Nuclear Vorticity in Electric Giant Resonances: Vortical, Toroidal and Compression Modes

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Motivation

Nuclei demonstrate both

- **irrotational flow (most of electric GR)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$
- **vortical flow (toroidal GR)** $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

Vorticity $\vec{w}(\vec{r})$ is a fundamental quantity:

- **does not contribute to the continuity equation,**
- **represents an independent part of charge-current distribution beyond the continuity equation.**

Vorticity is related to the exotic modes:

- **toroidal E1 mode (TM) ,**
- **compression E1 mode (CM),**

which are now of a keen interest .

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes and manifestations of vorticity:

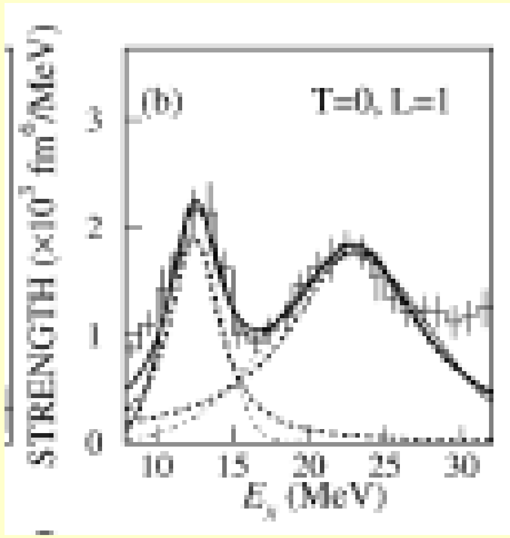
V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).
S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
N.Ryezayeva et al, PRL 89, 272502 (2002).
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
! **D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).**
V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
……….
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007). Recent review

Observation of ISGDR : CM and perhaps TM:

(α,α')

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)



(γ,γ')

N.Ryezayeva et al, PRL 89, 272502 (2002).

Operators of the modes:

Toroidal mode E1(T=0):

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).

$$\hat{M}_{tor}(E1\mu) = \frac{1}{20c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\underbrace{(r^3 - \frac{5}{3}r < r^2 >_0)}_{\text{cmc}}Y_{1\mu}]$$

$$= -\frac{i}{2\sqrt{3}c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\frac{\sqrt{2}}{5}r^2\vec{Y}^2_{1\mu} + (r^2 \underbrace{- < r^2 >_0}_{\text{cmc}})\vec{Y}^0_{1\mu}]$$

Compression mode E1(T=0):

S.F. Semenko, SJNP 34 356 (1981).

$$\hat{M}'_{com}(E1\mu) = \frac{1}{10}\int d\vec{r}\,\hat{\rho}(\vec{r})[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

The TM and CM operators are related.

Vortical mode E1(T=0): NO yet OPERATOR

Open problems

different conclusions
on CM vorticity

- definition of nuclear vorticity (HD vs Wambach),
- IS (T=0) and IV(T=1) branches of the modes,
- role of magnetization (spin) nuclear current,
- there is no the VM operator, VM vs TM/CM,

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, subm. to PRC, arXiv: 1105.0837[nucl-th]

We will show that the VM operator may be derived and related to TM and CM operators

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

This relation allows to understand better the connection between VM, TM, and CM

How to introduce the vorticity for nuclei?

In HD the vorticity is defined as

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$$

Nuclear quantum theory deals with the nuclear current $\vec{j}_{nuc}(\vec{r})$

The vorticity cannot be measured by the current curl

$$\vec{\nabla} \times \vec{j}_{nuc}(\vec{r})$$

because

$$\vec{v}(\vec{r}) = \frac{\delta \vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})} \Rightarrow \vec{\nabla} \times \vec{v}(\vec{r}) = \frac{\vec{\nabla} \times \delta \vec{j}_{nuc}(\vec{r}) - \left(\vec{\nabla}\rho_0(\vec{r})\right) \times \hat{\vec{v}}(\vec{r})}{\rho_0(\vec{r})}$$

$$\hat{M}(E\lambda\mu) \sim \int d\vec{r}\, (\vec{\nabla} \times \hat{\vec{j}}_{nuc})\, [....]$$

Then, how to introduce the nuclear vorticity?

Two definitions of vorticity in nuclear theory:

Definition 1
(from hydrodynamics):

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \qquad \vec{v}(\vec{r}) = \frac{\delta \vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$< \nu \,|\, \hat{M}(E\lambda\mu) \,|\, 0 >= \int d\vec{r}\, (\vec{\nabla} \times \delta \vec{j}^{\,\nu}_{nuc})\, [....]$$

$$\Downarrow$$

$$< \nu \,|\, \hat{M}_{vor}(E\lambda\mu) \,|\, 0 >= \int d\vec{r}\, \rho_0\, (\vec{\nabla} \times \vec{v}_{\nu})\, [....]$$

CM is irrotational:

$$\vec{v}_{com}(E1\mu) \propto \vec{\nabla}[r^3 - \frac{5}{3} r < r^2 >_0] Y_{1\mu} \qquad \vec{\nabla} \times \vec{v}_{com}(\vec{r}) = 0$$

Definition 2
(Ravenhall and Wambach)

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

By using expansion of the current transition

$$\delta \vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \mid \hat{\vec{j}}_{nuc}(\vec{r}) \mid j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \mid j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r) \vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r) \vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

one may construct the vorticity transition density

$$w_{\lambda\lambda}(r) = \sqrt{\frac{2\lambda+1}{\lambda}} \left(\frac{d}{dr} + \frac{\lambda+2}{r} \right) j_{\lambda\lambda+1}(r)$$

and strength

$$\nu^{(fi)}_{\lambda} = \int_0^{\infty} r^{\lambda+4} \, w^{(fi)}_{\lambda\lambda}(r) \, dr$$

expressed through the particular transverse current multipole $j^{(fi)}_{\lambda\lambda+1}(r)$, which, unlike $j^{(fi)}_{\lambda\lambda-1}(r)$, does not contribute to the continuity equation

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

So, $j^{(fi)}_{\lambda\lambda+1}(r)$ is an independent part of charge-current distribution.

This approach does not use the vortical operator. However, such operator could be useful for the comparison of vortical, toroidal and compression flows.

Wambach vorticity

Presence of $j^{(fi)}_{\lambda\lambda+1}(r)$ **is decisive to make the flow vortical**

$$\hat{M}'_{com}(E1\mu) = \frac{1}{10}\int d\vec{r}\,\hat{\rho}(\vec{r})[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{2c\sqrt{3}}\int d\vec{r}\;\; \hat{\vec{j}}_{nuc}(\vec{r})\,[r^2\frac{2\sqrt{2}}{5}\vec{Y}_{12\mu} + (r^2 - < r^2 >_0)\vec{Y}_{10\mu}]$$

CM involves $\vec{Y}_{12\mu}$ and so $\vec{j}_{12\mu}$. Hence CM is vortical despite its gradient flow !?

The reason of contradiction:

The Wambach vorticity $w_{\lambda\lambda}(r) \propto j_{\lambda\lambda+1}(r)$ was introduced mainly as a quantity fully unconstrained by the CE rather than the purely vortical value in the HD sense.

Thus an essential difference between HD and Wambach vorticity.

Derivation of the vortical operator (Wambach)

Multipole electric operator

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r} \quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})\,]$$

Main idea:

$$\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) \Rightarrow \underbrace{\rho_0(\vec{r})\vec{\nabla}\times\vec{v}(\vec{r})}_{\text{truly vortical}} \to \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - i\frac{kc}{\lambda}\vec{\nabla}\rho_0(\vec{r})\times\hat{\vec{r}}$$

Then

$$\hat{M}_{vor}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r} \;\; [j_\lambda(kr) \;\; \vec{Y}_{\lambda\lambda\mu}(\vartheta,\varphi)]\cdot$$

$$\cdot\left[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) \;-\; \frac{i}{\lambda}kc\vec{\nabla}\hat{\rho}(\vec{r})\times\hat{\vec{r}}\right] =$$

$$= \hat{M}(Ek\lambda\mu) - \hat{M}_S(k\lambda\mu)$$

$$\boxed{\begin{aligned}&\hat{\vec{v}} = \hat{\dot{\vec{r}}} = \frac{1}{i\hbar}[\vec{r},\hat{H}] = ikc\hat{\vec{r}},\\ &\hbar\omega_{if} = E_i - E_f = \hbar kc\end{aligned}}$$

Long-wave approximation:

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots]$$

$$\Downarrow$$

The second order term gives:

- **toroidal operator**
- **compression operator**
- **vortical operator**

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu) \qquad \hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^{\lambda}Y_{\lambda\mu}$$

$$\hat{M}_{tor}(E\lambda\mu) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\quad \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} + \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$\hat{M}_{S}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) - k\hat{M}_{com}(E\lambda\mu)$$

$$\hat{M}_{com}(E\lambda\mu) = \frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\quad \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} - \sqrt{\frac{\lambda+1}{\lambda}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$\hat{M}_{com}(E\lambda\mu) = -k\hat{M}'_{com}(E\lambda\mu) \qquad \hat{M}'_{com}(E\lambda\mu) = \frac{1}{2(2\lambda+3)}\int d\vec{r}\,\hat{\rho}(\vec{r})\,r^{\lambda+2}Y_{\lambda\mu}$$

$$\hat{M}_{vor}(Ek\lambda\mu) = \hat{M}(Ek\lambda\mu) - \hat{M}_{S}(Ek\lambda\mu) = k\left[\hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)\right]$$

$$\hat{M}_{vor}(E\lambda\mu) = -\frac{i}{c(2\lambda+3)}\sqrt{\frac{\lambda+1}{2\lambda+1}}\int d\vec{r}\quad \hat{\vec{j}}_{nuc}(\vec{r})\; r^{\lambda+1}\vec{Y}_{\lambda\lambda+1\mu}$$

$$\boxed{\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)}$$

$$\boxed{\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)}$$

$$\hat{M}_{tor}(E\lambda\mu) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} + \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$= -\frac{1}{2c}\sqrt{\frac{\lambda}{\lambda+1}}\frac{1}{2\lambda+3}\int d\vec{r}\ \ r^{\lambda+2}\vec{Y}_{\lambda\lambda\mu}[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\hat{M}_{com}(E\lambda\mu) = \frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} - \sqrt{\frac{\lambda+1}{\lambda}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$= \frac{i}{2c}\frac{\lambda}{2\lambda+3}\int d\vec{r}\ \ r^{\lambda+2}Y_{\lambda\mu}[\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\hat{M}_{vor}(E\lambda\mu) = -\frac{i}{c(2\lambda+3)}\sqrt{\frac{\lambda+1}{2\lambda+1}}\int d\vec{r}\ \hat{\vec{j}}_{nuc}(\vec{r})\ r^{\lambda+1}\vec{Y}_{\lambda\lambda+1\mu}$$

HD vortical operator

The HD vortical operator (matrix element) may be also constructed:

$$\hat{M}(Ek\lambda\mu) = i\frac{(2\lambda+1)!!}{ck^{\lambda+1}(\lambda+1)}\sqrt{\lambda(\lambda+1)}\int d\vec{r} \quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})$

Wambach:

$$\hat{\rho}(\vec{r})\vec{\nabla}\times\hat{\vec{v}}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - i\frac{kc}{\lambda}\vec{\nabla}\hat{\rho}(\vec{r})\times\hat{\vec{r}}$$

HD

$$\rho_0(\vec{r})\vec{\nabla}\times\hat{\vec{v}}(\vec{r}) = \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - \frac{1}{\rho_0(\vec{r})}\vec{\nabla}\rho_0(\vec{r})\times\hat{\vec{j}}_{nuc}(\vec{r})$$

$$\vec{v}_\nu(\vec{r}) = \frac{\delta\vec{j}_\nu(\vec{r})}{\rho_0(\vec{r})} \longrightarrow \quad \rho_0(\vec{r})\vec{\nabla}\times\vec{v}(\vec{r}) = \vec{\nabla}\times\delta\vec{j}_\nu(\vec{r}) - \frac{1}{\rho_0(\vec{r})}\vec{\nabla}\rho_0(\vec{r})\times\delta\vec{j}_\nu(\vec{r})$$

The numerical results:

- fully self-consistent Skyrme separable RPA (SRPA) ,
- SLy6 force
- ^{208}Pb

SRPA : ⇨

Electric GR

Phys. Rev. C: 66, 044307 (2002); 74, 064306 (2006); 78, 044313 (2008); IJMP(E): 16, 624 (2007); 17, 89 (2008); 18, 975 (2009).

Magnetic spin-flip GR

PRC, 80, 031302(R) (2009); JPG, 37, 064034 (2010); IJMP (E), 19, n.4, 558 (2010).

Strength function

$$S(E1;\omega) = \sum_{\nu \neq 0} |< \Psi_\nu \,|\, \hat{M}_{E\lambda} \,|\, 0 >|^2 \, \varsigma(\omega - \omega_\nu)$$

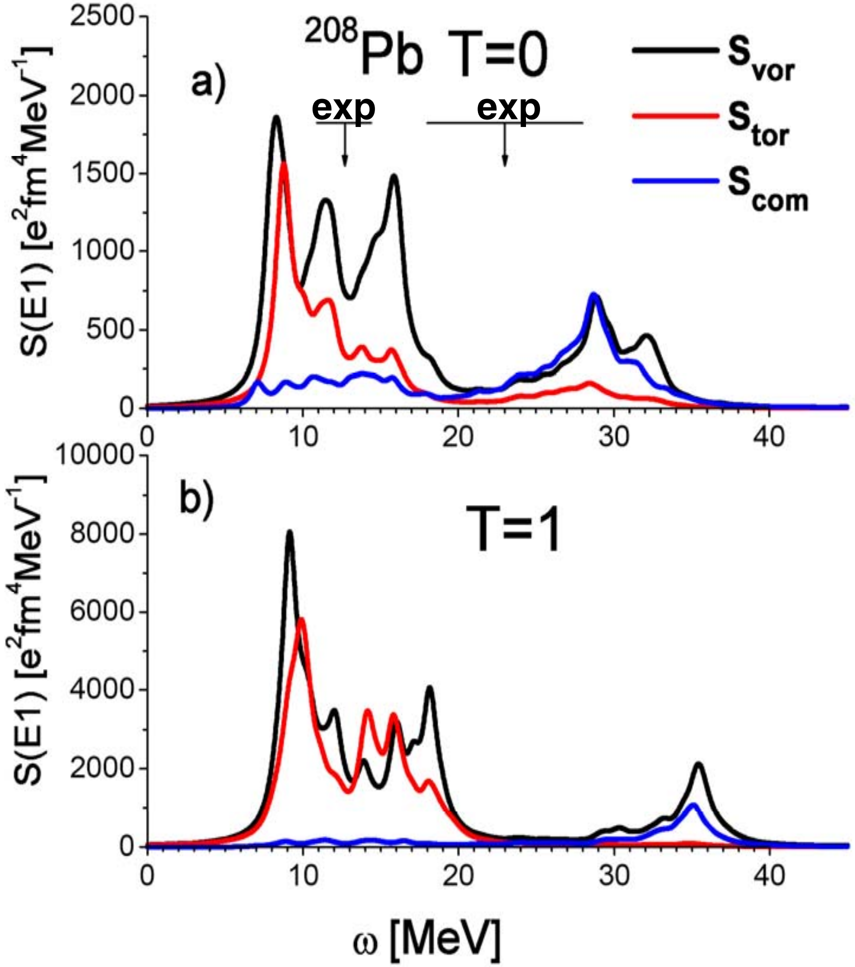
Toroidal, compressional, vortical operator

with the Lorentz weight

$$\varsigma(\omega - \omega_\nu) = \frac{1}{2\pi} \frac{\Delta}{[(\omega - \omega_\nu)^2 + \frac{\Delta^2}{4}]}$$

$\Delta = 1 \; MeV$

Comparison of VM, TM, and CM



- **Broad low-energy (LE) and high-energy (HE) bumps for VM, TM, and CM.**
- **LE strength is dominated by VM and TM**
- **HE strength is dominated by VM and CM**

- **General agreement for TM and CM with previous studies.**
- **Poor agreement with exper. of Ichida (like in previous studies).**

Uchida et al., 2003:

$E_1 = 12.7\,\text{MeV}, \quad \Gamma_1 = 3.5\,\text{MeV}$

$E_2 = 23.0\,\text{MeV}, \quad \Gamma_2 = 10.3\,\text{MeV}$

- Purely vortical VM does not coincide with partly vortical TM, especially at HE.

TM was previously considered as a typical example of the vortical flow.

- **Convection and magnetization (spin) parts of nuclear current,**
- **T=0 and T=1 channels**

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) + \hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{q}_{com}(\vec{r}) + \hat{\vec{j}}^{q}_{mag}(\vec{r}))$$

$$\hat{\vec{j}}^{q}_{con}(\vec{r}) = -ie^{q}_{eff}\sum_{k\ni q}\left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k)\right)$$

$$\hat{\vec{j}}^{q}_{mag}(\vec{r}) = \frac{g_s}{2}\sum_{k\ni q}\vec{\nabla}_k \times \hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k)$$

T=0: $e^{n}_{eff} = e^{p}_{eff} = 1, \quad g^{n,p}_{s}(T=0) = \frac{1}{2}(g^{n}_{s} + g^{p}_{s}) = 0.88\varsigma, \quad \varsigma = 0.64$

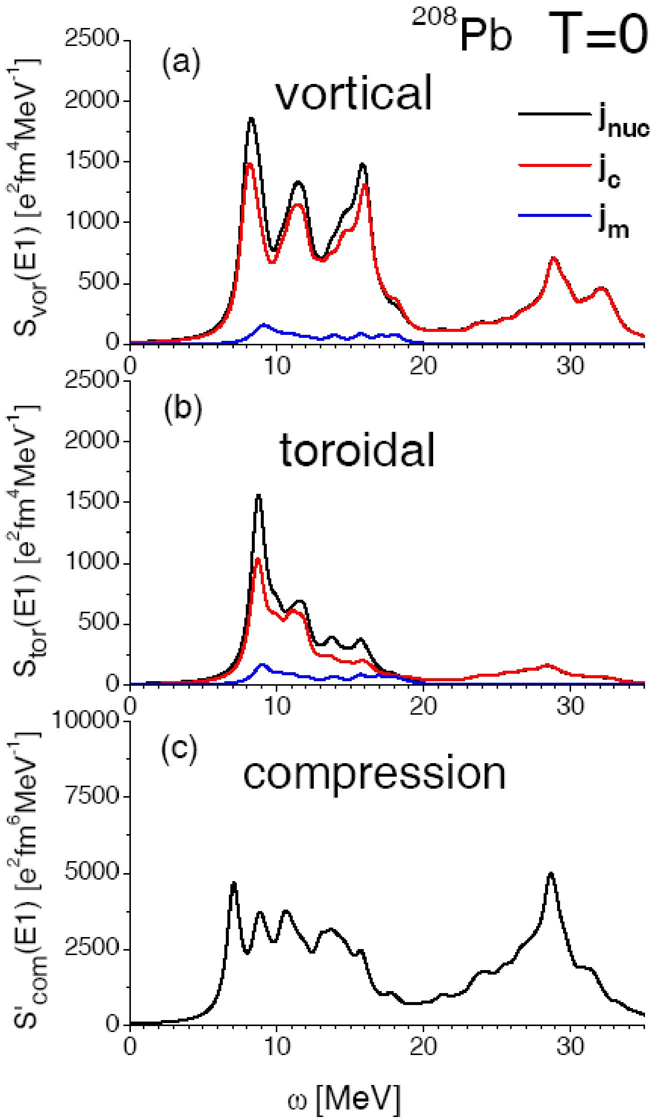
T=1: $e^{n}_{eff} = -e^{p}_{eff} = 1, \quad g^{n,p}_{s}(T=1) = \frac{1}{2}(g^{n}_{s} - g^{p}_{s}) = -4.70\varsigma, \quad \varsigma = 0.64$

$$g^{n,p}_{s}(T=0) \ll g^{n,p}_{s}(T=1)$$

Vortical, toroidal, and compressional T=0 strength

SLy6

$\Delta = 1\,\mathrm{MeV}$



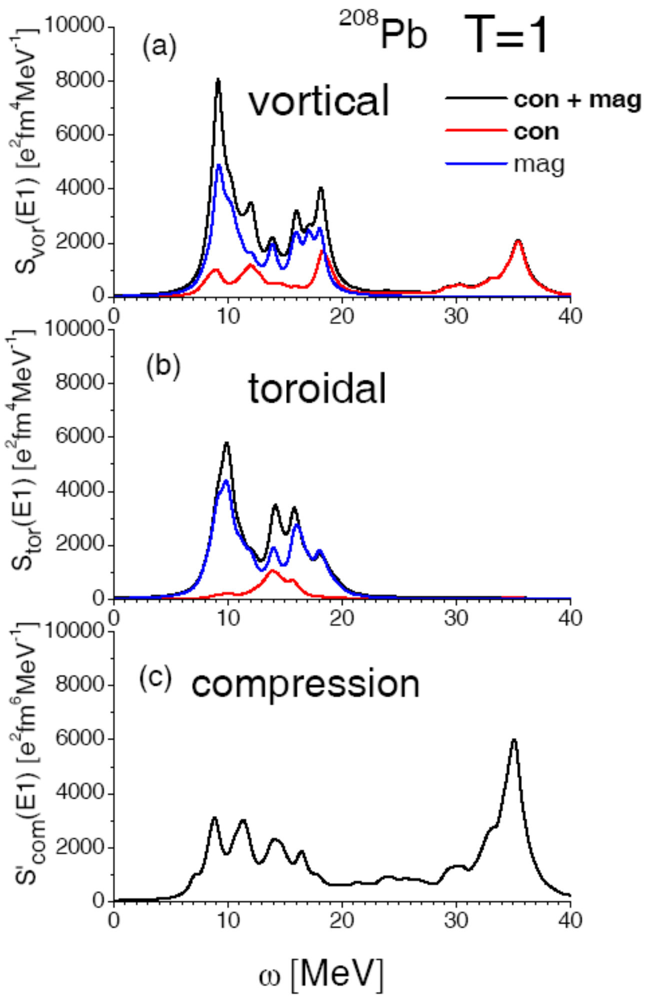
- dominant contribution of j_{con} to VM and TM

-no j_{mag} contribution to:
- CM
- HE strength

$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma$$
$$g_s^{T=0} = \frac{1}{2}(g_s^p + g_s^n) = 0.88\varsigma$$

Small T=0 g-factors!

Vortical, toroidal, and compressional **T=1** strength



SLy6

$\Delta = 1\,\mathrm{MeV}$

VM and TM:

- dominant contribution of j_{mag} !!

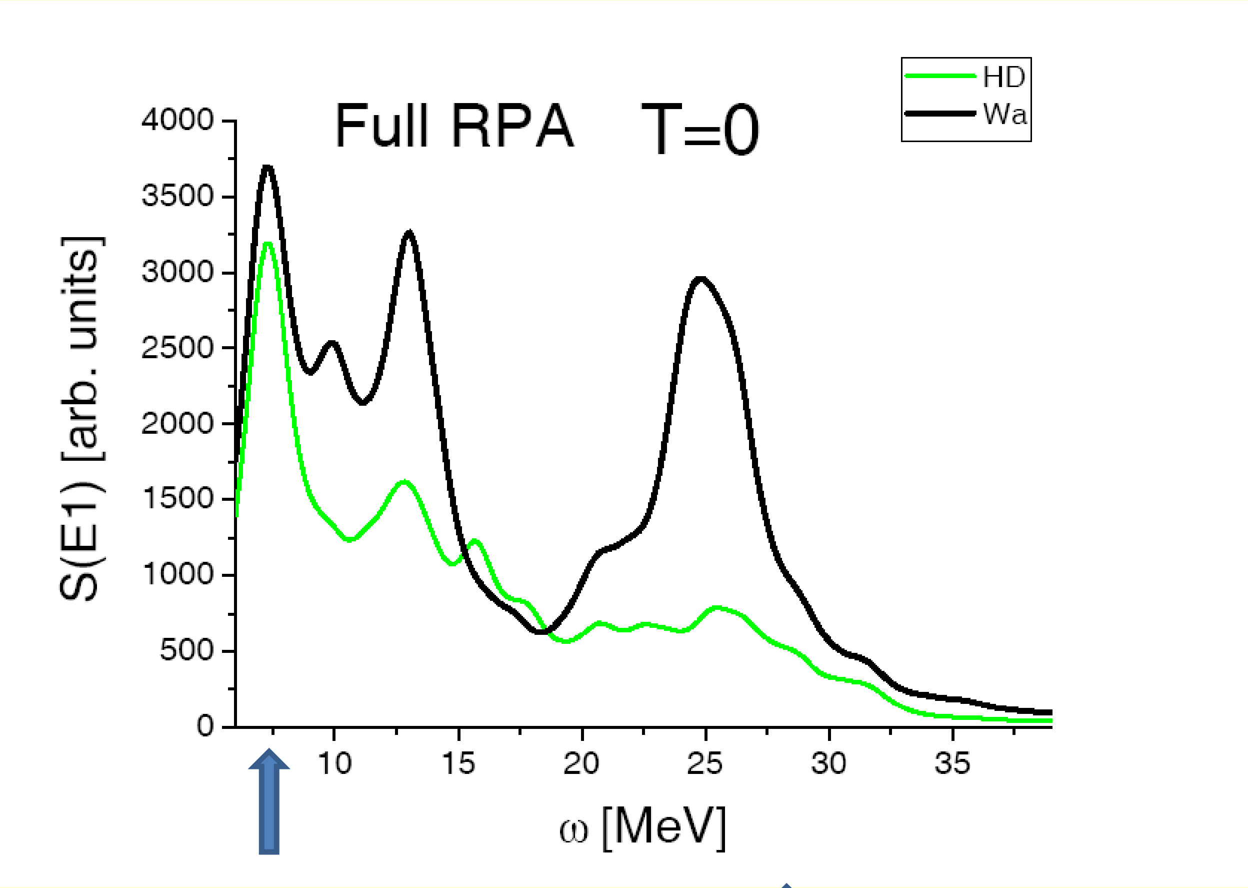
$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma,$$

$$g_s^{T=1} = \frac{1}{2}(g_s^p - g_s^n) = 4.7\varsigma$$

Large T=1 g-factors!

Vortical and toroidal modes in the T=1 channel are suitable to see the effect of j_{mag} in electric modes.

Wambach vs HD vorticity



W ~ HD ~T
vortical region

W ~ CM
HD is small
irrotational region

HD is modest

Conclusions

-The (Wambach) vortical operator has been derived.
This allows to treat and compare VM,TM, and CM
on the same theor. ground.

-The difference between Wambach and HD voirticities was discussed.

- Both T=0 and T=1 VM,TM, and CM were considered

- The dominant role of:
  - convection nuclear current in isoscalar VM and TM
  - magnetization nuclear current in isovector VM and TM
    was demonstrated.
  - E1(T=1) VM and TM represent a remarkable example
    of strong j_{mag} effect in electric GR.

Thank you for your attention!

Wambach does not use the vortical operator
though it would be useful for comparison of VM, TM, and CM.

We will derive the vortical operator and relate it with TM and CM ones:

$$\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

where for $\lambda = 1$ we have

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2c\sqrt{3}}\int d\vec{r} \quad \hat{\vec{j}}_{nuc}(\vec{r})\,[\frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} + (r^2 - < r^2 >_0)\vec{Y}_{10\mu}]$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{2c\sqrt{3}}\int d\vec{r} \quad \hat{\vec{j}}_{nuc}(\vec{r})\,[\frac{2\sqrt{2}}{5}r^2\vec{Y}_{12\mu} - (r^2 - < r^2 >_0)\vec{Y}_{10\mu}]$$

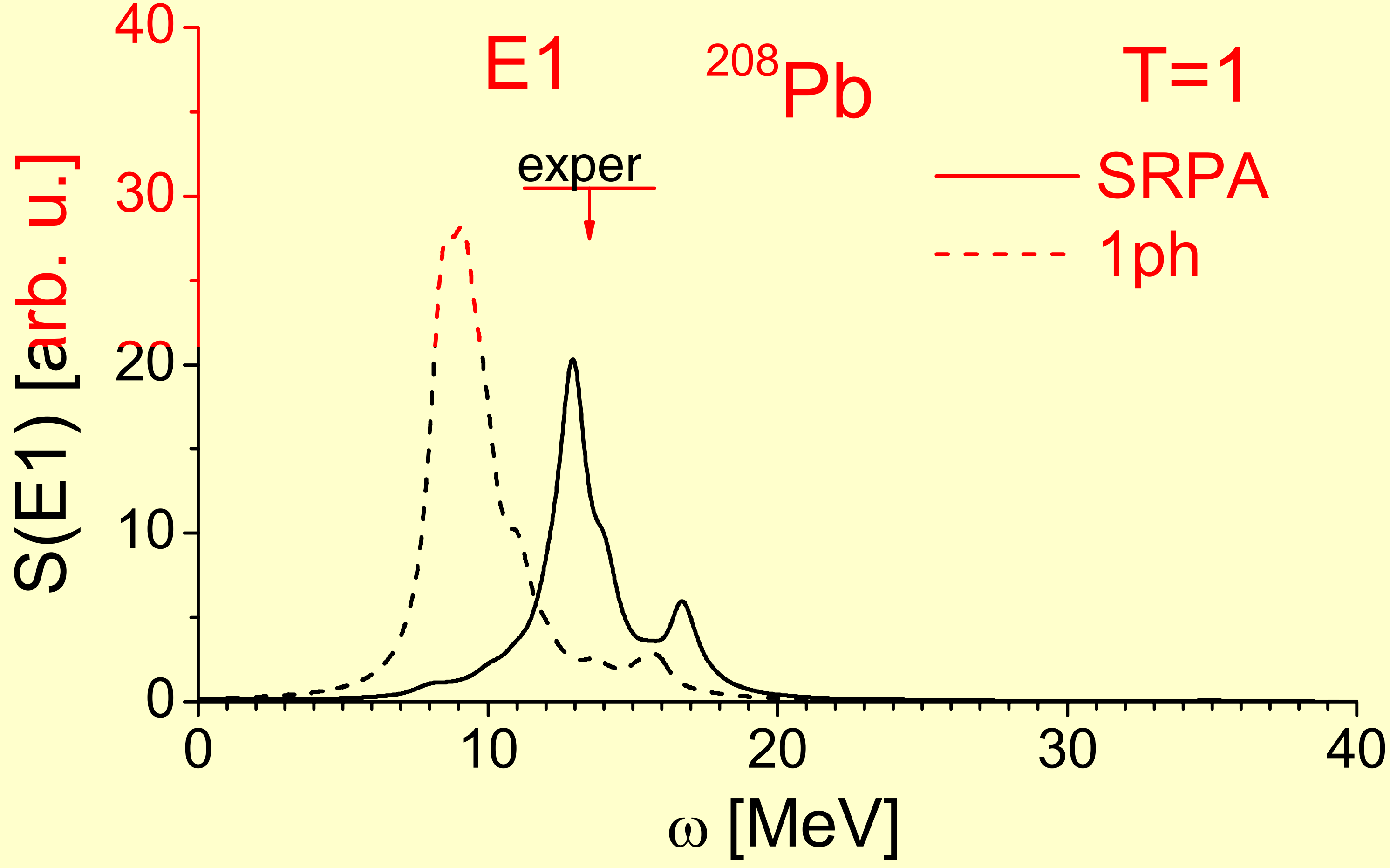
$$\hat{M}_{vor}(E1\mu) = -\frac{i}{5c}\sqrt{\frac{3}{2}}\int d\vec{r} \quad \hat{\vec{j}}_{nuc}(\vec{r})\, r^2\vec{Y}_{12\mu}$$

- involves $\vec{Y}_{12\mu}$
- no c.m.c.

E1(T=1) GR

SLy6

$\Delta = 1\,\mathrm{MeV}$



Center of mass corrections

$$\hat{O} = \sum_{k=1}^{A} o(\vec{r}_k) \quad \rightarrow \quad \hat{O} = \frac{1}{A}\sum_{k=1}^{A} z_k$$

$$\boxed{\begin{aligned}\delta\langle\hat{O}\rangle &= \int d\vec{r}\,\delta\rho(\vec{r})\,o(\vec{r}) \\ &= \int d\vec{r}\,\delta\vec{j}(\vec{r})\cdot\vec{\nabla}o(\vec{r}) = 0\end{aligned}}$$

$\longleftarrow$ **translation invariance:**
perturbation $\delta\rho$ does not change z-coordinate of the c.m.

$$\vec{\nabla}o(\vec{r}) = \vec{\nabla}(rY_{10}) = \sqrt{3}\,\vec{Y}_{100}$$

$$\sum_{\nu}\langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = \frac{1}{2mi}\rho_0(\vec{r})\vec{\nabla}f(\vec{r})$$

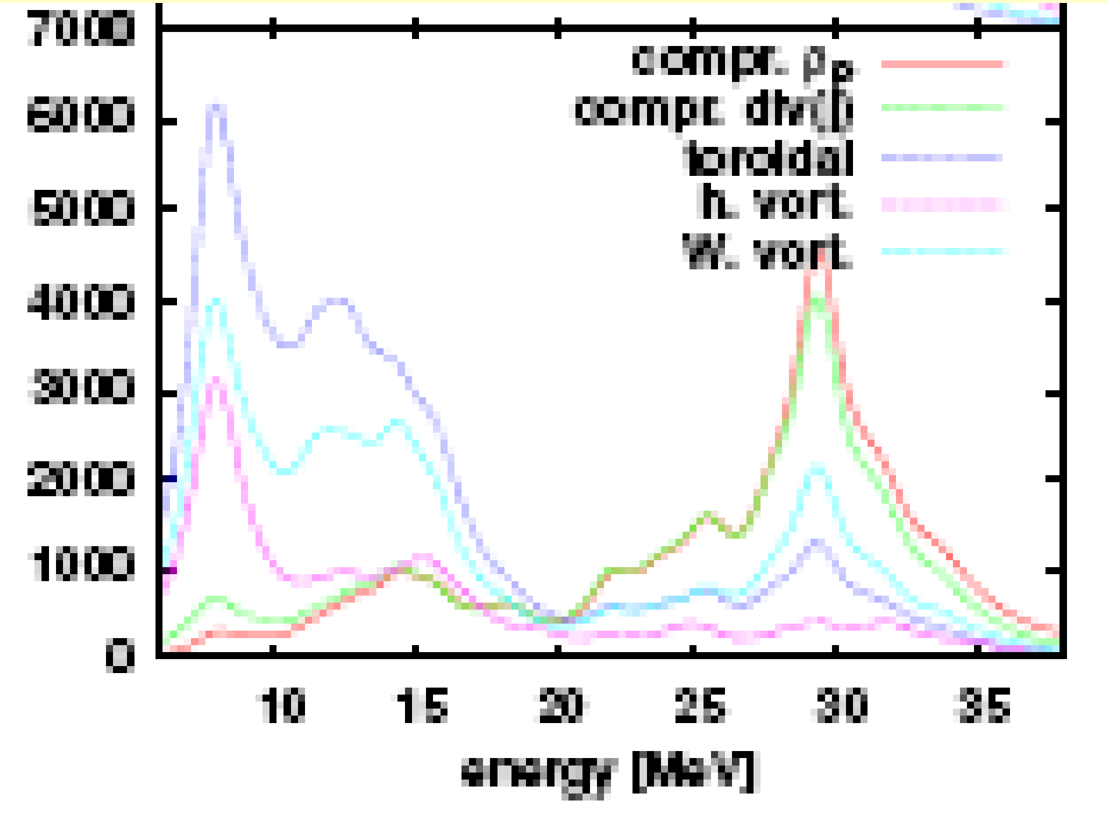
$$\sum_{\nu}\omega_{\nu}\langle 0|\hat{\rho}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = -\frac{1}{2m}\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})]$$

$$\delta\vec{j}_{\nu}(\vec{r}) = \langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle \propto \rho_0(\vec{r})\vec{\nabla}f(\vec{r}) \propto \rho_0(\vec{r})\vec{v}(\vec{r})$$

$$\delta\rho_{\nu}(\vec{r}) = \langle 0|\hat{\rho}(\vec{r})|\nu\rangle \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})] \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{v}(\vec{r})]$$

$$\boxed{\begin{aligned}\vec{v}_{vor} &= r^2\vec{Y}_{12\mu} + \eta\vec{Y}_{10\mu} \\ \vec{v}_{tor} &= \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} + (r^2-\eta)\vec{Y}_{10\mu} \\ \vec{v}_{com} &= \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} - (r^2-\eta)\vec{Y}_{10\mu}\end{aligned}} \quad \Rightarrow \quad \boxed{\begin{aligned}\eta_{vor} &= 0 \\ \eta_{tor} &= \eta_{com} = \langle r^2\rangle_0 \\ \eta'_{com} &= \frac{5}{3}\langle r^2\rangle_0\end{aligned}}$$

Wambach vs HD vorticity -1



Toroidal, compressional, and vortical operators for $\lambda = 1$:

$$\hat{M}_{tor}(E1\mu) = -\frac{i}{2c}\sqrt{\frac{1}{3}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\ [r^2(\frac{\sqrt{2}}{5}\vec{Y}_{12\mu} + \vec{Y}_{10\mu}) - \langle r^2 \rangle_0\ \vec{Y}_{10\mu}]$$

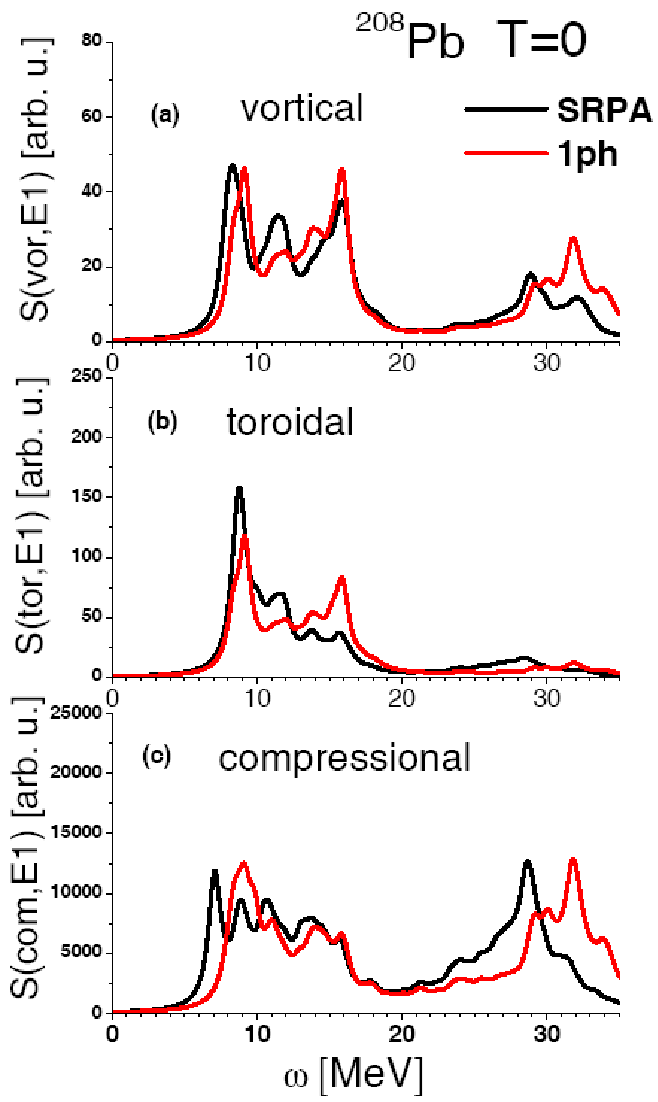
$$\hat{M}_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\ [r^3 - \frac{5}{3} \langle r^2 \rangle_0\ r]\,Y_{1\mu}$$

to be shown later:

$$\hat{M}_{vor}(E1\mu) = -\frac{i}{5c}\sqrt{\frac{3}{2}}\int d\vec{r}\ \ \hat{\vec{j}}_{nuc}(\vec{r})\ r^2\vec{Y}_{12\mu}$$

- no c.m.c. for the vortical operator

SRPA vs 1ph strength



- Collective down-shifts:
  - ~ 1-3 MeV for LE bump
  - ~ 4 MeV for HE bump

- Quite collective RPA states:

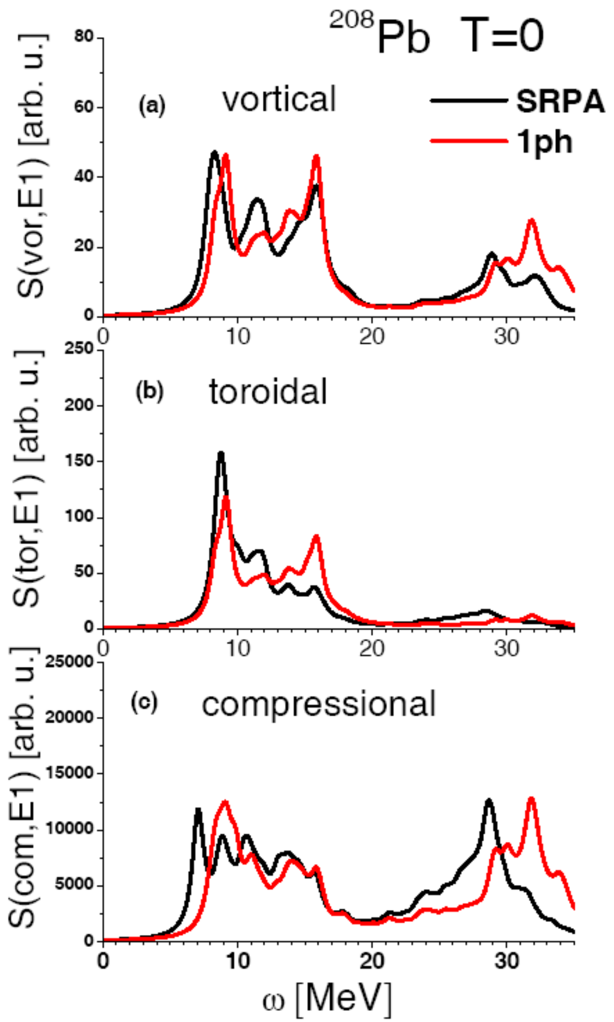
  state at 8.3 MeV:

- In LE bump the structure of VM, TM, and CM responses is mainly of 1ph origin

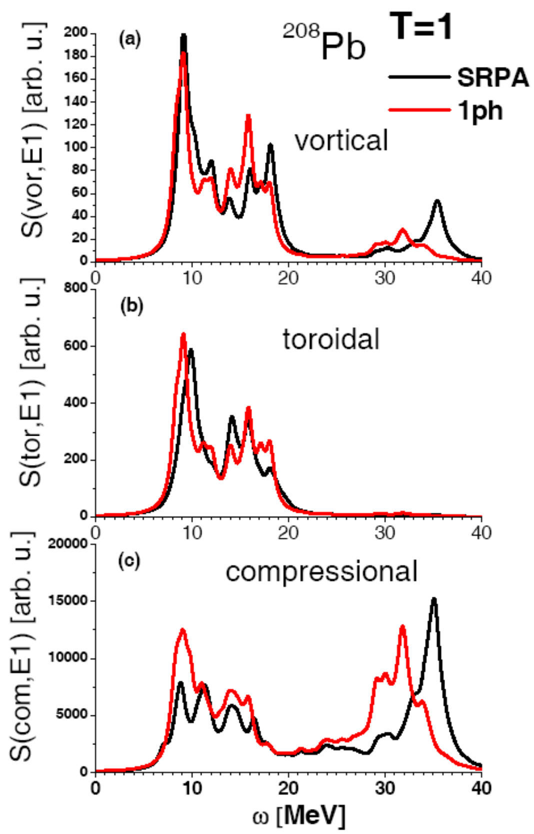
The 1ph origin of the vorticity ?

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

SRPA vs 1ph strength



- **Collective up-shifts:**
  - **~ 1-2 MeV for LE bump**
  - **~ 4 MeV for HE bump**

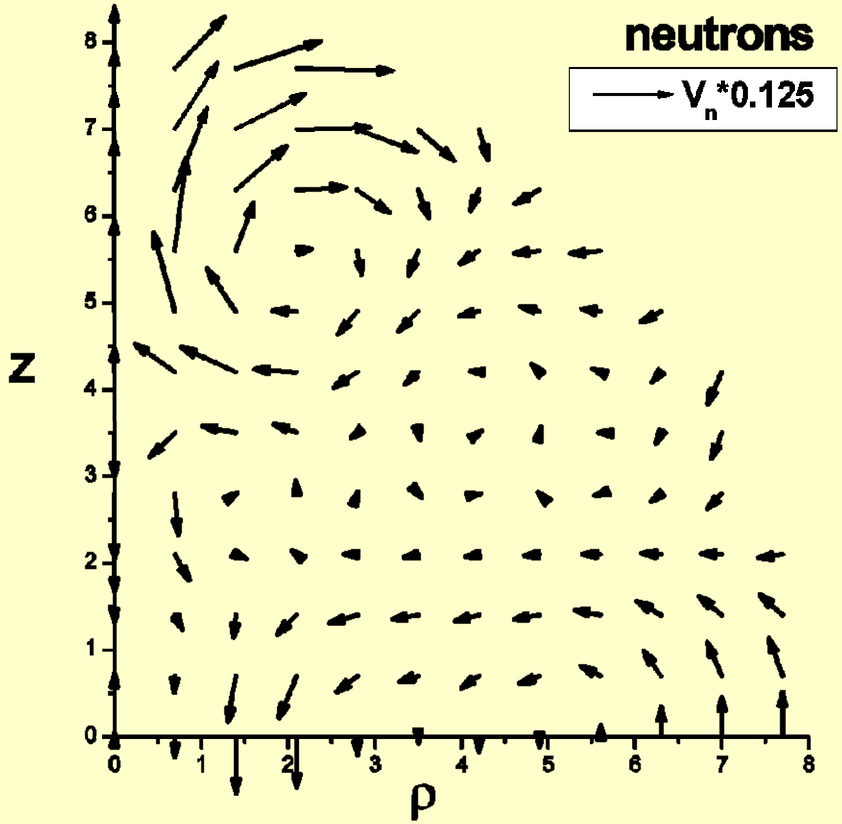


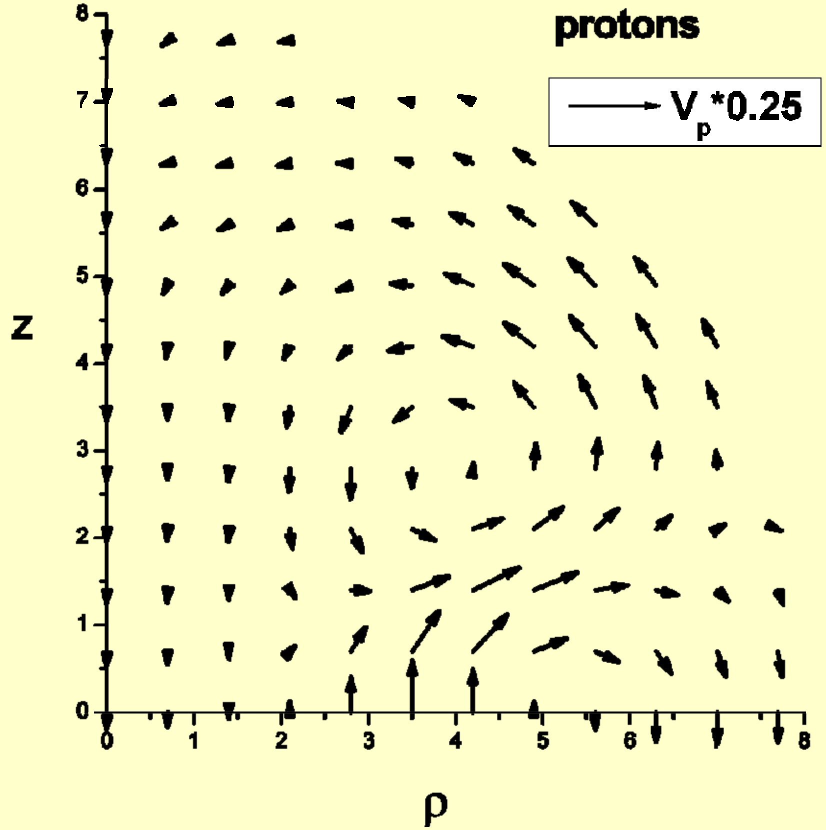
- **In LE bump the structure of VM, TM, and CM responses is mainly of 1ph origin**

The 1ph origin of the vorticity ?

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

Velocity fields for T=0 vortical state at 8.3 MeV



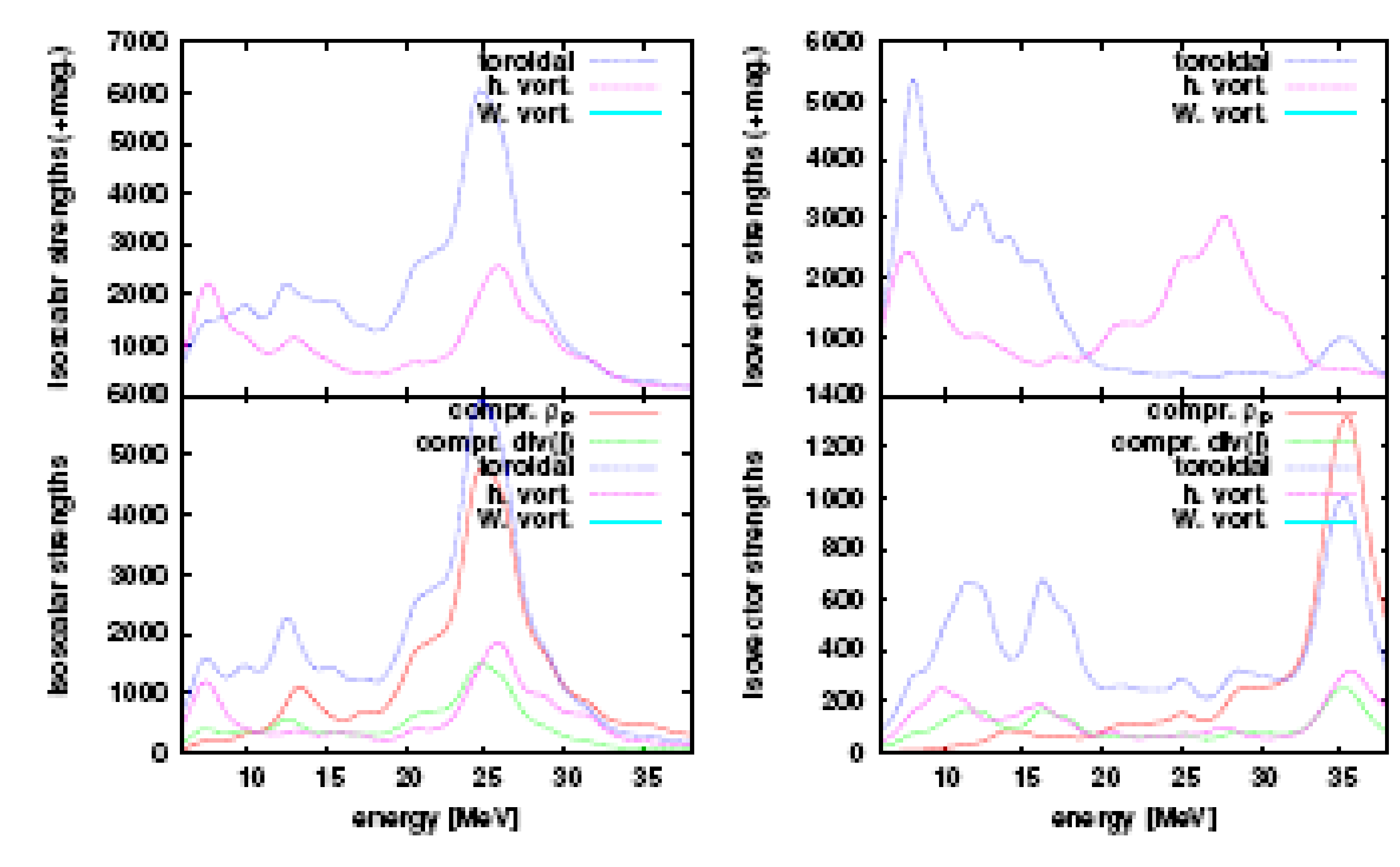


s-p manifestation of vorticity

nn[2g7/2 – 2f5/2] 36%

pp[1i13/2-1h11/2] 18%

Wambach vs HD vorticity -2



Motivation

Nuclei demonstrate both

- irrotational flow (most of electric GR) $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$

- vortical flow (toroidal GR) $\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$

Vorticity $\vec{w}(\vec{r})$ **is a fundamental quantity:**

- does not contribute to the continuity equation,

- represents an independent part of charge-current distribution beyond the continuity equation.

Vorticity is related to the exotic modes:

- toroidal E1 mode (TM) ,

- compression E1 mode (CM),

which are now of a keen interest .

different conclusions on CM vorticity

Open points:

- definition of nuclear vorticity (HD vs Wambach),

- the vortical mode (VM) and its operator field,

- relation between VM and TM/CM,

- IS (T=0) and IV(T=1) branches of the modes,

- role of magnetization (spin) nuclear current.

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, P. Vesely, subm. to PRC, arXiv: 1105.0837[nucl-th]

SRPA (1)

E. Lipparini and S. Stringari, NPA, 371, 430 (1981).

Time-dependent formulation:

$$E(J_\alpha(\vec{r},t)) = \langle \Psi | H | \Psi \rangle,$$

$$J_\alpha(\vec{r},t) \in \{\rho(\vec{r},t), \vec{j}(\vec{r},t), \ldots\} \qquad J_\alpha(\vec{r},t) = <\Psi | \hat{J}_\alpha | \Psi> \longleftarrow$$

T-even and T-odd densities

$$J_\alpha(\vec{r},t) = \bar{J}_\alpha(\vec{r}) + \delta J_\alpha(\vec{r},t) \longleftarrow$$

Linear regime: small time-dependent perturbation

$$\longleftarrow$$

Mean field hamiltonian: static g.s. + time-dependent response

$$h(\vec{r},t) = h_0(\vec{r}) + h_{res}(\vec{r},t)$$

$$= \sum_\alpha \left[\frac{\delta E}{\delta J_\alpha}\right]_{J=\bar{J}} \hat{J}_\alpha(\vec{r}) + \sum_{\alpha\alpha'} \left[\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}\right]_{J=\bar{J}} \delta J_\alpha(\vec{r},t) \hat{J}_\alpha(\vec{r})$$

$$\delta J_\alpha(t) = \langle \Psi(t) | J_\alpha | \Psi(t) \rangle - \langle 0 | J_\alpha | 0 \rangle \longleftarrow$$

The only unknowns

Now we have to specify the perturbed many-body wave function $\Psi(t)$

SRPA (2)

$$V_{res} \Rightarrow \sum_{k,k'=1}^{K} \{\kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_k\}$$

Macroscopic step:

Perturbed w.f. via scaling: $\Psi(t) = \prod_{k=1}^{K} \exp\{-q_k(t)\hat{P}_k\}\exp\{-p_k(t)\hat{Q}_k\}|0\rangle$,

both $\Psi(t), |0\rangle$ **are Slater determinants**

$$\hat{Q}_k = \hat{Q}_k^+, \quad \hat{T}\hat{Q}_k\hat{T}^{-1} = \hat{Q}_k$$
$$\hat{P}_k = i[\hat{H}, \hat{Q}_k]_{ph} = \hat{P}_k^+, \quad \hat{T}\hat{P}_k\hat{T}^{-1} = -\hat{P}_k$$

$$q_k(t) = \overline{q}_k \cos(\omega t)$$
$$p_k(t) = \overline{p}_k \sin(\omega t)$$

$$\hat{h}_{res}(t) = \sum_k \{-q_k(t)\hat{X}_k + p_k(t)\hat{Y}_k\} \overset{kk'}{=} 1/2 \sum_{kk'} \{\kappa_{kk'} \delta\hat{X}_k(t)\hat{X}_k + \eta_{kk'} \delta\hat{Y}_k(t)\hat{Y}_k\}$$

Microscopic step:

Perturbed w.f. via Thouless theorem: $\Psi_{Th}(t) = \{1 + \sum_{ph} c_{ph}(t)\hat{A}_{ph}^+)\}|0\rangle$

$$c_{ph}(t) = c_{ph}^+ e^{i\omega t} + c_{ph}^- e^{-i\omega t}$$

Merging step:

Both scaling and Thouless w.f. $\Psi(t)$ **must give equal variations:**

$$\delta\hat{X}_k(t)|_{sc} = \delta\hat{X}_k(t)|_{Th}, \quad \delta\hat{Y}_k(t)|_{sc} = \delta\hat{Y}_k(t)|_{Th}$$

SRPA (3)

Time-dependent HF equation

$$i\hbar \frac{d}{dt}\Psi(t) = (h_0 + h_{res}(t))\Psi(t)$$

Perturbed w. f. by Thouless theorem

$$\Psi(t) = (1 + \sum_{ph} c_{ph}(t)\hat{A}^{+}_{ph})\Psi_0$$

Response Hamiltonian

$$\hat{h}_{res}(t) = \sum_k \{-q_k(t)\hat{X}_k + p_k(t)\hat{Y}_k\}$$

Harmonic oscillations

$$\begin{cases} c_{ph}(t) = c^{+}_{ph}e^{i\omega t} + c^{-}_{ph}e^{-i\omega t}, \\ q_k(t) = \overline{q}_k \cos(\omega t) = \frac{1}{2}\overline{q}_k(e^{i\omega t} + e^{-i\omega t}), \\ p_k(t) = \overline{p}_k \cos(\omega t) = \frac{1}{2i}\overline{p}_k(e^{i\omega t} - e^{-i\omega t}) \end{cases}$$

TDHF gives the coupling

$$c^{\pm}_{ph} \leftrightarrow \overline{q}_k, \overline{p}_k$$

$$\boxed{c^{\pm}_{ph} = -\frac{1}{2}\frac{\sum_k \{\overline{q}_k \langle ph|\hat{X}_k|0\rangle \mp i\overline{p}_k \langle ph|\hat{Y}_k|0\rangle\}}{\varepsilon_{ph} \pm \omega}}$$

which allows to reduce a large set of 1ph apmlitudes $c^{\pm}_{ph}$ to a few unknowns $\overline{q}_k, \overline{p}_k$

So high-rank RPA problem can be reduced to low-rank one!

However, $\overline{q}_k, \overline{p}_k$ still remain to be unknow

SRPA (4)

Physical requirement:

Variations of operators of the residual interaction must be the same for both macroscopic (scaling) and microscopic (Thouless) many-body wave functions.

$$\boxed{\delta\hat{X}_k(t)|_{sc} = \delta\hat{X}_k(t)|_{Th}, \quad \delta\hat{Y}_k(t)|_{sc} = \delta\hat{Y}_k(t)|_{Th}}$$

$$\delta\hat{X}_k(t)|_{sc} = \sum_{k'} q_{k'}(t) \ \kappa^{-1}_{kk'}$$

$$\delta\hat{X}_k(t)|_{Th} = \sum_{ph} (c_{ph}(t)^* < ph|\hat{X}_k|0> + c_{ph}(t) < 0|\hat{X}_k|ph>^*)$$

$$\boxed{\begin{aligned} &\sum_k \{\overline{q}_k (d_{kk'}(XX) - \kappa^{-1}_{kk'}) + \overline{p}_k d_{kk'}(XY)\} = 0 \\ &\sum_k \{\overline{q}_k d_{kk'}(YX) + \overline{p}_k (d_{kk'}(YY) - \eta^{-1}_{kk'})\} = 0 \end{aligned}}$$

final RPA equations for

$$\overline{q}_k, \overline{p}_k$$

$$d_{kk'}(XY) = \sum_{ph} [\frac{\langle ph|\hat{X}_k|0\rangle^* \langle ph|\hat{Y}_{k'}|0\rangle}{(\varepsilon_{ph} - \omega)} + \frac{\langle ph|\hat{X}_k|0\rangle \langle ph|\hat{Y}_{k'}|0\rangle^*}{(\varepsilon_{ph} + \omega)}]$$

SRPA (3)

Final RPA equations:

$$\sum_k \{\bar{q}_k (d_{kk'}(XX) - \kappa^{-1}_{kk'}) + \bar{p}_k d_{kk'}(XY)\} = 0$$
$$\sum_k \{\bar{q}_k d_{kk'}(YX) + \bar{p}_k (d_{kk'}(YY) - \eta^{-1}_{kk'})\} = 0$$

$$H = h_0 + 1/2 \sum_{kk'} \{\kappa_{kk'} \overbrace{\hat{X}_k \hat{X}_{k'}}^{\text{T-even}} + \eta_{kk'} \overbrace{\hat{Y}_k \hat{Y}_{k}}^{\text{T-odd}}\}$$

$\det[\omega_j] = 0 \Rightarrow$ **RPA spectrum**

where e.g. $d_{kk'}(XY) = \sum_{ph} [\frac{\langle ph|\hat{X}_k|0\rangle^* \langle ph|\hat{Y}_{k'}|0\rangle}{(\varepsilon_{ph} - \omega)} + \frac{\langle ph|\hat{X}_k|0\rangle \langle ph|\hat{Y}_{k'}|0\rangle^*}{(\varepsilon_{ph} + \omega)}]$

$$\hat{X}_k(\vec{r}) = i \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}] \langle 0|[J_\alpha, \hat{P}_k]|0\rangle \hat{J}_{\alpha'}$$
$$\hat{Y}_k(\vec{r}) = i \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_\alpha \delta J_{\alpha'}}] \langle 0|[J_\alpha, \hat{Q}_k]|0\rangle \hat{J}_{\alpha'}$$

$$\kappa^{-1}_{kk'} = i \langle 0|[\hat{X}_k, \hat{P}_{k'}]|0\rangle$$
$$\eta^{-1}_{kk'} = i \langle 0|[\hat{Y}_k, \hat{Q}_{k'}]|0\rangle$$

$$C^+ = \sum_{ph} [c^-_{ph} a^+_p a_h - c^+_{ph} a^+_h a_p]$$

RPA phonon

$$c^\pm_{ph} = -\frac{1}{2} \frac{\sum_k \{\bar{q}_k \langle ph|\hat{X}_k|0\rangle \mp i \bar{p}_k \langle ph|\hat{Y}_k|0\rangle\}}{\varepsilon_{ph} \pm \omega}$$

- Rank of RPA matrix is 4K. For giant resonances usually K=2 is enough.

Very low rank!

SRPA (5): detailed expressions with isospin indices s={n,p}

$$h_{res}(\vec{r},t) = \sum_{ss'} \sum_{\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}]_{J=\bar{J}} \delta J_{s\alpha}(\vec{r},t) \hat{J}_{s'\alpha'}(\vec{r})$$

$$= \sum_{ss'} \sum_{k} \{q_{qs}(t) \hat{X}^{s'}_{sk} + p_{qs}(t) \hat{Y}^{s'}_{sk}\} = \sum_{ss'} \sum_{kk'} \{\kappa^{s'k'}_{sk} \delta \hat{X}_{sk}(t) \hat{X}_{s'k'} + \eta^{s'k'}_{sk} \delta \hat{Y}_{sk}(t) \hat{Y}_{s'k'}\}$$

$$\delta J_{s\alpha}(t) = i \sum_{k} \{q_{s\alpha}(t) \langle [\hat{P}_{sk}, J_{s\alpha}] \rangle + p_{s\alpha}(t) \langle [\hat{Q}_{sk}, J_{s\alpha}] \rangle\}$$

$$\hat{X}_{sk} = \sum_{s'} \hat{X}^{s'}_{sk} = i \sum_{s'\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}] \langle [\hat{P}_{sk}, \hat{J}_{s\alpha}] \rangle \hat{J}_{s'\alpha'}$$

$$\hat{Y}_{sk} = \sum_{s'} \hat{Y}^{s'}_{sk} = i \sum_{s'\alpha\alpha'} [\frac{\delta^2 E}{\delta J_{s\alpha} \delta J_{s'\alpha'}}] \langle [\hat{Q}_{sk}, \hat{J}_{s\alpha}] \rangle \hat{J}_{s'\alpha'}$$

$$[\kappa^{-1}]^{s'k'}_{sk} = [\kappa^{-1}]^{sk}_{s'k'} = -i \langle [\hat{P}_{s'k'}, \hat{X}^{s'}_{sk}] \rangle$$

$$[\eta^{-1}]^{s'k'}_{sk} = [\eta^{-1}]^{sk}_{s'k'} = -i \langle [\hat{Q}_{s'k'}, \hat{Y}^{s'}_{sk}] \rangle$$

If $\hat{A}^{\dagger} = \hat{A}, \quad \hat{T}^{-1} \hat{A}_{\pm} \hat{T} = \pm \hat{A}_{\pm}$ **then**

$$\langle [\hat{A}_{+}, \hat{B}_{+}] \rangle = \langle [\hat{A}_{-}, \hat{B}_{-}] \rangle = 0$$

$$\langle [\hat{A}_{+}, \hat{B}_{-}] \rangle \neq 0$$

Calculation of average commutators via s-p matrix elements $\longrightarrow$ $\langle [A, B] \rangle = \sum_{ph} \{ \langle | \hat{A} | ph \rangle \langle ph | \hat{B} | \rangle - \langle | \hat{B} | ph \rangle \langle ph | \hat{A} | \rangle \}$

SRPA (6): strength function

Lorentz weight

$$\xi(\omega-\omega_\nu)=\frac{1}{2\pi}\frac{\Delta}{(\omega-\omega_\nu)^2+(\Delta/2)^2}$$

L=0,1,3

$$S_L(D_{X\lambda\mu},\omega)=\sum_\nu \omega_\nu^L <\nu|\hat{D}_{X\lambda\mu}|0>^2\ \xi(\omega-\omega_\nu)=$$

$$\beta = sk\tau \qquad \tau=\begin{cases}\mathbf{X}\\ \mathbf{Y}\end{cases} \qquad A_\beta^\nu=\sum_{ph}\frac{\varepsilon_{ph}\langle ph|\hat{X}_\beta|0\rangle\langle ph|\hat{D}|0\rangle}{\varepsilon_{ph}^2-z^2}$$

$$=\sum_\nu \mathrm{Res}\Big[z_\nu^L\frac{\sum_{\beta\beta'}F^\nu_{\beta\beta'}(z)A^\nu_\beta(z)A^\nu_{\beta'}(z)}{\det F^\nu(z)}\zeta(\omega-z)\Big]_{z=\pm\omega_\nu} \longleftarrow$$ **heavily calculated**

$$\approx \mathrm{Res}[\ldots]_{z=\omega\pm i\Delta/2}+\sum_{ph}\mathrm{Res}[\ldots]_{z=\pm\varepsilon_{ph}} \longleftarrow$$ **easily calculated**

$$\boxed{=\underbrace{\frac{1}{\pi}\Im\Big[\frac{z^L\sum_{\beta\beta'}F_{\beta\beta'}(z)A_\beta(z)A_{\beta'}(z)}{F(z)}\Big]_{z=\omega+i\Delta/2}}_{\textbf{Contribution of residual inter.}}+\sum_{ph}\underbrace{\varepsilon_{ph}^L<ph|\hat{D}_{X\lambda\mu}|0>^2\ \xi(\omega-\omega_\nu)}_{\textbf{Unperturbed 2qp strength}}}$$

Justification: sum of all residues at the complex plane is ZERO. Then

$$\sum_\nu \mathrm{Res}f(z=\omega_\nu)=-\cancel{\mathrm{Res}f(z=\pm\infty)}-\cancel{\sum_\nu \mathrm{Res}f(z=-\omega_\nu)}-\sum_{ph}\mathrm{Res}f(z=\pm\varepsilon_{ph})-\mathrm{Res}f(z=\omega\pm i\Delta/2)$$

$$\approx-\mathrm{Res}f(z=\omega\pm i\Delta/2)-\sum_{ph}\mathrm{Res}f(z=\pm\varepsilon_{ph})$$