Calculation of charge-exchange neutrino-nucleus reactions with the relativistic random phase approximation

T. Marketin[1,2] N. Paar[2] T. Nikšić[2] D. Vretenar[2] P. Ring[3]

[1]GSI Helmholtzzentrum für Schwerionenforschung, Germany
[2]Physics Department, Faculty of Science, University of Zagreb, Croatia
[3]Physik-Department der Technischen Universität München, Germany

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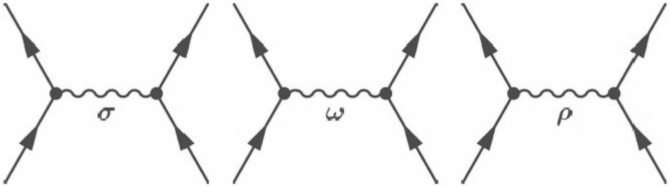
Introduction

Most of the fundamental tests of weak-interaction theory have involved nuclei:

- It is essential that the nuclear physics of these processes be well understood.
- Weak-interaction processes are also sensitive to the axial-vector current.
- These processes are critical in astrophysical applications $\rightarrow$ precise data is required for a large number of nuclei, most of which are outside the reach of current experimental setups.

Nuclear structure models must be able to describe both stable and unstable nuclei within a unique theoretical framework.

Relativistic mean-field theory



- σ scalar, isoscalar meson, attractive component of the interaction
- ω vector, isoscalar meson, repulsive component of the interaction
- ρ vector, isovector meson, isospin component of the interaction

The model is defined by the Lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

Variation leads to the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \varphi} = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \varphi \right)} \right)$$

Dirac equation for the nucleons

$$\left[\gamma^\mu \left(i\partial_\mu + V_\mu \right) + m + S \right] \psi = 0$$

and the Klein-Gordon equations for the mesons

$$-\Delta \phi_m + m_{\phi_m}^2 = \pm \langle \bar{\psi} \Gamma_m \psi \rangle$$

D. Vretenar, A. V. Afanasjev, G. A. Lalazissis and P. Ring, Phys. Rep. 409, 101 (2005)

The interaction part of the Lagrangian density reads

$$\mathcal{L}_{int} = -g_\sigma \bar{\psi}\sigma\psi - g_\omega \bar{\psi}\gamma_\mu \omega^\mu \psi - g_\rho \bar{\psi}\gamma_\mu \vec{\rho}^\mu \vec{\tau}\psi - e\bar{\psi}\gamma_\mu A^\mu \frac{1-\tau_3}{2}\psi.$$

Dependence of meson-nucleon vertices on baryon density

$$\rho_v = \sqrt{j_\mu j^\mu}, \qquad j_\mu = \bar{\psi}\gamma_\mu\psi\,,$$

produces the rearrangements terms

$$\Sigma_\mu^R = \frac{j_\mu}{\rho_v}\left(\frac{\partial g_\omega}{\partial \rho_v}\bar{\psi}\gamma^\nu\psi\omega_\nu + \frac{\partial g_\rho}{\partial \rho_v}\bar{\psi}\gamma^\nu\vec{\tau}\psi\cdot\vec{\rho}_\nu + \frac{\partial g_\sigma}{\partial \rho_v}\bar{\psi}\psi\sigma\right).$$

Parameters of the model are adjusted to reproduce properties of nuclear matter and finite nuclei.

Nucleon pairing correlations are treated within the relativistic Hartree-Bogoliubov model

$$\hat{\rho}_{kl} = \langle \Phi | c_l^\dagger c_k | \Phi \rangle \qquad \hat{\kappa}_{kl} = \langle \Phi | c_l c_k | \Phi \rangle$$

where the generalized density matrix is defined by

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho^* \end{pmatrix} \quad \mathcal{H} = \frac{\delta E}{\delta \mathcal{R}} \rightarrow i\partial_t \mathcal{R} = [\mathcal{H}, \mathcal{R}]$$

which leads to the RHB equations. D1S parametrization of the Gogny force is employed in the model.

$$V^{pp}(1,2) = \sum_{i=1}^{2} e^{-\left(\frac{\mathbf{r}_1 - \mathbf{r}_2}{\mu_i}\right)^2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)$$

Random phase approximation

In an external field oscillating with a small amplitude

$$\hat{F}(t) = \hat{F}e^{-i\omega t} + \text{h.c.}$$

the equation of motion for the density operator reads

$$i\partial_t\hat{\rho} = \left[\hat{h}(\hat{\rho}) + \hat{f}(t), \hat{\rho}\right] \qquad \hat{\rho}(t) = \hat{\rho}^{(0)} + \delta\hat{\rho}^{(0)}$$

leading to the final form of the RPA equations

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix} = E_\lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix}$$

Residual interaction is derived from the Lagrangian density

$$\mathcal{L}_{\rho+\pi} = -g_\rho\bar{\psi}\gamma_\mu\vec{\rho}^\mu\vec{\tau}\psi - \frac{f_\pi}{m_\pi}\bar{\psi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\vec{\tau}\psi$$

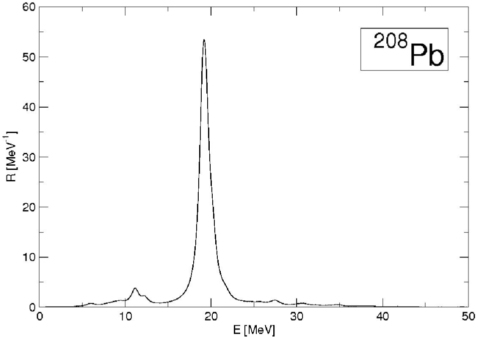
N. Paar, D. Vretenar, E. Khan and G. Colò, Rep. Prog. Phys. 70, 691 (2007)

Pion-nucleon coupling requires the inclusion of the contact Landau-Migdal term

$$V_{\delta\pi} = g' \left(\frac{f_\pi}{m_\pi} \right)^2 \vec{\tau}_1 \vec{\tau}_2 \mathbf{\Sigma}_1 \cdot \mathbf{\Sigma}_2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$m_\pi = 138\,\mathrm{MeV}$$

$$\frac{f_\pi^2}{4\pi} = 0.08$$



The weak-interaction Hamiltonian

$$\hat{H}_W = -\frac{G}{\sqrt{2}} \int d\boldsymbol{x} \mathcal{J}_\lambda(\boldsymbol{x}) j^\lambda(\boldsymbol{x}),$$

with the matrix element given by

$$\left\langle f \left| \hat{H}_W \right| i \right\rangle = -\frac{G}{\sqrt{2}} l_\lambda \int d\boldsymbol{x} e^{-i\boldsymbol{qr}} \left\langle f \left| \mathcal{J}_\lambda(\boldsymbol{x}) \right| i \right\rangle .$$

Employing the multipole expansion of the leptonic part

$$e^{i\boldsymbol{qr}} = \sum_{J=0}^{\infty} \left[4\pi(2J+1)\right]^{1/2} i^J j_J(\kappa r) Y_{J0}(\Omega_r)$$

it is possible to examine the contribution of transitions with a particular value of the angular momentum.

μ^- capture

The capture of a negative muon from the atomic 1s orbit

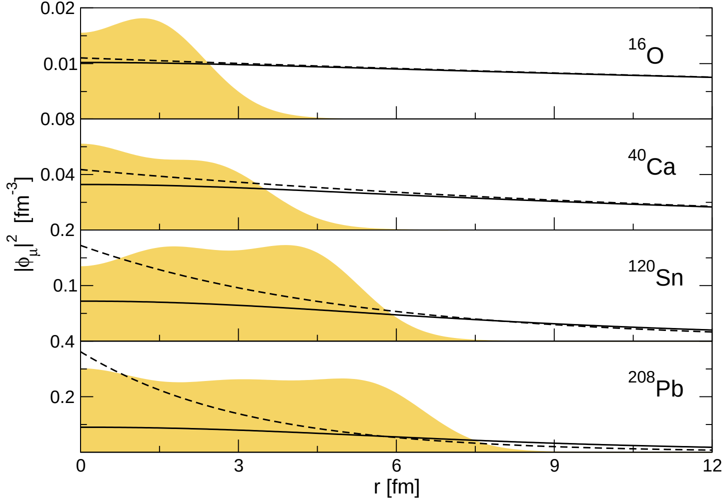
$$\mu^- + (Z, N) \rightarrow (Z-1, N+1)^* + \nu_\mu$$

cannot be treated in the long wavelength limit. The capture rate

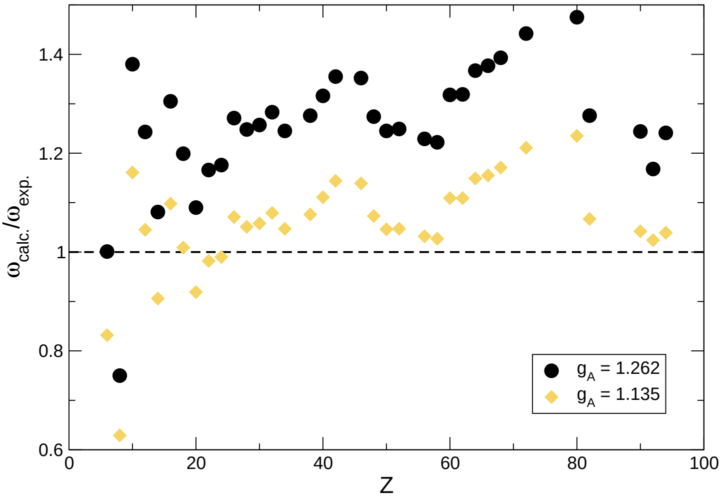
$$\omega_{fi} = \frac{V^2\nu^2}{2\pi} \sum_{\text{lepton spins}} \frac{1}{2J_i+1} \sum_{M_i, M_f} \left| \left\langle f \left| \hat{H}_W \right| i \right\rangle \right|^2$$

can be written as

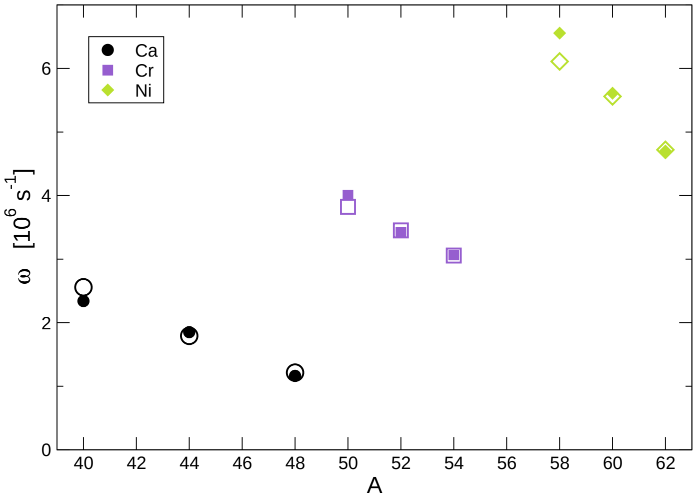
$$\begin{aligned} \omega_{fi} &= \frac{2G^2\nu^2}{1+\nu/M_T} \frac{1}{2J_i+1} \left\{ \sum_{J=0}^{\infty} \left| \left\langle J_f \left\| \phi_{1s} \left(\hat{\mathcal{M}}_J - \hat{\mathcal{L}}_J \right) \right\| J_i \right\rangle \right|^2 \right. \\ &+ \left. \sum_{J=1}^{\infty} \left| \left\langle J_f \left\| \phi_{1s} \left(\hat{\mathcal{T}}_J^{el} - \hat{\mathcal{T}}_J^{mag} \right) \right\| J_i \right\rangle \right|^2 \right\} \end{aligned}$$

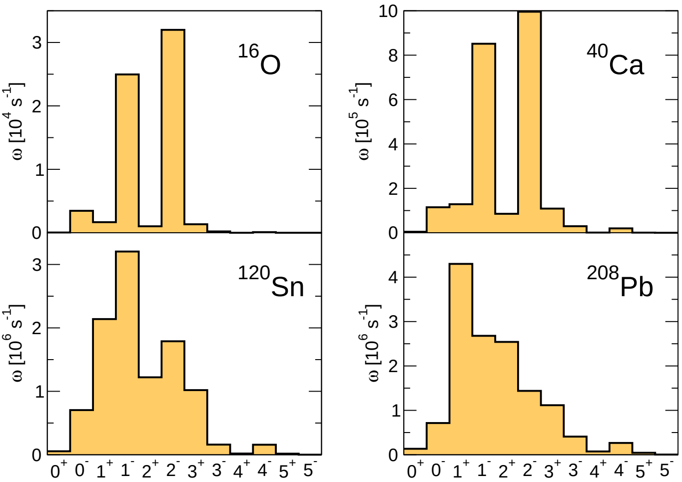


0.02
0.01
0.08
0.04
0.2
0.1
0.4
0.2
$|\phi_\mu|^2$ [fm^{-3}]
^{16}O
^{40}Ca
^{120}Sn
^{208}Pb
0
3
6
9
12
r [fm]



$\omega_{calc.}/\omega_{exp.}$
1.4
1.2
1
0.8
0.6
0
20
40
60
80
100
Z
g_A = 1.262
g_A = 1.135


Ca
Cr
Ni
ω [10^6 s^-1]
0
2
4
6
40
42
44
46
48
50
52
54
56
58
60
62
A



16O
40Ca
120Sn
208Pb
ω [10^4 s^-1]
ω [10^5 s^-1]
ω [10^6 s^-1]
ω [10^6 s^-1]
0+ 0- 1+ 1- 2+ 2- 3+ 3- 4+ 4- 5+ 5-
0+ 0- 1+ 1- 2+ 2- 3+ 3- 4+ 4- 5+ 5-

ν capture

Charged-current neutrino-nucleus reaction

$$\nu_l + (Z, N) \rightarrow (Z + 1, N - 1)^* + l^-,$$

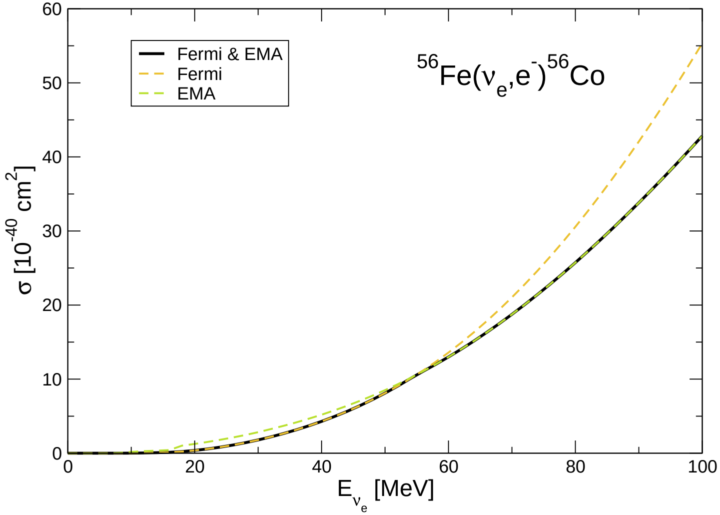
with the cross section

$$\left(\frac{d\sigma_\nu}{d\Omega}\right) = \frac{V^2 p_l E_l}{(2\pi)^2} \sum_{\text{lepton spins}} \frac{1}{2J_i + 1} \sum_{M_i, M_f} \left| \left\langle f \left| \hat{H}_W \right| i \right\rangle \right|^2$$

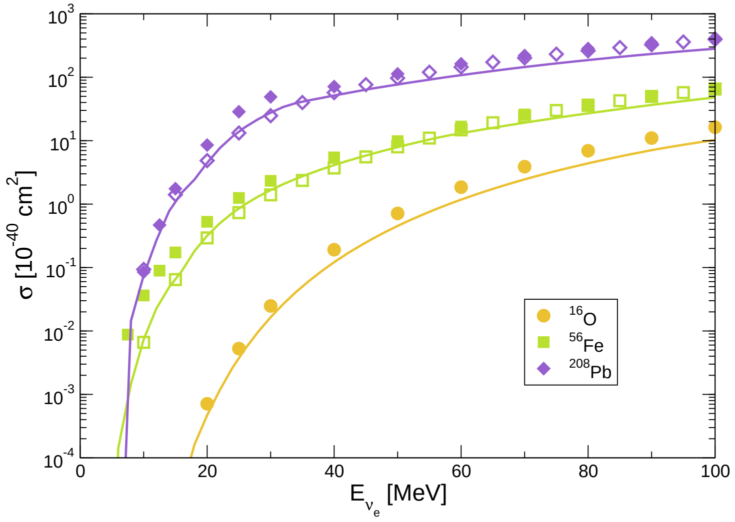
Distortion of the outgoing lepton wave function

- Fermi function
- Effective momentum approximation

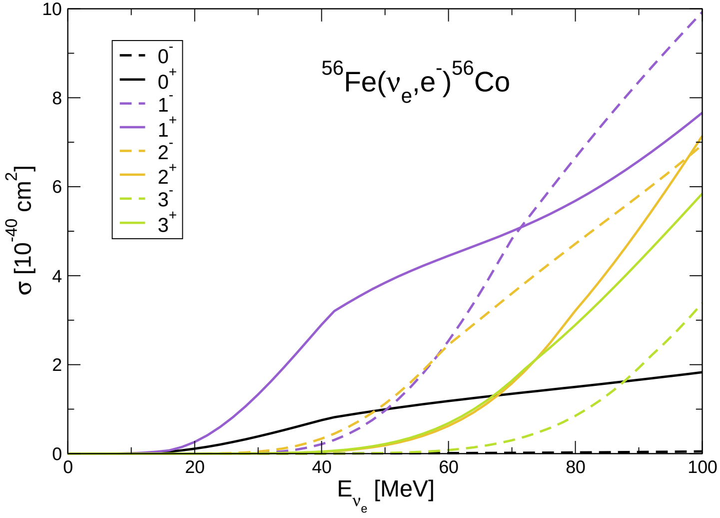
$$p_{\text{eff},l} = \sqrt{E_{\text{eff},l}^2 - m_l^2}, \qquad E_{\text{eff},l} = E_l - V_{\text{eff},C}$$



Fermi & EMA
Fermi
EMA
$^{56}Fe(\nu_e,e^-)^{56}Co$
$\sigma\ [10^{-40}\ cm^2]$
E_{ν_e} [MeV]
0
10
20
30
40
50
60
0
20
40
60
80
100

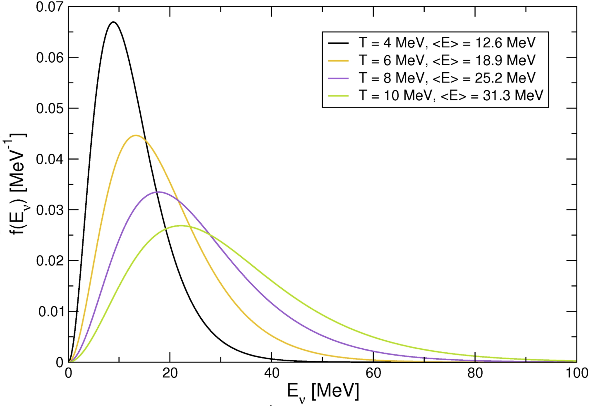


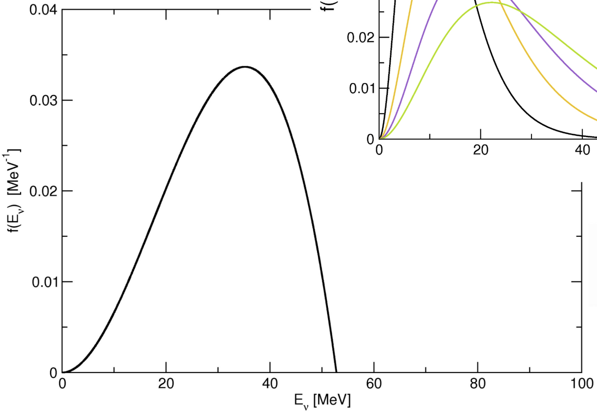
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\sigma\ [10^{-40}\ cm^2]$
0
20
40
60
80
100
E_{ν_e} [MeV]
^{16}O
^{56}Fe
^{208}Pb



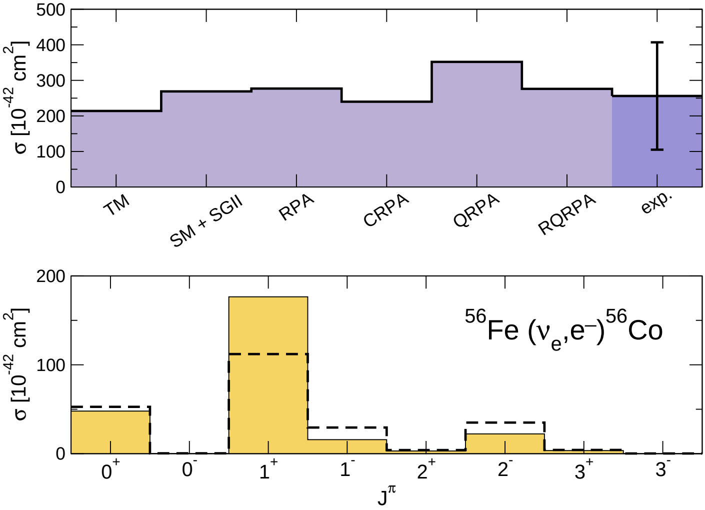
$^{56}Fe(\nu_e,e^-)^{56}Co$
0^-
0^+
1^-
1^+
2^-
2^+
3^-
3^+
$\sigma\ [10^{-40}\ cm^2]$
E_{ν_e} [MeV]
0
2
4
6
8
10
0
20
40
60
80
100

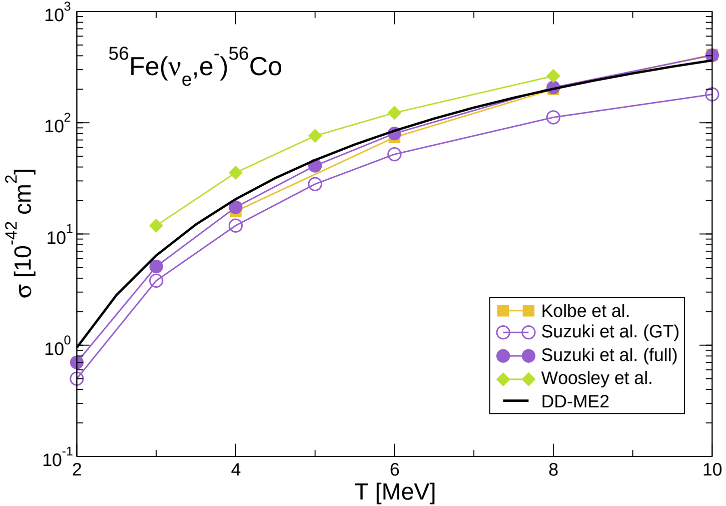
$$f_{SN}(E_\nu) = \frac{1}{T^3} \frac{E_\nu^2}{e^{E_\nu/T-\alpha}+1}$$

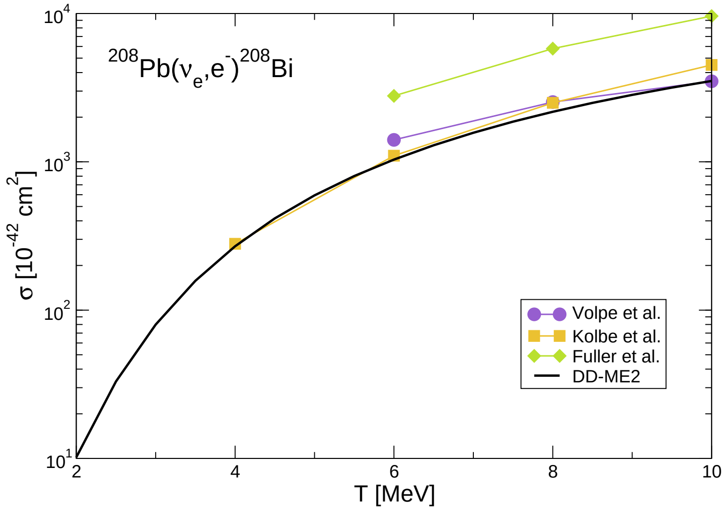




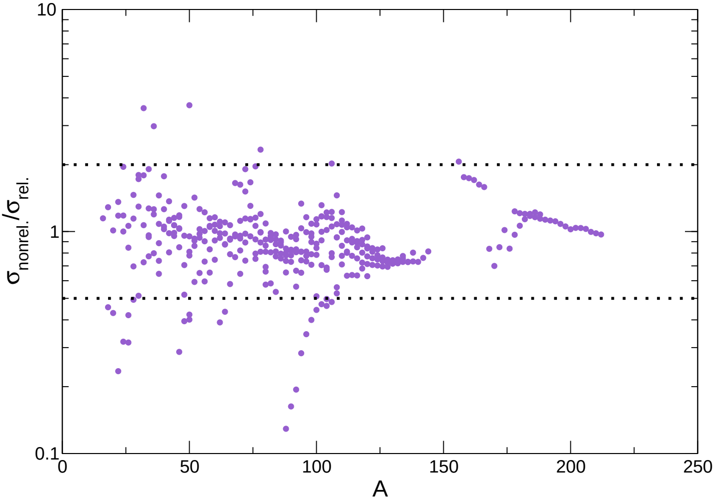
$$f_{DAR}(E_\nu) = \frac{96 E_\nu^2}{m_\mu^4}(m_\mu - 2E_\nu)$$


500
400
300
200
100
0
σ [10^-42 cm^2]
TM
SM + SGII
RPA
CRPA
QRPA
RQRPA
exp.
200
100
0
σ [10^-42 cm^2]
56Fe (νe,e−)56Co
0+
0-
1+
1-
2+
2-
3+
3-
Jπ


$^{56}Fe(\nu_e,e^-)^{56}Co$
σ [10^{-42} cm^2]
T [MeV]
10^{-1}
10^0
10^1
10^2
10^3
2
4
6
8
10
Kolbe et al.
Suzuki et al. (GT)
Suzuki et al. (full)
Woosley et al.
DD-ME2



$^{208}Pb(\nu_e,e^-)^{208}Bi$
10^4
10^3
10^2
10^1
$\sigma\ [10^{-42}\ cm^2]$
2
4
6
8
10
T [MeV]
Volpe et al.
Kolbe et al.
Fuller et al.
DD-ME2



K. Langanke and E. Kolbe, Atomic Data and Nuclear Data Tables 79, 293 (2001)