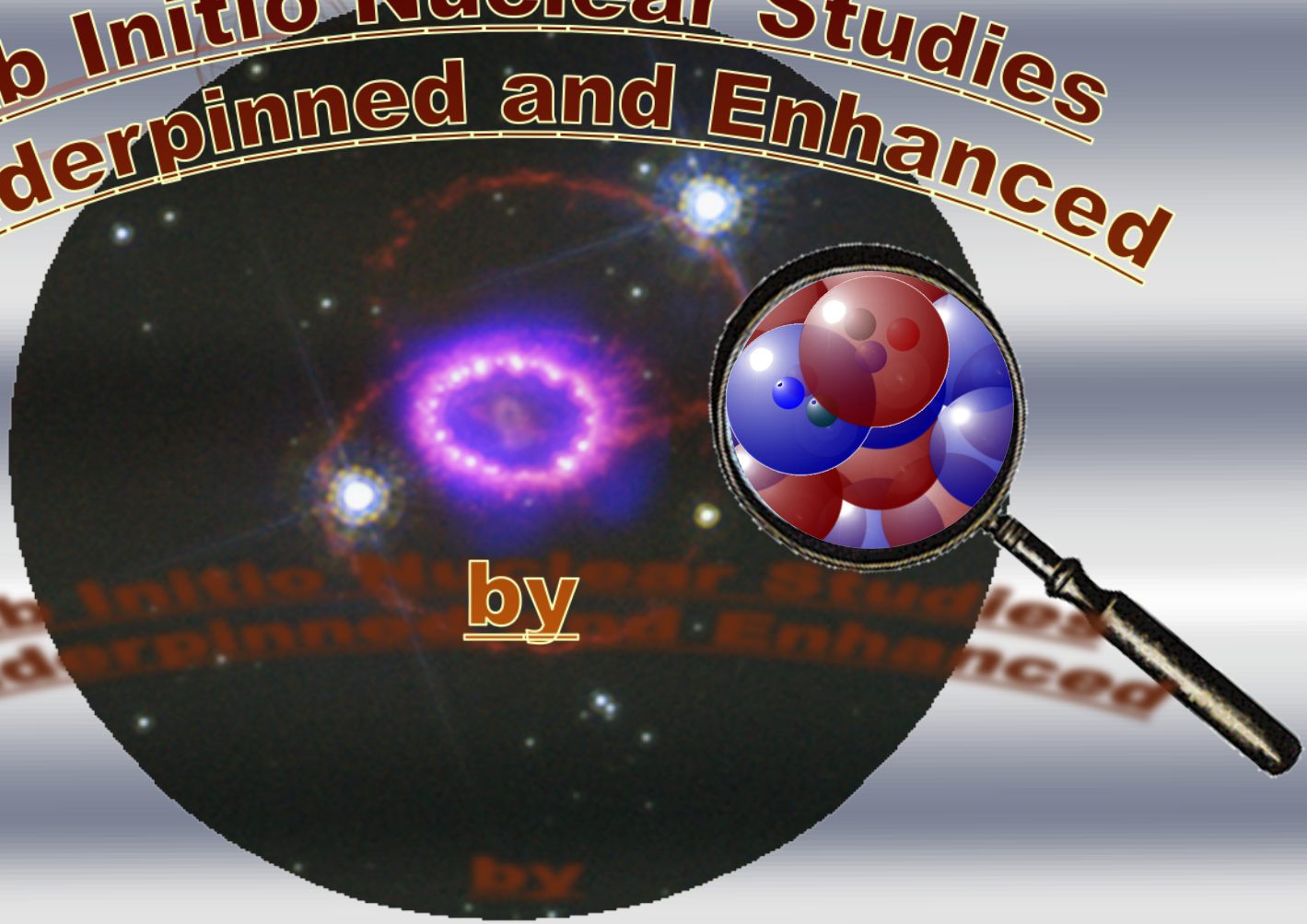
Ab Initio Nuclear Studies Underpinned and Enhanced by Symmetry Considerations



J. P. Draayer, T. Dytrych, K. D. Launey
Daniel Langr (Czech Tech U)

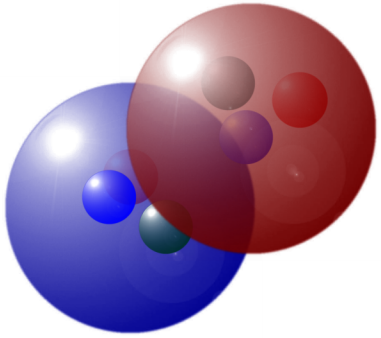
Advances in Nuclear Many-body Theory 7-10
June 2011, Primosten, Croatia (Peter Ring's 70th)

Louisiana State University
Baton Rouge, Louisiana

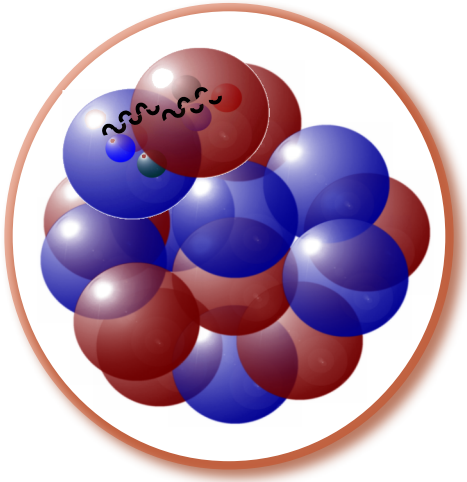
LSU

Ab Initio Vision

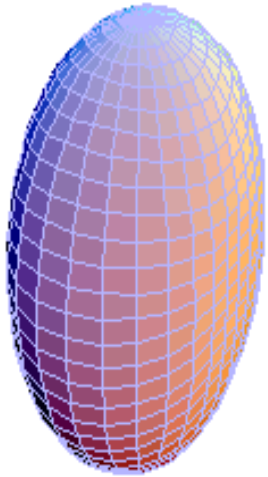
Fundamental Principles



Many-body Dynamics



Collective Properties



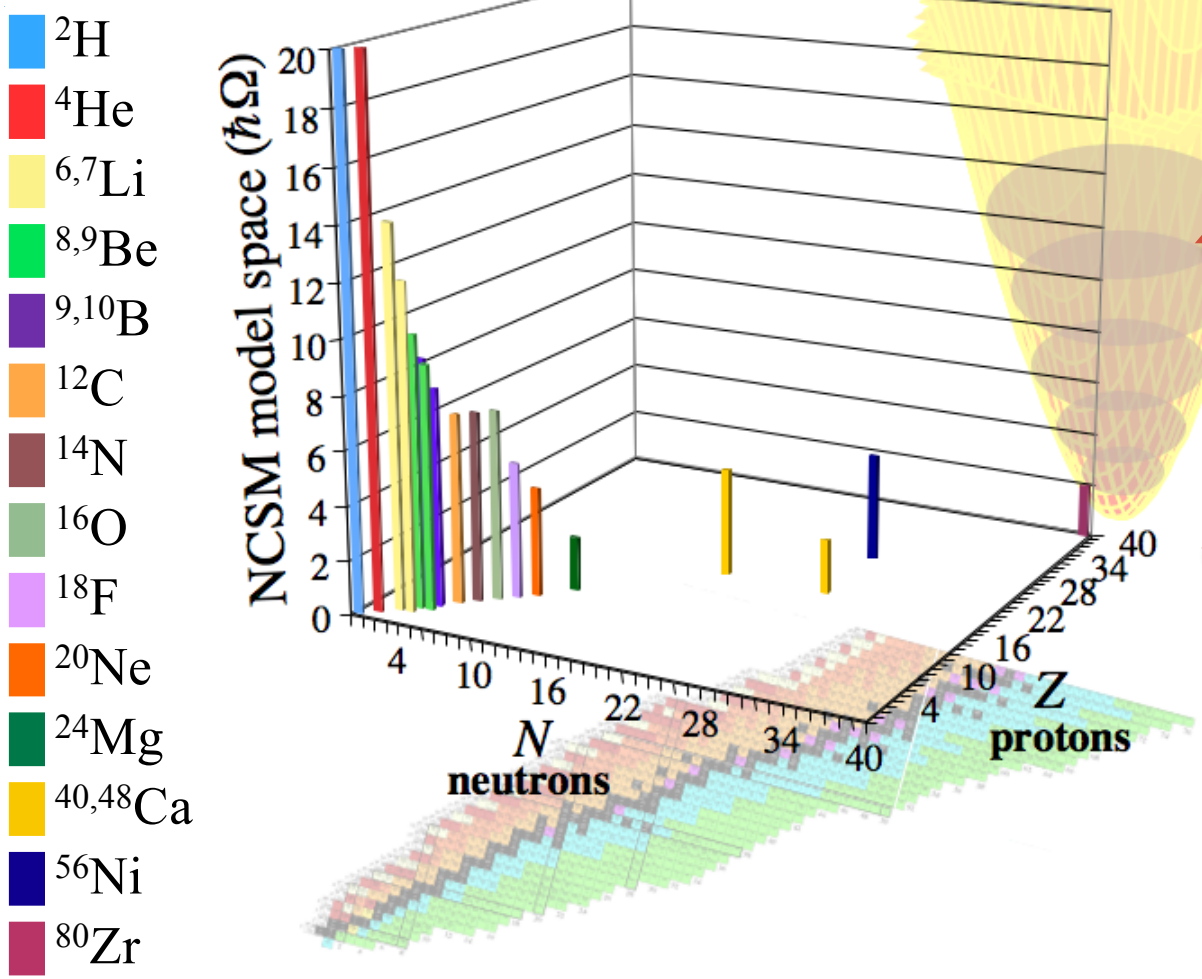
nucleon-nucleon interaction (NN, NNN, ...)

quarks/gluons

symmetry adapted NCSM

structure and reactions

Computational Limits of No Core Shell Model



Highly deformed modes, Cluster-like configurations, B(E2) w/o effective charges

'higher' spaces

'heavier' nuclei

… enables…

Symmetry Adapted NCSM

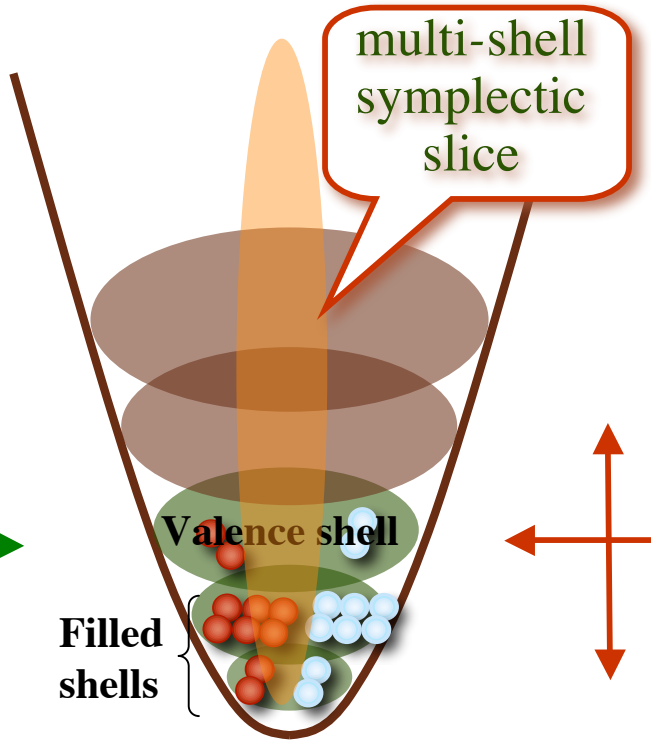
NCSM versus 'Symmetry Adapted' NCSM

NCSM
(multi-$\hbar\omega$)

Horizontal discs
(all configurations)

N_{max}

$(0\hbar\omega)$ limit



SA-NCSM*
(multi-$\hbar\omega$)

Vertical slices
(monopole & quadrupole
collective excitations)

$0\hbar\omega$ SU(3) limit

***Complete (= NCSM)**
if all irreps included

- Realistic interaction (local/nonlocal;NN,NNN,…)
- In principle, exact solutions
- Reproduction of binding energies and spectral features of light nuclei

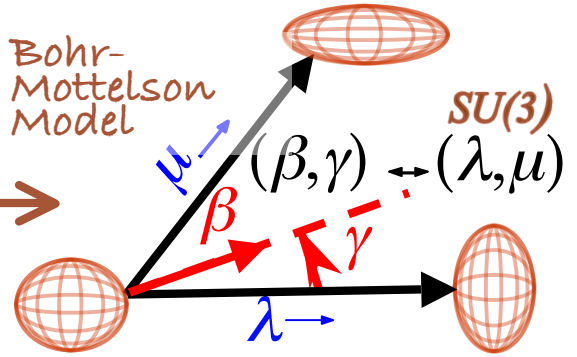
- Manage spurious center-of-mass motion
- Relation to the NCSM: fully microscopic!
- Reproduce rotational energy and EM transition rates without effective charge
- Extensible: spaces; 3&4 body enabled, etc.

Symmetry Adapted Theory

basis classification scheme

space				*spin*
$U(N) \supset SU(3) \supset SO(3)$				$U(2)$
$[f]$	α	(λ,μ)	κ L	S
		deformation		*(LS) JM*

Elliott and Symplectic Model



Step 1

Generate distributions of nucleons over HO shells for a given Nmax model space

N_{max} = pf, sd, p, s ⊕ ⊕ ⊕ · · ·

np coupling scheme

Step 2

for each set of nucleons in a HO shell determine antisymmetric representations of U(N)xU(2)

Step 3

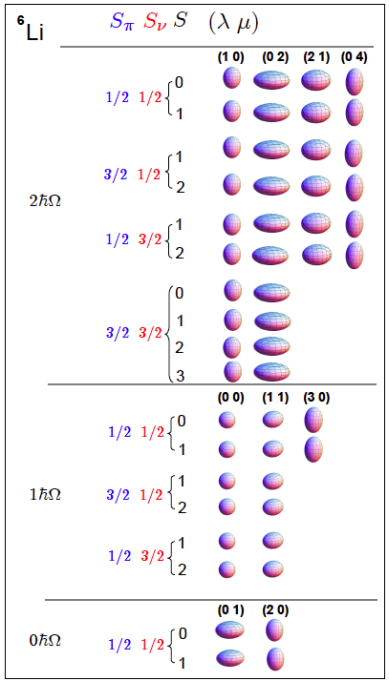
decompose each U(N) irrep into a complete set of SU(3) irreps

'seed' configurations

	U(2)	⊗	U(10)	⊃	SU(3)
quantum labels:	S		$[f]$		α $(\lambda\ \mu)$

- Example: 4 nucleons in pf

S=0			⊃	{(8 2) (7 1) (4 4) (5 2) (0 6) (6 0) (3 3) (1 4) (4 1) (2 2) (1 1)}
S=1			⊃	{(9 0) (6 3) (7 1) (4 4) (2 5) (5 2) (3 3) (1 4) (4 1) (2 2) (0 3) (3 0) (1 1)}
S=2			⊃	{(5 2) (0 6) (3 3) (2 2) (3 0)}



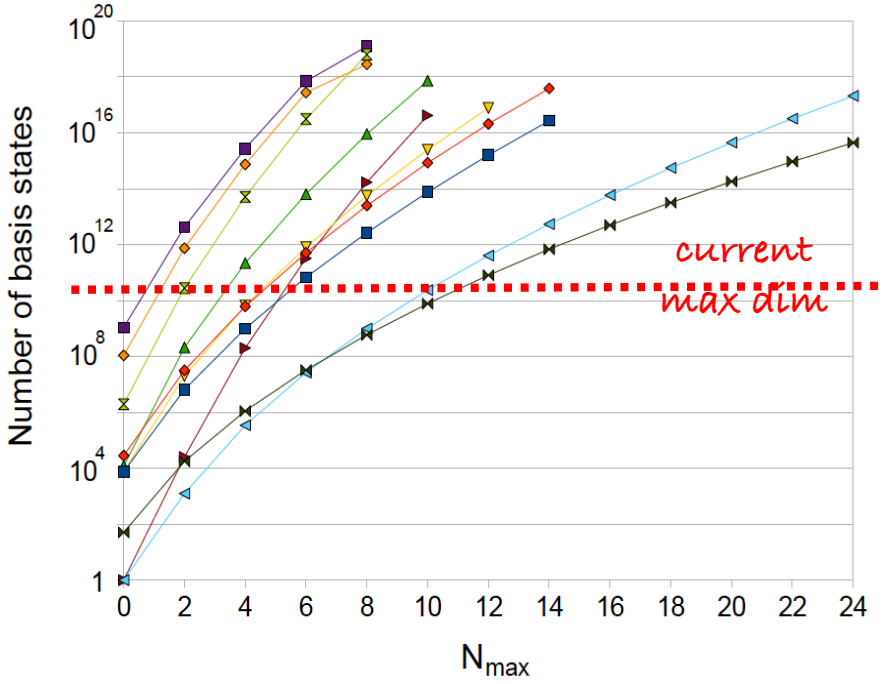
NCSM versus SA-NCSM

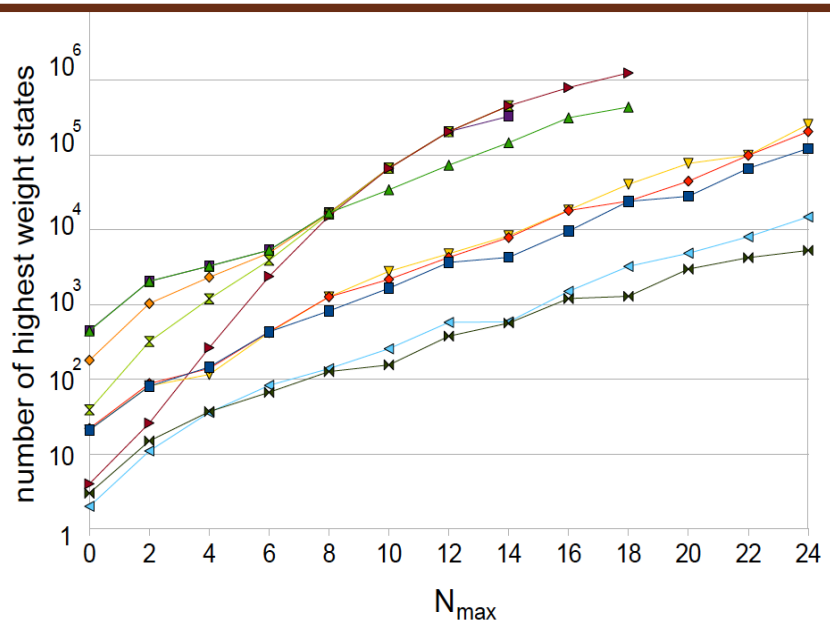
64Ge
68Se
72Kr
60Ca
48Ca
32Ar
28S
24Si
16O
12C

NCSM

Number of basis states

"Binominal Counting"





SA-NCSM

Number of 'seeds' required for complete calculation

"Cluster Bookkeeping"

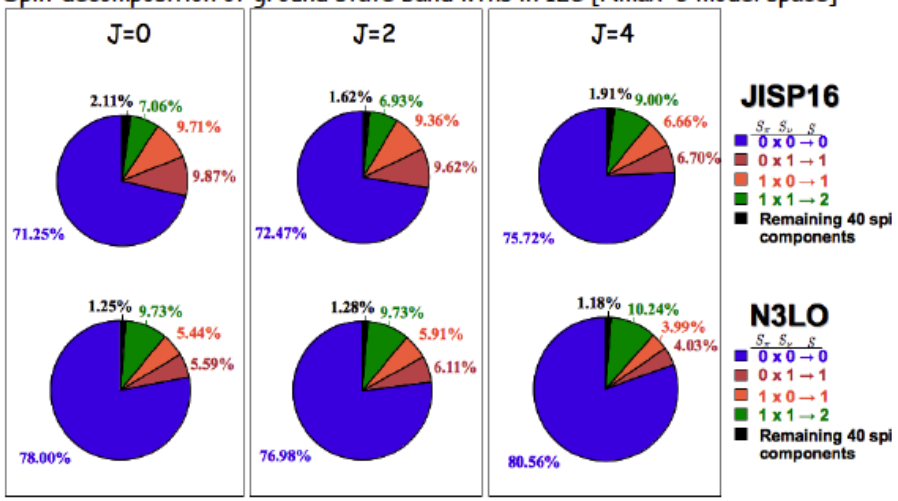
Low Spin and High Deformation Dominance

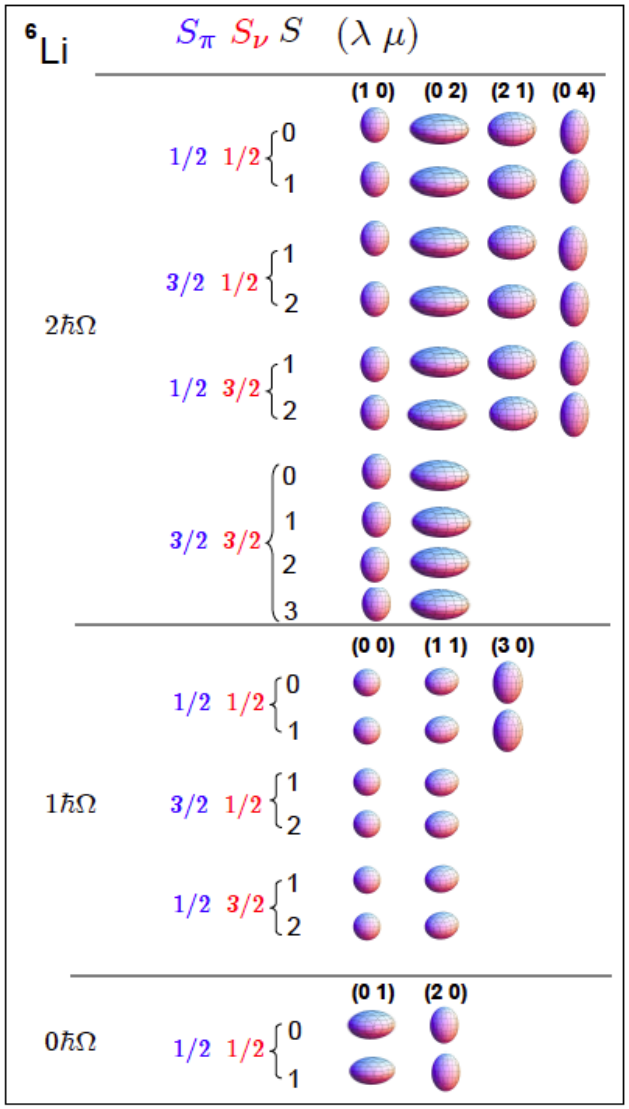
- SU(3)-scheme decomposes Nmax model space into subspaces of states labeled by $S_\pi \;\; S_\nu \;\; (\lambda\,\mu)S$
- the center-of-mass HO does not mix $S_\pi \;\; S_\nu \;\; (\lambda\,\mu)S$

c.m. spurious states can be removed from each subspace exactly

- truncation according to intrinsic spin $S_\pi \;\; S_\nu \;\; S$

Spin-decomposition of ground state band wfns in 12C [Nmax=6 model space]

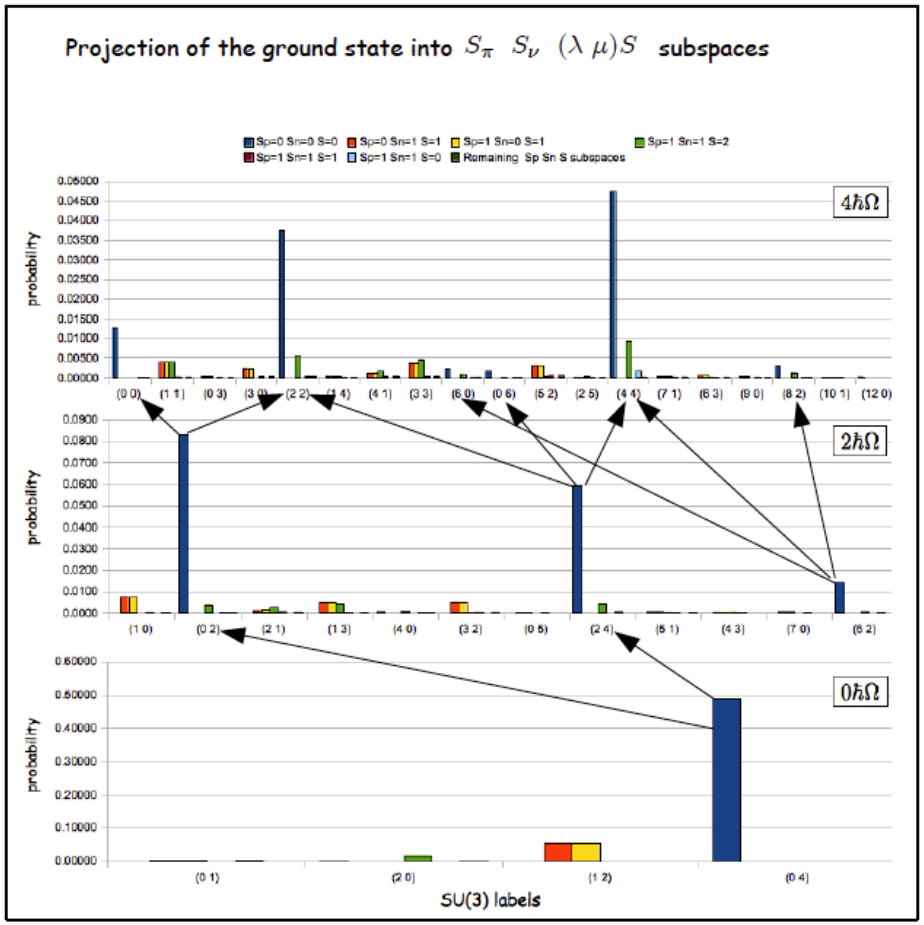




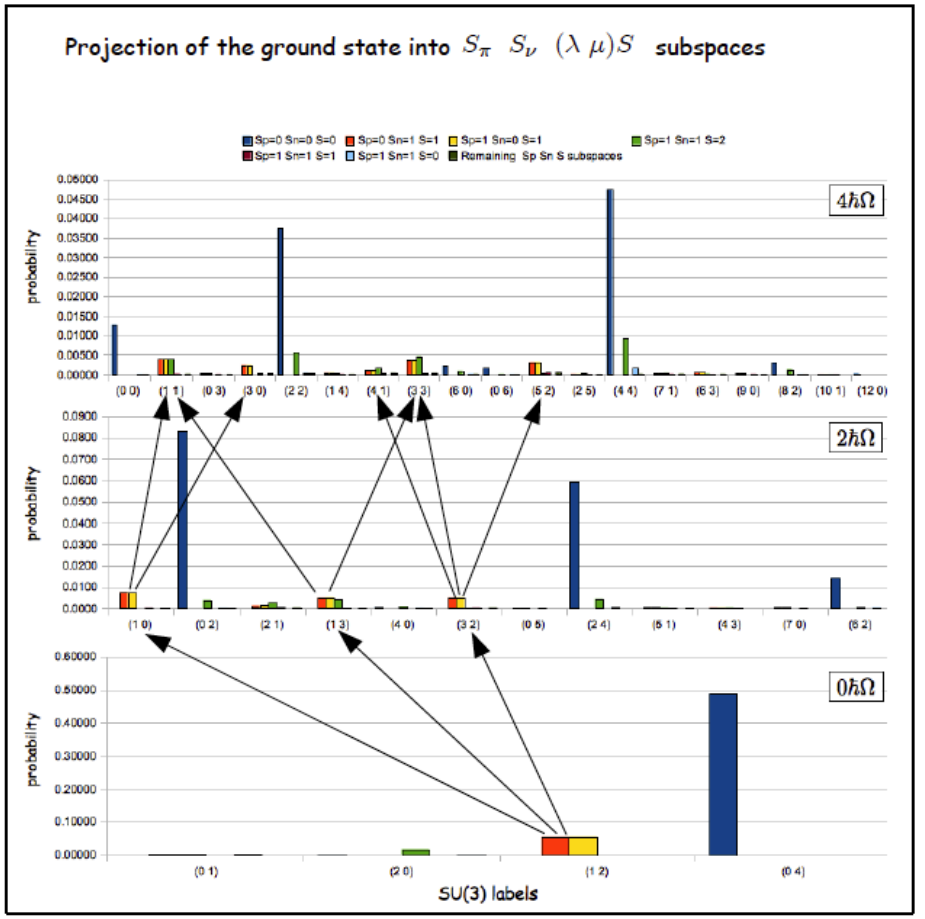
Symplectic (1p-1h, 2ħω) Raising Operator

Action of raising operator that has (2,0) SU(3) tensor character (symplectic generator) →

^{12}C (L=0, S=0, $J_{gs} = 0^+$)

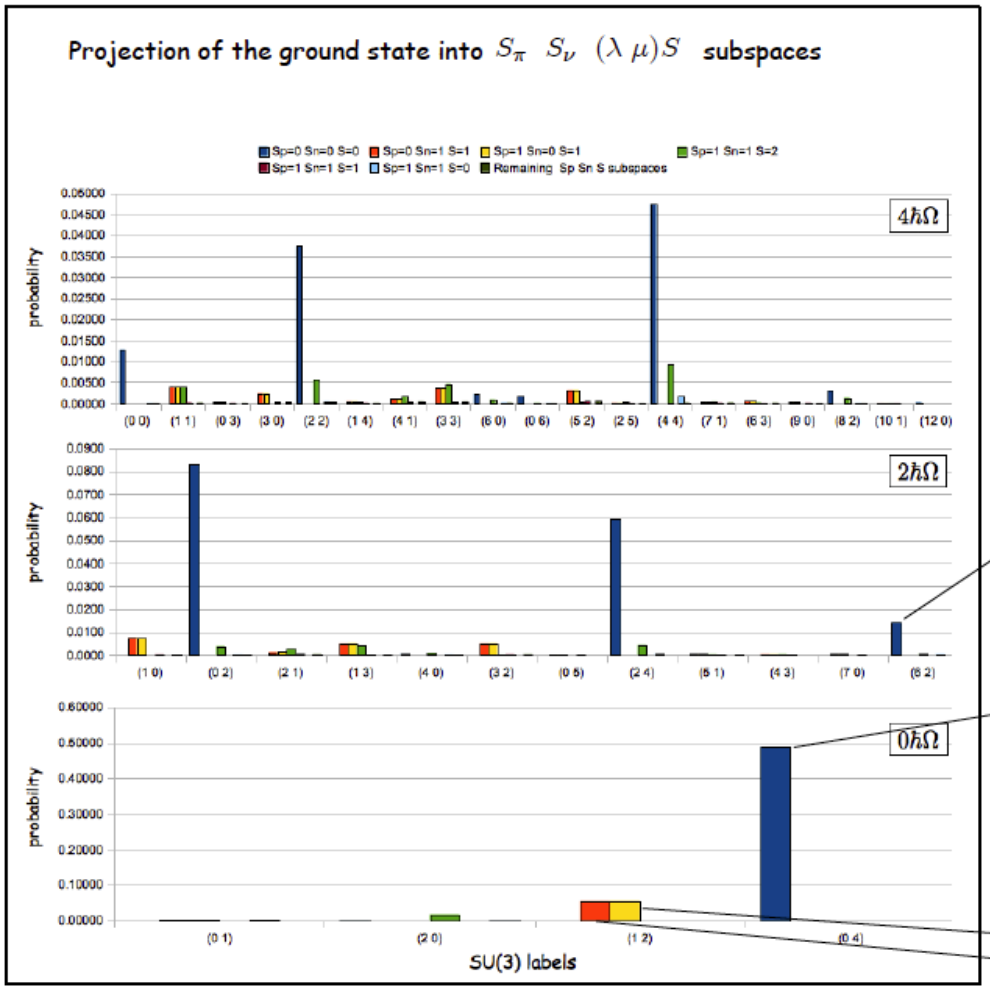


^{12}C (L=1, S=1, $J_{gs} = 0^+$)

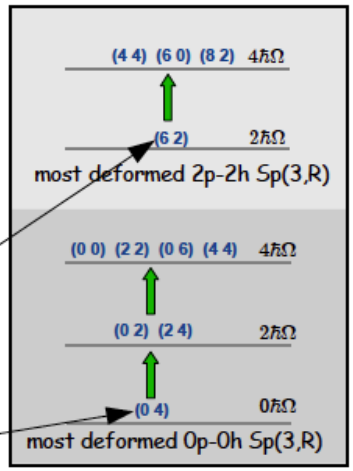


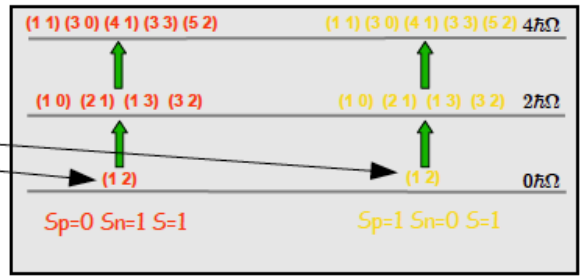
Dominate Modes - (1p-1h, 2ħω & 2p-2h, 2ħω)

$^{12}C\ (J_{gs} = 0^+)$

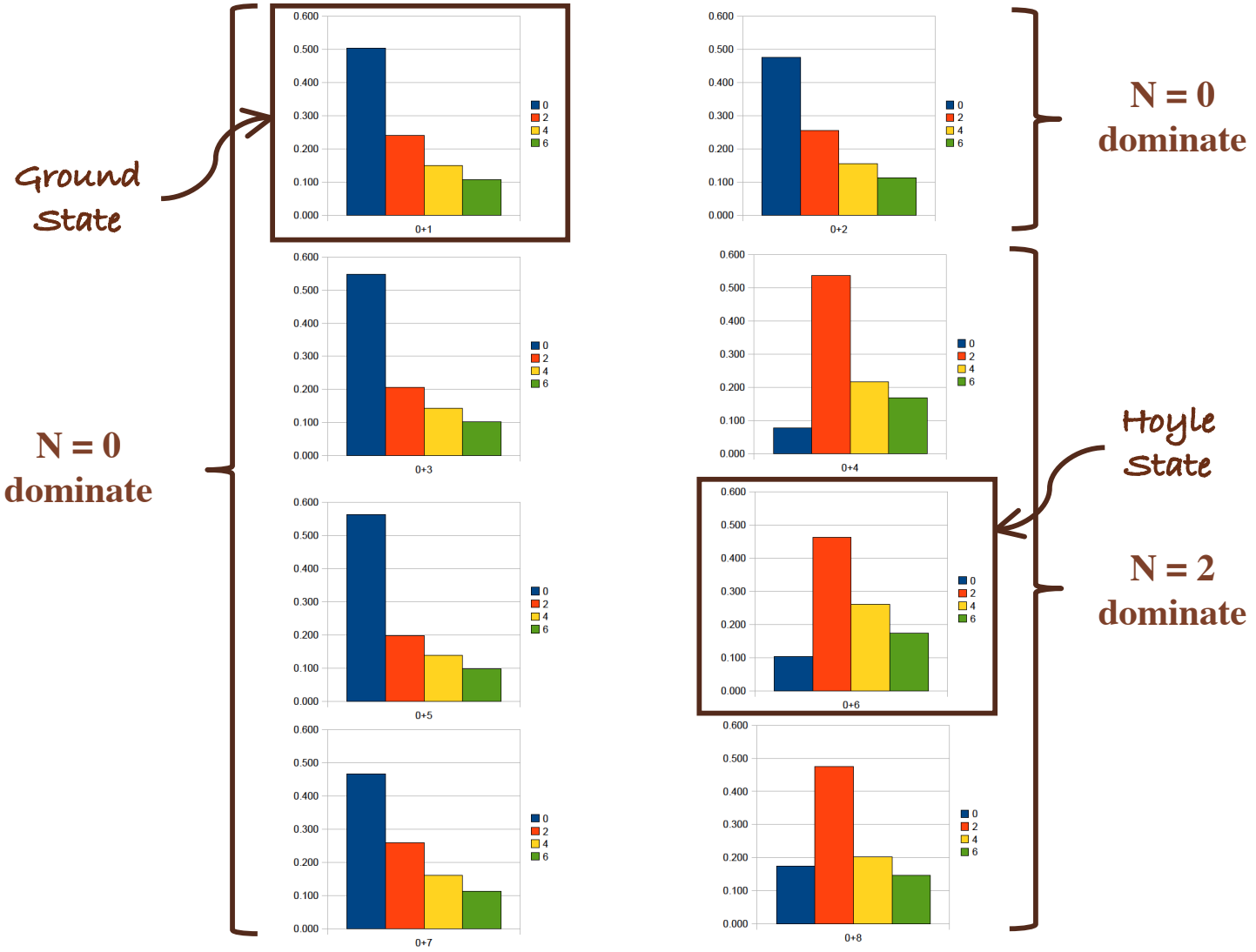


- most important subspaces contain states of the three leading Sp(3,R) irrep
- Significant contribution from the most deformed 2p-2h Sp(3,R) irrep

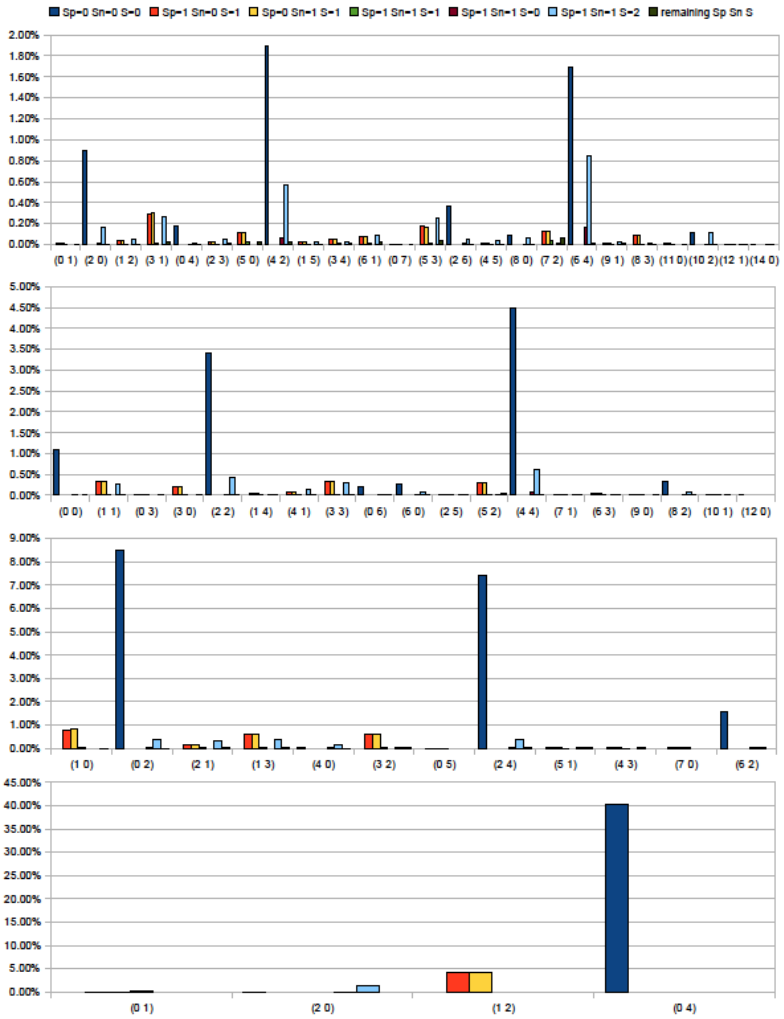


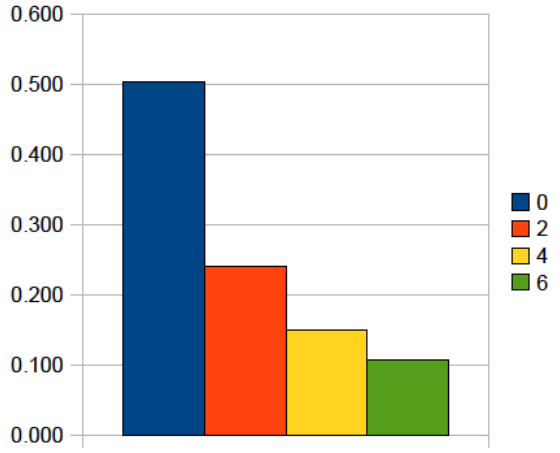


… low-lying 0^+ states in ^{12}C (N_{max}=6) …

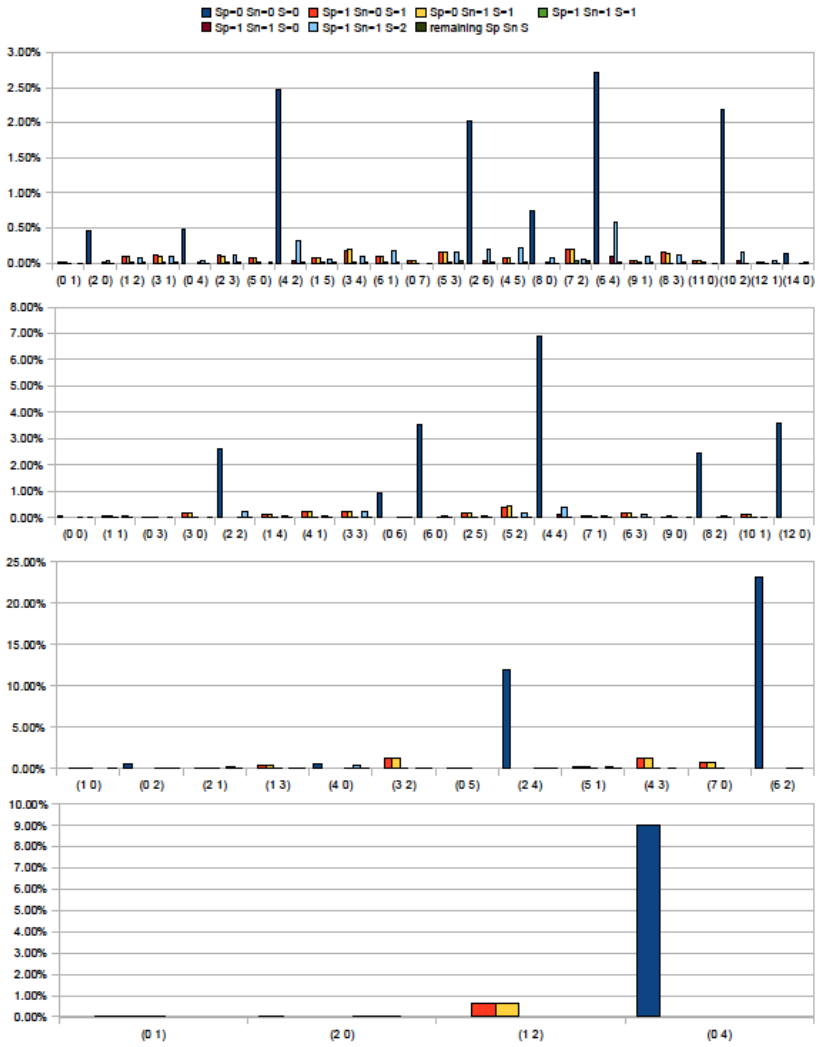


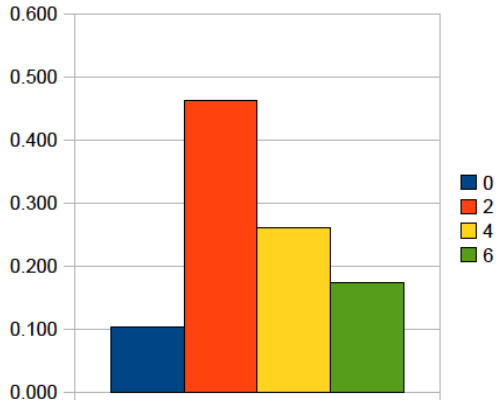
… Structure of 1^{st} 0^+ in ^{12}C …





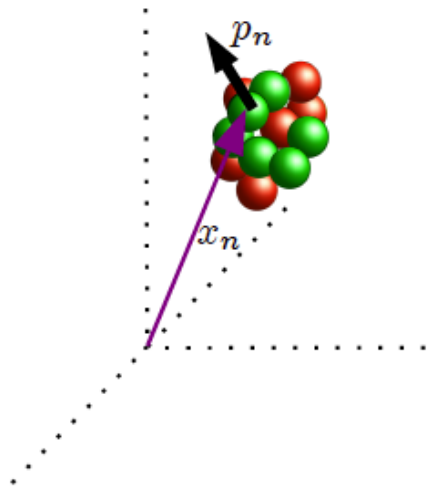
… Structure of 6^{th} 0^+ in ^{12}C …





Sp(3,R) ⊃ SU(3) Structure

- Sp(3,R): symmetry of the nuclear collective dynamics



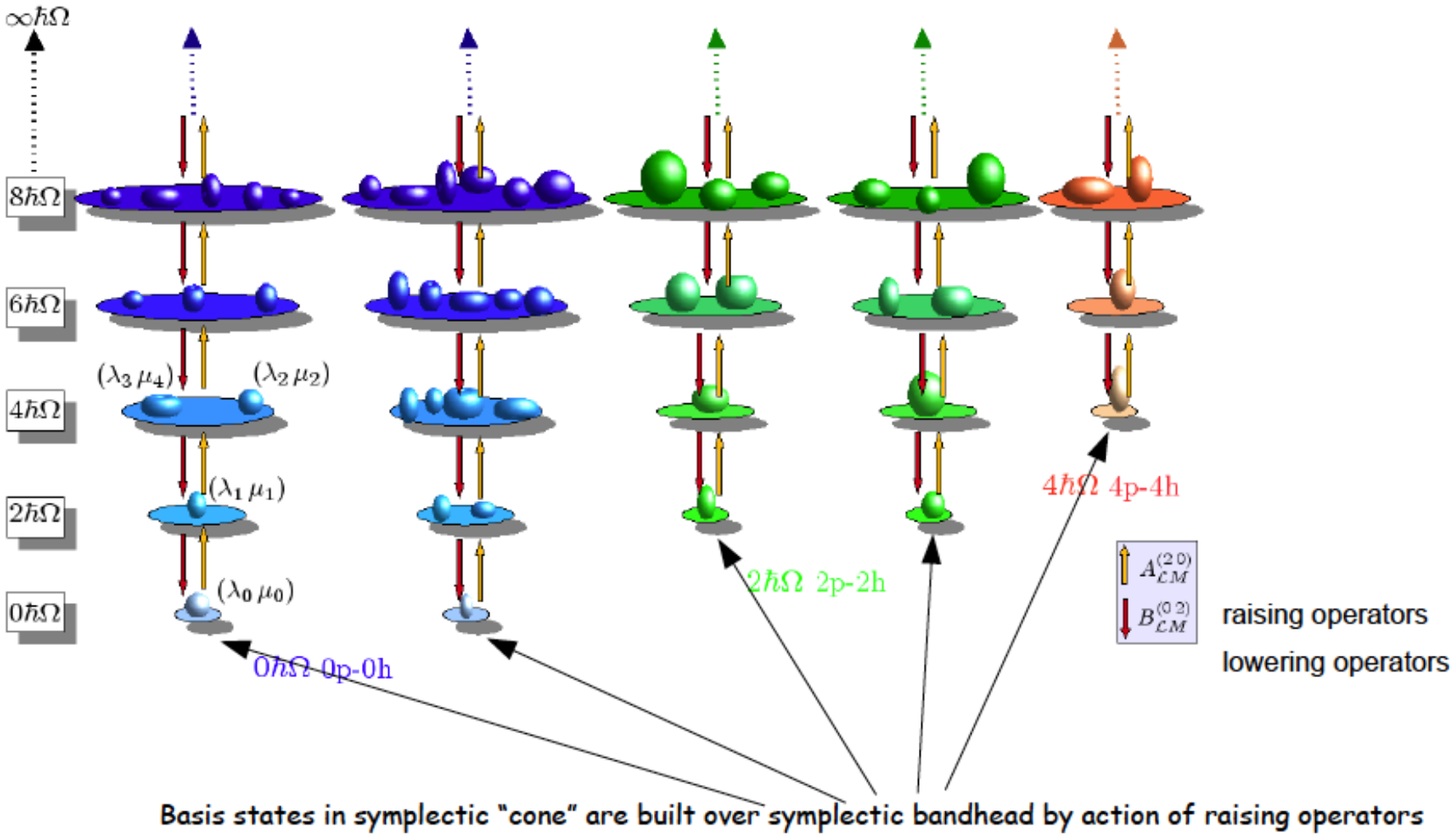
6	$\sum_n x_{ni}x_{nj}$	mass monopole and quadrupole moments
9	$\sum_n x_{ni}p_{nj} \pm x_{nj}p_{ni}$	(-) angular momentum (+) monopole and quadrupole deformations
6	$\sum_n p_{ni}p_{nj}$	quadrupole flow tensor (kinetic energy)
21 generators		

- quadrupole and monopole vibrations and deformations
- rotational dynamics from rigid rotor to irrotational flow

- SU(3) is a subgroup of Sp(3,R) ⇒ Symplectic basis states are labeled by $(\lambda\ \mu)$ and also by $S_\pi\ \ S_\nu\ \ S$

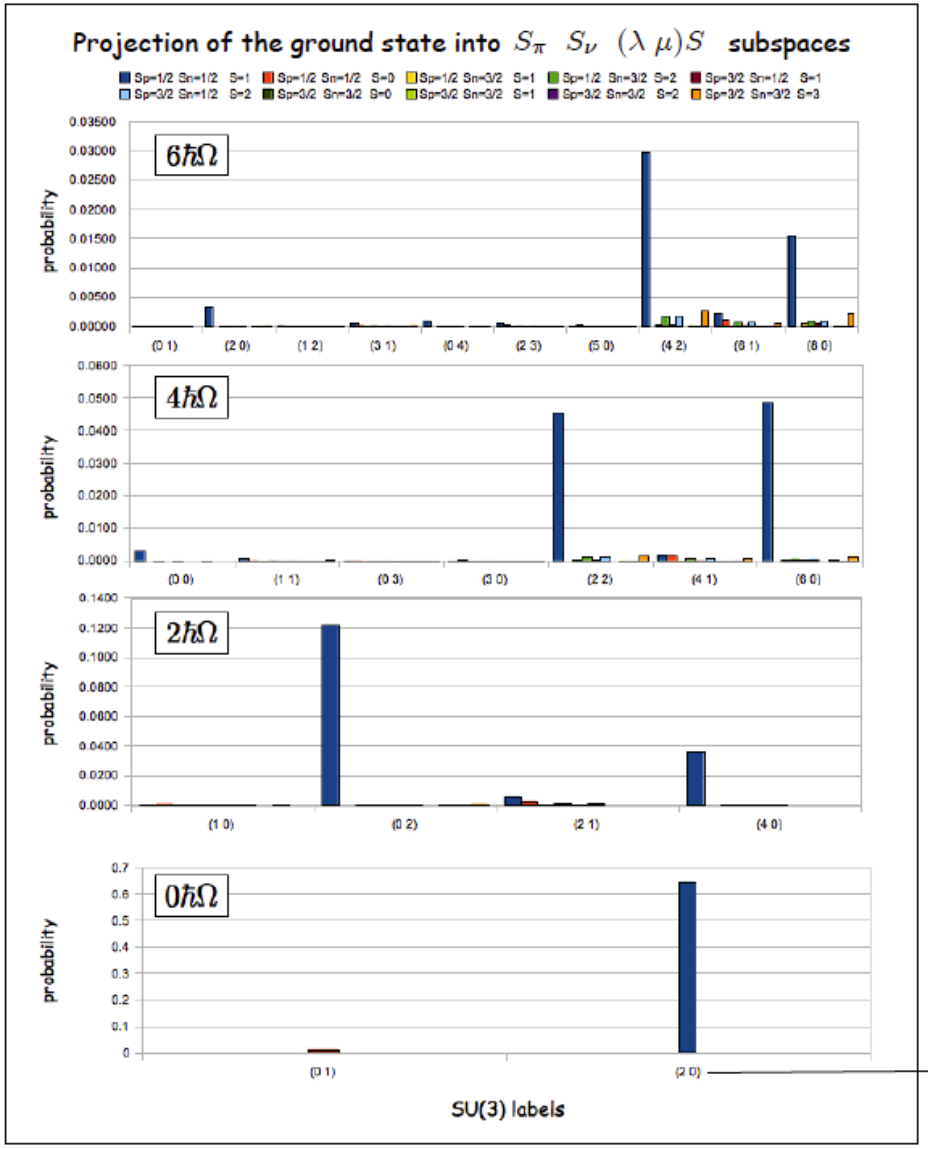
- Symplectic Sp(3,R) symmetry matches deformed geometry [SU(3)] with the various modes of the nuclear collective dynamics

Physical / Algebraic Structure



Basis states in symplectic "cone" are built over symplectic bandhead by action of raising operators

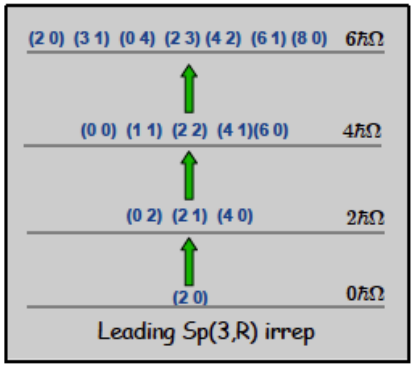
^{6}Li Ground State with $N_{max} = 6$



^{6}Li ($J_{gs} = 1^+$)

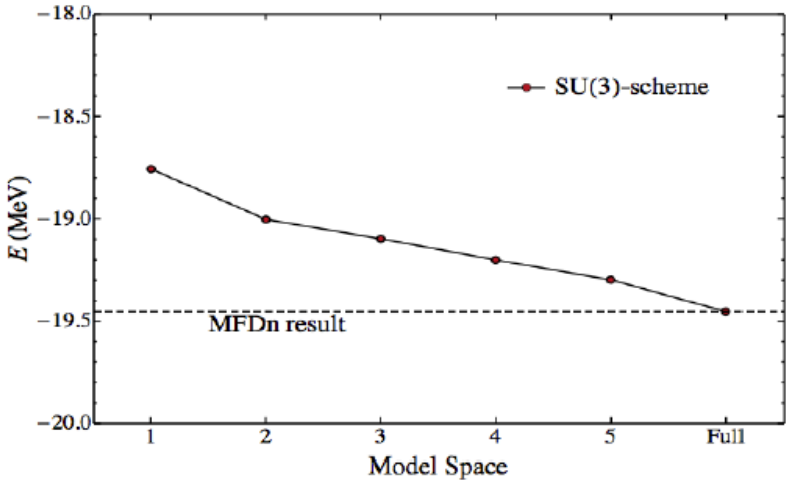
$N_{max} = 6$

most important subspaces contain states of the leading Sp(3,R) irrep



^{6}Li Binding Energy with $N_{max} = 6$

Binding energy for different model space cutoffs



^{6}Li ($J_{gs} = 1^+$)
$N_{max} = 6$

Definition of model space:

$0\hbar\Omega$
$2\hbar\Omega$ } full space
$4\hbar\Omega$
$\oplus$
$6\hbar\Omega$ restricted set of $S_\pi\ S_\nu S\ (\lambda\,\mu)$

			1			2		3		4		5
1/2	1/2	1	(2 0)	(4 2)	(6 1)	(8 0)		(0 4)				
3/2	1/2	2	(4 2)					(8 0)		(6 1)		
1/2	3/2	2	(4 2)					(8 0)		(6 1)		
3/2	3/2	3	(4 2)	(8 0)			$\oplus$		$\oplus$		$\oplus$	(6 1)	$\oplus$	
1/2	1/2	0						(6 1)				
3/2	1/2	1										(8 0)		(4 2)
1/2	3/2	1										(8 0)		(4 2)
S_π	S_ν	S						$(\lambda\,\mu)$						

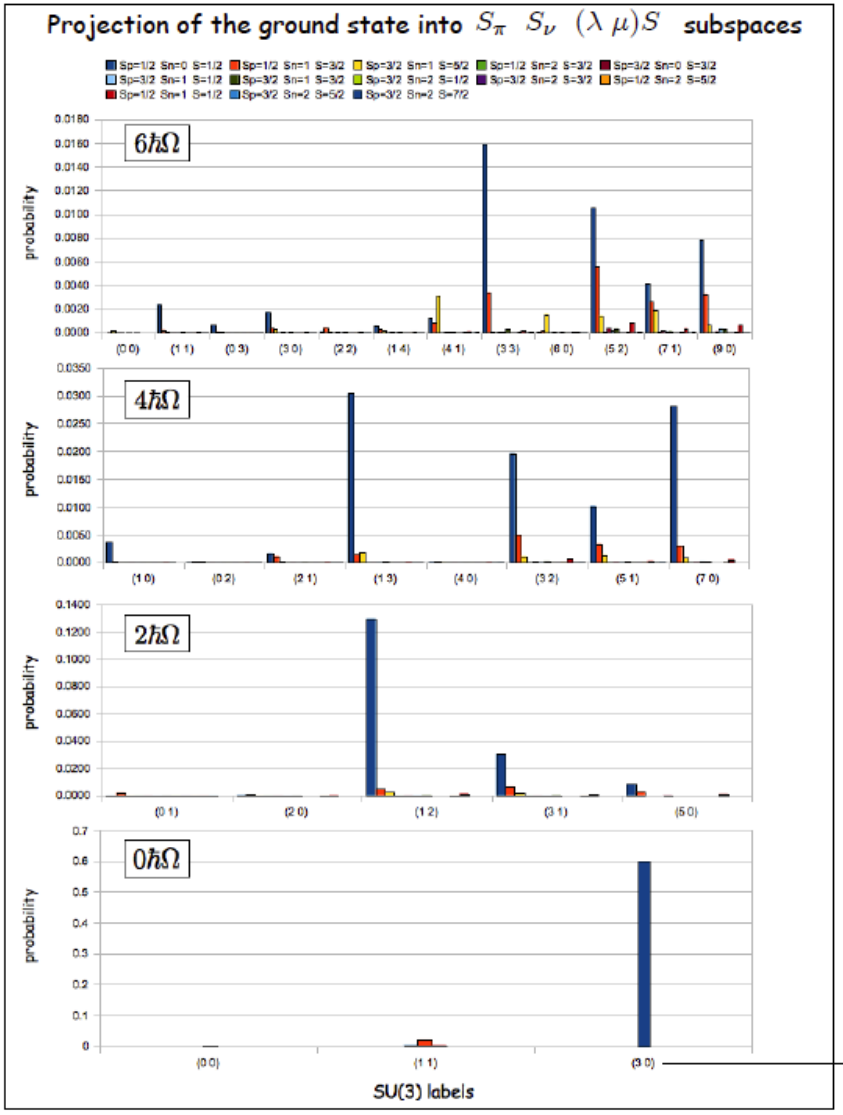
99.6% overlap with the ground state
98.7% binding energy

$(\lambda\,\mu)$ - subspaces included in the model space 5

full model space

Sp=1/2	Sn=1/2	S=0	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) (4 2) **(6 1)** (8 0)
Sp=1/2	Sn=1/2	S=1	(0 1) **(2 0)** (1 2) (3 1) **(0 4)** (2 3) (5 0) **(4 2)** **(6 1)** **(8 0)**
Sp=1/2	Sn=3/2	S=1	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) **(4 2)** (6 1) **(8 0)**
Sp=1/2	Sn=3/2	S=2	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) **(4 2)** **(6 1)** **(8 0)**
Sp=3/2	Sn=1/2	S=1	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) **(4 2)** (6 1) **(8 0)**
Sp=3/2	Sn=1/2	S=2	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) **(4 2)** **(6 1)** **(8 0)**
Sp=3/2	Sn=3/2	S=0	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) (4 2) (6 1) (8 0)
Sp=3/2	Sn=3/2	S=1	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) (4 2) (6 1) (8 0)
Sp=3/2	Sn=3/2	S=2	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) (4 2) (6 1) (8 0)
Sp=3/2	Sn=3/2	S=3	(0 1) (2 0) (1 2) (3 1) (0 4) (2 3) (5 0) **(4 2)** **(6 1)** **(8 0)**

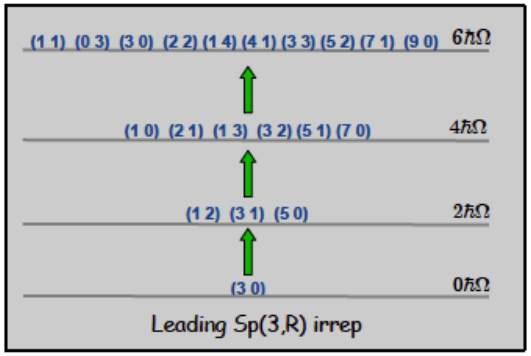
^{7}Li Ground State with $N_{max} = 6$



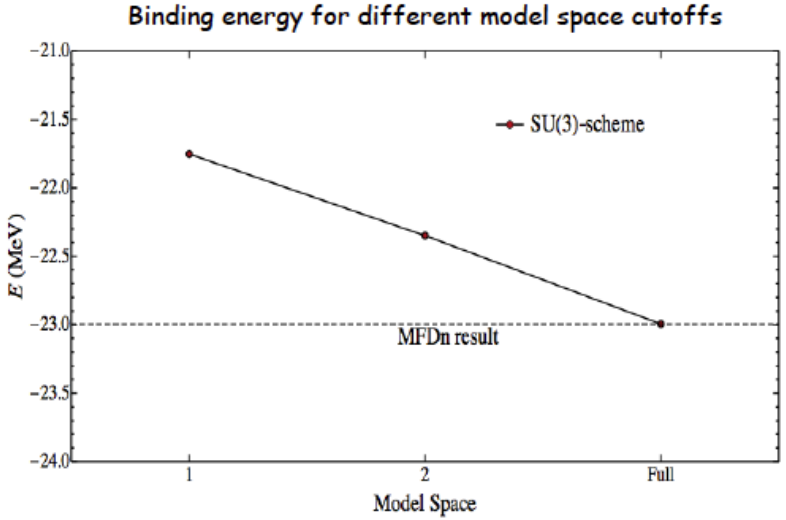
^{7}Li ($J_{gs} = 3/2^-$)

$N_{max} = 6$

- most important subspaces contain states of the leading Sp(3,R) irrep
- most important spin components have similar deformations



^{7}Li Binding Energy with $N_{max} = 6$



^{7}Li ($J_{gs} = 3/2^-$)

$N_{max} = 6$

Definition of model space:

$0\hbar\Omega$
$2\hbar\Omega$ } full space
$4\hbar\Omega$
$\oplus$
$6\hbar\Omega$ restricted set of $S_\pi\ S_\nu S\ (\lambda\,\mu)$

S_π	S_ν	S	1		2
1/2	0	1/2	(1 1)(3 3)(5 2)(9 0)(7 1)	$\oplus$	(3 0)(4 1)
1/2	1	3/2	(3 3)(5 2)(9 0)(7 1)		
3/2	1	5/2	(3 3)		(5 2)(7 1)(9 0)

99.1% overlap with the ground state
97.2% binding energy

$(\lambda\,\mu)$ - subspaces included in the model space 2

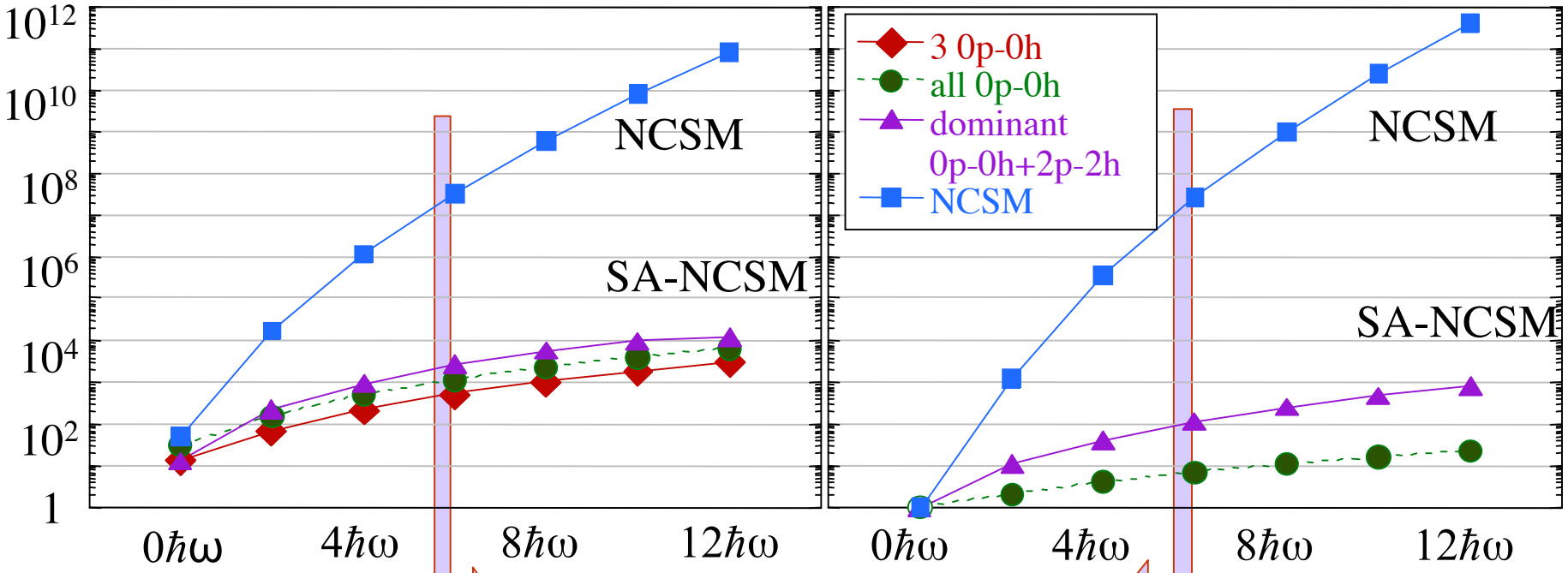
full model space
Sp=1/2 Sn=0 S=1/2 (0 0)**(1 1)**(0 3)**(3 0)**(2 2)(1 4)**(4 1)(3 3)**(6 0)**(5 2)(7 1)(9 0)**
Sp=1/2 Sn=1 S=3/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)**(4 1)(3 3)**(6 0)**(5 2)(7 1)(9 0)**
Sp=1/2 Sn=1 S=1/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)**(4 1)(3 3)**(6 0)**(5 2)(7 1)(9 0)**
Sp=1/2 Sn=2 S=3/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=1/2 Sn=2 S=5/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=0 S=3/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=1 S=1/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=1 S=3/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=1 S=5/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=2 S=1/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=2 S=3/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=2 S=5/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)
Sp=3/2 Sn=2 S=7/2 (0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)(7 1)(9 0)

Major Reductions in Model Space

^{12}C

^{16}O

Dimension of Model Space



NCSM vs SA-NCSM

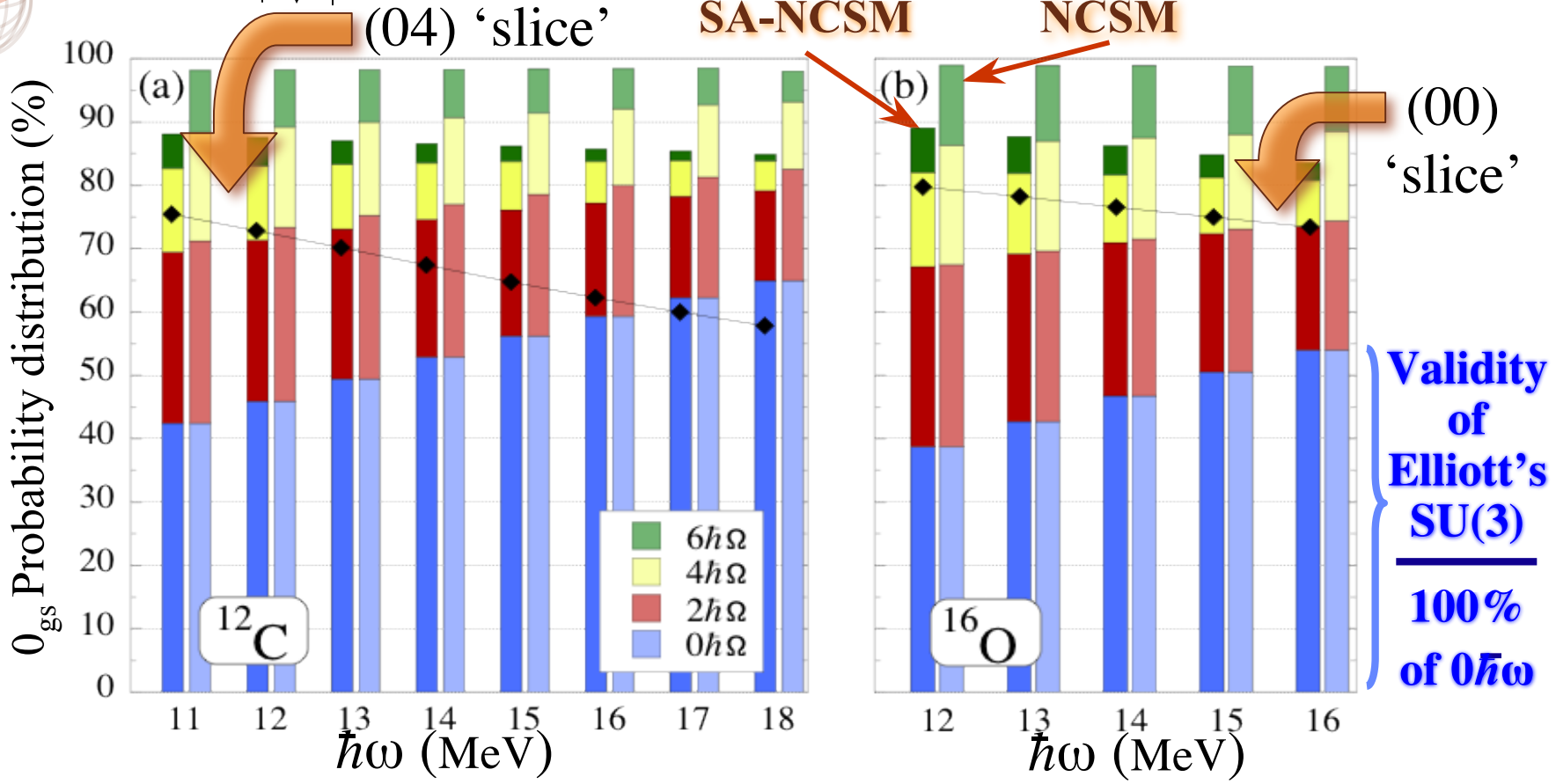
model space dimension

0.009% for ^{12}C

0.0004% for ^{16}O

T. Dytrych. K.D. Sviracheva, C. Bahri, J.P. Draayer, J.P. Vary, Phys. Rev. Lett. 98 (2007) 162503

Probability Distribution: 2^+ and 4^+



Validity of Elliott's SU(3)

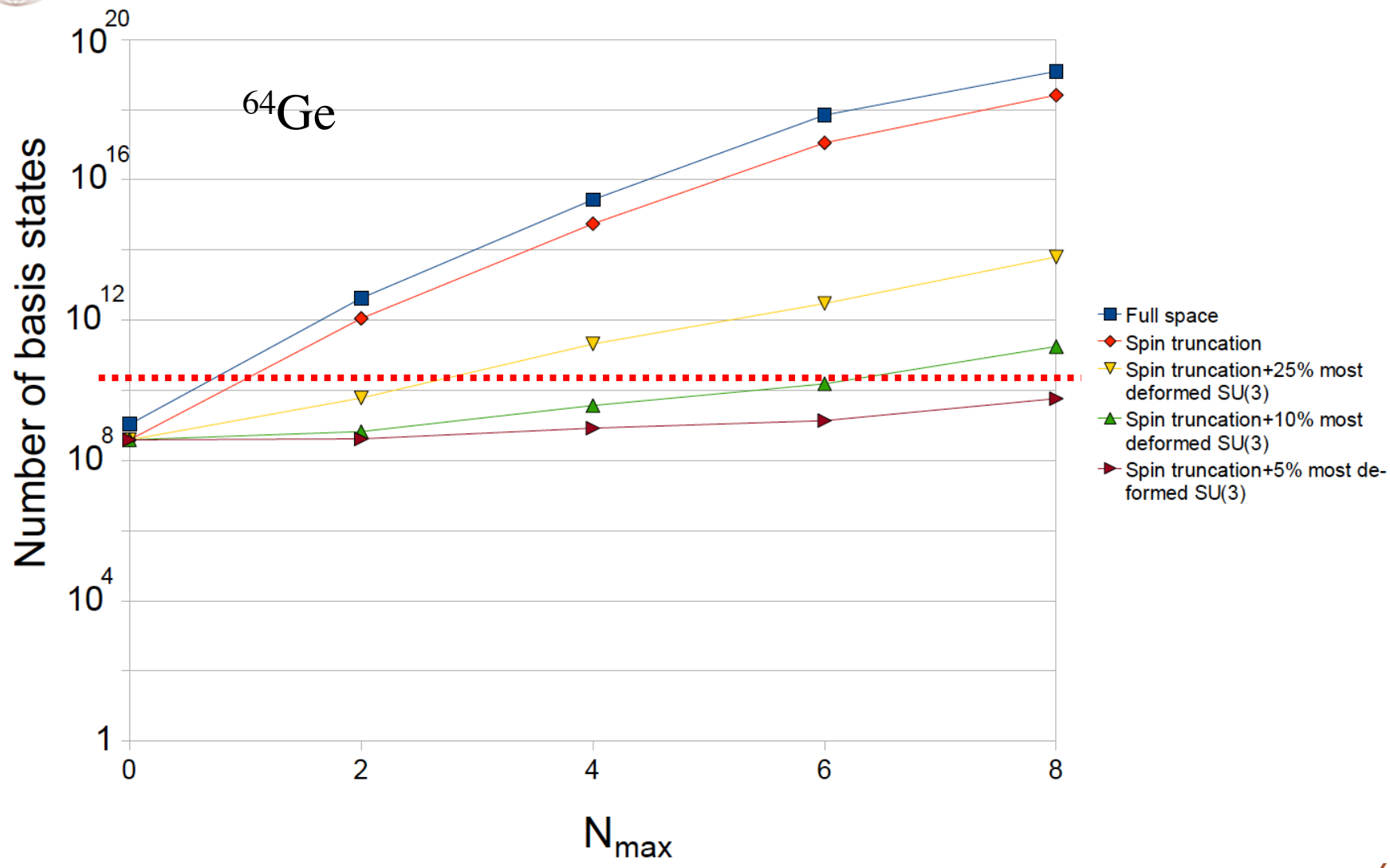
100% of $0\hbar\omega$

Only **3** 0p-0h representations: ~85%

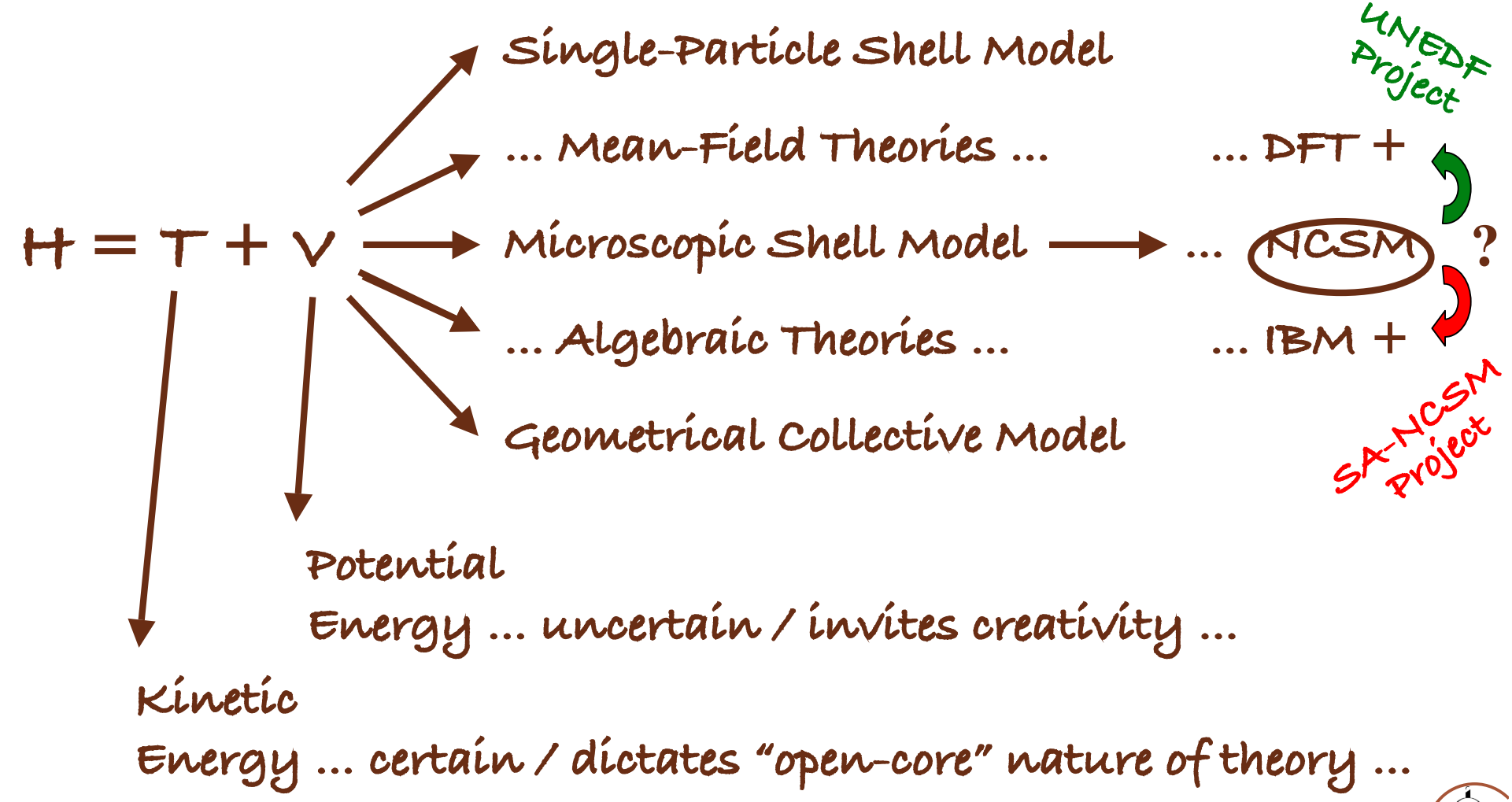
$N_\sigma(\lambda\mu)$
- 24.5(04) : most deformed
- $24.5(12)^2$: spin one states

$2\hbar\omega$ 2p-2h irreps:
~ 4% (^{12}C)
~10% (^{16}O)

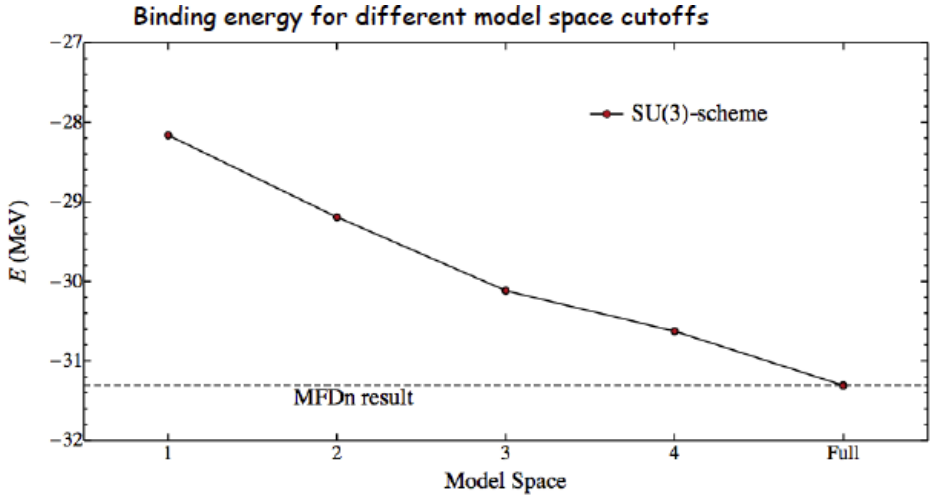
Reaching Higher / Symmetry Winnowing



… Emerging 'Shell Model' Philosophy …



Physical / Algebraic Structure



Definition of model space:

$0\hbar\Omega$
$2\hbar\Omega$ } full space
$\oplus$
$4\hbar\Omega$ restricted set of $S_\pi\ S_\nu S\ (\lambda\ \mu)$

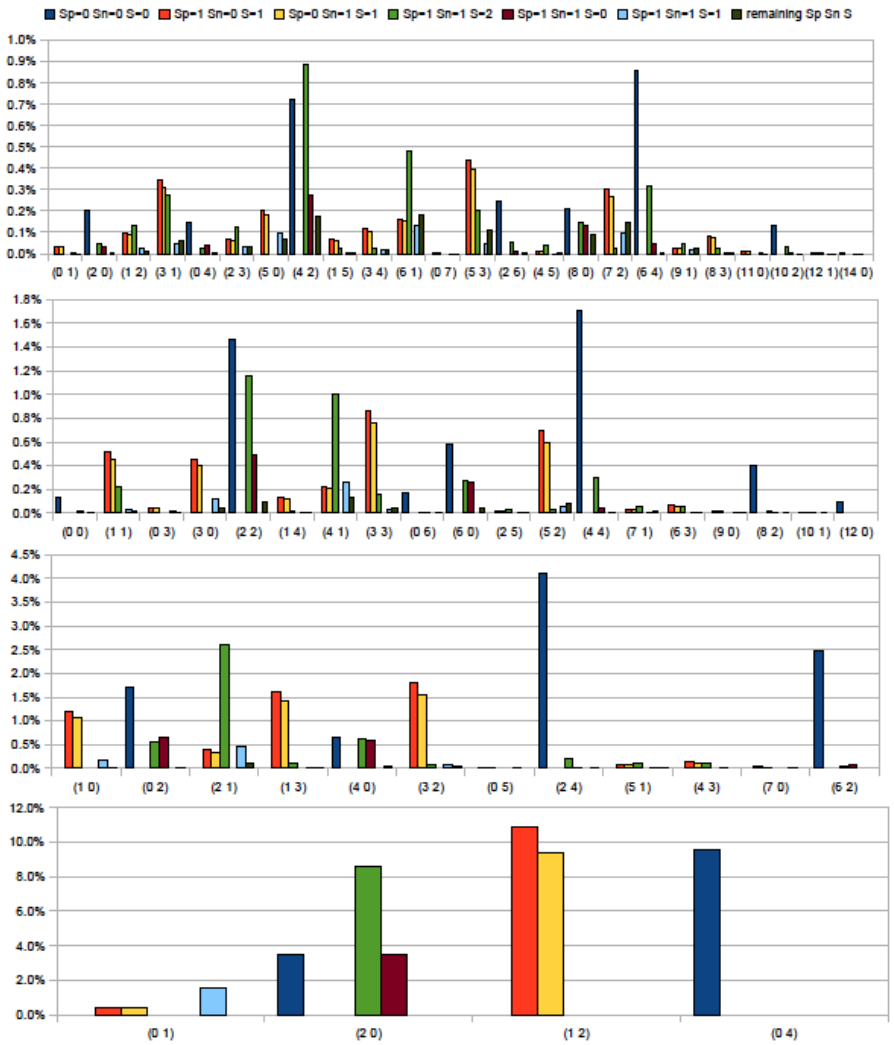
	1		2		3		4
0 0 0	(4 4) (2 2) (0 0)				(8 2) (6 0)		(0 6)
1 1 2	(4 4) (2 2) (3 3)		(1 1)				(4 1) (8 2) (6 0)
1 0 1		$\oplus$	(1 1) (3 3)	$\oplus$	(5 2) (3 0)	$\oplus$	(4 1)
0 1 1			(1 1) (3 3)		(5 2) (3 0)		(4 1)
1 1 0							(4 4)
$S_\pi\ S_\nu\ S$							

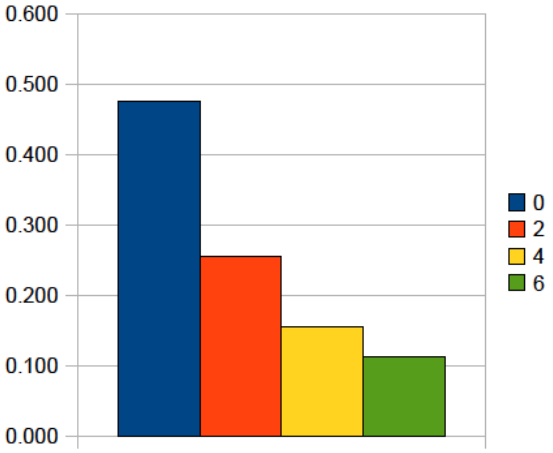
98.8% overlap with the ground state
98.1% binding energy

$(\lambda\ \mu)$ - subspaces included in the model space 4 — full $4\hbar\Omega$ model space

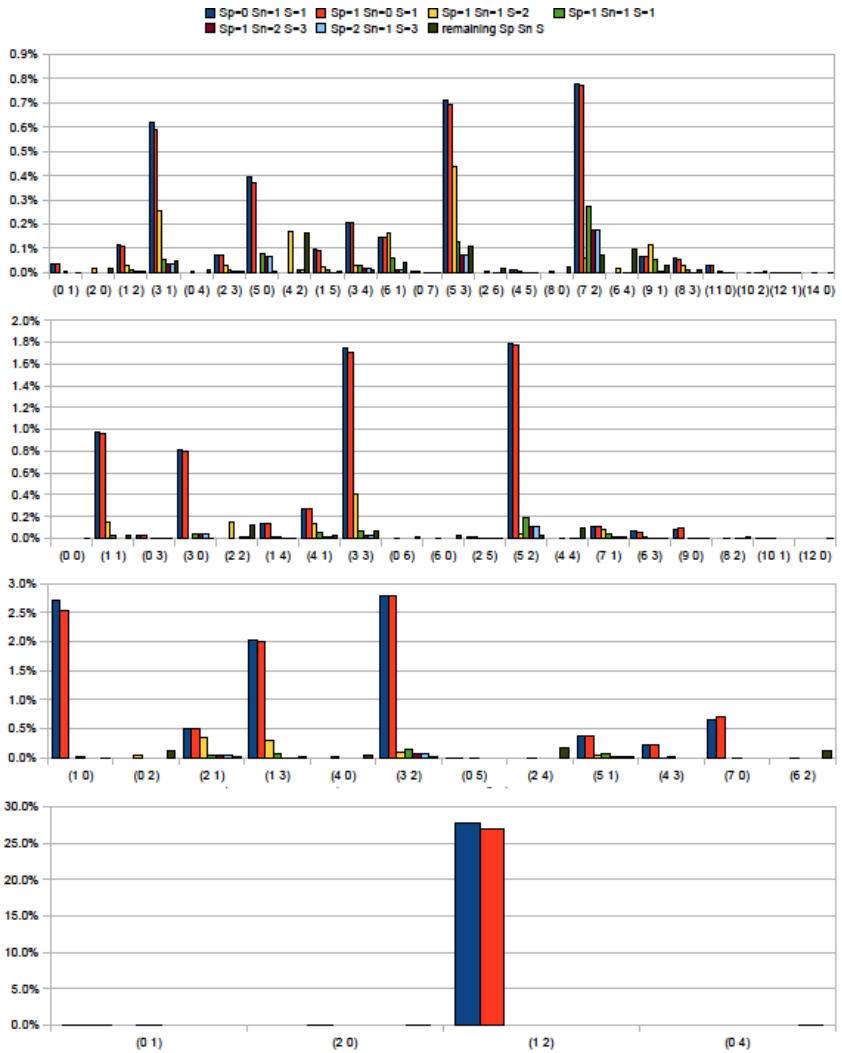
Sp=0 Sn=0 S=0	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)(12 0)	Sp=0 Sn=3 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(5 2)
Sp=1 Sn=1 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)	Sp=3 Sn=0 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(5 2)
Sp=0 Sn=1 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)	Sp=3 Sn=1 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=1 Sn=0 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)	Sp=3 Sn=1 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=1 Sn=0 S=0	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)	Sp=3 Sn=1 S=4	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=1 Sn=1 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)(10 1)	Sp=1 Sn=3 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=0 Sn=2 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=1 Sn=3 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=1 Sn=2 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=1 Sn=3 S=4	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(6 0)(5 2)
Sp=1 Sn=2 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=2 Sn=3 S=1	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=1 Sn=2 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=2 Sn=3 S=2	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=0 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=2 Sn=3 S=3	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=1 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=2 Sn=3 S=4	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=1 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=2 Sn=3 S=5	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=1 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)(6 3)(9 0)(8 2)	Sp=3 Sn=2 S=1	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=2 S=0	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)	Sp=3 Sn=2 S=2	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=2 S=1	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)	Sp=3 Sn=2 S=3	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=2 S=2	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)	Sp=3 Sn=2 S=4	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=2 S=3	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)	Sp=3 Sn=2 S=5	(1 1)(0 3)(3 0)(2 2) (4 1)
Sp=2 Sn=2 S=4	(0 0)(1 1)(0 3)(3 0)(2 2)(1 4)(4 1)(3 3)(0 6)(6 0)(2 5)(5 2)(4 4)(7 1)		

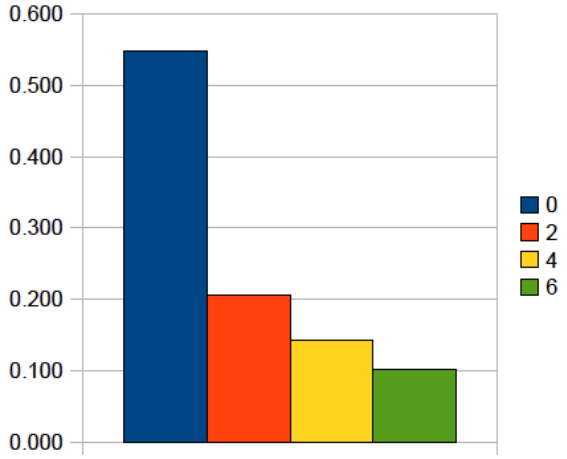
… Structure of 2^{nd} 0^+ in ^{12}C …



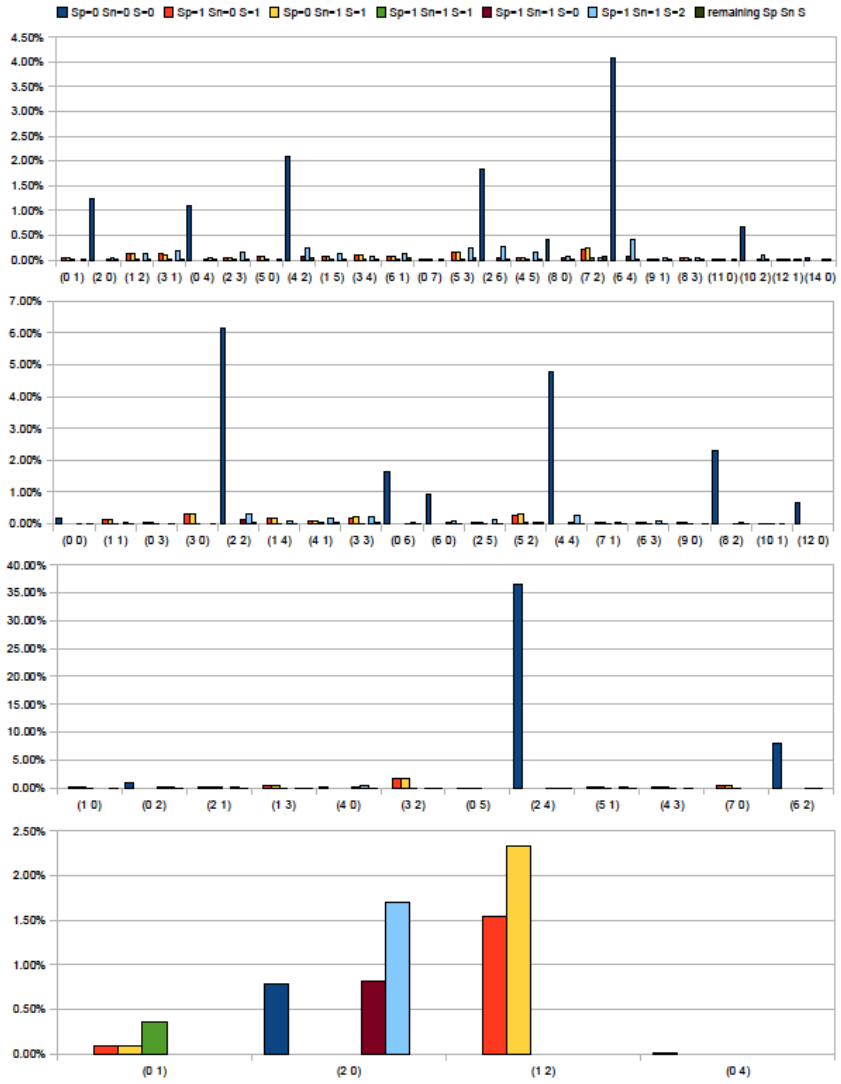


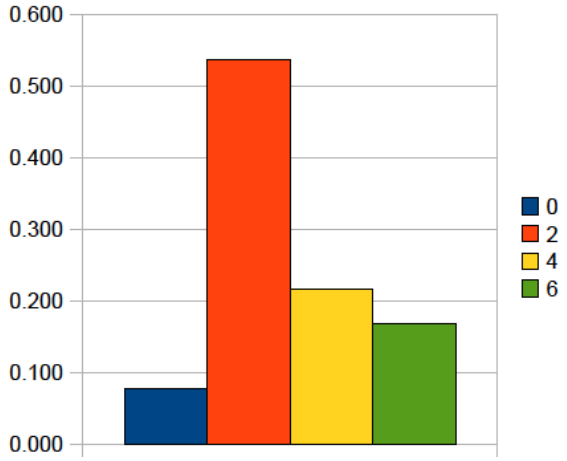
… Structure of 3^{rd} 0^+ in ^{12}C …





… Structure of 4^{th} 0^+ in ^{12}C …





… Structure of 5^{th} 0^+ in ^{12}C …