June, 2011 @ Primošten

Microscopic formulation of IBM for rotational nuclei

Kosuke NOMURA (U. Tokyo)

東京大学
THE UNIVERSITY OF TOKYO

Introduction

Quadrupole Collectivity:

vibrational, rotational & transitional shapes governed by multi-fermion dynamics

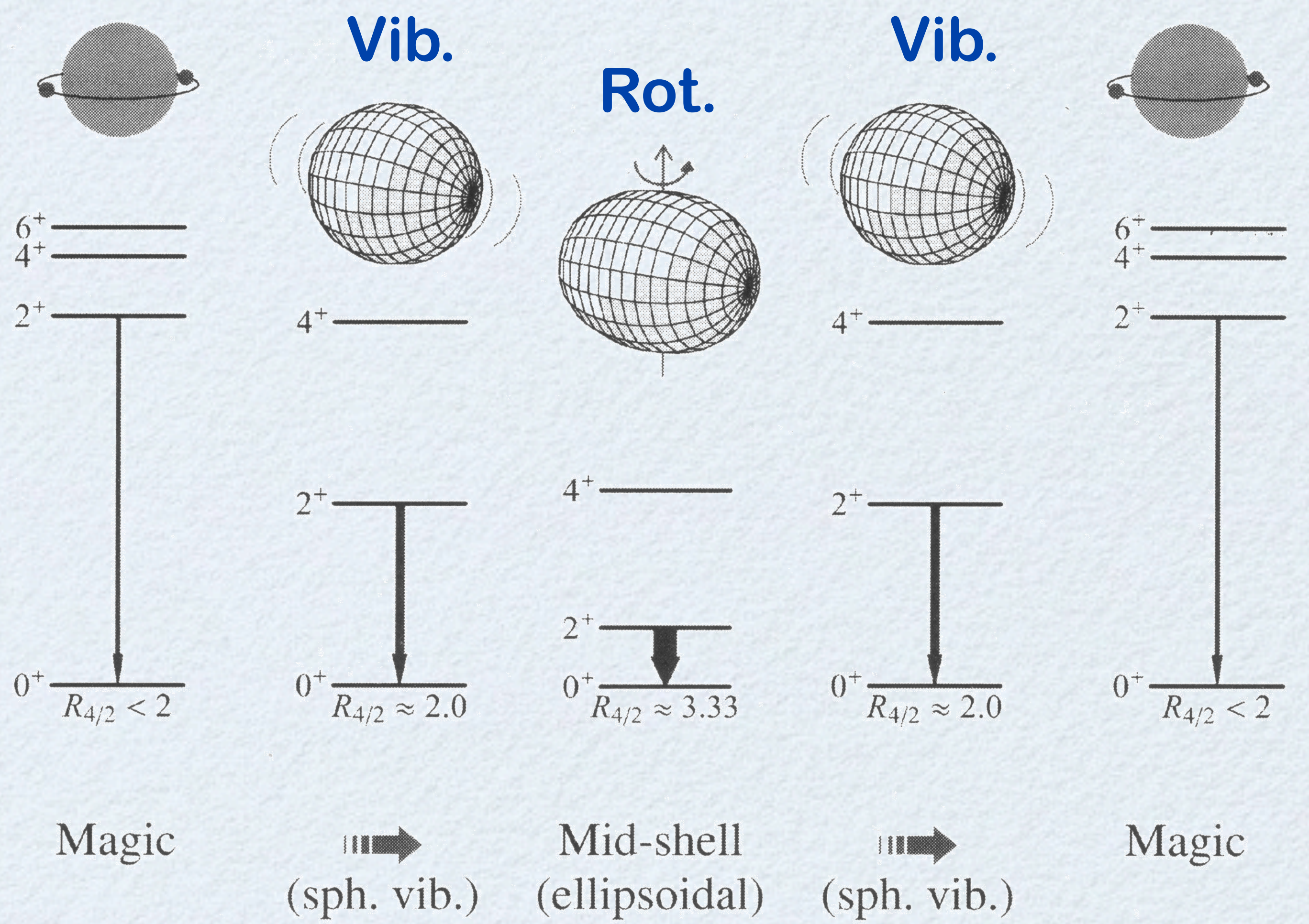
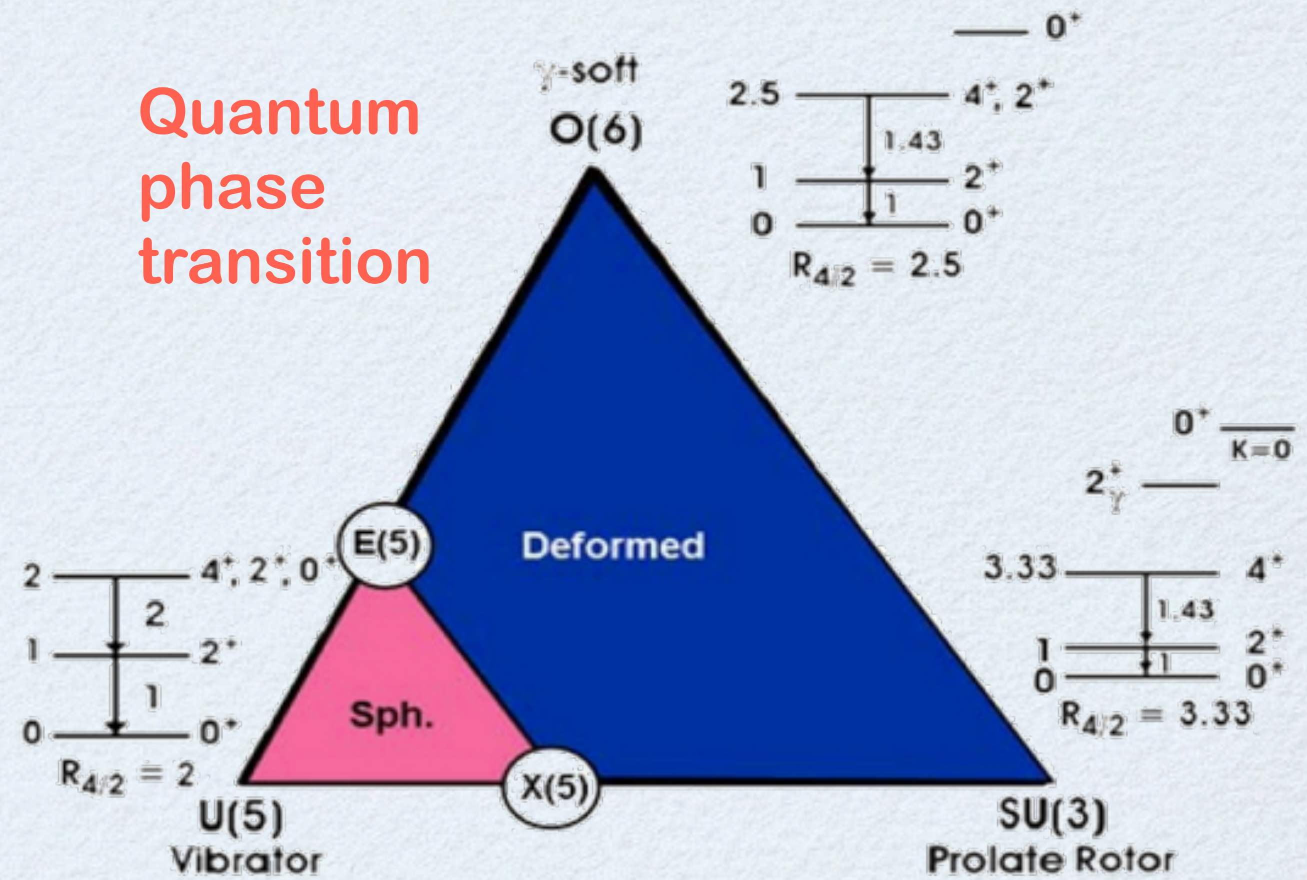


Fig. from Casten, “Nuclear Structure from a simple perspective”

- **Derived from nucleons ?**
- **Prediction ?**

Quantum phase transition



This talk will focus mainly on rotational limit.

Mean-field theory with energy density functional (EDF)

- ☑ **Skyrme, Gogny, RMF, etc. for nuclear properties. Universal.**
- ☑ **Spectroscopy with good J & N, fluctuation, though complicated.**

  - Pedagogical: Ring & Schuck (1985); Review: Bender et al. (2003)
  - Skyrme: Bender & Heenen (2008)
  - Gogny: T.R.Rodríguez & Egido (2010)
  - RMF: Nikšić et al. (2007)
  - Many others …

Alternatives of symmetry restoration & config. mixing

- ☑ **Solution of 5D collective Hamiltonian**

  - Gogny: Delaroche et al. (2010); RMF: Li et al. (2010)

- ☑ **Mapping from nucleons to interacting bosons**

  - ex) interacting boson model of atomic nuclei (1974)

The interacting boson model (IBM) and the microscopic basis

☑ Collective pairs of valence nucleons

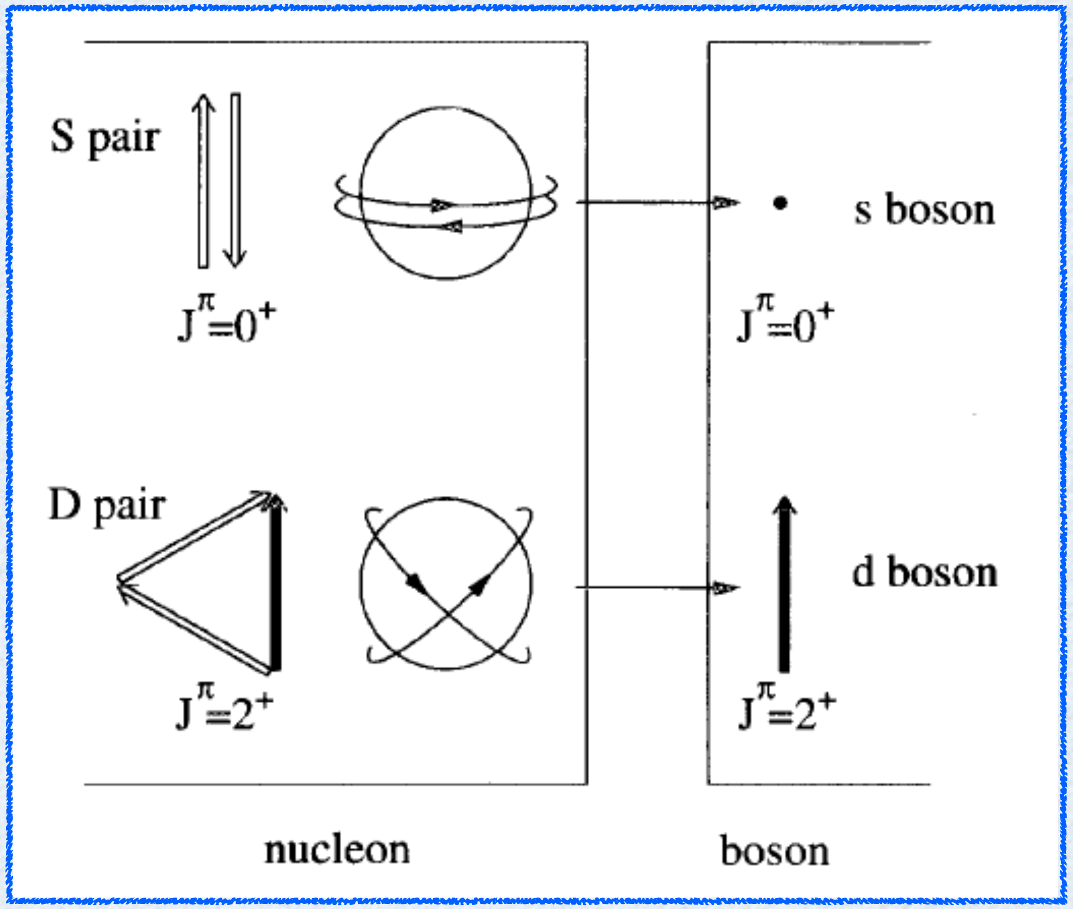
☑ Microscopic basis by shell model for moderate deformation

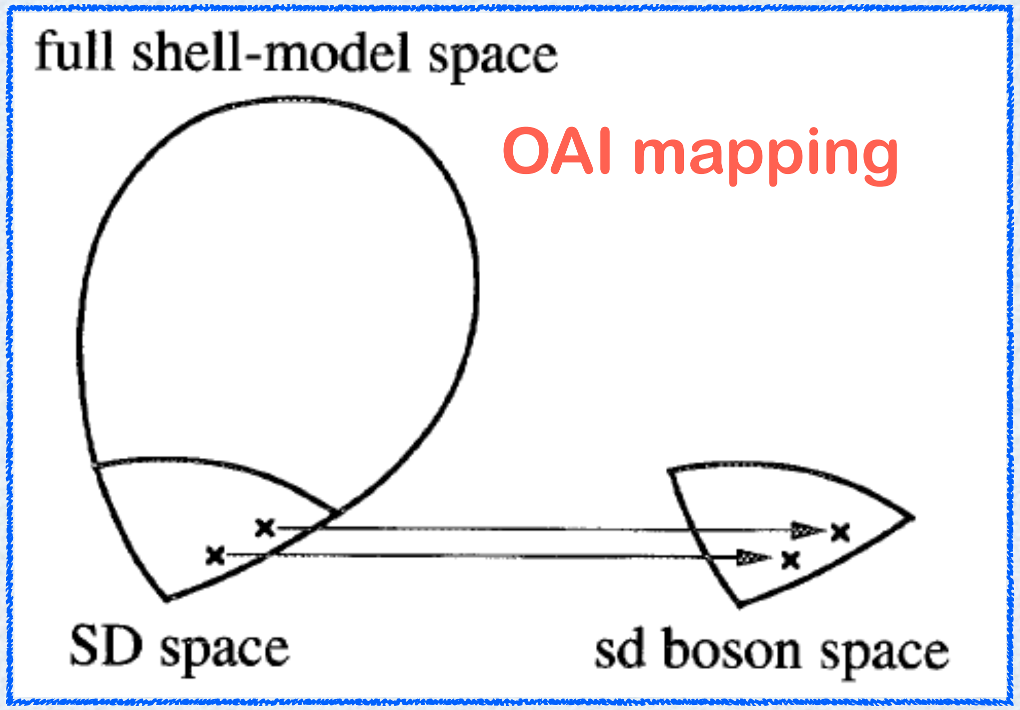
- A. Arima & F. Iachello (1974)
- T. Otsuka, A. Arima & F. Iachello (1978)
- T. Otsuka in “Algebraic approaches to Nuclear Structure” ed. by R. F. Casten (1993)
- T. Mizusaki and T. Otsuka (1997)

Strong deformation ???

☐ A new formulation

- K.N., N. Shimizu & T. Otsuka (2008)





Outline

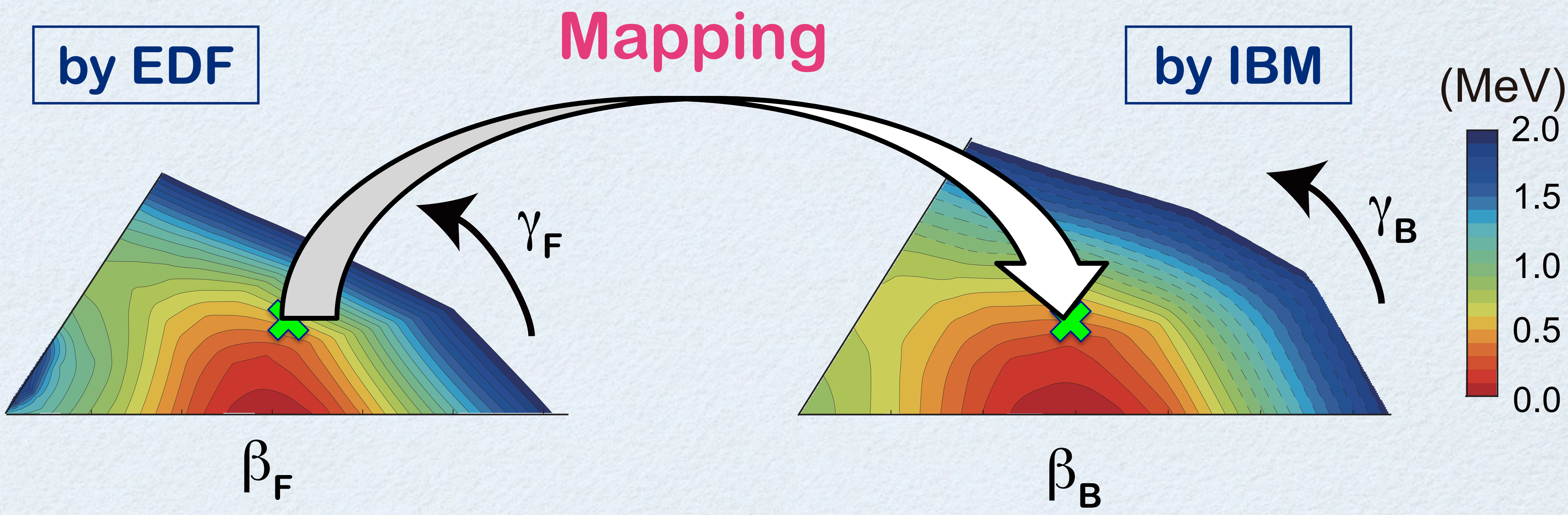
- [x] Introduction
- [ ] Basics and underlying physics
- [ ] Boson mapping for a rotor
- [ ] (Relevant topics)
- [ ] Summary

Analysis of energy surface with quadrupole degrees of freedom can be a good starting point for spectroscopy (e.g., in GCM, 5D collective Hamiltonian).

Also, IBM Hamiltonian could be constructed by energy-surface mapping.

Spectra with good J & N, quantum fluctuation... ???

Energy surfaces for quadrupole deformation



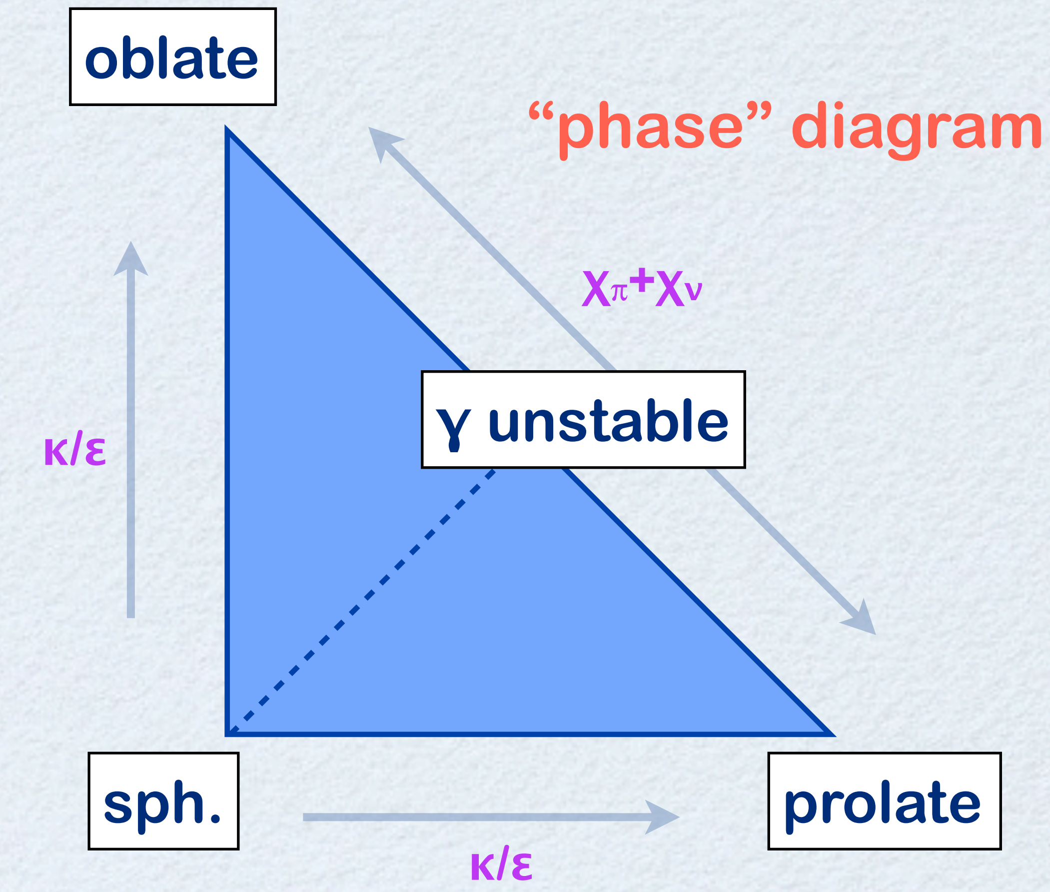
- total energy from constrained Hartree-Fock (with pairing)
- total energy for a boson condensation

This process gives IBM parameters. Diagonalize boson Hamiltonian → Spectra with good J & N

IBM-2 Hamiltonian and coherent state formalism

$$\hat{H}_B = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu$$

Sph. Driving Def. Driving



$$\hat{n}_{d\rho} = d^\dagger_\rho \tilde{d}_\rho \qquad \hat{Q}_\rho = s^\dagger_\rho \tilde{d}_\rho + d^\dagger_\rho \tilde{s}_\rho + \chi_\rho [d^\dagger_\rho \tilde{d}_\rho]^{(2)}$$

Coherent state

$$|\Phi\rangle = \prod_{\rho=\pi,\nu} \frac{1}{\sqrt{N_\rho!}} (\lambda^\dagger_\rho)^{N_\rho} |0\rangle \qquad \lambda^\dagger_\rho = \frac{1}{\sqrt{1+\beta_\rho{}^2}} \left[s^\dagger_\rho + d^\dagger_{\rho 0} \beta_\rho \cos\gamma_\rho + \frac{1}{\sqrt{2}} (d^\dagger_{\rho+2} + d^\dagger_{\rho-2}) \beta_\rho \sin\gamma_\rho \right]$$

Energy surface

$$E(\beta_B, \gamma_B) = \langle \Phi | \hat{H}_B | \Phi \rangle \qquad \begin{cases} \beta_\pi = \beta_\nu \equiv \beta_B \\ \gamma_\pi = \gamma_\nu \equiv \gamma_B \end{cases}$$

Some formulas for IBM energy surface

Boson parameters are calculated by adjusting IBM to nucleon energy surfaces around the minimum, using Wavelet.

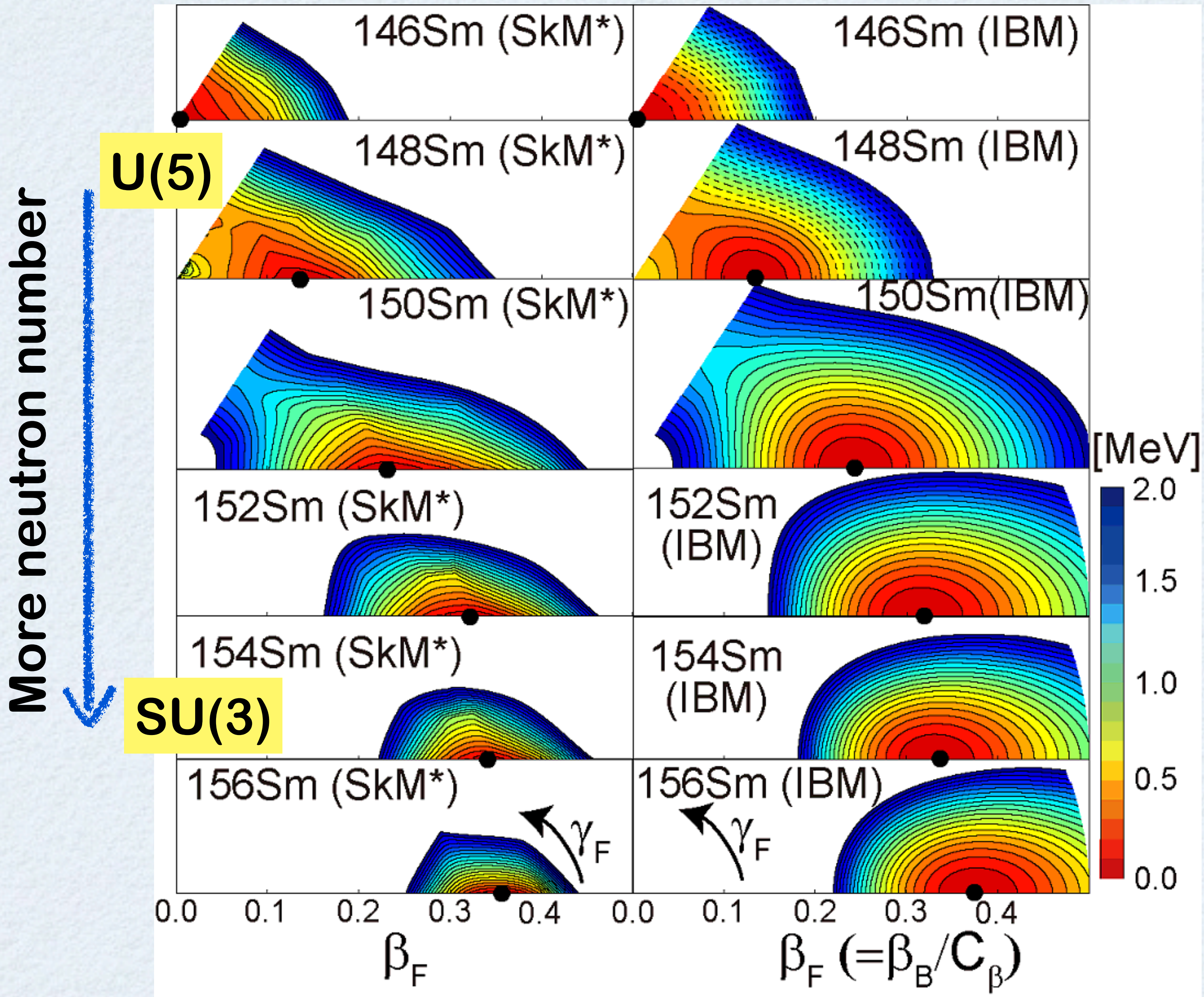
$$E(\beta_B, \gamma_B) = \frac{\epsilon(N_\pi + N_\nu)\beta_B^2}{1+\beta_B^2} + N_\pi N_\nu \kappa \frac{\beta_B^2}{(1+\beta_B^2)^2} \times \left[4 - 2\sqrt{\frac{2}{7}}(\chi_\pi + \chi_\nu)\beta_B \cos 3\gamma_B + \frac{2}{7}\chi_\pi \chi_\nu \beta_B^2\right]$$

Relationship between IBM and geometrical (β,γ):

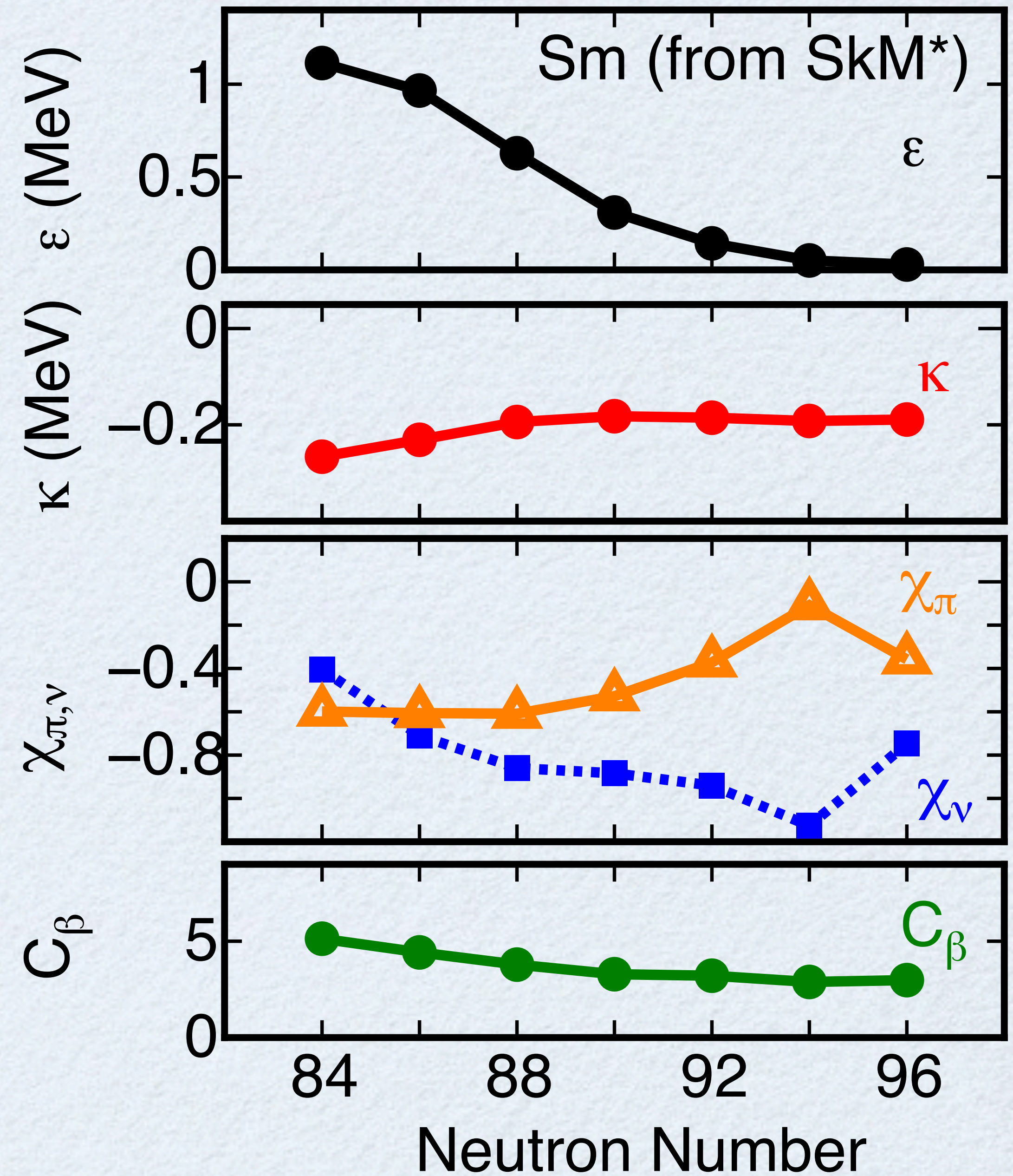
$$\beta_B = C_\beta \beta, \quad \gamma_B = \gamma$$

Spherical-to-deformed shape transition

Total energies in βγ plane



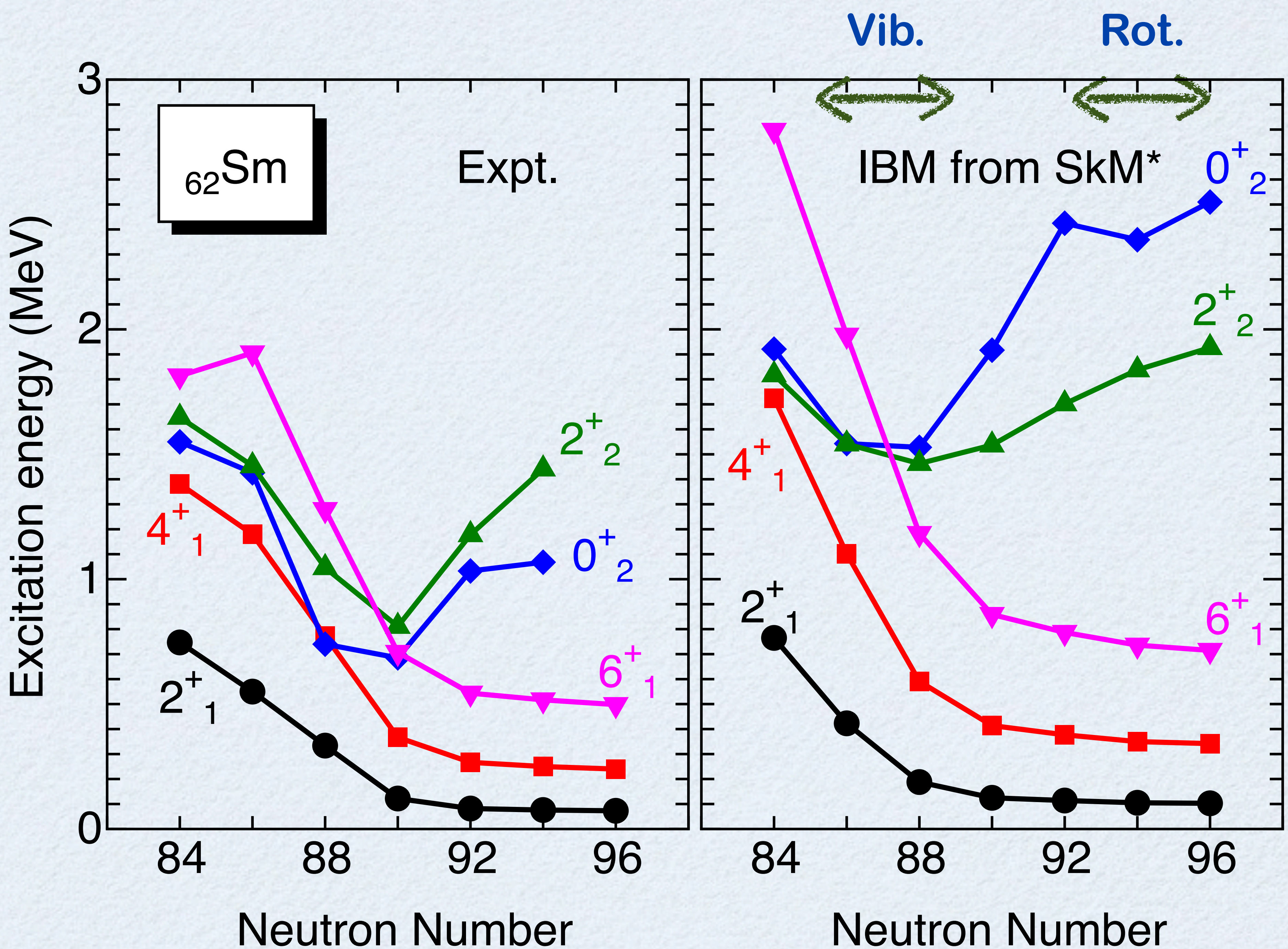
IBM parameters determined microscopically



K.N. et al., PRC81, 044307 (2010)

Excitation spectra (IBM from Skyrme SkM*)

Rotational spectra are overestimated. Origin ?



Outline

- ☑ Introduction
- ☑ Basics and underlying physics
- ☐ Boson mapping for a rotor
- ☐ (Relevant topics)
- ☐ Summary

Boson mapping for a rotational nucleus

Bohr & Mottelson (1980):

"SD truncation is far from sufficient to describe the intrinsic state of rotational nucleus"

- A. Bohr & B. R. Mottelson, Phys. Scr. 22, 468 (1980)

Debates over the validity of SD-pair truncation:

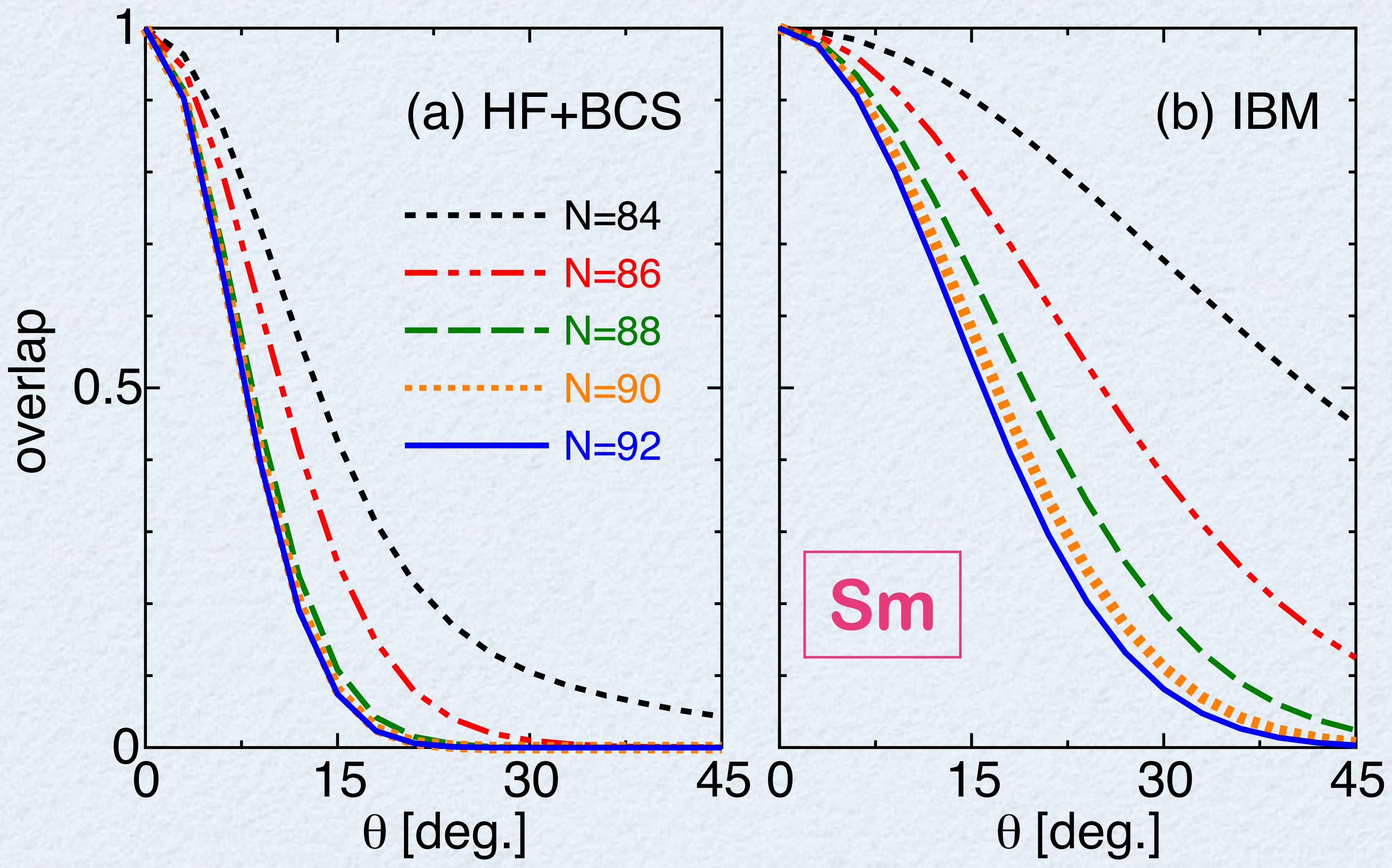
Renormalization of J=4 (G) pair, sdg-IBM ..., still not conclusive.

- T. Otsuka, A. Arima & N. Yoshinaga (1982)
- M. R. Zirnbauer (1984)
- N. Yoshinaga, A. Arima & T. Otsuka (1984)
- T. Otsuka & J. N. Ginocchio (1985)
- T. Otsuka & M. Sugita (1988)

In the energy-surface analysis, both vibrational and rotational kinetic energies are incorporated into boson system.

This is basically true for modest deformation...

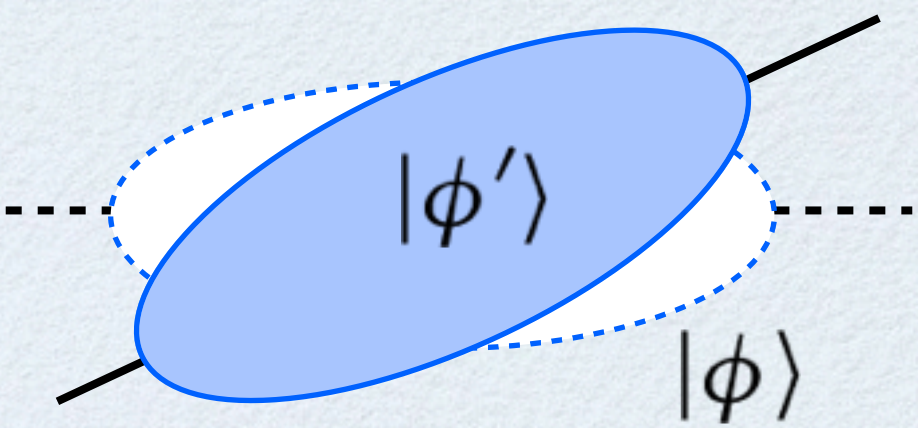
For strong deformation, bosonic intrinsic wave function differs notably from nucleonic one.



Mapping the rotational “response”

$$\delta\langle\phi_F|H_F|\phi'_F\rangle \mapsto \delta\langle\phi_B|H_B|\phi'_B\rangle$$

intr. state — Rotated intr. state



$$\hat{H}_B = \epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa\hat{Q}_\pi\cdot\hat{Q}_\nu + \alpha\hat{L}\cdot\hat{L}$$

This part does not change. — Rot. kinetic term

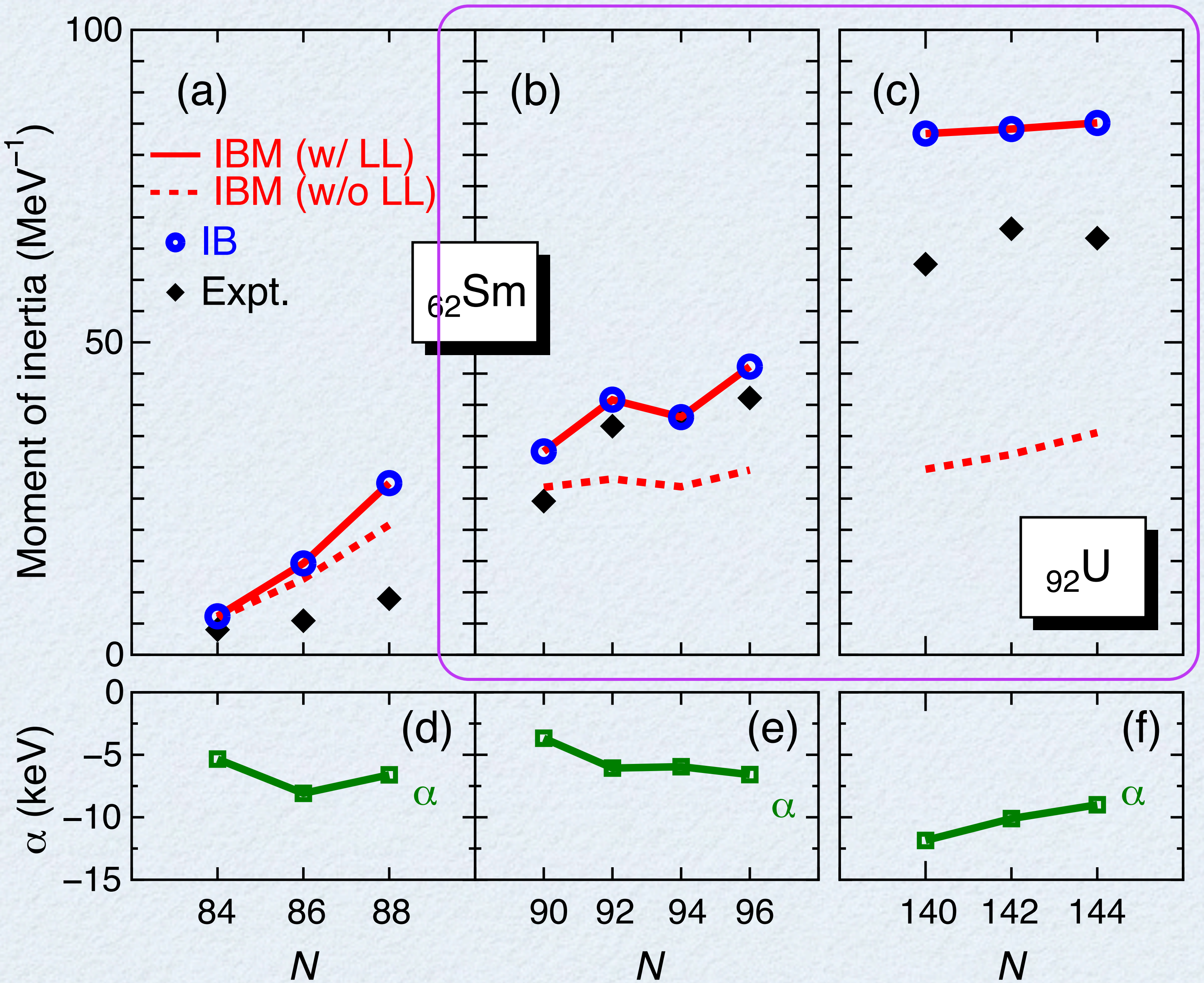
Basic idea:

- Response to the infinitesimal rotation should be reproduced.
- Intrinsic state and deformation (shape) can be the same.
- Do not include additional bosons.

- K.N., T. Otsuka, N. Shimizu & L. Guo, PRC83, 041302(R) (2011)

Cranking moment of inertia

Large difference between fermion and boson systems



Inglis-Belyaev (IB) formula

$$\mathscr{I}_F = 2 \cdot \sum_{i,j>0} \frac{|\langle i|L_k|j\rangle|^2}{E_i + E_j}(u_i v_j - u_j v_i)^2,$$

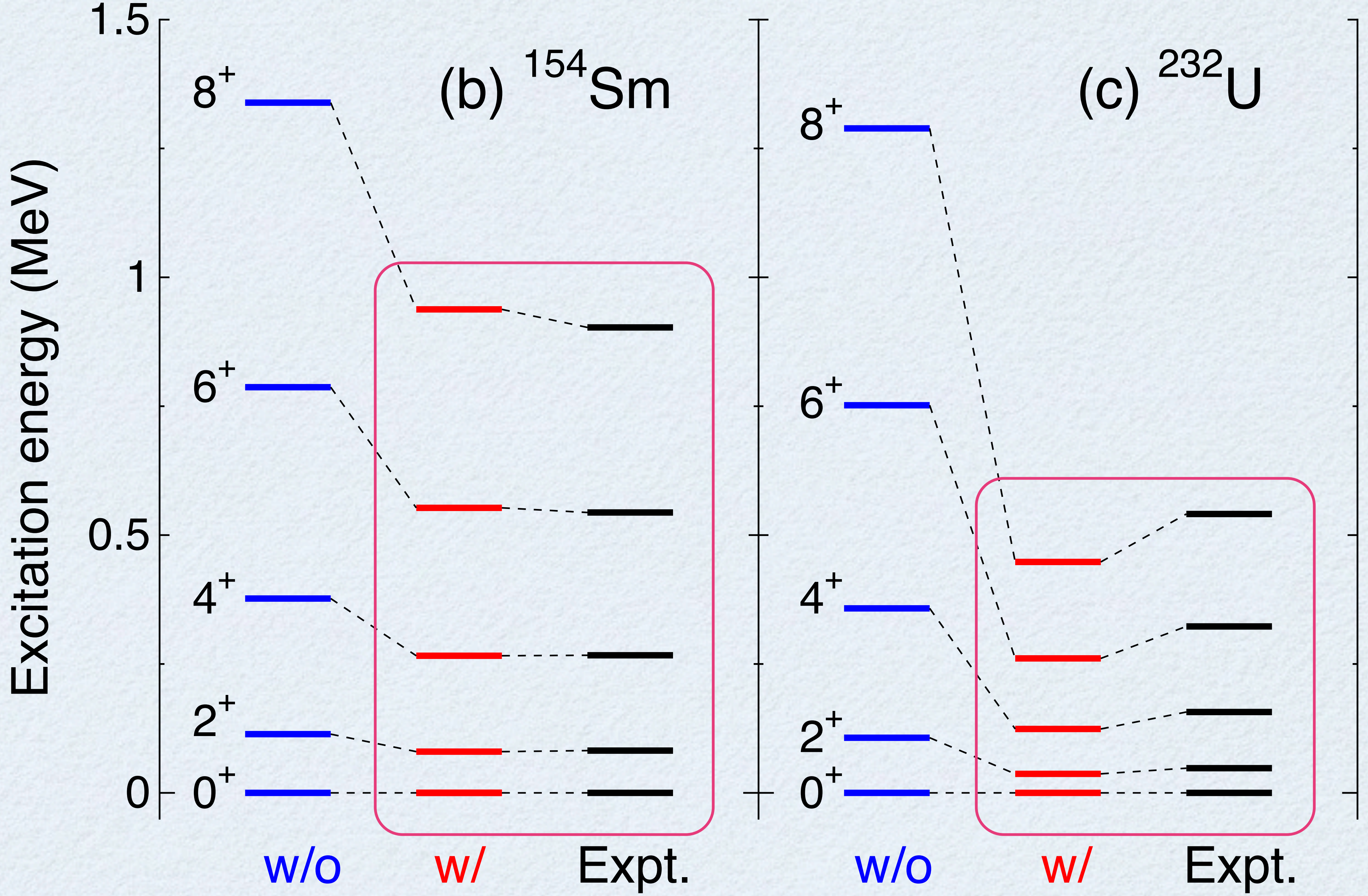
Mom. of inertia for IBM

$$\mathscr{I}_B = \lim_{\omega \to 0} \frac{1}{\omega} \frac{\langle \phi_B | L_k | \phi_B \rangle}{\langle \phi_B | \phi_B \rangle},$$

$\mathscr{I}_B$ **is adjusted to** $\mathscr{I}_F$

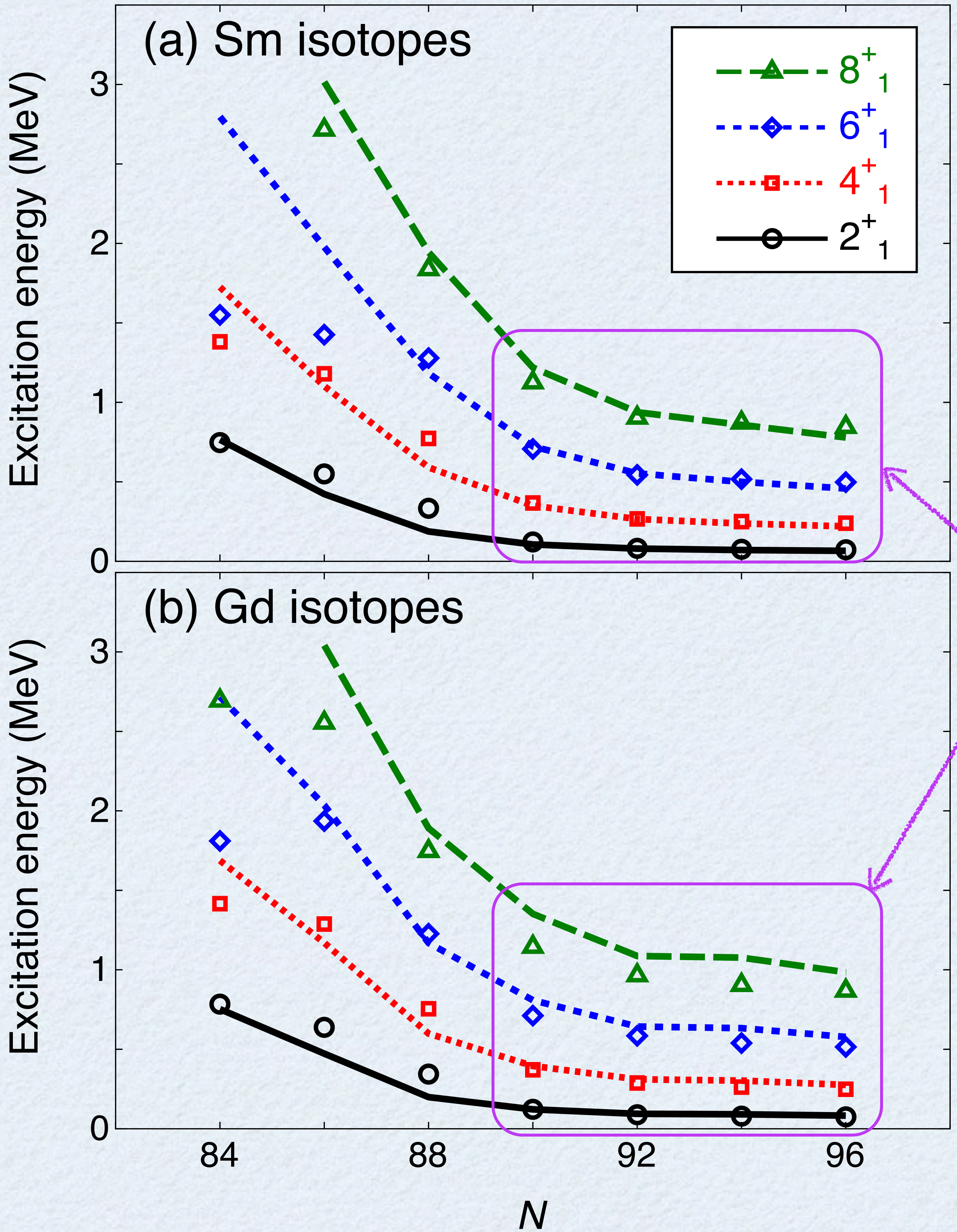
⇒ α value

Some examples of the rotational bands



w/o: no LL, w/: with LL

Evolution of yrast spectra in Sm and Gd



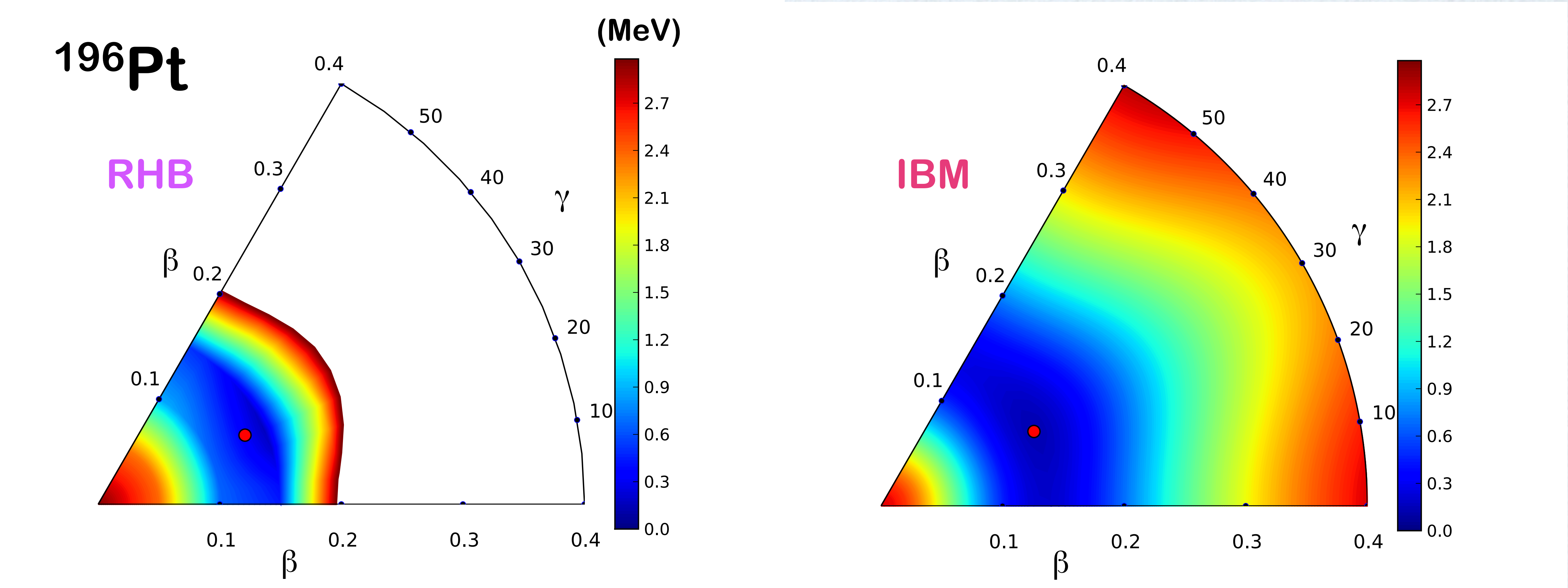
LL term is included for nuclei with $R_{4/2}$>3.2

Outline

- ☑ Introduction
- ☑ Basics and underlying physics
- ☑ Boson mapping for a rotor
- ☐ (Relevant topics)
- ☐ Summary

Comparison with 5D collective Hamiltonian

RMF and mapped IBM energy surfaces with DD-PC1 EDF

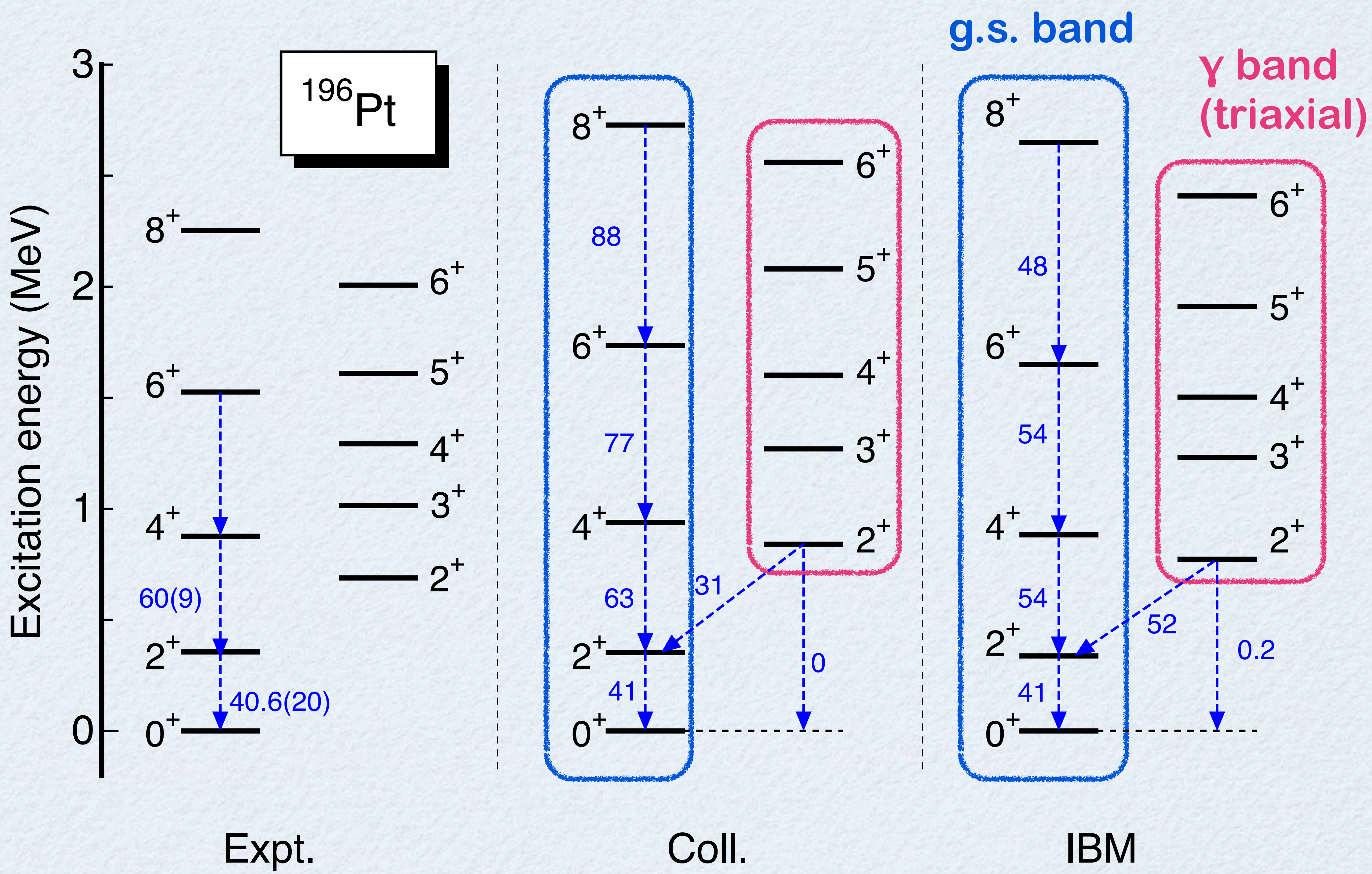


Triaxiality is reproduced by three-body term of boson Hamiltonian

- K.N., T. Nikšić, T. Otsuka, N. Shimizu & D. Vretenar, PRC in press [arXiv:1104.4667]
- K.N., T. Nikšić, T. Otsuka, N. Shimizu & D. Vretenar, in preparation

How does IBM mimic collective spectra ?

Coll. Hamiltonian and IBM, from DD-PC1 EDF; no scaling



- K.N., T. Nikšić, T. Otsuka, N. Shimizu & D. Vretenar, PRC in press [arXiv:1104.4667]
- K.N., T. Nikšić, T. Otsuka, N. Shimizu & D. Vretenar, in preparation

Summary

Bridge over the gap between IBM and DFT

- ☑ Spectroscopy with good J & N, quantum fluctuation
- ☑ IBM is valid for a rotor

  - Main part ← energy surface with varying quadrupole deformation
  - Rotational (LL) term ← rotational response for a fixed shape

- ☐ Comparison with full config. mixing calc.
- ☐ Specific isovector excitation (mixed symmetry state)

Collaborators:

T. Otsuka (U. Tokyo)
N. Shimizu (U. Tokyo)

L. Guo (RIKEN)

D. Vretenar (U. Zagreb)
T. Nikšić (U. Zagreb)

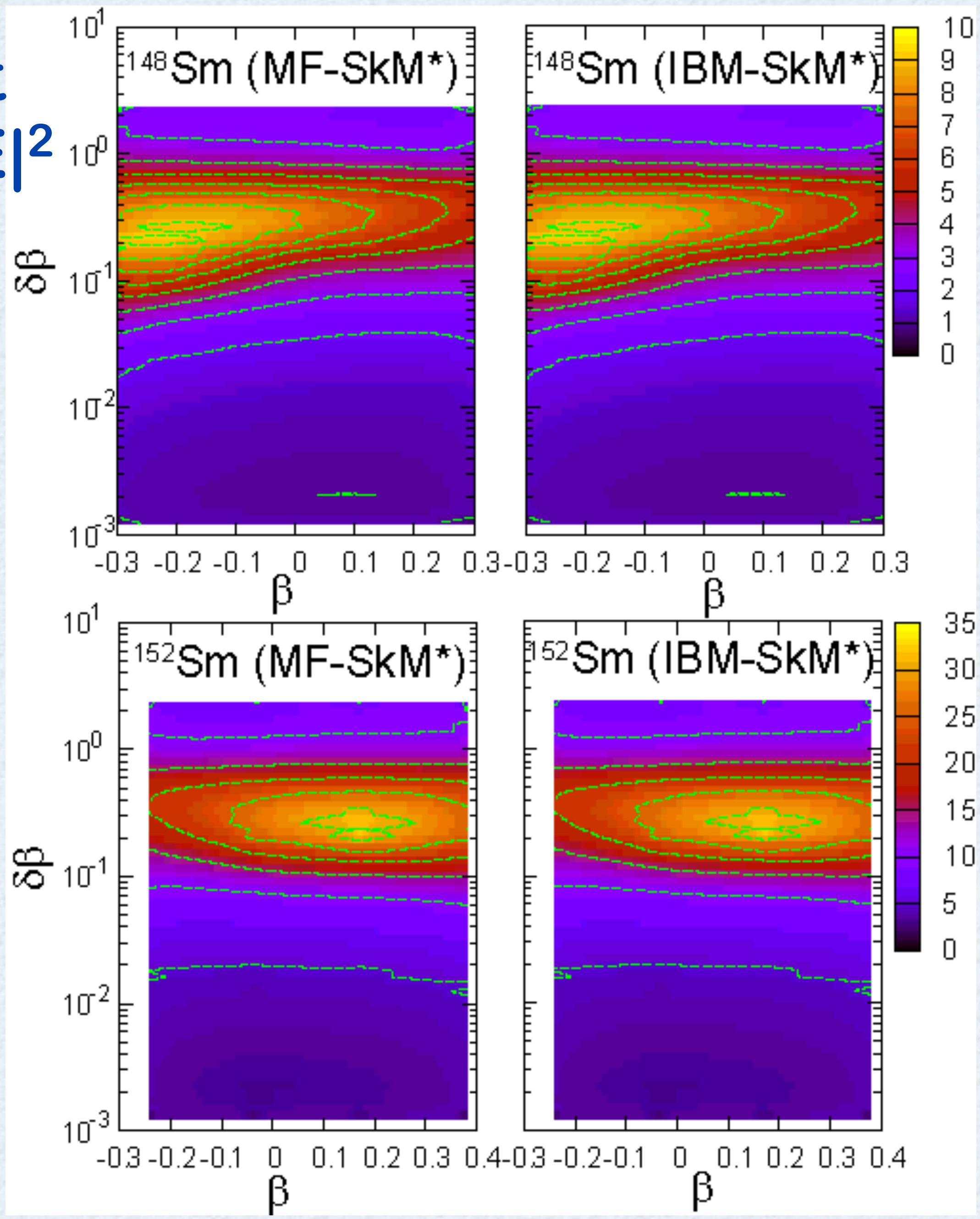
Thank you / Hvala.

Backup slides

Fit by Wavelet Transform

allows unique choice of the IBM parameters

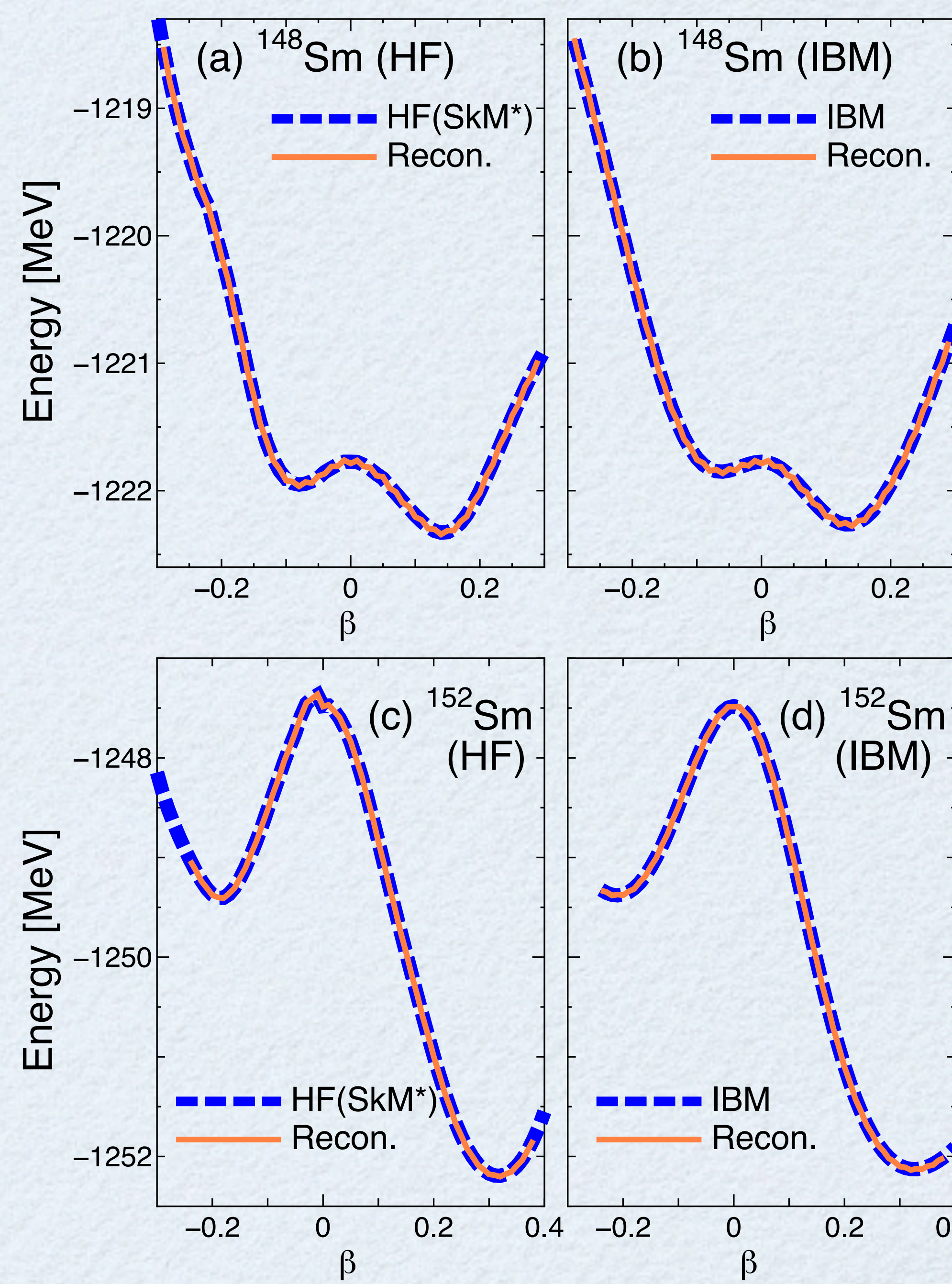
χ^2 fit of $|E|^2$



WT of energy surface for axial sym.

$$\tilde{E}(\delta\beta, \beta) = \frac{1}{\sqrt{\delta\beta}} \int E(\beta') \Phi^{\star}\left(\frac{\beta - \beta'}{\delta\beta}\right) d\beta'$$

- Scale (frequency): $\delta\beta$
- Position: β
- Energy surface: $E(\beta')$
- Basis (wavelet): $\Phi^{\star}$



K.N., Shimizu & Otsuka, PRC81, 044307 (2010)

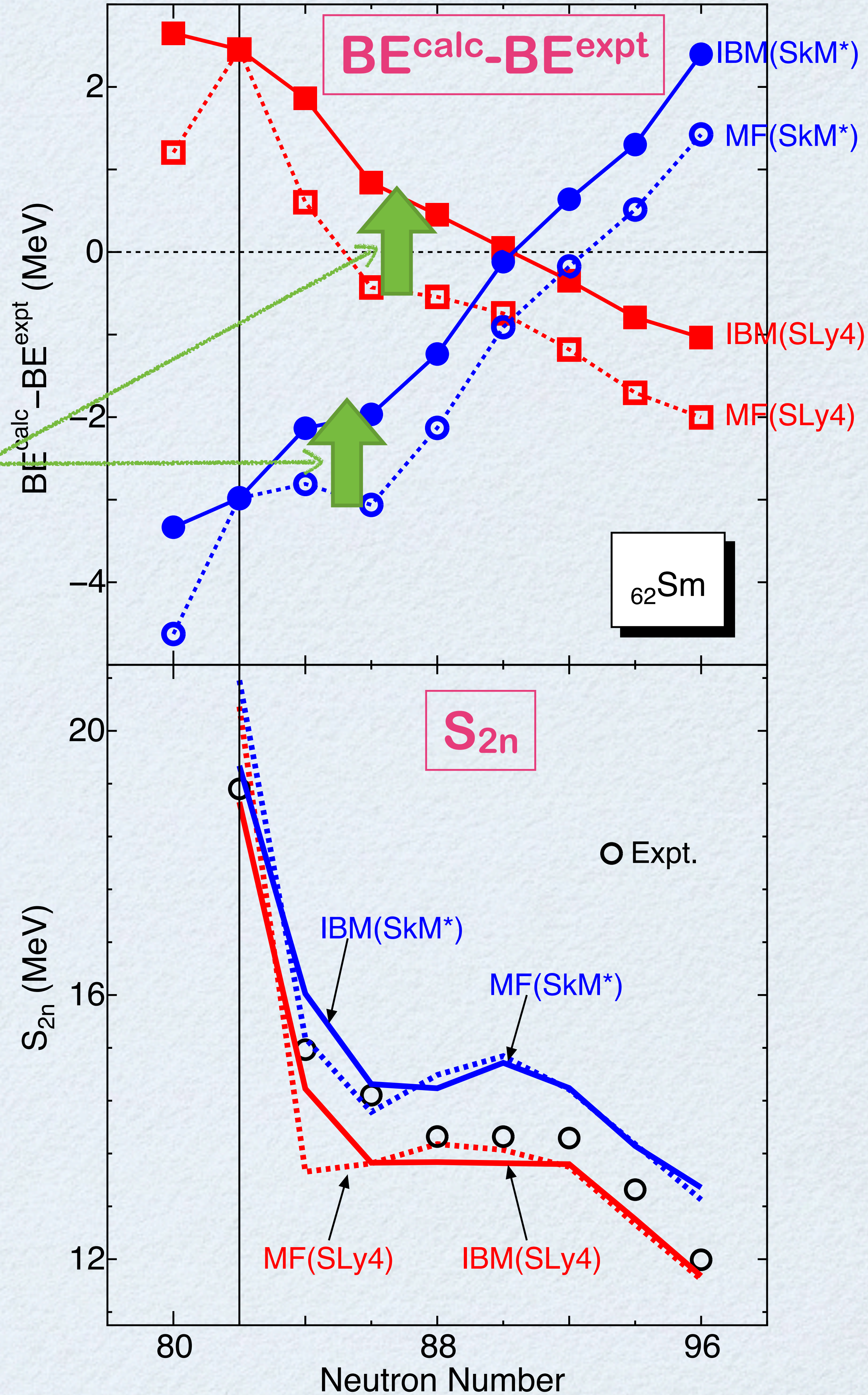
Correlation energy

Ref: K.N., N. Shimizu & T. Otsuka, PRC81, 044307 (2010)

Correlation energy included by the IBM hamiltonian

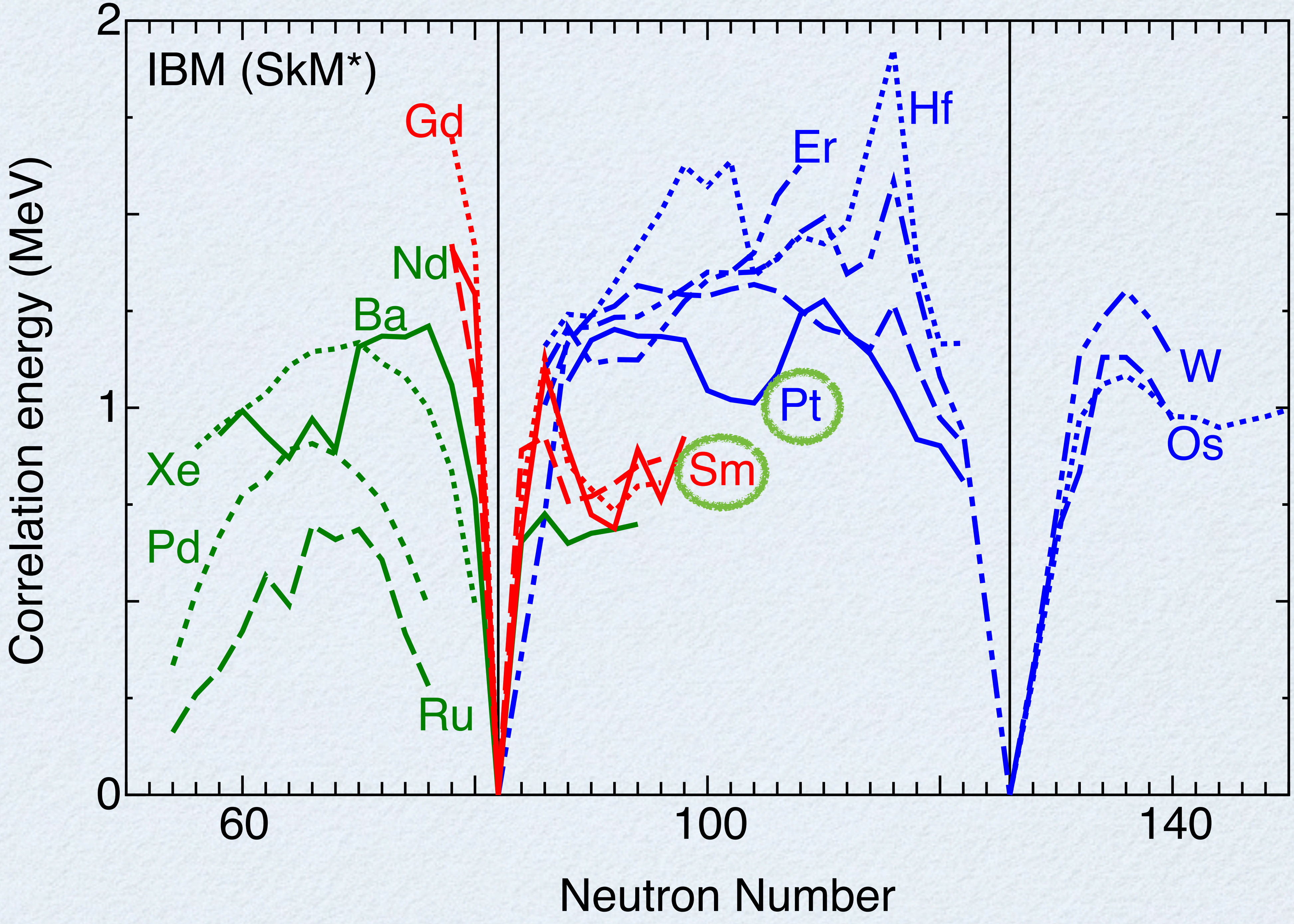
- BE^{IBM}: eigenenergy of H_{IBM}
- BE^{MF}: mean-field solution

Similar arguments by
- Skyrme+GCM: Bender et al. (2006)
- Gogny+5DCH: Delaroche et al. (2010)



Systematics of correlation energy

Maximal in the transitional regions, e.g., Sm and Pt isotopes



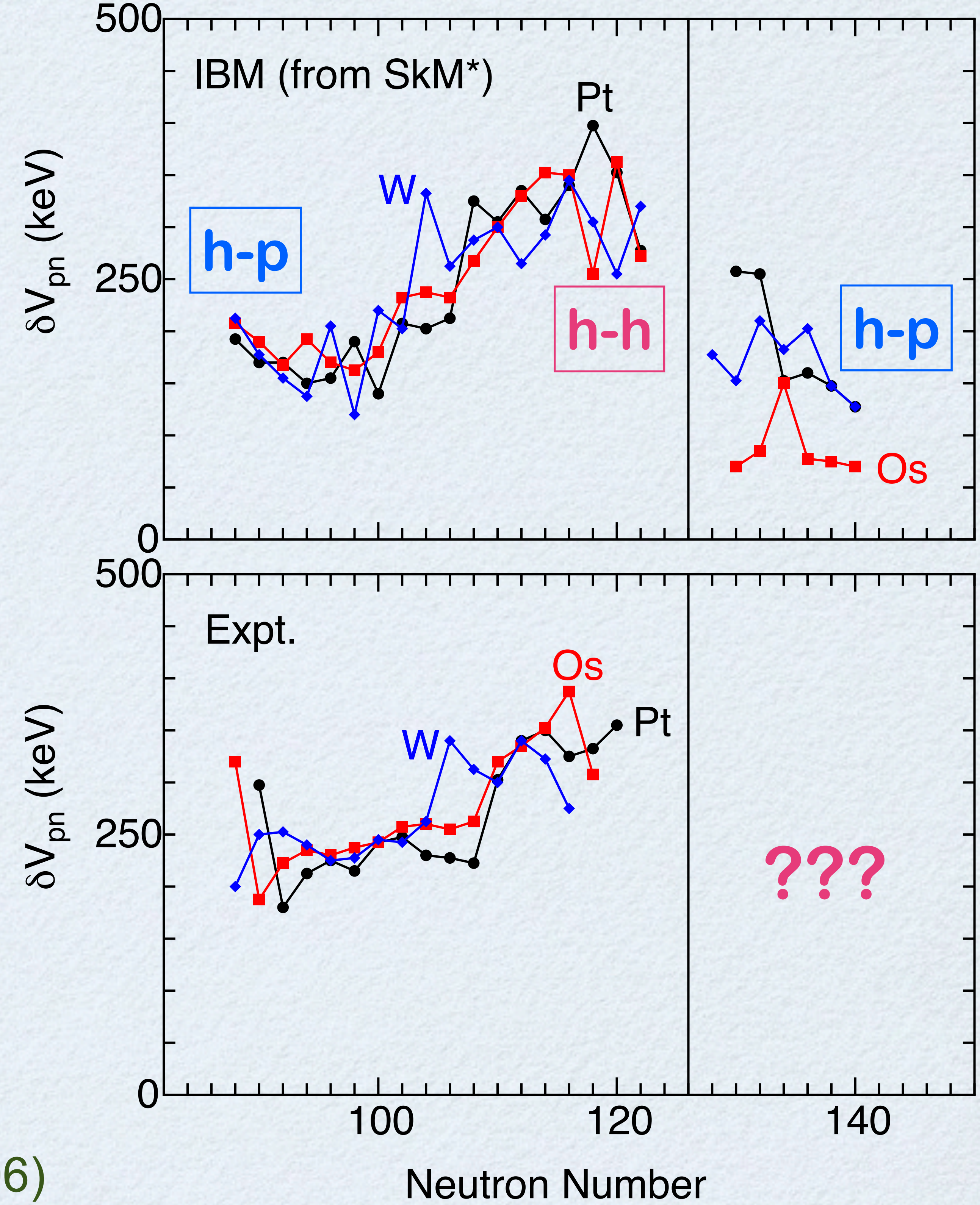
δV_{pn}: Empirical average p-n interaction

Double difference of BE(Z,N):

$$\delta V_{pn} = \frac{1}{4}[\{BE(Z,N) - BE(Z,N-2)\} - \{BE(Z-2,N) - BE(Z-2,N-2)\}]$$

Collectivity, deformation, shell structure, ...

Larger δV_{pn} value for p-p and h-h than p-h and h-p configs.
This trend is predicted also for right-lower quadrant of ^{208}Pb.



Refs:

- Federman & Pittel (1978)
- J.-Y. Zhang et al. (1989)
- Cakirli et al. (2005) ; Cakirli & Casten (2006)