Constraining the neutron skin from collective nuclear motion

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ISOVECTOR GIANT DIPOLE RESONANCES AND NEUTRON SKIN

Theoretically it has been shown that the cross section for GDR excitation depends strongly on the neutron-proton relative radius difference.
S. Shlomo et al., Phys. Rev. C 36, 1317 (1987)
G. R. Satchler, Nucl. Phys. A 472, 215 (1987)
K. Nakayama and G. Bertch, Phys. Rev. Lett. 59, 1053 (1987)

Krasznahorkay *et al. have used the excitation of the* giant dipole resonance to extract the neutron-skin thickness of nuclei.
Krasznahorkay et al., Phys. Rev. Lett. 66, 1287 (1991).

The cross section of the isovector giant dipole resonance (GDR) excited by inelastic scattering of 120-MeV α particles for $0° \le \Theta_{\alpha'} \le 3°$ has been measured using the $^{116,124}Sn(\alpha,\alpha'\gamma_0)$ and $^{208}Pb(\alpha,\alpha'\gamma_0)$ reactions. The results are compared with distorted-wave Born-approximation calculations with a GDR form factor depending on the relative difference of proton and neutron radii $(\Delta R_{pn}/R_0)$. Within the model used, the $\Delta R_{pn}/R_0$ values for ^{116}Sn, ^{124}Sn, and ^{208}Pb deduced from the comparison of measured and calculated cross sections are $(0.7^{+2.3}_{-0.7})\%$, $(4.1^{+2.1}_{-2.5})\%$, and $(3.0 \pm 1.3)\%$, respectively.

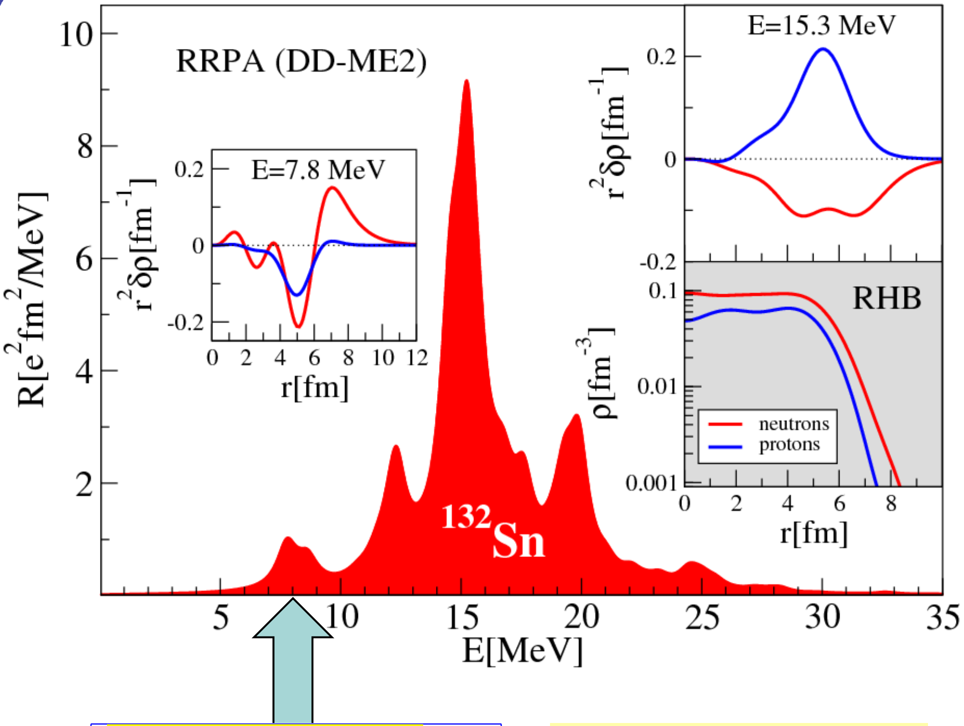
SPIN-DIPOLE RESONANCES AND NEUTRON SKIN

There is a predictable correlation between the SDR cross section and the difference between the rms radii of the neutron and proton density distributions. By normalizing the results in the case of ^{120}Sn, data on neutron-skin thickness along the stable Sn isotopic chain were obtained, in good agreement with theoretical predictions.

A. Krasznahorkay et al., Phys. Rev. Lett. 82, 3216 (1999)

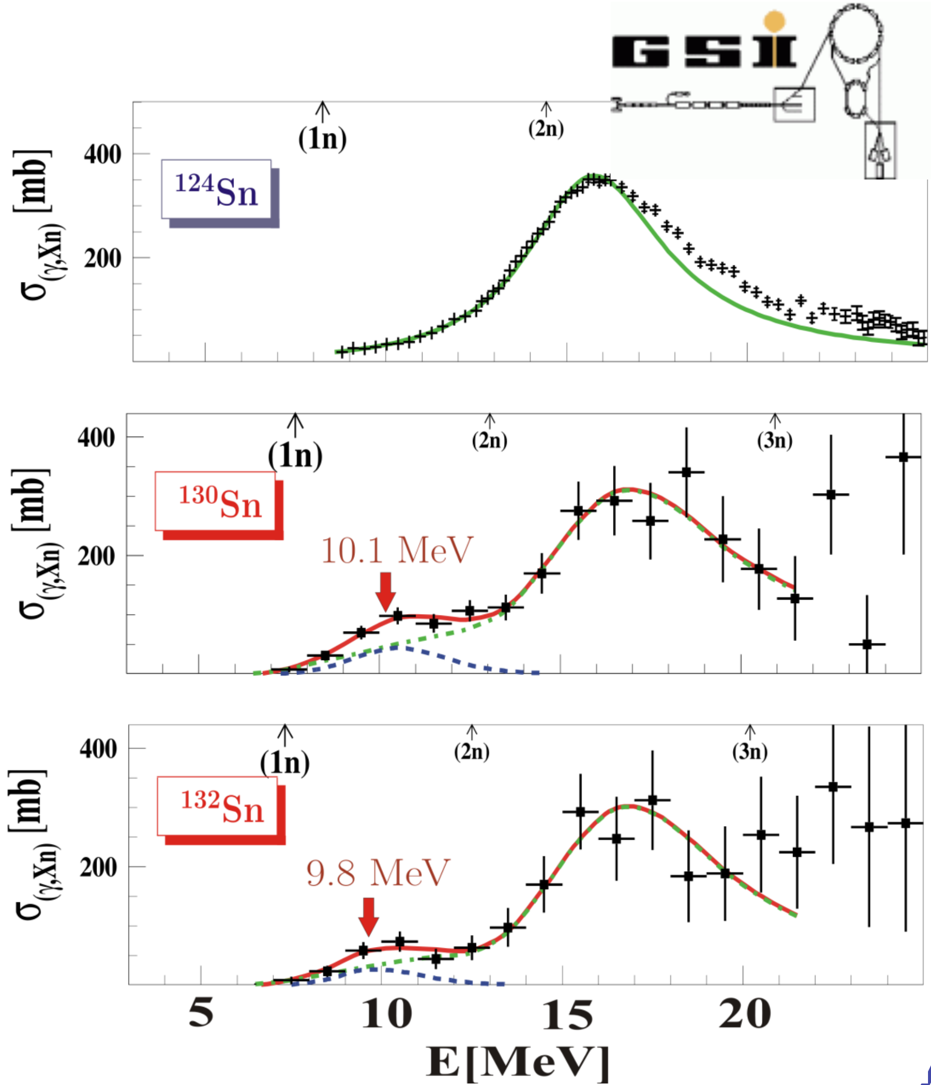
The cross sections of the isovector spin-dipole resonances (SDR) in the Sb isotopes excited by the (^{3}He, t) charge-exchange reaction at 450 MeV for $0° \leq \theta_t \leq 1.15°$ have been measured. The results are compared with the predicted sum-rule strengths related to the difference of neutron and proton root-mean-square radii. Within the model used, the radii differences (neutron-skin thicknesses) deduced for ^{114}Sn, ^{116}Sn, ^{118}Sn, ^{122}Sn, and ^{124}Sn are $\leq$0.09, 0.12 $\pm$ 0.06, 0.13 $\pm$ 0.06, 0.22 $\pm$ 0.07, and 0.19 $\pm$ 0.07 fm, respectively. This result suggests a new method for the measurement of the neutron-skin thicknesses of unstable nuclei. [S0031-9007(99)08966-8]

PYGMY DIPOLE RESONANCES AND NEUTRON SKIN



^{132}Sn at 7.6 MeV	
28.2%	$2d_{3/2} \to 2f_{5/2}$
21.9%	$2d_{5/2} \to 2f_{7/2}$
19.7%	$2d_{3/2} \to 3p_{1/2}$
10.5%	$1h_{11/2} \to 1i_{13/2}$
3.5%	$2d_{5/2} \to 3p_{3/2}$
1.9%	$1g_{7/2} \to 2f_{5/2}$
1.5%	$1g_{7/2} \to 1h_{9/2}$
0.6%	$1g_{7/2} \to 2f_{7/2}$
0.6%	$2d_{3/2} \to 3p_{3/2}$

Distribution of the neutron particle-hole configurations for the peak at 7.6 MeV



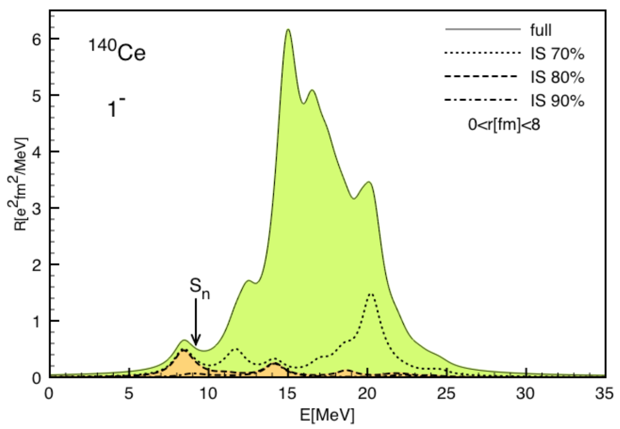
P.Adrich et al., Phys. Rev. Lett. 95, 132501(2005)

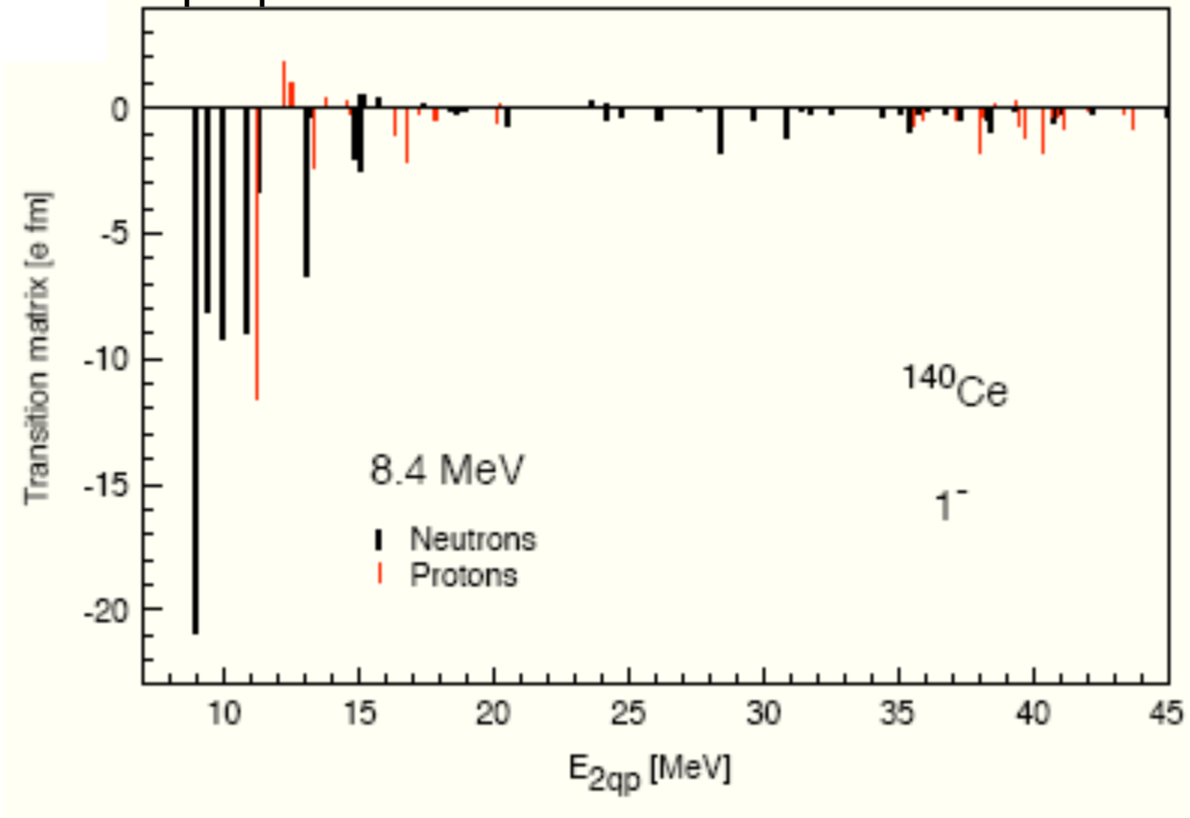
PYGMY DIPOLE RESONANCES AND NEUTRON SKIN

$$B_J^T(E_\lambda) = \frac{1}{2J_i+1}\left|\sum_{\mu\mu'}\left\{X_{\mu\mu'}^{\lambda,J0}\langle\mu||\hat{Q}_J^T||\mu'\rangle + (-1)^{j_\mu - j_{\mu'}+J}\,Y_{\mu\mu'}^{\lambda,J0}\,\langle\mu'||\hat{Q}_J^T||\mu\rangle\right\}(u_\mu v_{\mu'} + (-1)^J v_\mu u_{\mu'})\right|^2 ,$$

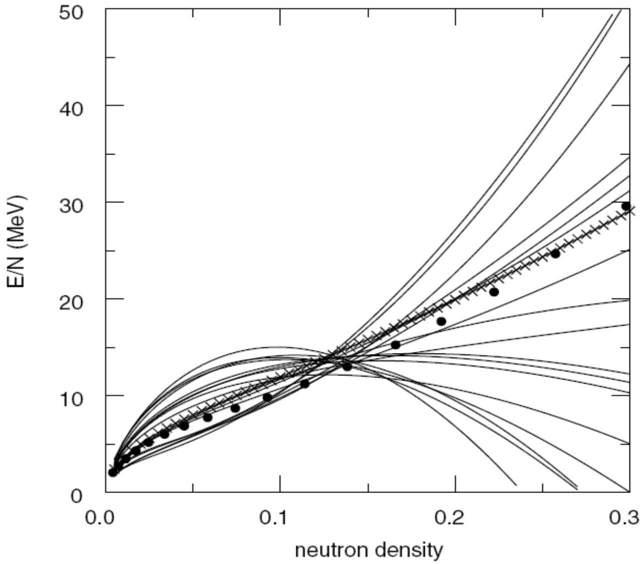
PDR corresponds to collective excitation mainly dominated by neutrons from the neutron skin. Therefore, the PDR should provide direct insight into the neutron skin properties.

Coherent contributions to the transition strength B(E1) from several neutron configurations





Symmetry energy $S_2(\rho)$ and neutron skin in ^{208}Pb

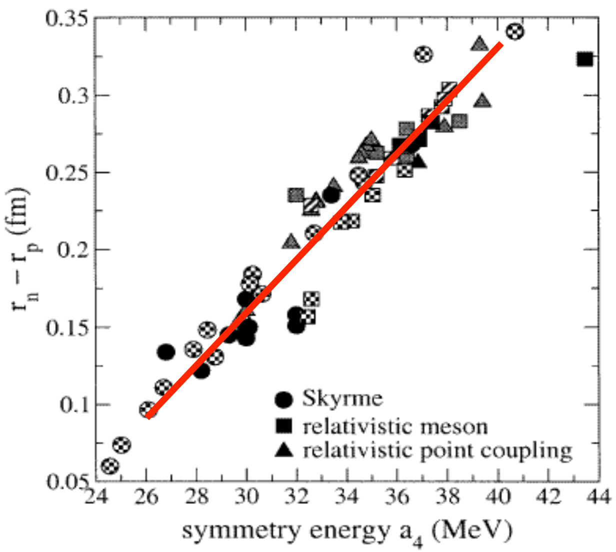


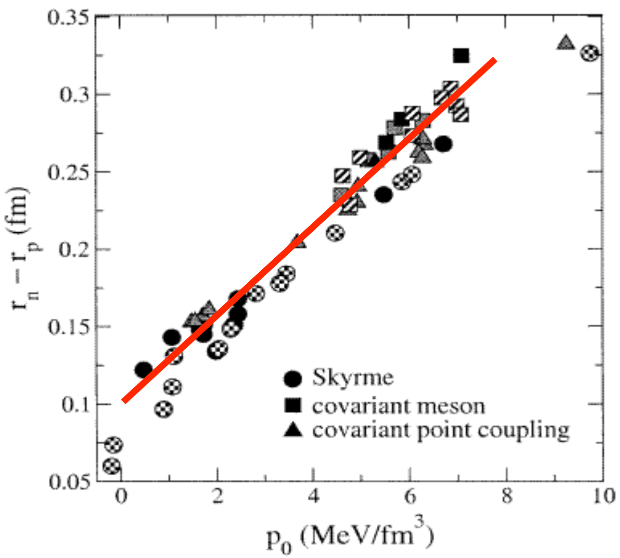
Alex Brown,
PRL 85 (2000) 5296

$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + O(\alpha^4)\,,\ \alpha = \frac{N-Z}{A}$$

$$S_2(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0} =$$

$$= a_4 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K_0}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots$$





R.J.Furnstahl
NPA 706(2002)85-110

- strong linear correlation between neutron skin thickness and parameters a_4, p_0

NUCLEAR SYMMETRY ENERGY AND NEUTRON SKINS DERIVED FROM PYGMY DIPOLE RESONANCE

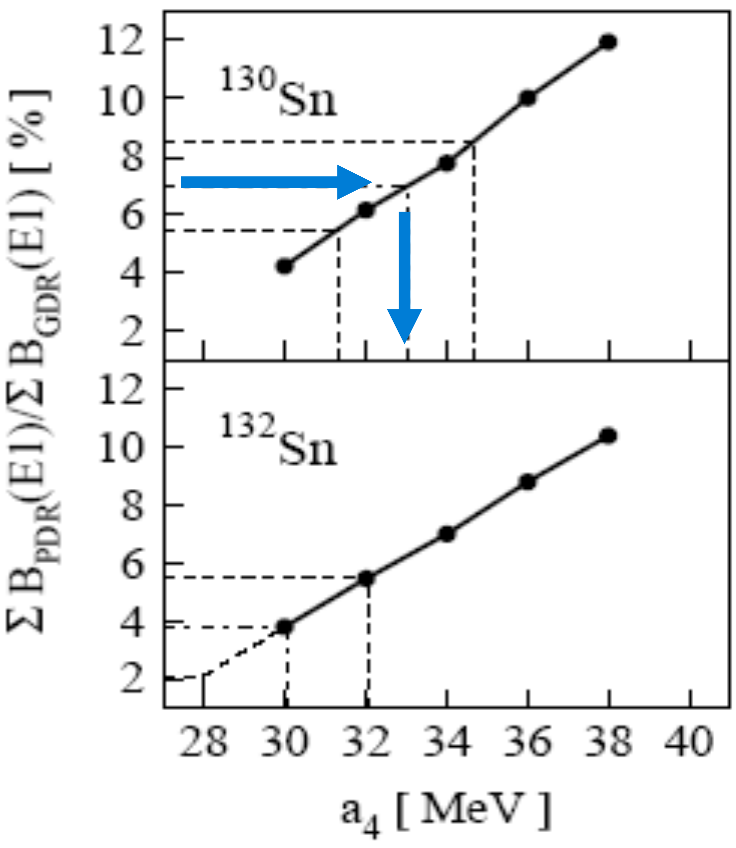
Precise knowledge of neutron-skin thickness could constrain the density dependence of $S(\rho)$

Pygmy-Strength (since related to skin) should do the same job, but, experimentally, is accessed much easier !

→ Quantitative attempt to determine the neutron skin thickness by means of RHB + RQRPA, using various density-dependent effective interactions and recent experimental data on PDR

A. Klimkiewicz, N. Paar, et al., (LAND collaboration), Phys. Rev. C 76, 051603(R) (2007)

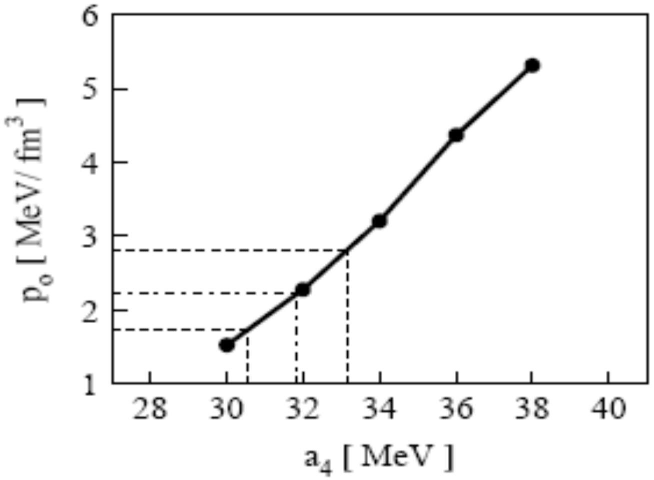
PDR strength versus a_4, p_o



RHB+RQRPA calculations and exp. data (GSI) for PDR strength

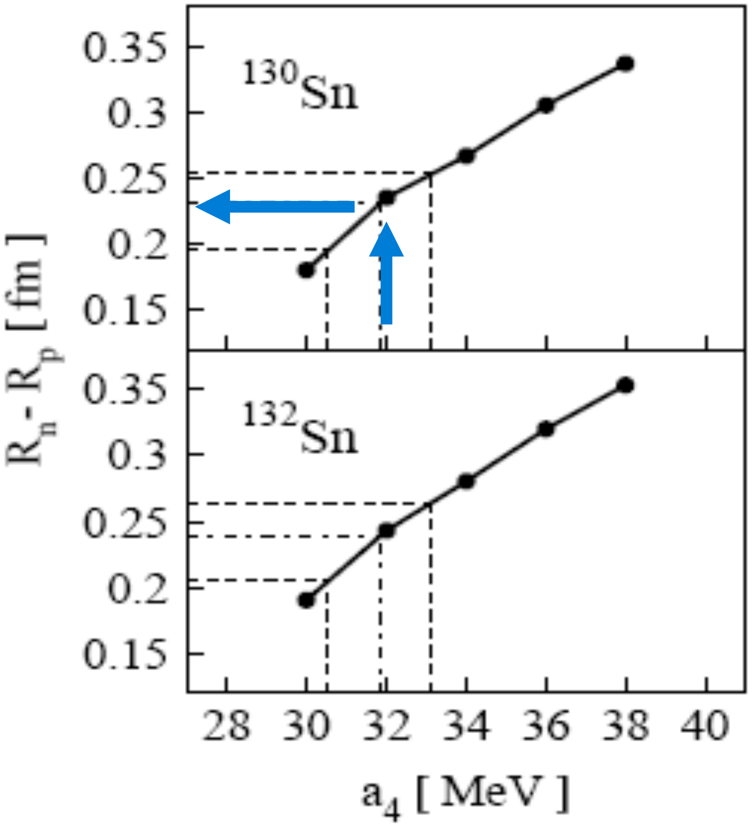
Result (*averaged ^{130,132}Sn) :*

a_4 = 32.0 ± 1.8 MeV



p_o = 2.3 ± 0.8 MeV/fm^3

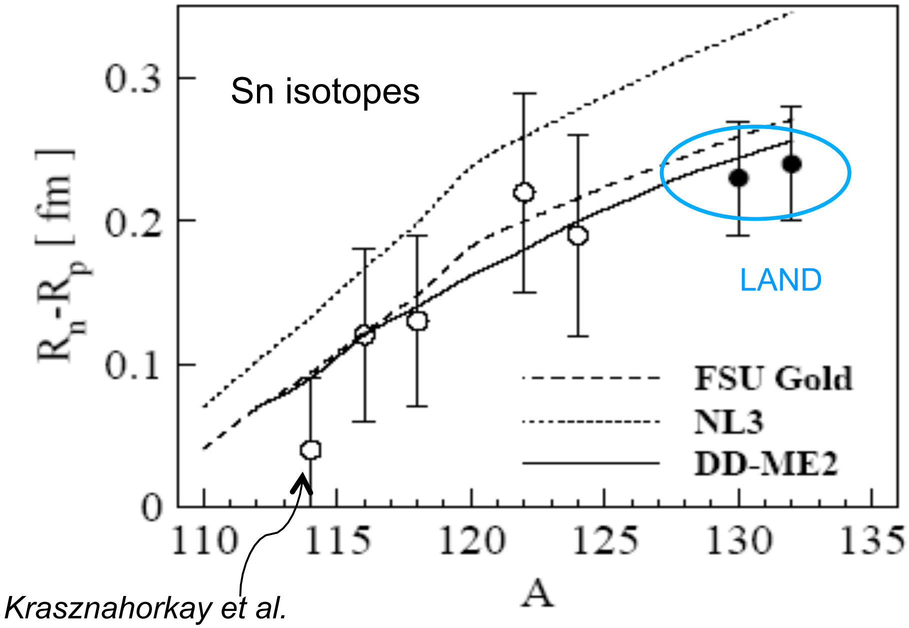
Neutron skin thickness



R_n – R_p :

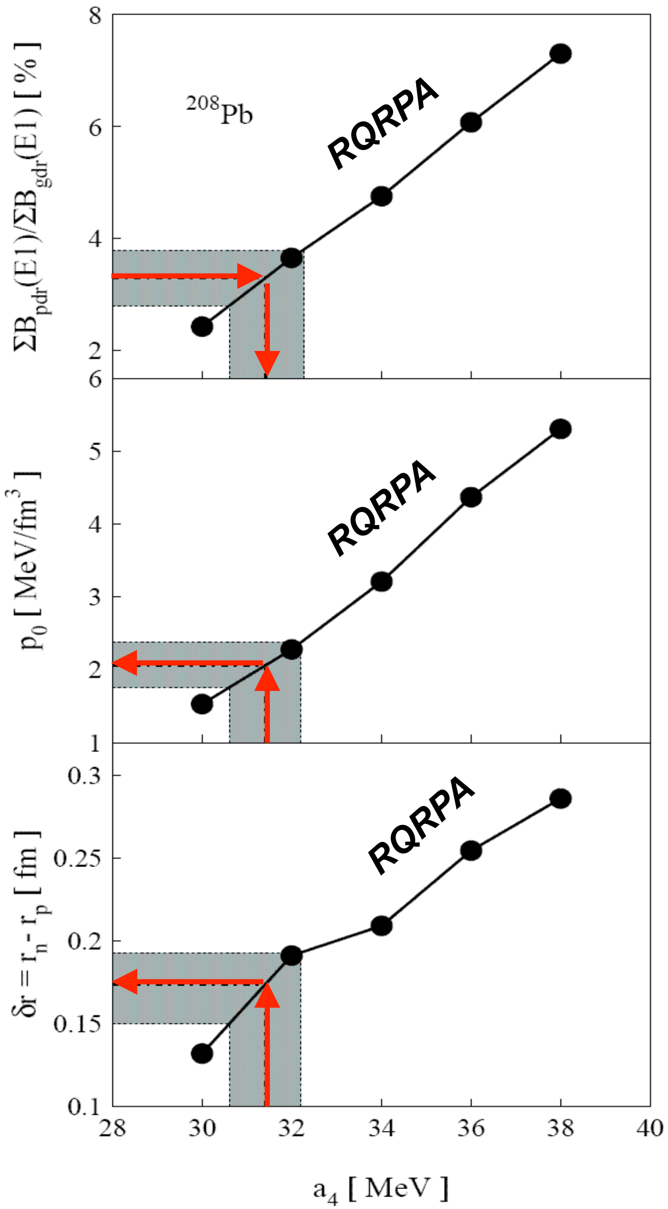
^{130}Sn: 0.23 ± 0.04 fm

^{132}Sn: 0.24 ± 0.04 fm



A.Krasznahorkay et al.
PRL 82(1999)3216

^{208}Pb analysis



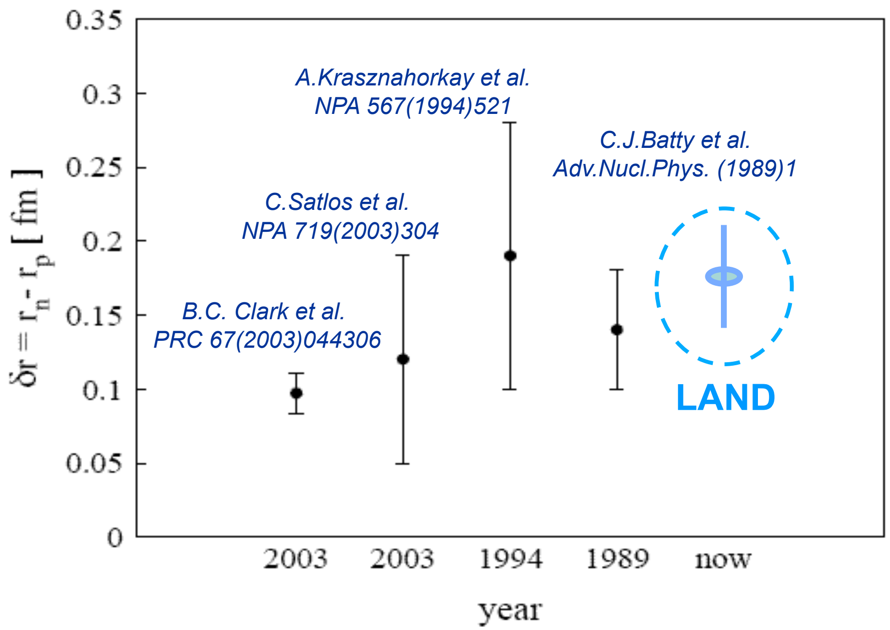
$\sum B_{pdr}(E1)=1.98\ e^2\ fm^2$

from N.Ryezayeva et al., PRL 89(2002)272501

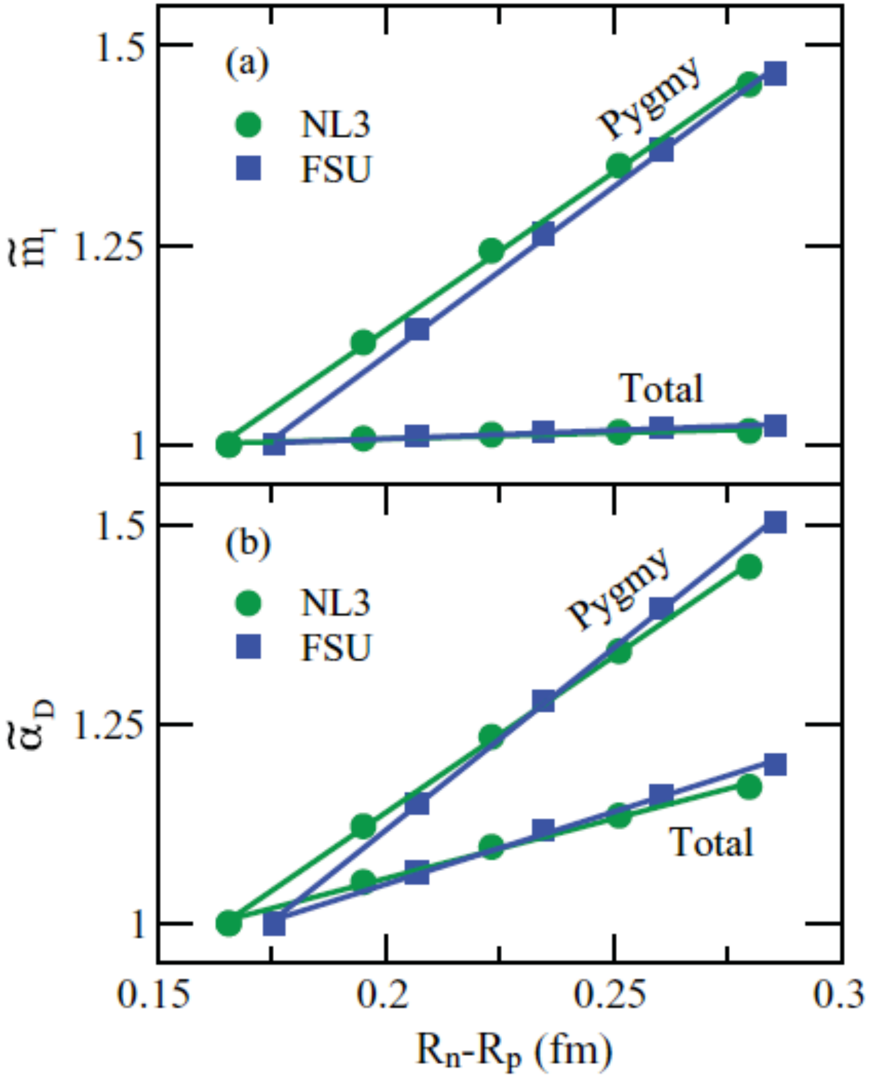
$\sum B_{gdr}(E1)=60.8\ e^2\ fm^2$

from A.Veyssiere et al.,NPA 159(1970)561

$$R_n - R_p = 0.18 \pm 0.035\ \text{fm}$$



Pygmy resonances and neutron skins



Energy weighted sum and dipole polarizability for ^{68}Ni as a function of the neutron-skin thickness of ^{208}Pb.

Families of NL3 and FSUGold parameterizations are obtained by changing the isovector parameters (for the isoscalar-isovector mixing term and the isovector term).

J. Piekarewicz, Phys. Rev. C 83, 034319 (2011)

PHYSICAL REVIEW C **81**, 051303(R) (2010)

Information content of a new observable: The case of the nuclear neutron skin

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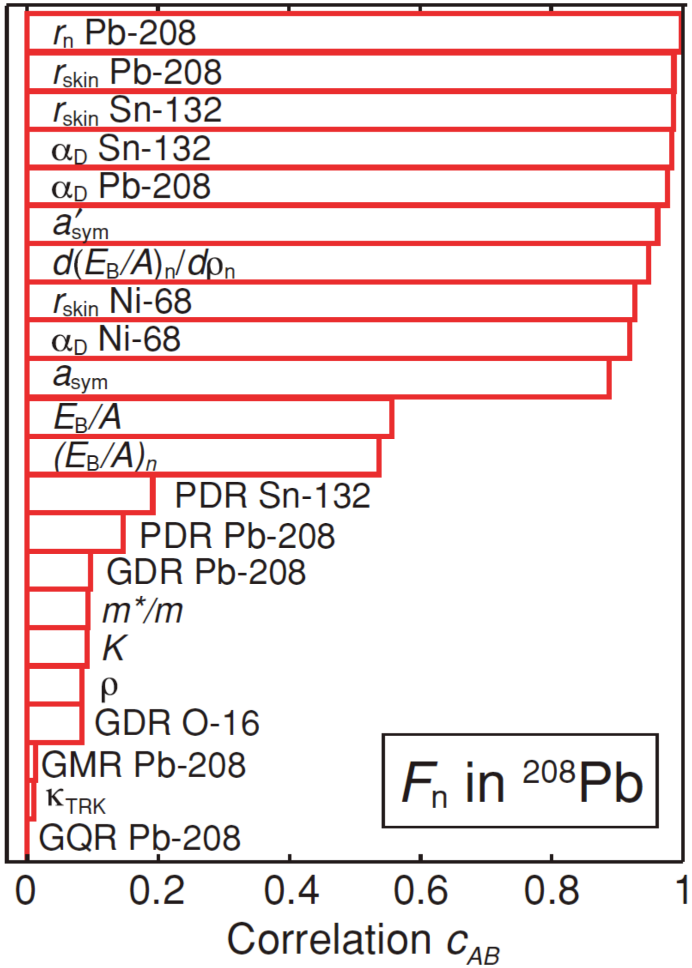
We address two questions pertaining to the uniqueness and usefulness of a new observable: (i) Considering the current theoretical knowledge, what novel information does new measurement bring in? (ii) How can new data reduce uncertainties of current theoretical models? We illustrate these points by studying the radius of the neutron distribution of a heavy nucleus, a quantity related to the equation of state for neutron matter that determines properties of nuclei and neutron stars. By systematically varying the parameters of two theoretical models and studying the resulting confidence ellipsoid, we quantify the relationships between the neutron skin and various properties of finite nuclei and infinite nuclear matter. Using the covariance analysis, we identify observables and pseudo-observables that correlate, and do not correlate, with the neutron skin. By adding the information on the neutron radius to the pool of observables determining the energy functional, we show how precise experimental determination of the neutron radius in ^{208}Pb would reduce theoretical uncertainties on the neutron matter equation of state.

Correlation of various observables with neutron form factor in ^{208}Pb, related to PREX experiment



SV-min (Skyrme type parameterization) embraces nuclear bulk properties (binding energies, surface thicknesses, charge radii, spin-orbit splittings, and pairing gaps) for selected semimagic nuclei.

lack of correlation between *Fn (or neutron* skin) and PDR strength; GMR, GDR, and GQR energies; and isoscalar and isovector effective mass, incompressibility, and saturation Density

Strong correlation *Fn* with dipole polarizability:

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$

P.-G. Reinhard, W. Nazarewicz, Phys. Rev. C 81, 051303 (2010)

Covariance analysis and correlations between observables

Microscopic models based on NEDF include number of parameters

$$\mathbf{p} = (p_1, p_2, p_3, \dots, p_N)$$

The optimal set of parameters $\mathbf{p_0}$ is obtained using a set of observables in a least square fit with the quality measure:

$$\chi^2(\mathbf{p}) = \sum_{\mathcal{O}} \left(\frac{\mathcal{O}^{(\mathrm{th})}(\mathbf{p}) - \mathcal{O}^{(\mathrm{exp})}}{\Delta \mathcal{O}} \right)^2$$

Near the minimum, the *χ*2 landscape is given by a confidence ellipsoid

$$\chi^2(\mathbf{p}) - \chi_0^2 \approx \sum_{i,j=1}^{N} (p_i - p_{i,0}) \mathcal{M}_{ij} (p_j - p_{j,0})$$

$$\mathcal{M}_{ij} = \tfrac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2 |_{\mathbf{p}_0}$$

The physically reasonable domain **p** is defined as that multitude of parameters around $\mathbf{p_0}$ that fall inside the covariance ellipsoid $\chi^2 = \chi_0^2 + 1$

Covariance analysis and correlations between observables

Consider two observables A, B calculated within the model, they are also functions of the model parameters

$$A = A(\mathbf{p}) \qquad\qquad B = B(\mathbf{p})$$

Assuming they are smoothly varying with **p**, the covariance between the observables A and B:

$$\overline{\Delta A \, \Delta B} = \sum_{ij} \partial_{p_i} A (\hat{\mathcal{M}}^{-1})_{ij} \partial_{p_j} B$$

Variance $\overline{\Delta^2 A}$ and $\overline{\Delta^2 B}$ define uncertainties of each observable.

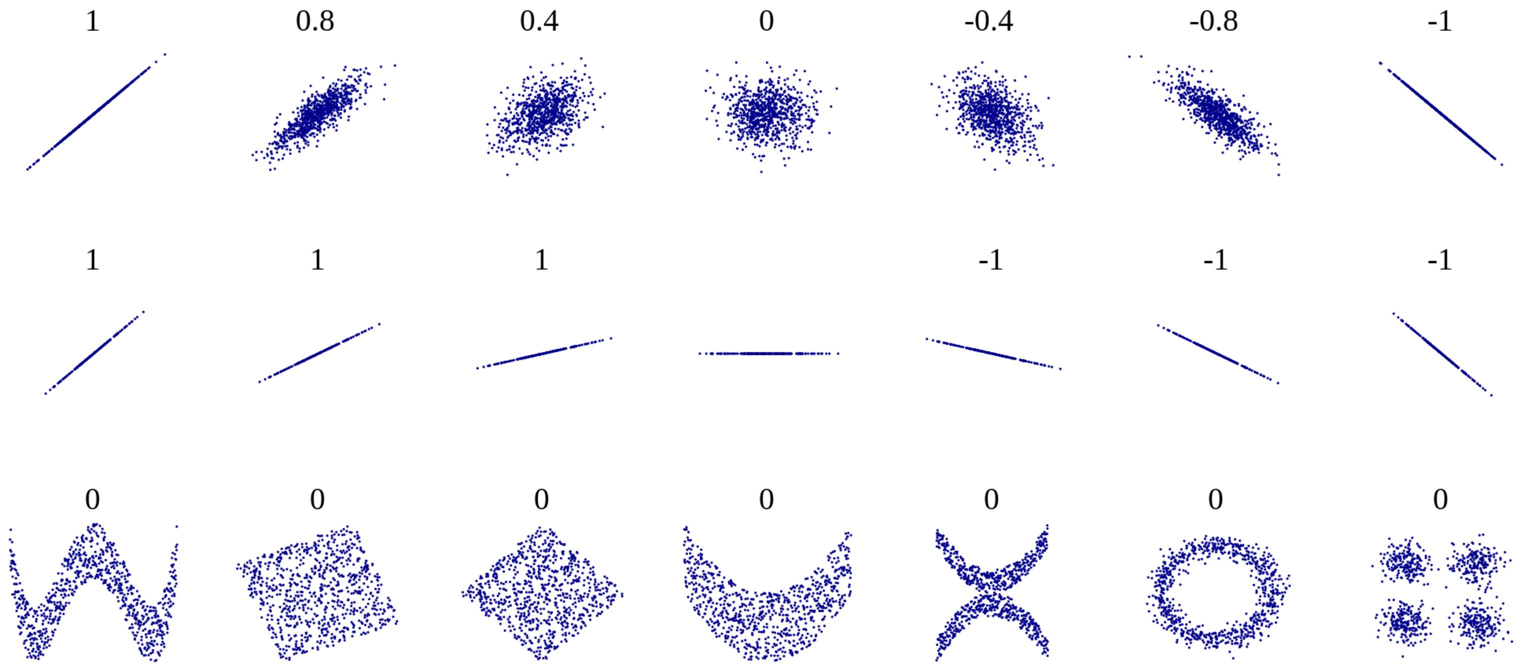
Pearson product-moment correlation coefficient

$$c_{AB} = \frac{|\overline{\Delta A \, \Delta B}|}{\sqrt{\overline{\Delta A^2} \; \overline{\Delta B^2}}}$$

provides a measure of the correlation (linear dependence) between two variables A and B, giving a value between +1 and −1 inclusive.

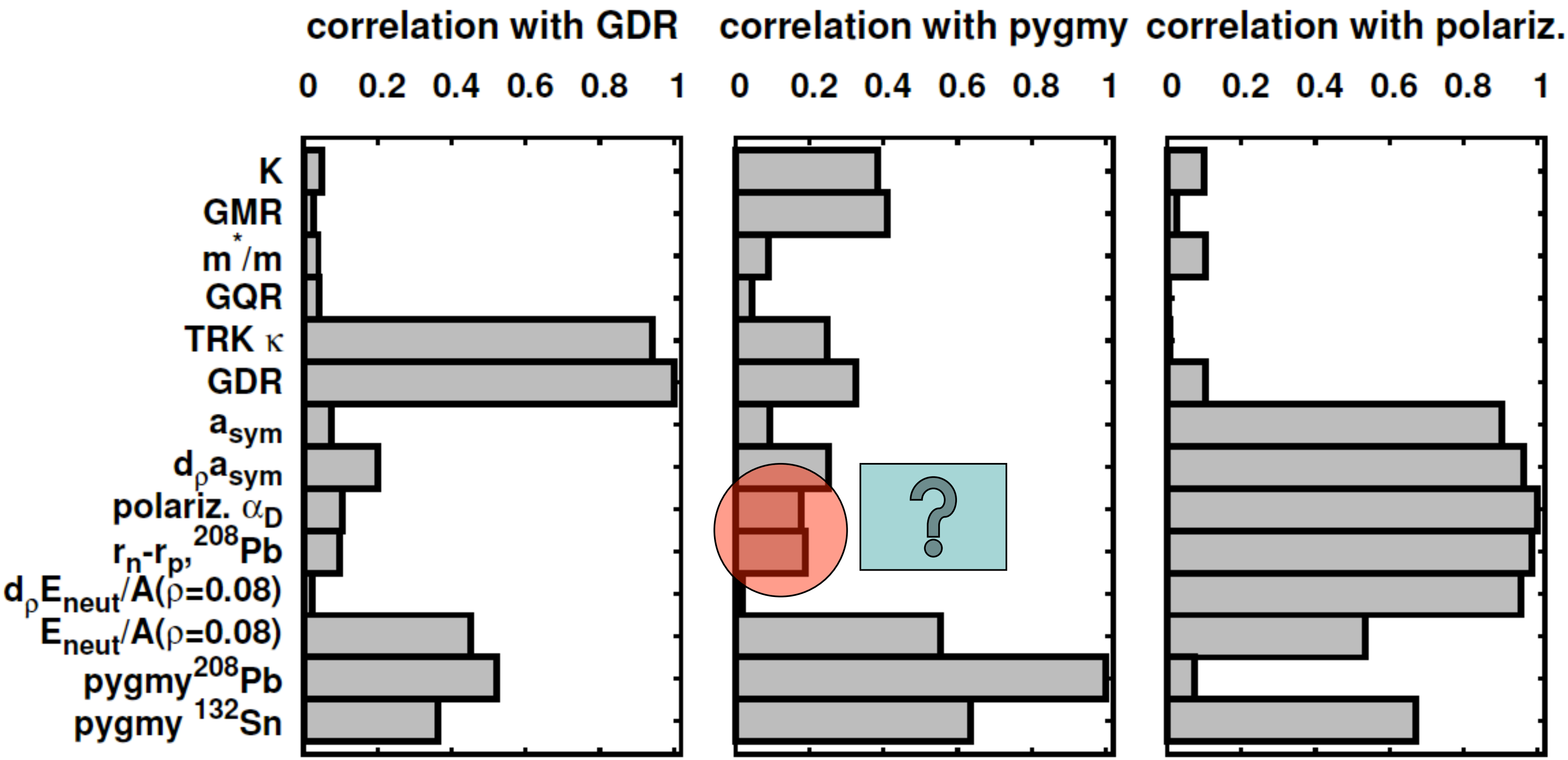
Covariance analysis and correlations between observables

In general, the Pearson product-moment correlation reflects the noisiness and direction of a linear relationship (top row), but not the slope of that relationship (middle), nor many aspects of nonlinear relationships (bottom)



$c_{AB} > 0.5$ is considered as strong correlation

Covariance analysis and correlations between observables



P.-G. Reinhard, W. Nazarewicz, “Co-Variances in connection to χ^2 fitting” (2011)

Covariance analysis based on RNEDF

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int}$$

- the Lagrangian of the free nucleon: $\mathcal{L}_N = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi$

- the Lagrangian of the free meson fields and the electromagnetic field:

$$\begin{aligned}\mathcal{L}_m &= \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &\quad -\frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

- minimal set of interaction terms:

$$\mathcal{L}_{int} = -\bar{\psi}\Gamma_\sigma\sigma\psi - \bar{\psi}\Gamma_\omega^\mu\omega_\mu\psi - \bar{\psi}\vec{\Gamma}_\rho^\mu\vec{\rho}_\mu\psi - \bar{\psi}\Gamma_e^\mu A_\mu\psi.$$

with the vertices:

$$\Gamma_\sigma = g_\sigma, \quad \Gamma_\omega^\mu = g_\omega\gamma^\mu, \quad \vec{\Gamma}_\rho^\mu = g_\rho\vec{\tau}\gamma^\mu, \quad \Gamma_e^m = e\frac{1-\tau_3}{2}\gamma^\mu$$

Density dependence of the model

A) an effective density dependence introduced through a non-linear potential:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

model parameters: meson masses m_σ, m_ω, m_ρ, meson-nucleon coupling constants g_σ, g_ω, g_ρ, nonlinear self-interactions coupling constants g_2, g_3

PARAMETERIZATION: NL3

B) the medium dependent meson-nucleon couplings g_σ, g_ω, g_ρ

-> functions of vector density: $\rho_v = \sqrt{j_\mu j^\mu}$ $\quad j_\mu = \bar{\psi}\gamma_\mu\psi$

$$g_i(\rho) = g_i(\rho_{\mathrm{sat}})f_i(x)$$

$$f_i(x) = a_i\frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2}$$

$$i = \sigma, \omega$$

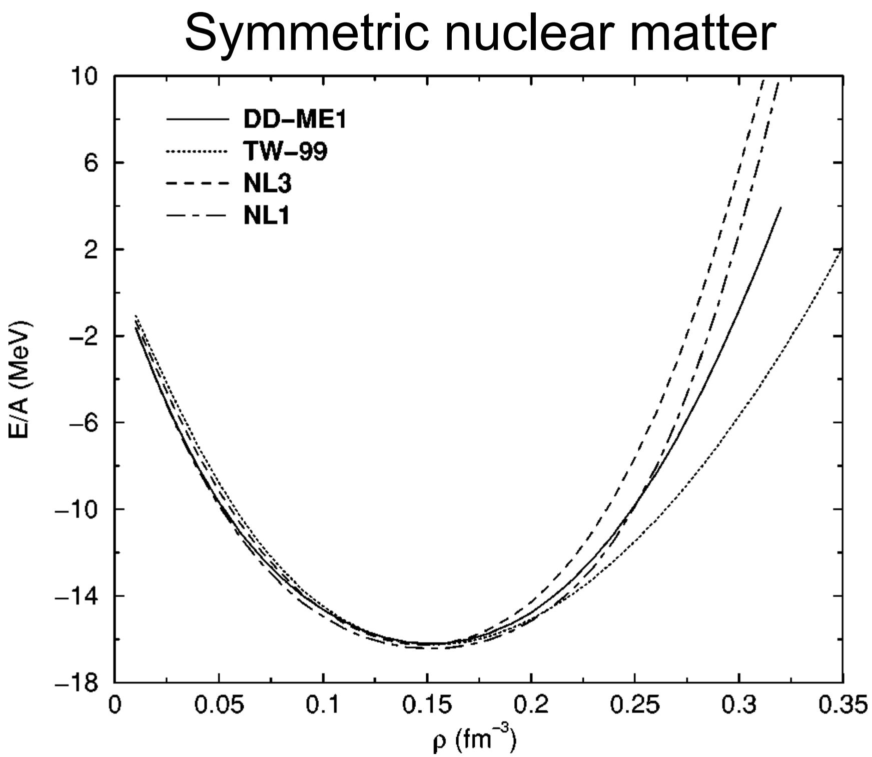
$$g_\rho(\rho) = g_\rho(\rho_{\mathrm{sat}})\ \mathrm{e}^{-a_\rho(x-1)}$$

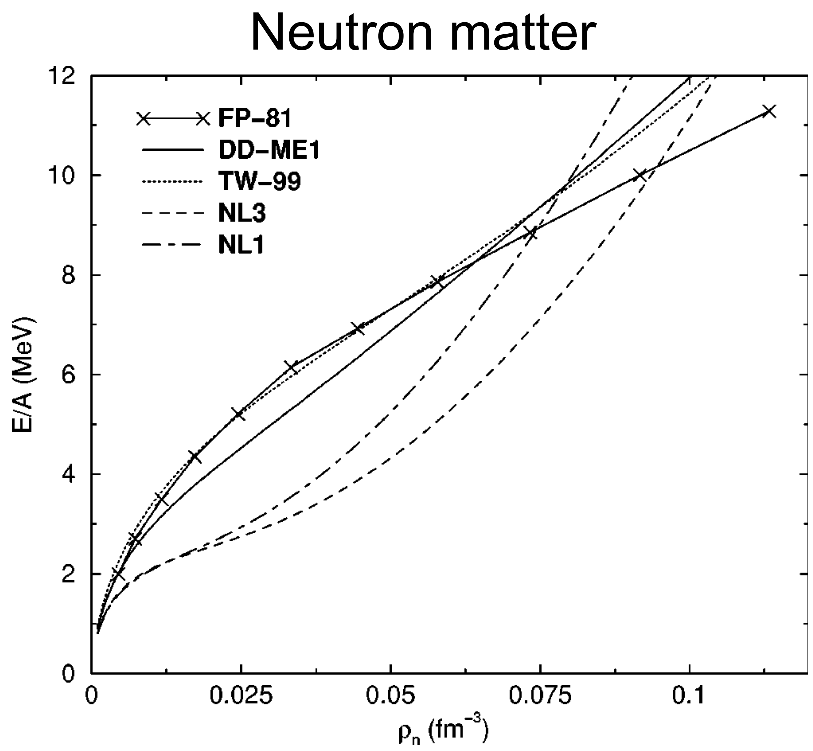
$$x = \rho/\rho_{\mathrm{sat}}$$

model parameters: meson masses + parameters of vertex functions

PARAMETERIZATION: DD-ME2

Nuclear matter properties for the covariance analysis





	DD-ME1	TW-99	NL3	NL1
ρ_{sat} (fm^{-3})	0.152	0.153	0.149	0.153
E/A(MeV)	−16.20	−16.25	−16.25	−16.42
K_0 (MeV)	244.5	240.0	271.8	211.3
m*	0.578	0.556	0.60	0.57
a_4 (MeV)	33.1	32.5	37.9	43.7
p_0 (MeV/fm^3)	3.26	3.22	5.92	7.0
ΔK_0 (MeV)	−128.5	−126.5	52.1	67.3

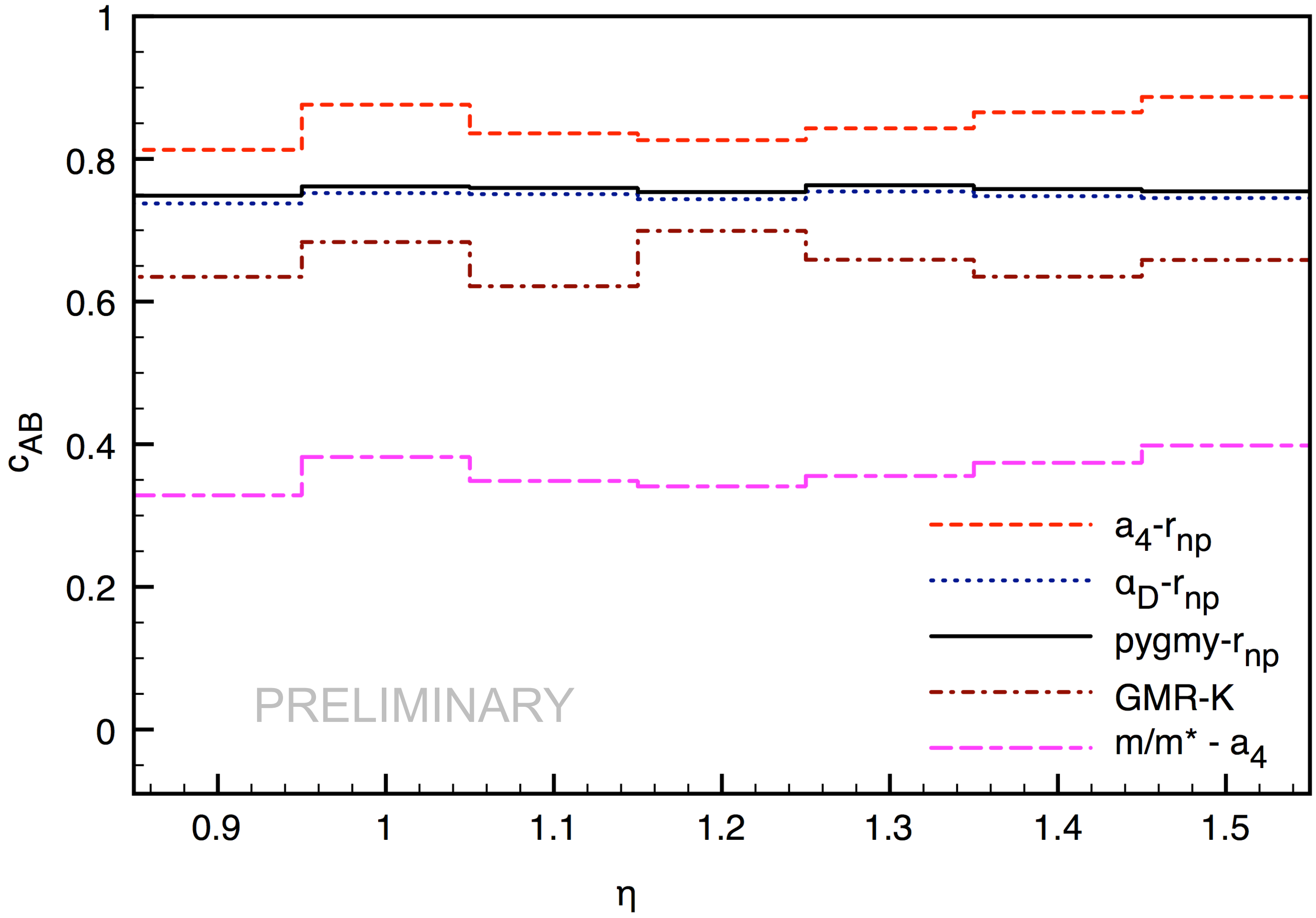
	DD-ME2	DD-ME1
ρ_{sat} (fm^{-3})	0.152	0.152
E/A (MeV)	−16.14	−16.20
K_0 (MeV)	250.89	244.5
m^*	0.572	0.578
a_4 (MeV)	32.3	33.1

Correlations c_{AB} between various quantities

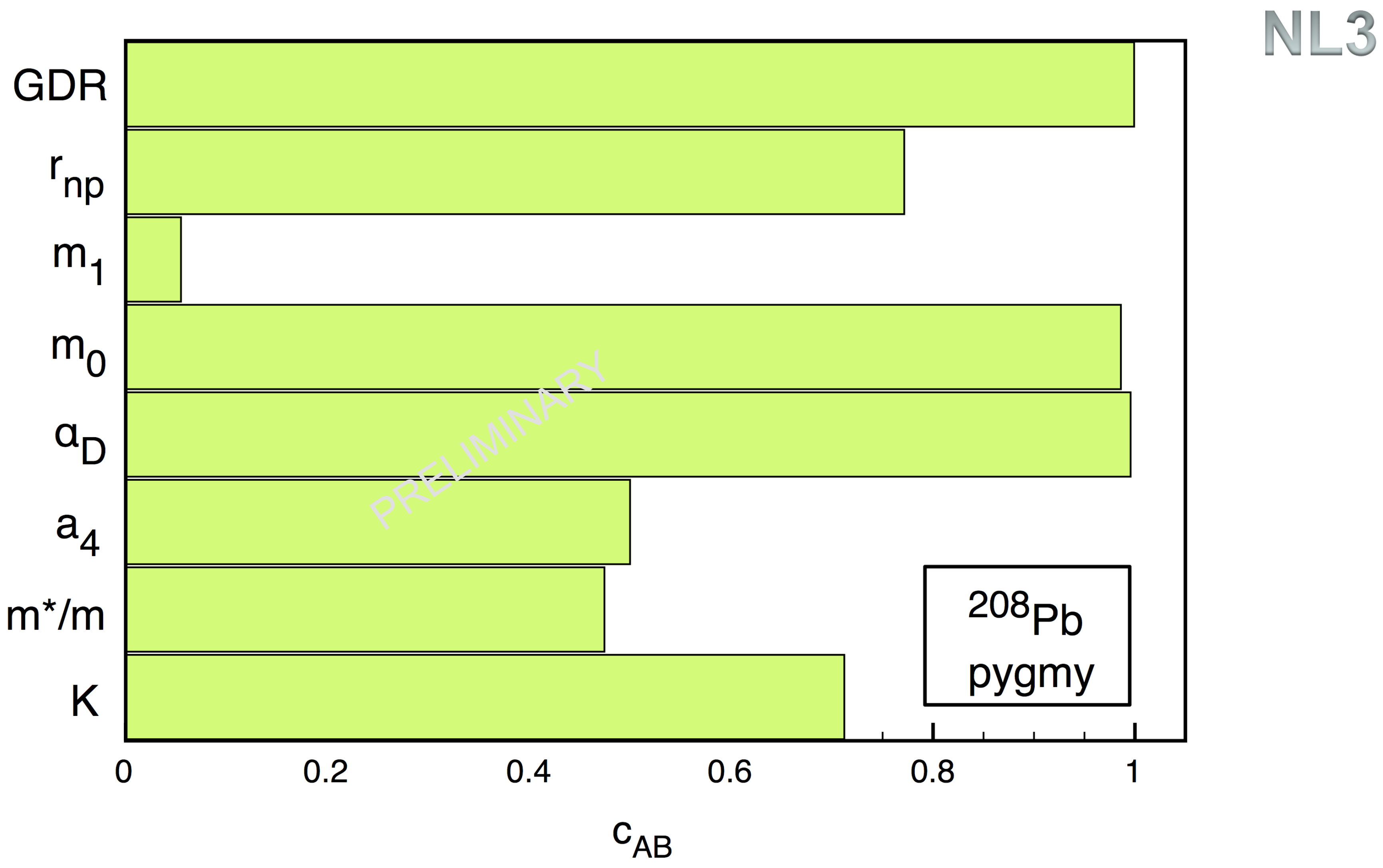
$$p_i = p_{0,i} + \eta \delta p_i$$

$$\chi^2(\mathbf{p}) - \chi_0^2 = 1 \longrightarrow \eta = 1$$

NL3

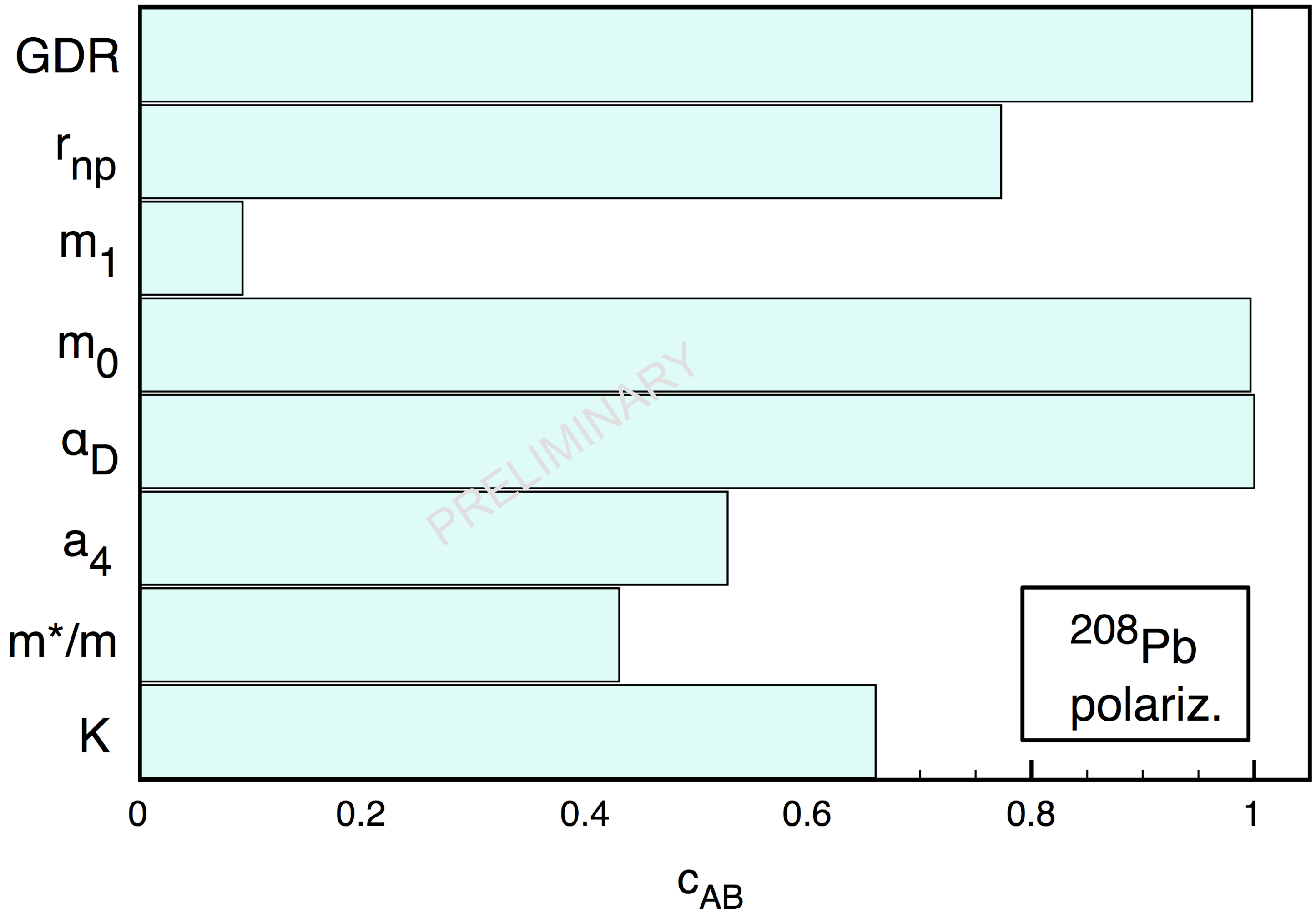


Correlations c_{AB} between pygmy strength and various quantities



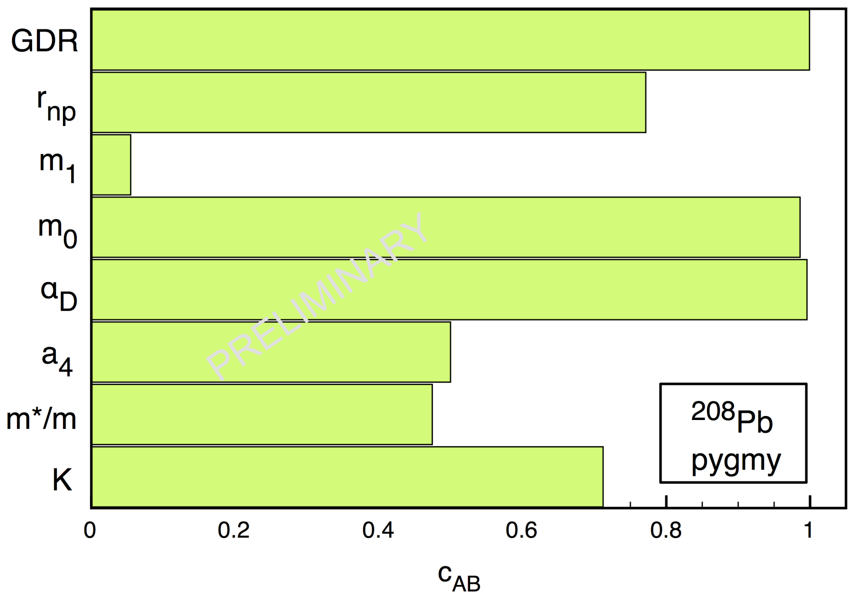
Correlations c_{AB} between dipole polarizability and various quantities

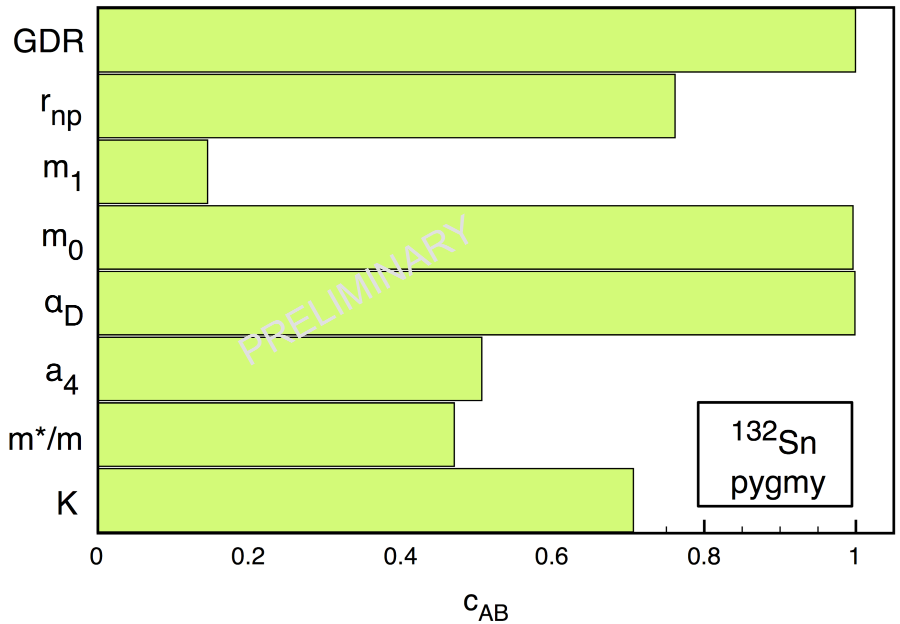
NL3



Correlations c_{AB} for the pygmy strength in ^{132}Sn and ^{208}Pb

NL3





CONCLUSIONS

Correlations between the pygmy dipole strength and neutron skin:

Physical arguments show the link between pygmy dipole resonances and the neutron skin (supported by this symposium talks).

On the other hand, statistical analysis based on covariances and Skyrme SV-min interaction indicates that dipole polarizability represents an observable correlated to the neutron skin, while correlations between pygmy strength and neutron skin are absent (P.-G. Reinhard, W. Nazarewicz).

Preliminary NL3-based covariance analysis indicates the same correlations between the pygmy strength and dipole polarizability with respect to the size of the neutron skin. DD-ME2 results seem rather unstable, work in progress.

How reliable are the covariance analysis and correlations? Model dependence?

Physical arguments vs. statistical analysis?