化的乘积来计算。

Chasman 工作



标记：$Z = N$ 原子核中，最外层 4 个核子的贡献取平均值，标记为 Δ。
束缚能的二阶差分：

$$\begin{aligned} \delta V(Z,N) &= [B(Z,N) - B(Z,N-2)] - [B(Z-2,N) - B(Z-2,N-2)] \\ &= \delta\tilde{S}_{2n}(Z,N) + 4\Delta, \end{aligned} \tag{16}$$

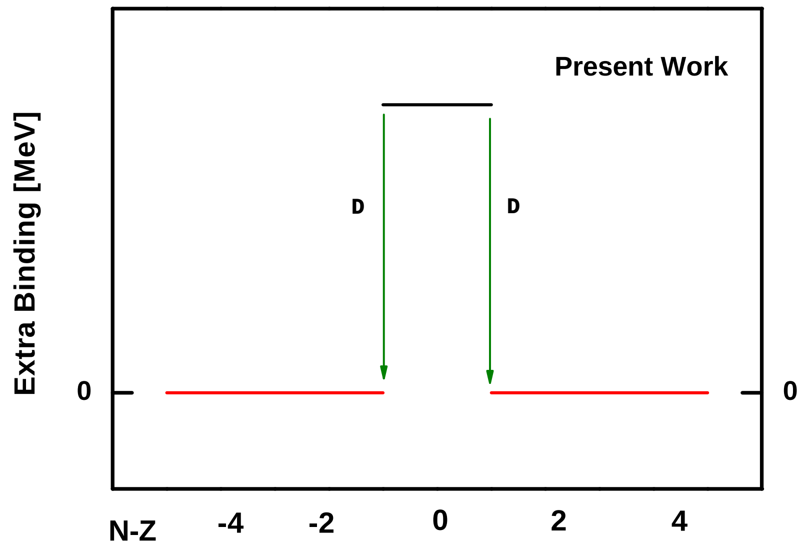
平滑部分：

$$\begin{aligned} \overline{\delta V}(Z,N) &= \frac{1}{4}[\delta V(Z-2,N) + \delta V(Z+2,N) + \delta V(Z,N-2) + \delta V(Z,N+2)] \\ &= \overline{\delta\tilde{S}_{2n}}(Z,N), \end{aligned} \tag{17}$$

额外束缚能：

$$\Delta = \frac{1}{4}\left[\delta V(Z,N) - \overline{\delta V}(Z,N)\right]. \tag{18}$$

本工作



标记：$Z = N$ 原子核中，质子数与中子数相等引入的额外束缚能，标记为 Δ。
束缚能的二阶差分：

$$\begin{aligned} \delta V(Z,N) &= [B(Z,N) - B(Z,N-2)] - [B(Z-2,N) - B(Z-2,N-2)] \\ &= \delta\tilde{S}_{2n}(Z,N) + 2\Delta, \end{aligned} \quad (19)$$

平均值：

$$\begin{aligned} \overline{\delta V}(Z,N) &= \frac{1}{4}[\delta V(Z-2,N) + \delta V(Z+2,N) + \delta V(Z,N-2) + \delta V(Z,N+2)] \\ &= \overline{\delta\tilde{S}_{2n}}(Z,N) - \Delta, \end{aligned} \quad (20)$$

出额外束缚能：

$$\Delta = \frac{1}{3}\left[\delta V(Z,N) - \overline{\delta V}(Z,N)\right]. \quad (21)$$

中子分离能之差与双中子分离能之差

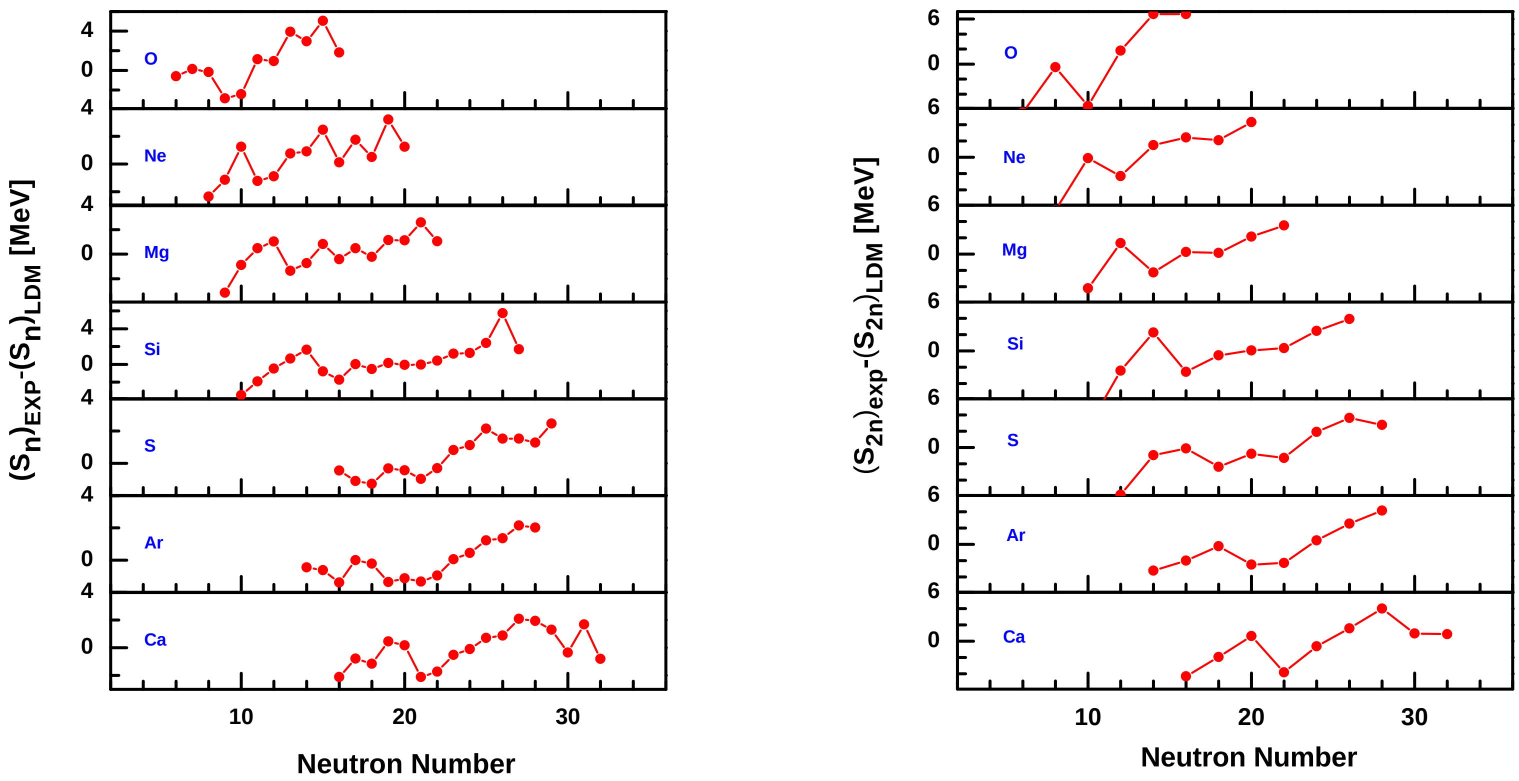


Figure: The values of $(S_n)_{\mathrm{EXP}} - (S_n)_{\mathrm{LDM}}$ and $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ as a function of neutron number.

Subtract extra binding energy for $N = Z$ even-even nuclei

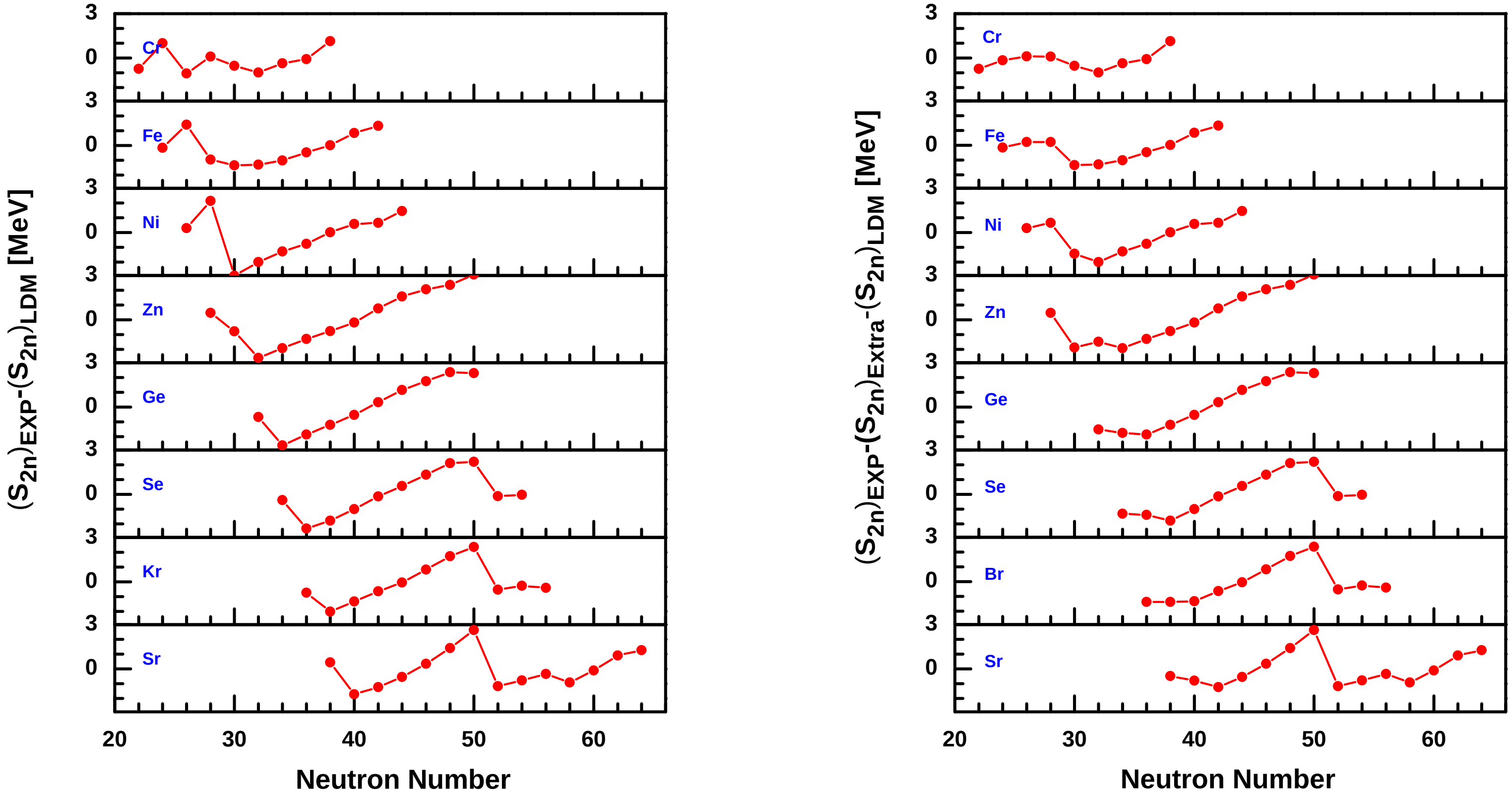


Figure: Experimental values of $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ and $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{Extra}} - (S_{2n})_{\mathrm{LDM}}$ for Cr to Sr isotopes as a function of neutron number.

- 在 $N = Z$ 的非幻数处，跳变消失，
- 在 $N = Z$ 幻数处，跳变在 ^{56}Ni 处依然存在；
- 在 $N = Z + 2$ 处，新的跳变在 Fe, Ni, Zn, Se 及 Sr 同位素链产生；

⇒ 在 $N = Z$ 非传统幻数处，跳变不是新幻数的信号，而是由额外束缚能导致的；

⇒ 通过这个方法提取的额外束缚能包含了其他的效应。

奇偶核

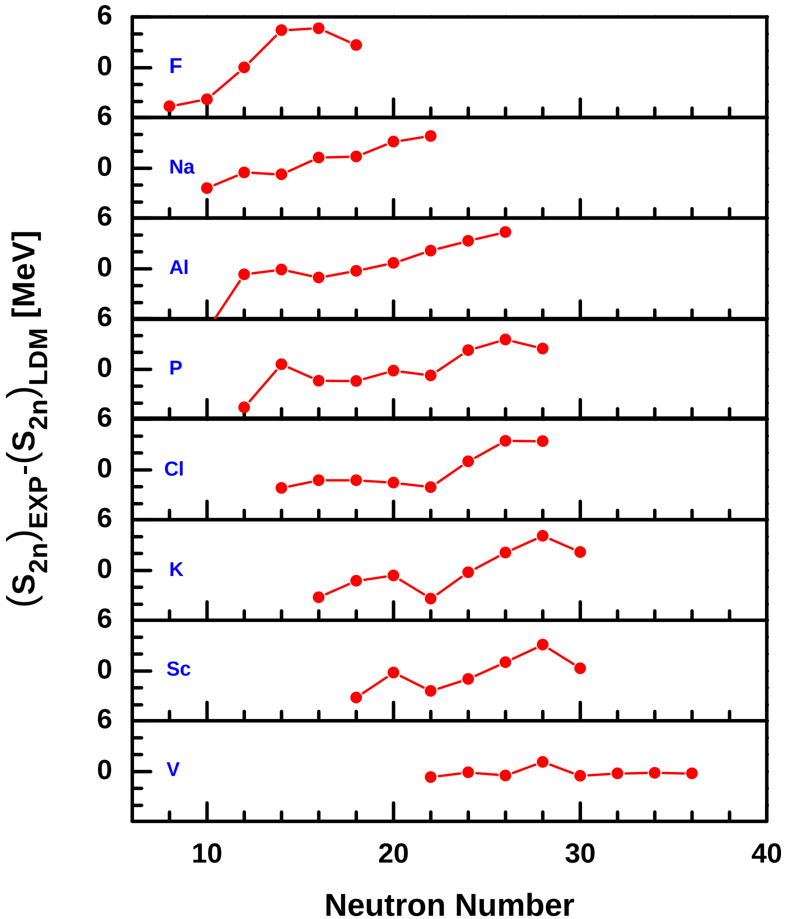
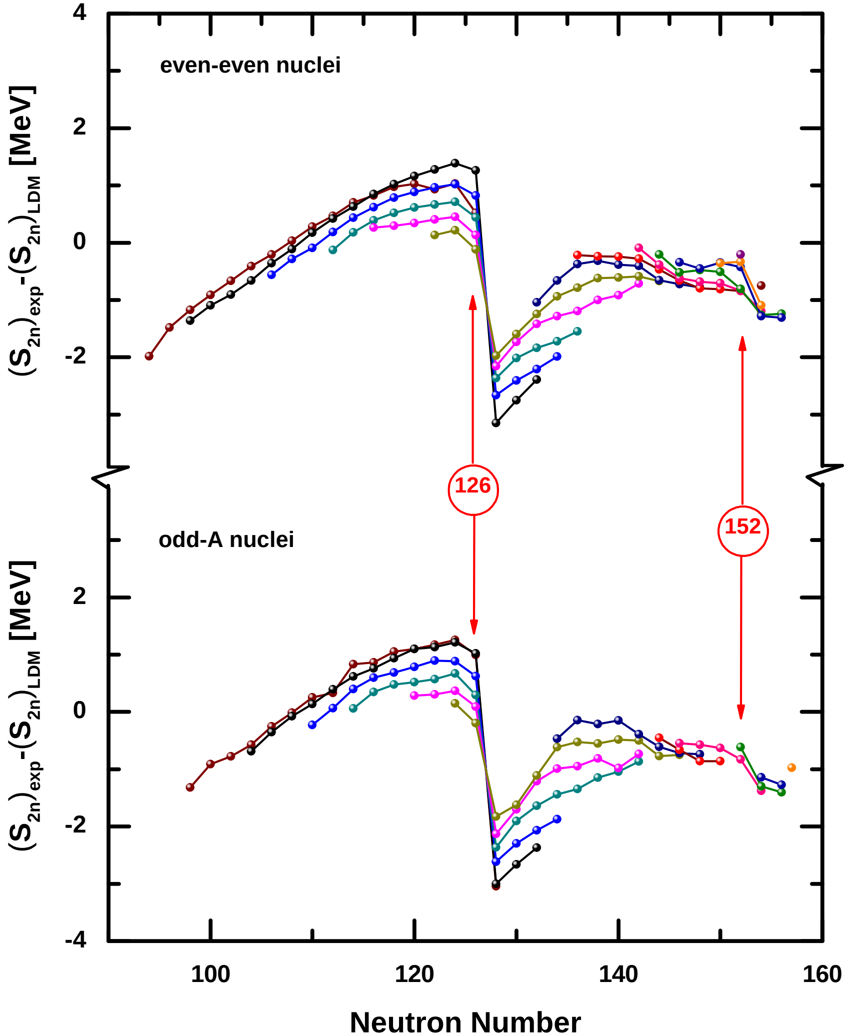


Figure: The values of $(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ for odd-even nuclei as a function of neutron number.

- 传统幻数：
  - $N = 28$ 处，K 到 V 同位素链出现明显跳变，其大小依次为：1.92MeV, 2.80MeV, 1.65MeV;
  - $N = 20$ 处，P 到 Sc 同位素链出现跳变，其大小依次为：0.55MeV, 0.51MeV, 2.76MeV, 2.19MeV；但是，对 Na 和 Al 同位素链，却没有跳变出现，可能反映此传统幻数消失了；
  - $N = 8$ 处，F 同位素链没有出现跳变，同样可能反映了幻数的消失；
- 可能的新幻数：
  - $N = 14$ 处，Al 与 P 同位素链出现跳变，大小依次为：0.96MeV, 1.94MeV；
  - $N = 16$ 处，对F 同位素链出现跳变，大小为 2.00MeV；
  - $N = 26$ 处，P 同位素链出现跳变，大小为 1.08MeV；

此处的跳变可能意味着新幻数的产生。

New magic number in heavy nuclei

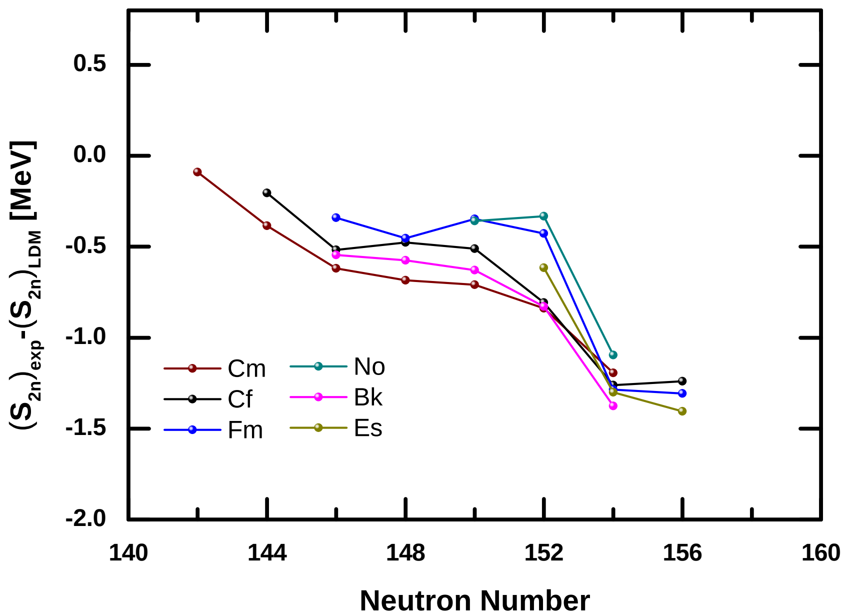


- The theoretical fission thresholds reveal the magicity of the neutron number 152.

  H. C. Pauli and T. Ledergerber, NPA 175 (1971) 545.

- Deformed shell closures are found at $N = 152$ through a systemic study of global properties of superheavy nuclei in the framework of macroscopic-microscopic method.

  A. Baran and K. Sieja et al, PRC 72 (2005) 044310.



References

- The halo structure of a light nucleus is shown to be associated with the closed shell effects of its core nucleus, and new magic numbers at $N = 6$ and 14 or 16 and/or $Z = 6$ and 14 are predicted on the basis of the potential energy surfaces calculated within the cluster-core model.

  Raj K Gupta, M Balasubramaniam et al, J. Phys. G: Nucl. Part. Phys. 32 (2006) 565.

- Theoretical, experimental and estimated single-neutron separation energies and two-neutron separation energies of Mg isotopes.

  Qijun Zhi, Zhongzhou Ren, PLB 638 (2006) 166.

- For nuclei having approximately equal numbers of neutrons and protons, there are large changes in binding energy due to many-body effects-even for closed shell nuclei. There are also large correlation energy effects in the even-odd and odd-even nuclei, with one neutron or one proton less than the $N = Z$ even-even nucleus. A previously derived result, that the diagonal correlation energy in the last occupied orbital for the latter nuclei is 1/2 of the equivalent correlation energy for closed shell nuclei, is tested against observed binding energies and found to be fairly accurate. R. R. Chasman PRL 99 (2007) 082501.

Jumps for even-even and odd-even nuclei

Table: Jumps (in MeV) in each isotope chain

N	6	8	10	12	14	16	18	20	26	28
C	3.24									
N		2.36								
O		5.19			0.00					
F						2.00				
Ne			2.19			0.33				
Na				0.23						
Mg				3.60		0.13				
Al					0.96					
Si					4.84					
P					1.94			0.55	1.08	
Cl							0.30	0.51		
S						2.28		0.51	0.86	
Ar							2.25			
K								2.76		1.92
Ca								4.47		3.07

Jumps for even-even and odd-even nuclei

N	20	22	24	26	28	30	32	34	36	38	40	50	54	56
Sc	2.19				2.80									
Ti		1.64			2.05		0.53							
V			0.39		1.65									
Cr			2.04		0.61									
Mn			0.78	0.68	0.31									
Fe				2.36										
Co					2.80									
Ni					5.05									
Cu					3.25	0.38								
Zn					1.24	1.80								
Ga						0.35	0.48							
Ge							1.92							
As								0.53				3.32		
Se								1.91				2.32		

Jumps for even-even and odd-even nuclei

N	20	22	24	26	28	30	32	34	36	38	40	42	50	54	56
Br									1.46				2.87	0.59	
Kr									1.27				2.89		
Rb										1.14			3.76		0.37
Sr										2.15			3.79		0.58
Y											1.48		4.43		0.61
Zr											2.39		3.81		0.98
Nb												1.08	3.73		0.72

There is no experimental data about nuclei with $N = Z$ when $Z > 40$.

Jumps (in MeV) for even-even and odd-even nuclei

N	50	64	82
Mo	3.33		
Tc	3.48	0.25	
Ru	2.90		
Rh	3.14		
Pd	3.01		
Ag	2.96		
Cd	2.90		
Sn			5.36
Sb			5.36
Te			4.66
I			4.33
Xe			3.70
Cs			3.92
Ba			3.46

N	82	92	94	96	98	100	102	104
La	3.51							
Ce	3.13							
Pr	3.20							
Nd	2.95	0.16						
Pm	3.17	0.21						
Sm	2.97	0.30						
Eu	3.21		0.28					
Gd	3.15		0.11	0.15				
Tb	3.11							
Dy	2.82				0.15			
Ho	2.87				0.38			
Er	2.47				0.15	0.20		
Tm	2.45						0.17	0.29
Yb	2.55						0.1	0.43

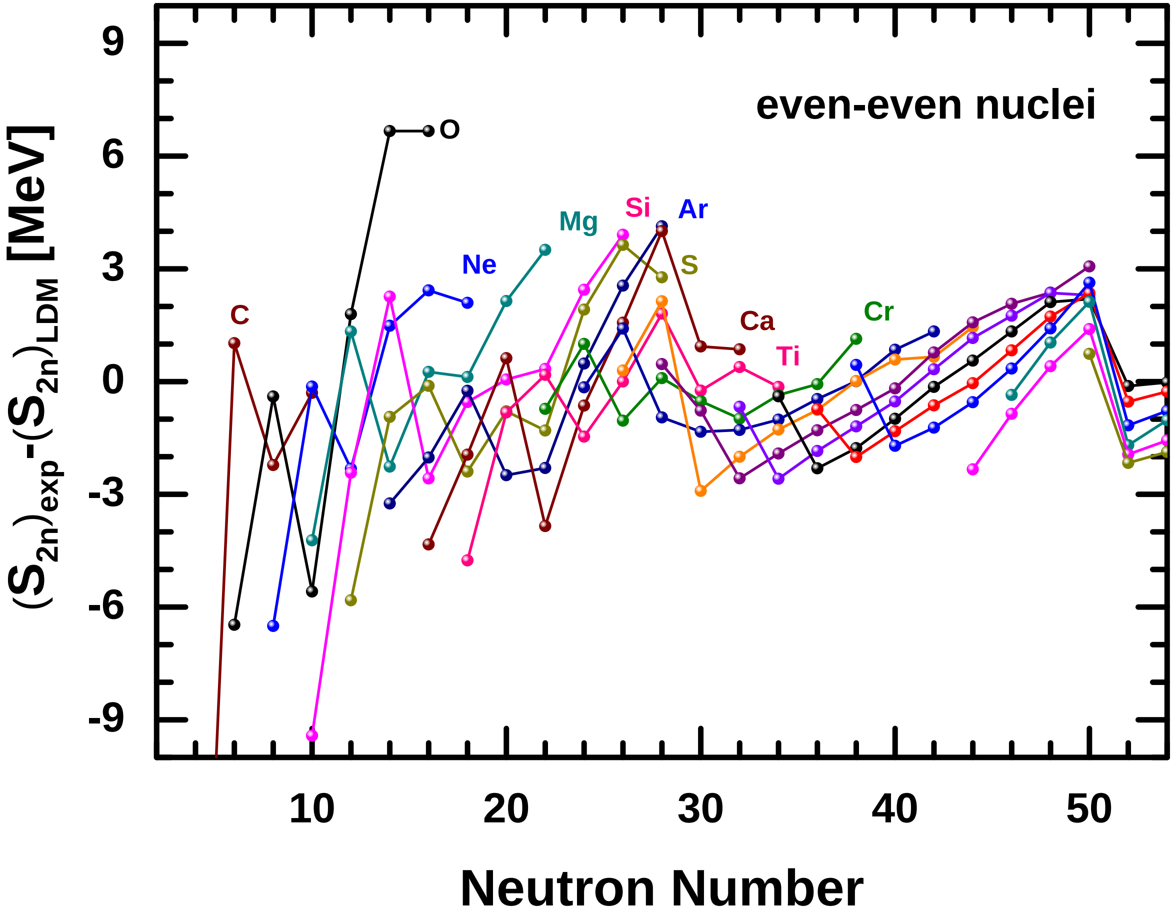
Jumps for even-even and odd-even nuclei

N	104	108	110	124	126
Lu	0.35				
Hf		0.39	0.17		
Ta		0.53			
W		0.45			
Re		0.59			
Os		0.19			
Hg				0.51	
Ti				0.26	4.04
Pb				0.12	4.41
Bi				0.19	4.02
Po				0.20	3.48
At				0.26	3.24
Rn				0.27	2.80
Fr				0.38	2.66

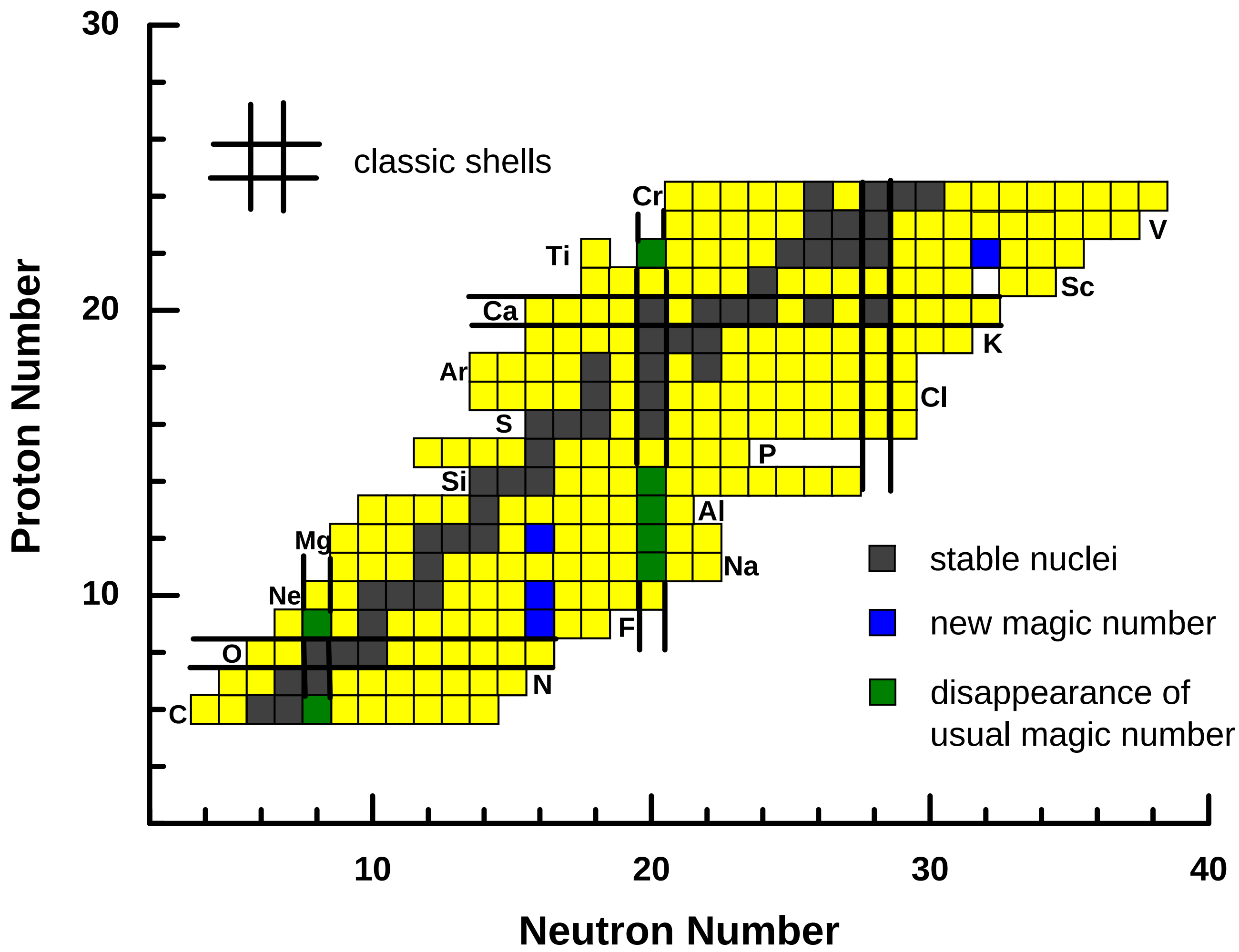
N	124	126	138	140	142	144	146	150	152
Ra	0.32	2.29							
Ac	0.27	2.22	0.17						
Th	0.33	1.86							
Pa	0.35	1.63			0.27				
U					0.25				
Np				0.24	0.22				
Pu					0.19	0.19	0.13		
Am						0.21	0.20		
Cm					0.22				0.36
Bk								0.20	0.55
Cf					0.22			0.30	0.36
Es									0.69
Fm									0.86
No									0.76

Two-neutron separation energy difference in light mass region

1. Even-even nuclei:



Changes of magic number in light neutron-rich nuclei



Contribution of Wigner term

In order to take account of the excessive binding of nuclei for which $N \simeq Z$, a phenomenological Wigner term of the form

$$E_W = V_W e^{(-\lambda|N-Z|/A)} \tag{22}$$

was also include, where the parameters read: $V_w = -2.90$ MeV, $\lambda = 28.0$.

M. Samyn and S. Goriely et al, NPA 700 (2002) 142.

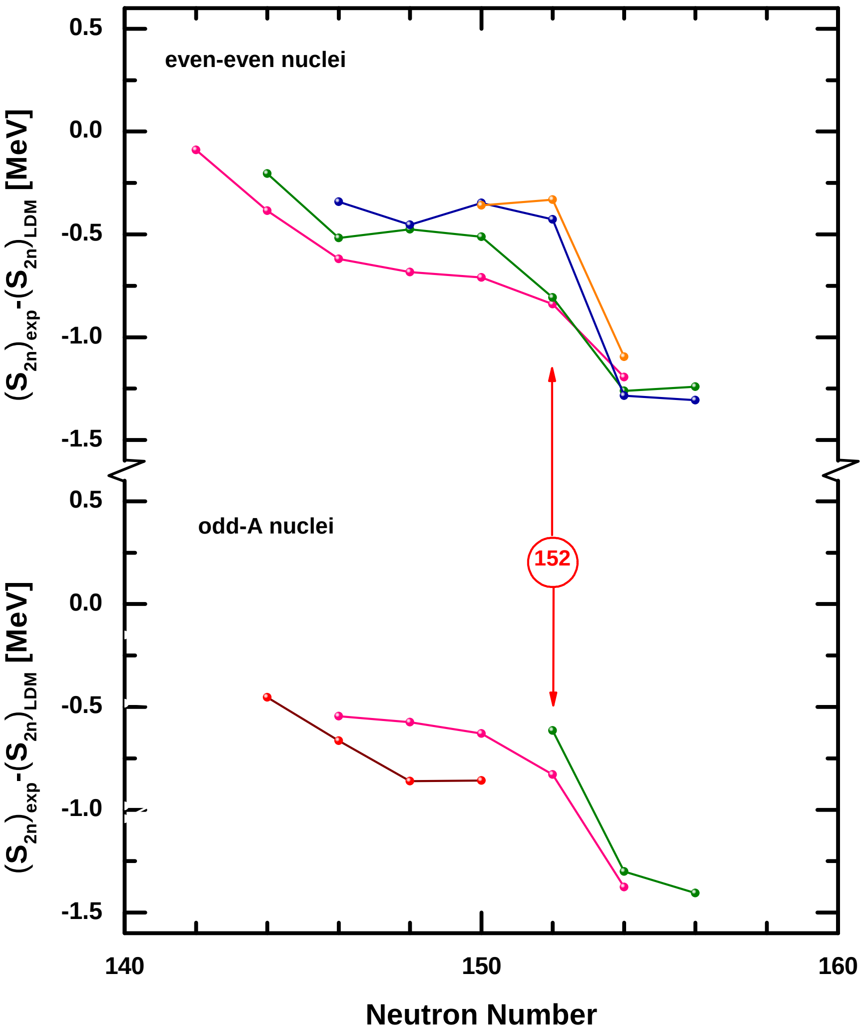
The binding energy including Wigner term reads:

$$BE = BE_{\rm LDM} + BE_{\rm Wig}. \tag{23}$$

The two-neutron separation energy including Wigner term reads:

$$S_{2n} = (S_{2n})_{\rm LDM} + (S_{2n})_{\rm Wig}. \tag{24}$$

Gap for nuclei in heavy mass region



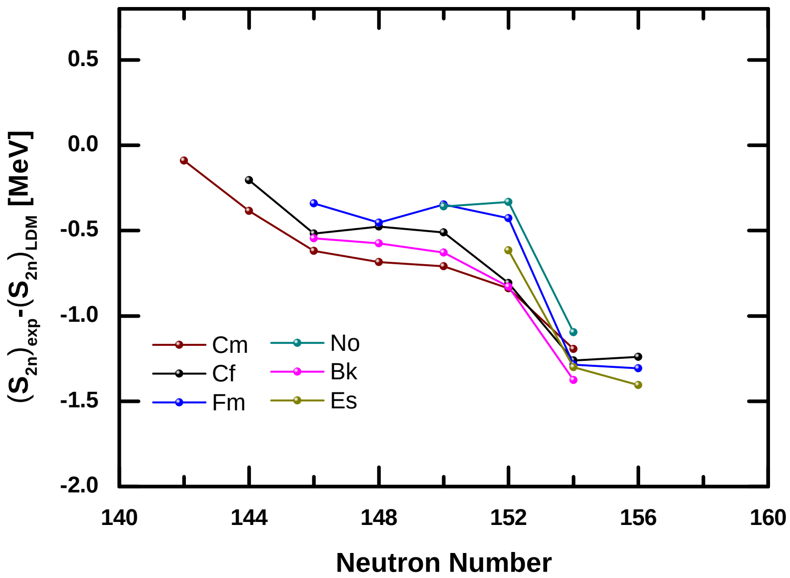
- The theoretical fission thresholds reveal the magicity of the neutron number 152.

  H. C. Pauli and T. Ledergerber, NPA 175 (1971) 545.

- Gap on approach of N=152:

nuclei	proton number	gap (MeV)
^{248}Cm	96	0.35517
^{249}Bk	97	0.54681
^{250}Cf	98	0.4537
^{251}Es	99	0.68517
^{252}Fm	100	0.85816
^{254}No	102	0.76326

Gap for nuclei in heavy mass region



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$(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ for all nuclei

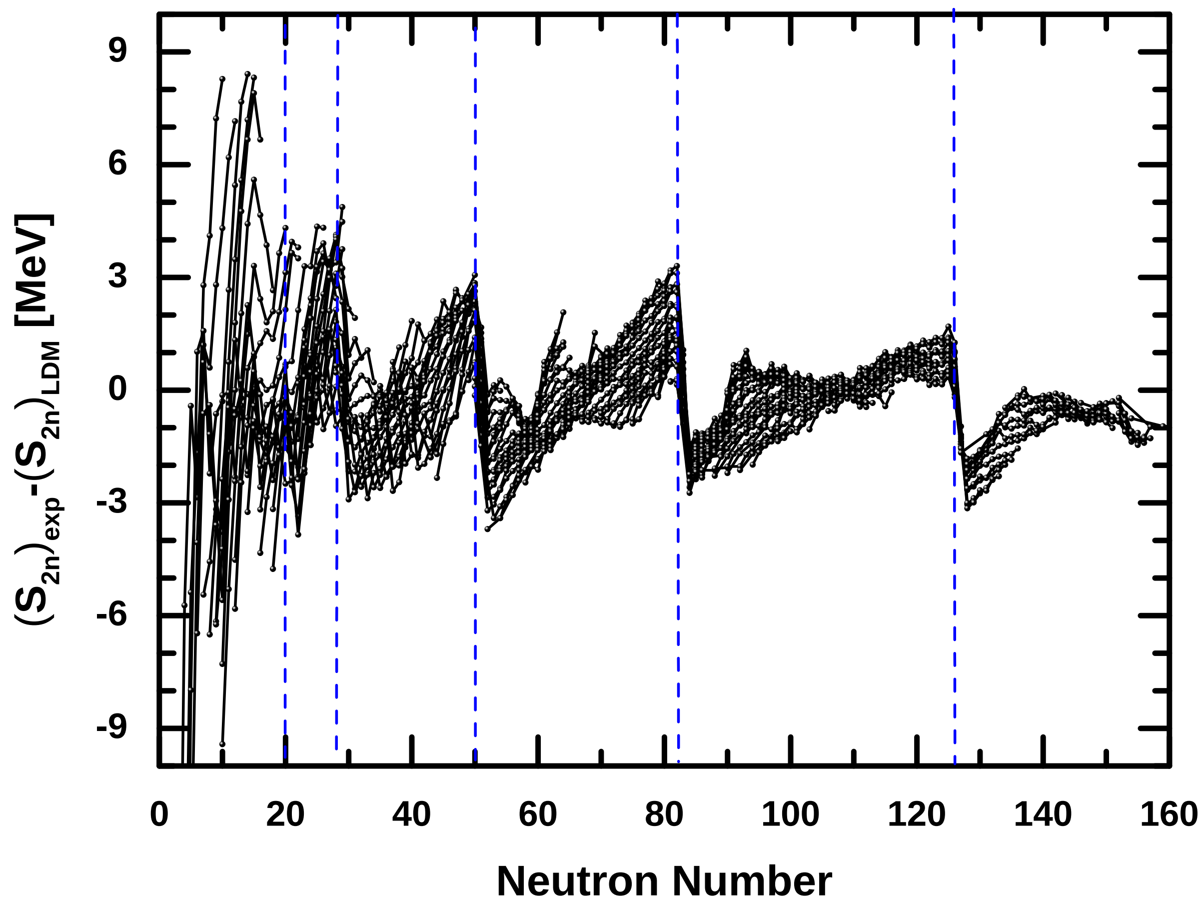

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for all nuclei as a function of neutron number.

$(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ for odd-A nuclei

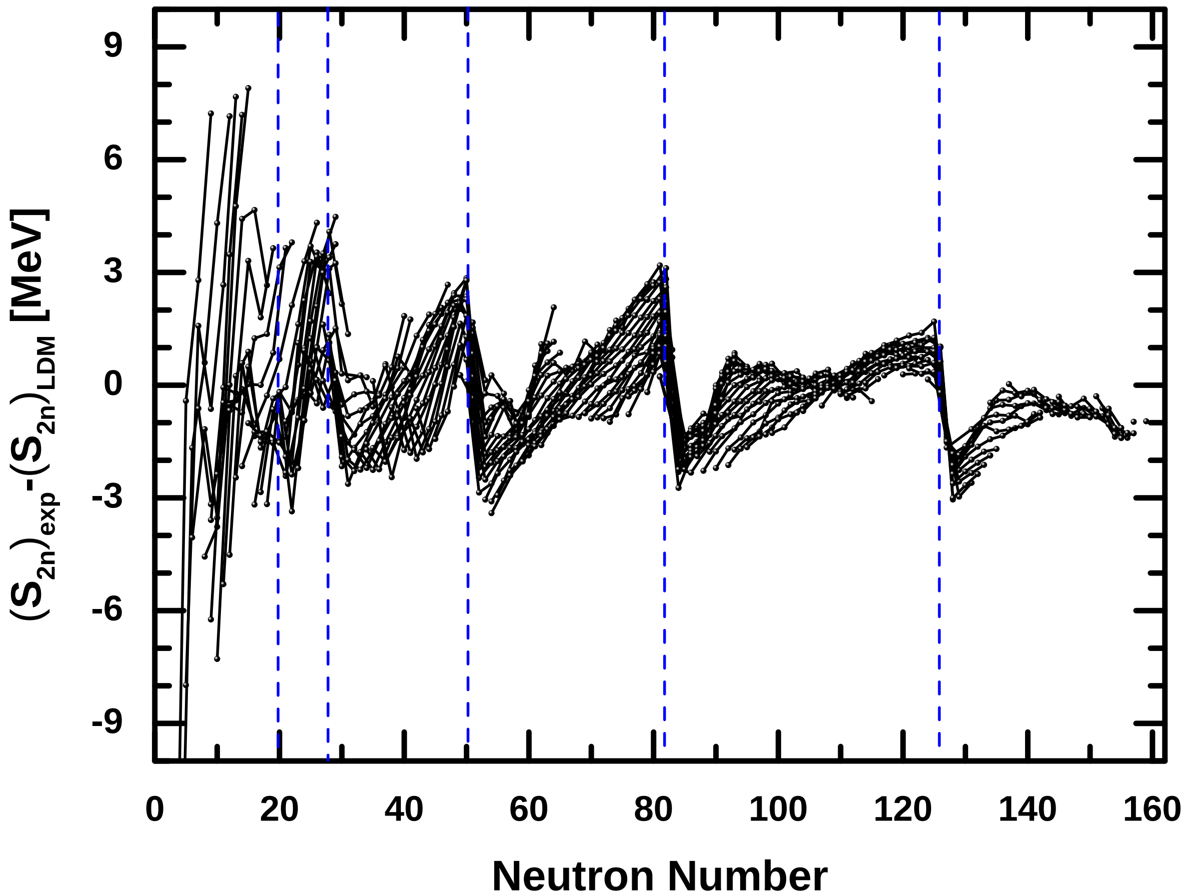

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for odd-A nuclei as a function of neutron number.

$(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ for odd-odd nuclei

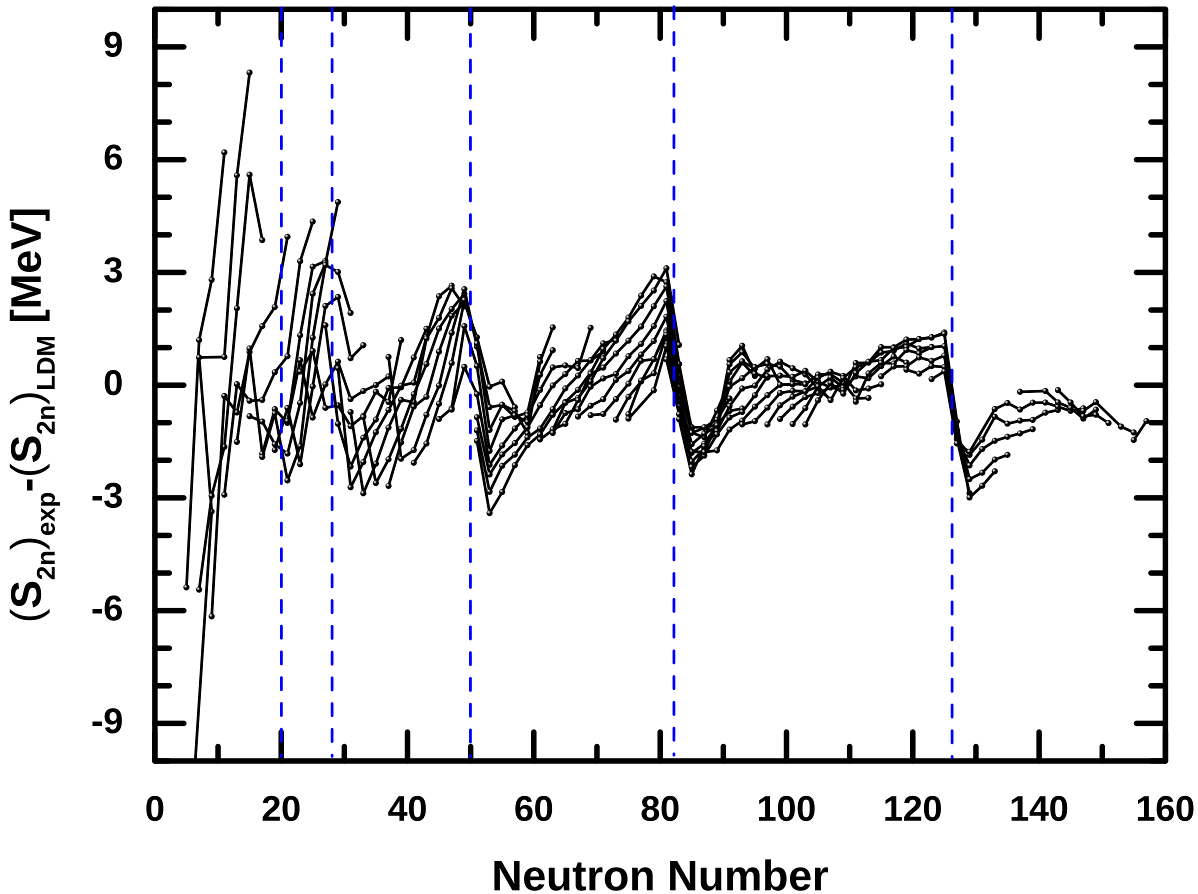

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for odd-odd nuclei as a function of neutron number.

$(S_{2n})_{\mathrm{EXP}} - (S_{2n})_{\mathrm{LDM}}$ for even-even nuclei

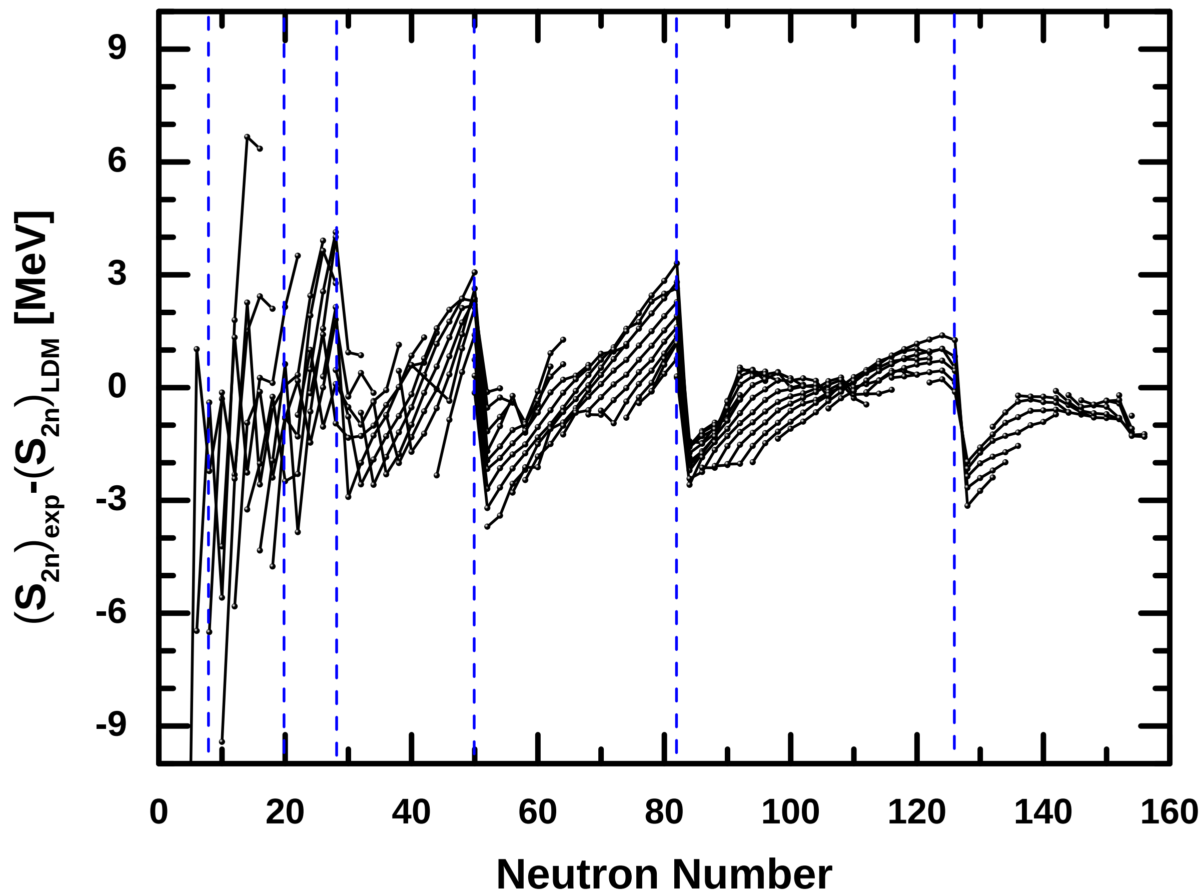

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for even-even nuclei as a function of neutron number.

Experimental Evidence for Shell Effects in N=4～32 Region

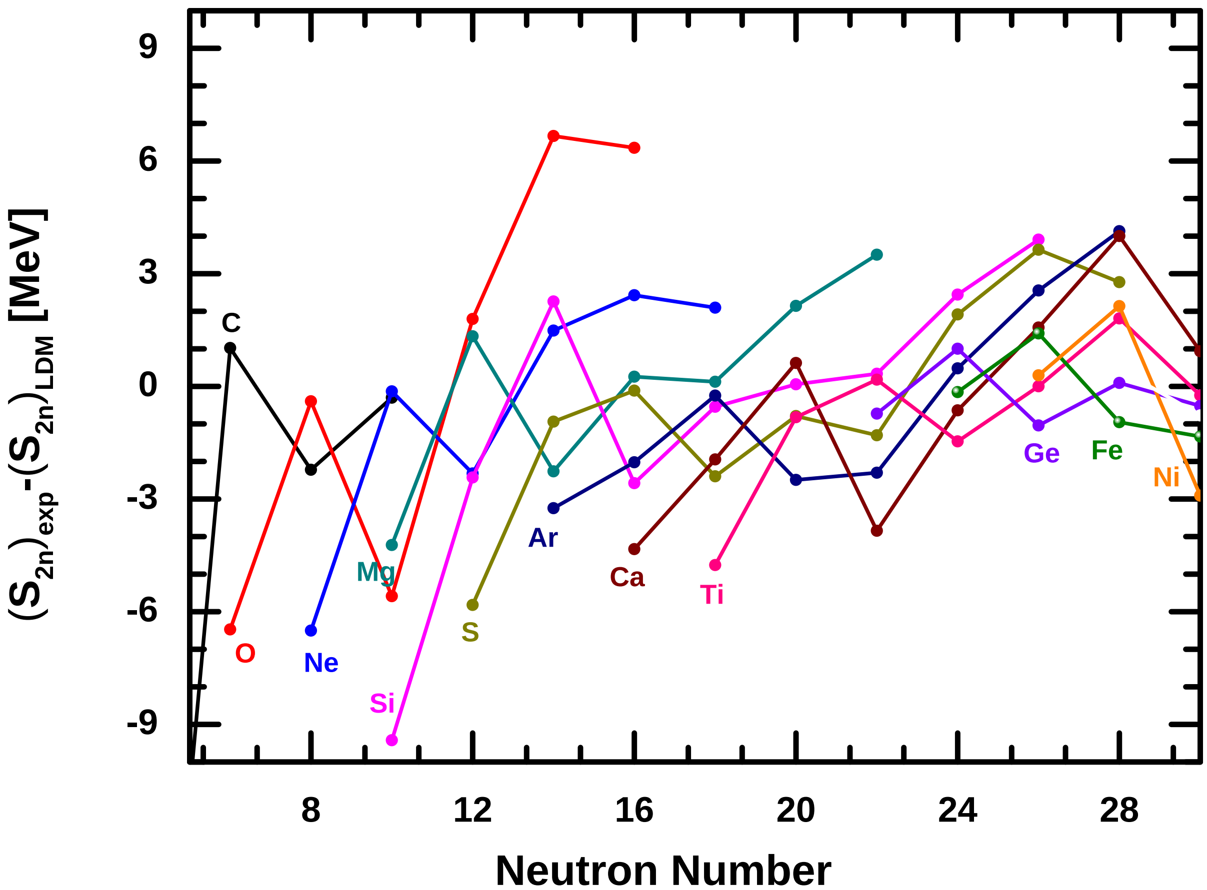

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for even-even nuclei as a function of neutron number in N=4～32 region.

Experimental Evidence for Shell Effects in N=4～32 Region

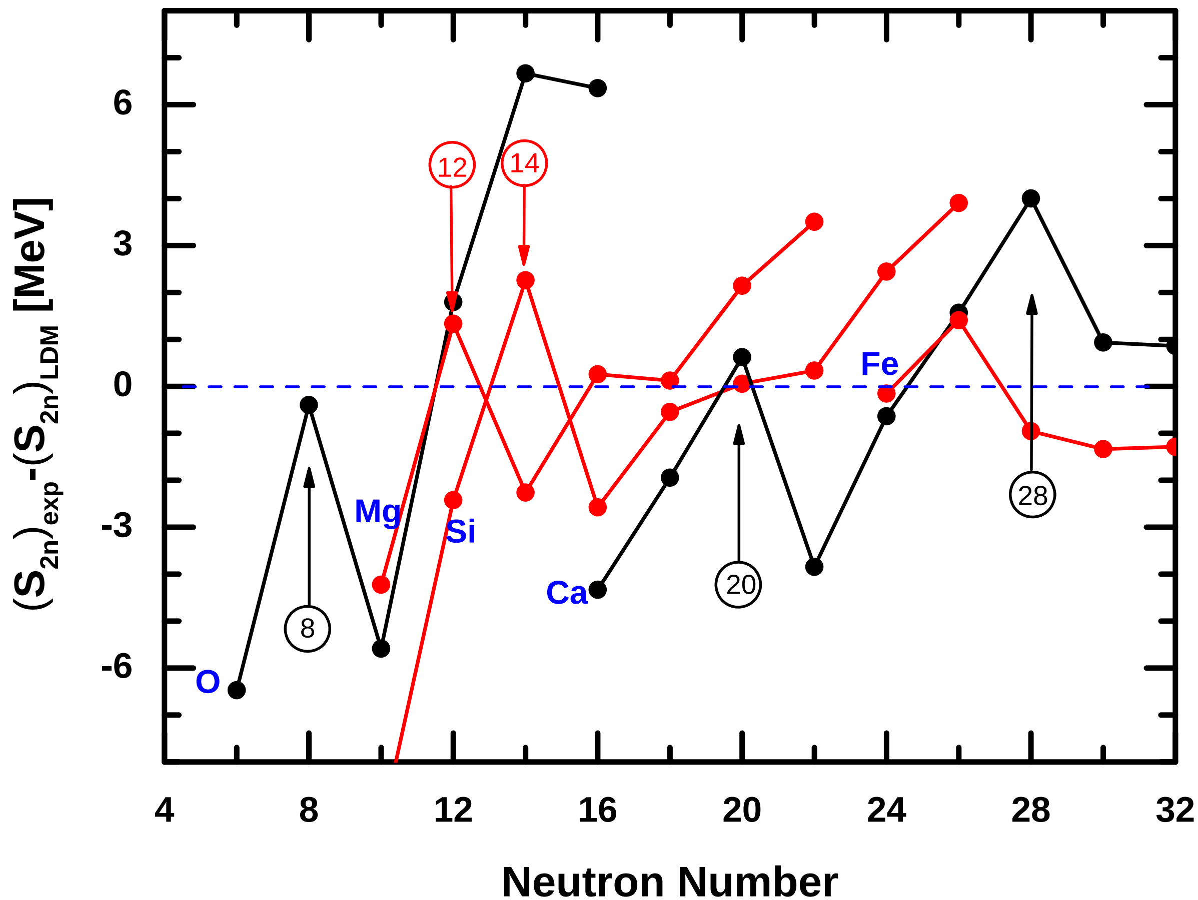

Figure: The differences between two-neutron separation energies obtained from experiment and liquid drop model calculation for even-even nuclei as a function of neutron number in N=4～32 region.

$(S_{2p})_{\mathrm{EXP}} - (S_{2p})_{\mathrm{LDM}}$ for all nuclei

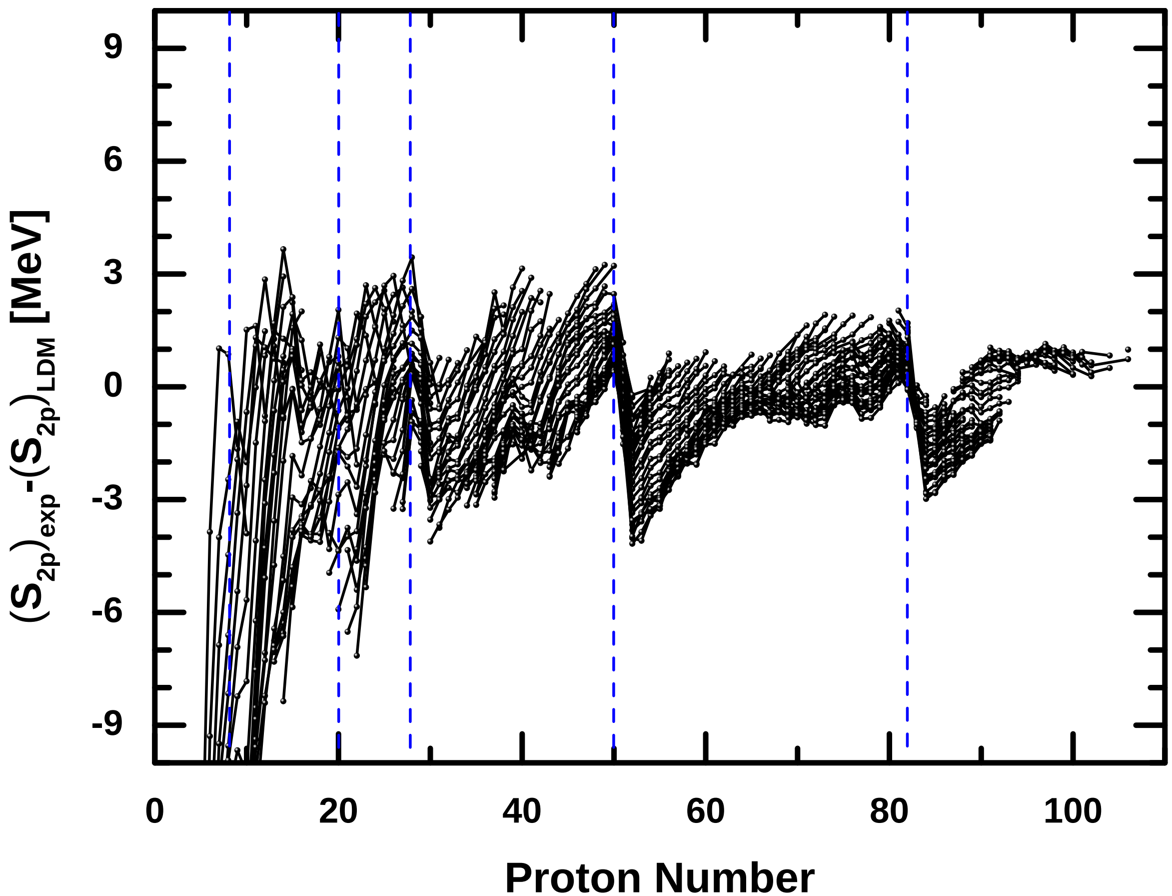

Figure: The differences between two-proton separation energies obtained from experiment and liquid drop model calculation for all nuclei as a function of proton number.

$(S_{2p})_{\mathrm{EXP}} - (S_{2p})_{\mathrm{LDM}}$ for even-even nuclei

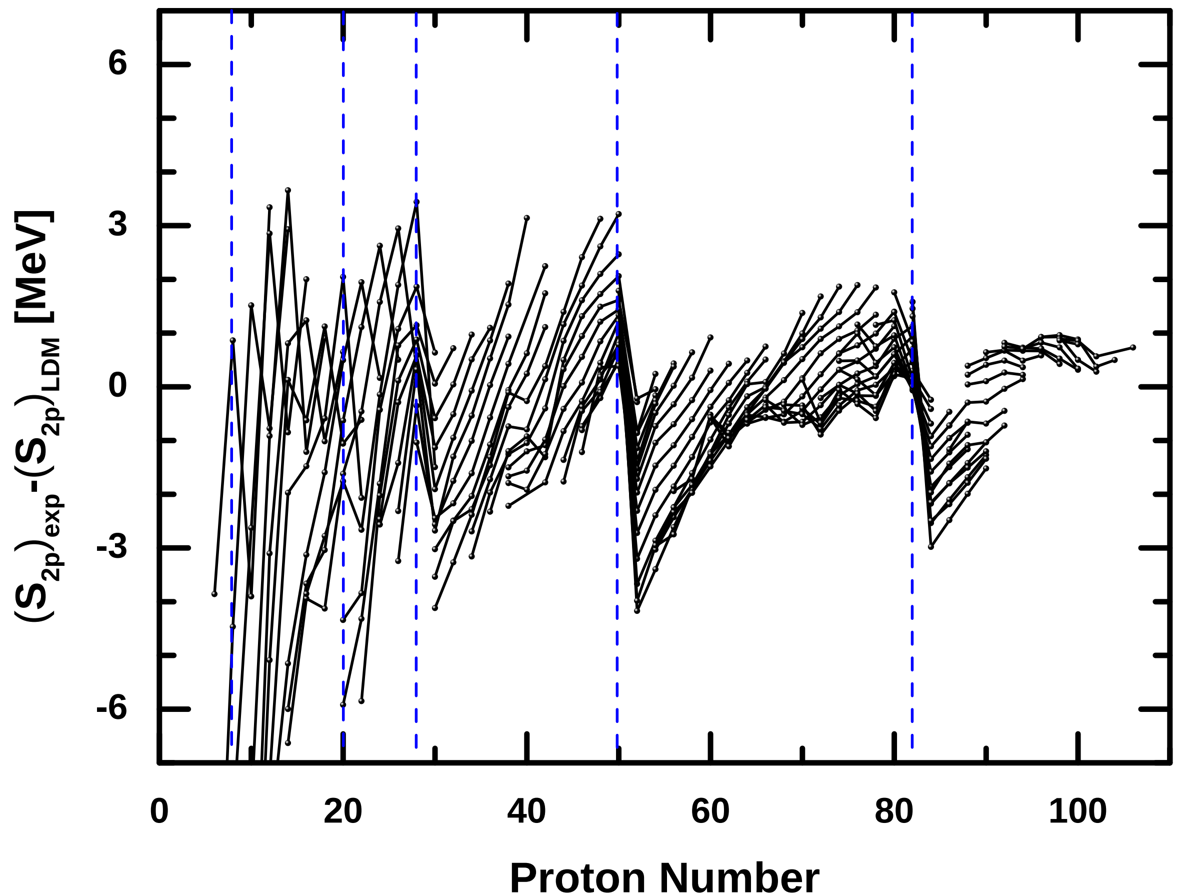

Figure: The differences between two-proton separation energies obtained from experiment and liquid drop model calculation for even-even nuclei as a function of proton number.

Experimental Evidence for Shell Effects in Z=4∼32 Region

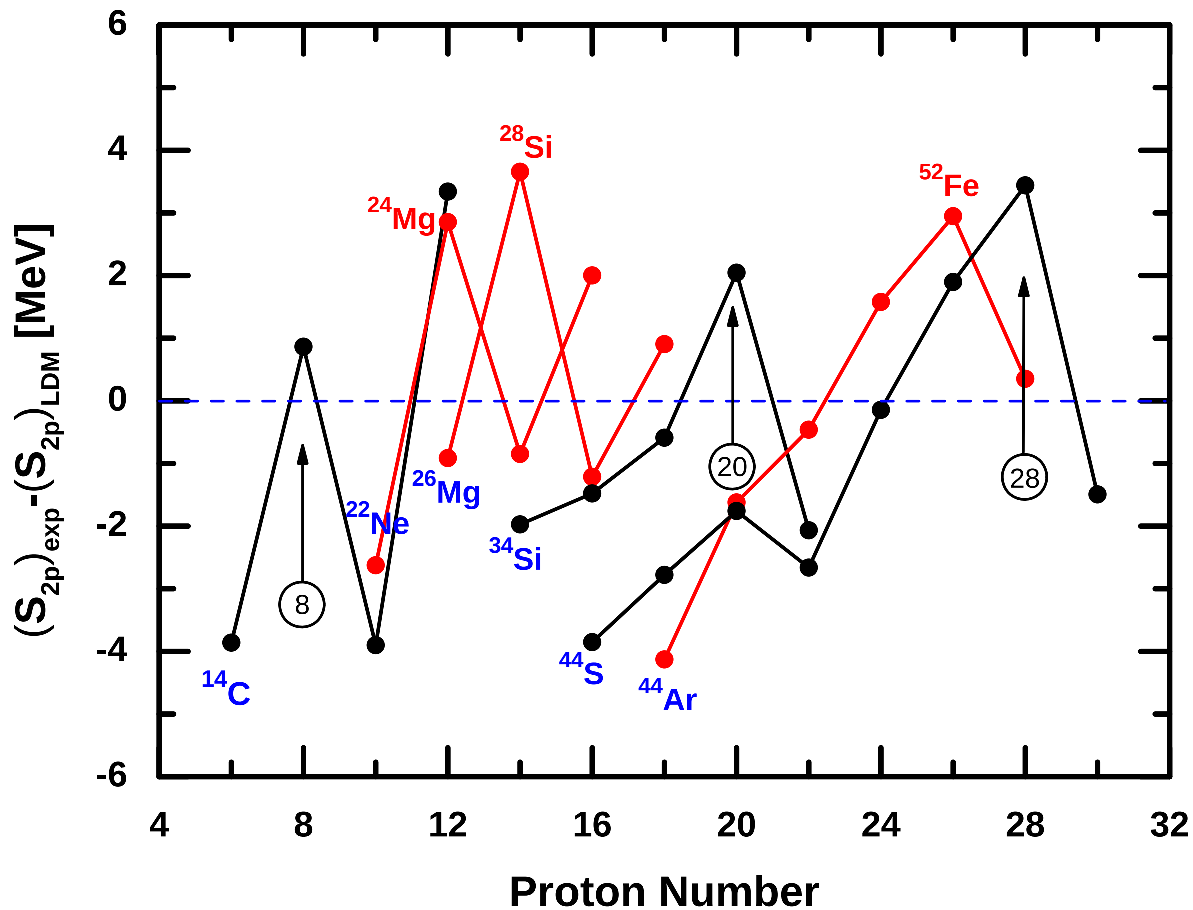

Figure: The differences between two-proton separation energies obtained from experiment and liquid drop model calculation for even-even nuclei as a function of proton number in Z=4∼32 region.