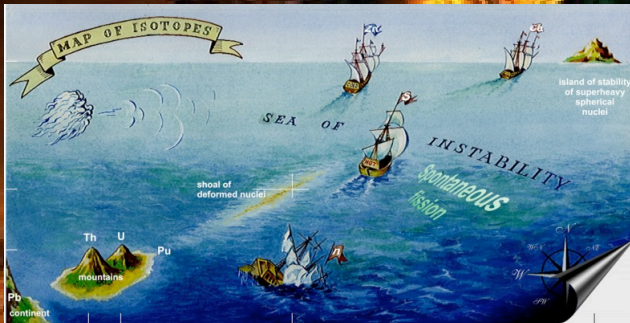


Exploring the region of SuperHeavy nuclei with NEDF



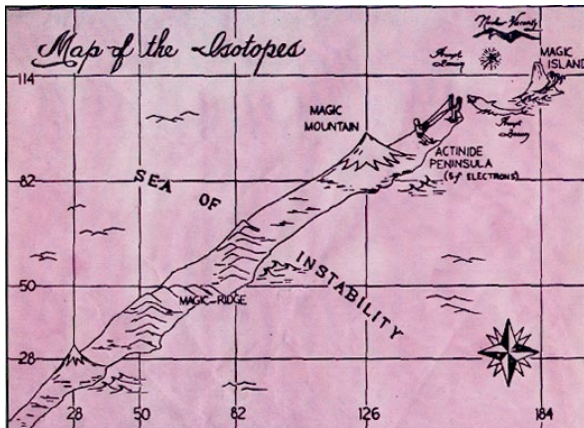
Vaia D. Prassa

Aristotle University of Thessaloniki

Department of Physics

Outline

- 1 **Motivation**
- 2 **Introduction**
 - NEDF
 - RHB
- 3 **Results**
 - Binding Energies
 - Axial Symmetry
 - Triaxial Symmetry & Beyond Mean Field
- 4 **Summary & Conclusions**



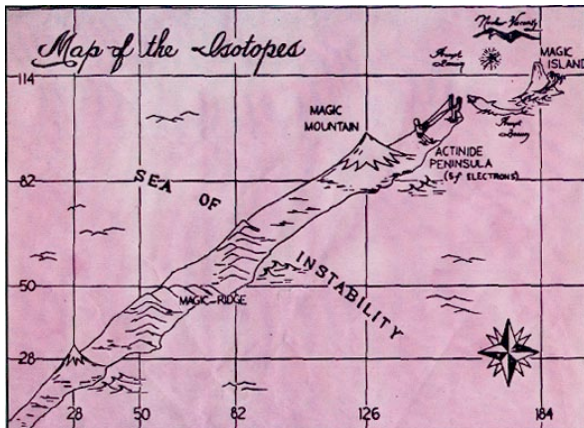
Which is the limit of the nuclear mass and charge?

SHE: Exist due to subtle quantum mechanical effects

Balance between nuclear force and coulomb field

Experimental status: Synthesis of Cn ($Z=112$) and $Z=114$ @GSI, elements up to $Z=118$ @Dubna in fusion reactions. Laboratories: GSI-FAIR, GANIL-SPIRAL2, Dubna, Jyvaskyla, Livermore, RIKEN.

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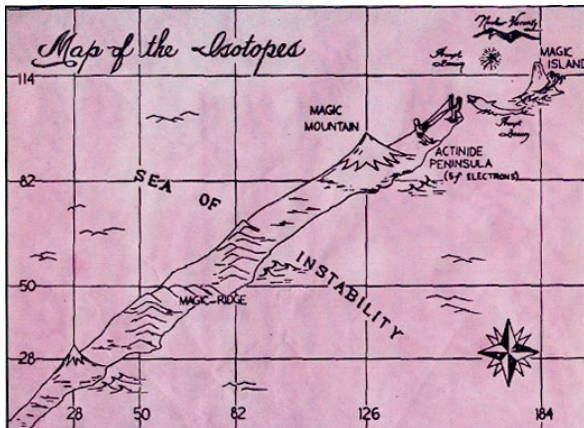
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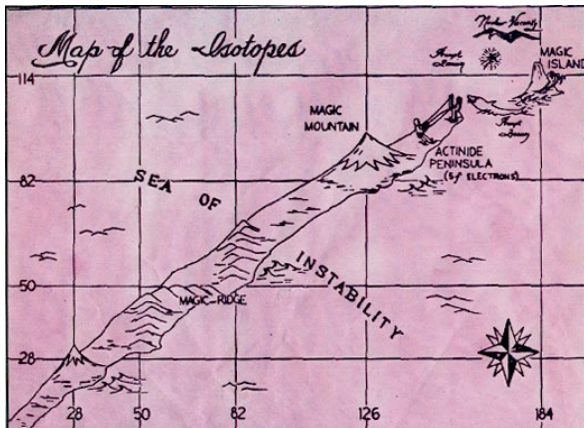
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Nuclear Energy density functionals

DFT provides a way to systematically map the many-body problem onto a one-body problem without explicitly involving inter-nucleon interactions. **Fundamental entity: Energy Functional** that depends on one-body densities and currents.

Relativistic Mean Field

In conventional QHD a nucleus is described as a system of Dirac nucleons coupled to exchange mesons through an effective Lagrangian. In MF approximation the meson-field operators are replaced by their expectation values in the nuclear ground state.

In analogy to the meson-exchange RMF phenomenology, an effective Lagrangian that includes the isoscalar-scalar, isoscalar vector and isovector-vector **four-fermion interactions**, reads

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma\partial - m)\psi - \frac{1}{2}\alpha_S(\rho)(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\rho)(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \quad (1) \\ & - \frac{1}{2}\alpha_{TV}(\rho)(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma A\frac{(1-\tau_3)}{2}\psi \end{aligned}$$

Free nucleon Lagrangian, **point-coupling interaction terms** and **coupling of the protons to the electromagnetic field**. Derivative terms accounts for the leading effects of *finite-range interactions*.

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Analysis of **open-shell nuclei** \Rightarrow **correlations** in the self-consistent RMF.
Unified treatment of the nuclear MF (particle-hole (ph)) and pairing (particle-particle (pp)) correlations. Crucial for an accurate description of ground states and properties of excited states in weakly bound nuclei.

$$E_{RMF}[\hat{\rho}, \hat{k}, \phi_m] = E_{RMF}[\hat{\rho}, \phi_m] + E_{pair}[\hat{k}], E_{pair}[\hat{k}] = \frac{1}{4} Tr[\hat{k}^* V^{pp} \hat{k}] \quad (2)$$

* Separable pairing interaction is used: $\langle k | V^{1S_0} | k' \rangle = -Gp(k)p(k')$.

Beyond MF approximation

Nuclear structure far from stability (i.e. spectra and transition probabilities..)
Restoration of the Hamiltonian symmetries that are broken on the MF level and fluctuations of collective coordinates.

Collective Bohr Hamiltonian:

$$\hat{H} = \hat{T}_{vib} + \hat{T}_{rot} + V_{coll} \quad (3)$$

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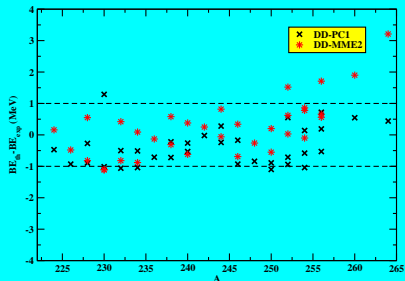
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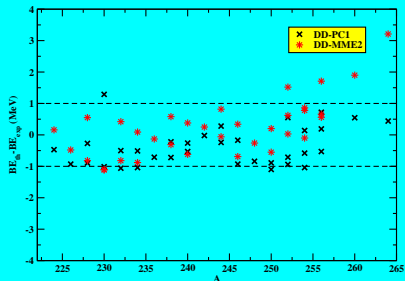


Binding energies for $A = 224 - 264$.

In most cases the r.m.s. error less than 900 keV.

Support the extension of the calculation to heavier nuclei.

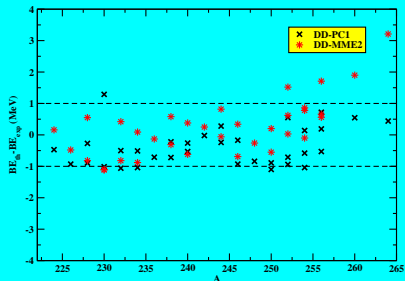
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Q_α -values

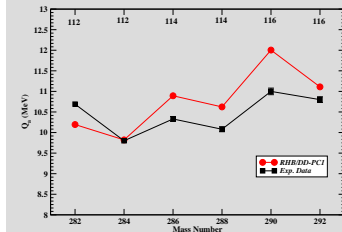
Identification of SHE by their α -decay chains.

$$Q_\alpha = BE(\text{parent}) + BE(^4\text{He}) - BE(\text{daughter})$$

Q_α -values

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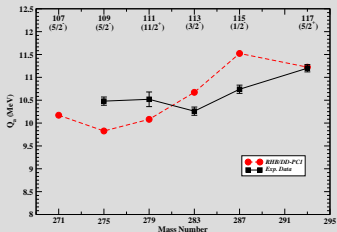
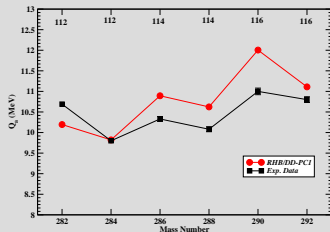
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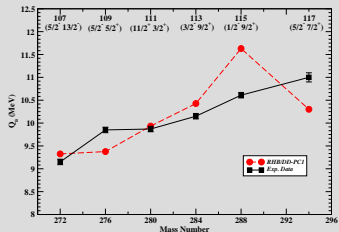
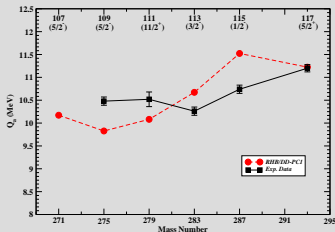
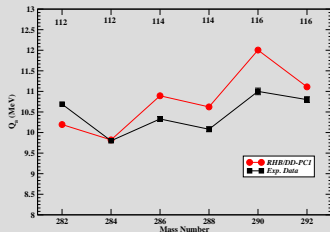
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T_a -values

The Viola-Seaborg-Sobiczewski (VSS) formula.

$$\log_{10} T_{1/2} = [aZ + b][Q/\text{MeV}]^{-1/2} + cZ + h_{\log},$$

$a=1.66175$, $b=-8.5166$, $c=-0.20228$, $d=-33.9069$

$h_{\log} = 0$, even-even

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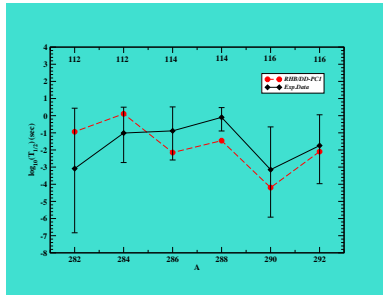
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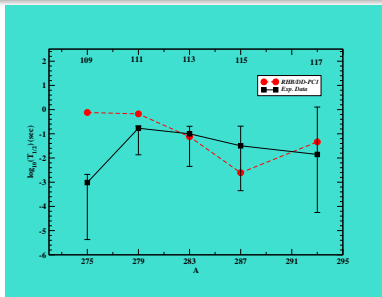
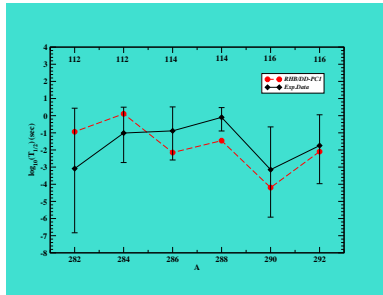
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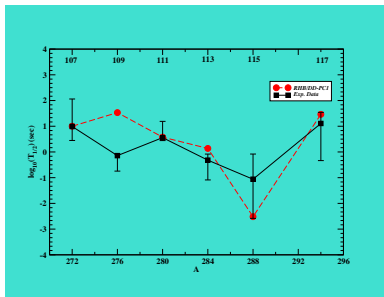
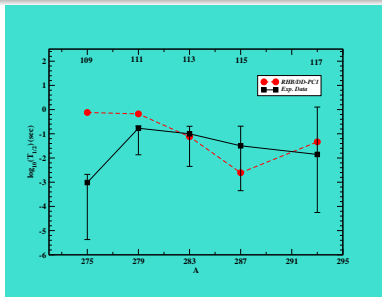
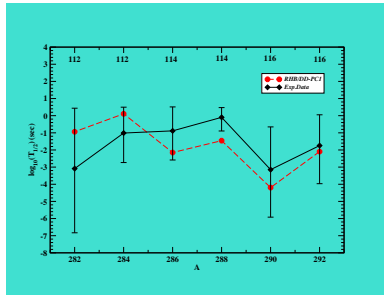
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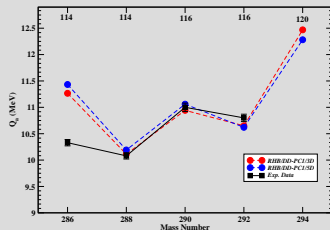
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Q_α -values

Good agreement with the experimental data.
 Maximum discrepancy 1MeV for $^{286}114$.

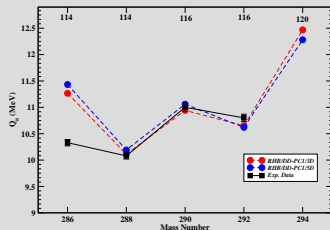
Prediction for $^{294}120$, $Q_\alpha \approx 12.4$ MeV.



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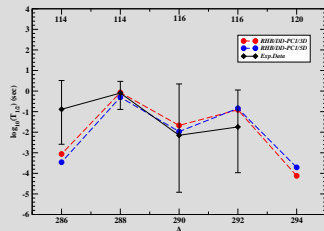
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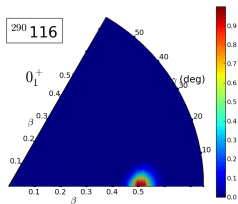
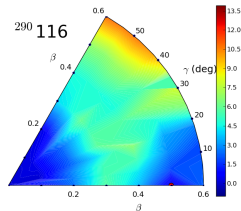
T_α -values

Sensitive to small changes of Q_α -values.
 Good agreement with exp. data.

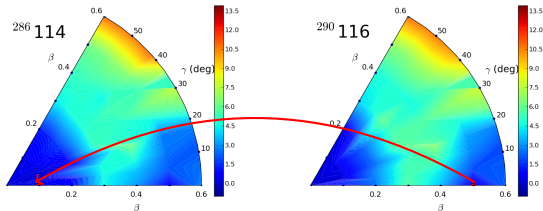


Decay chain of $^{290}_{116}$
PES- 3D MF

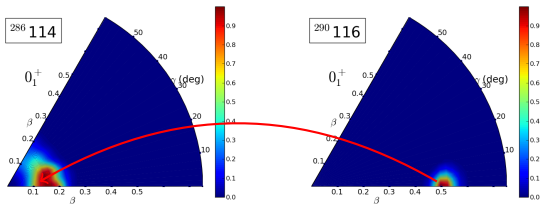
0^+ states - Beyond MF



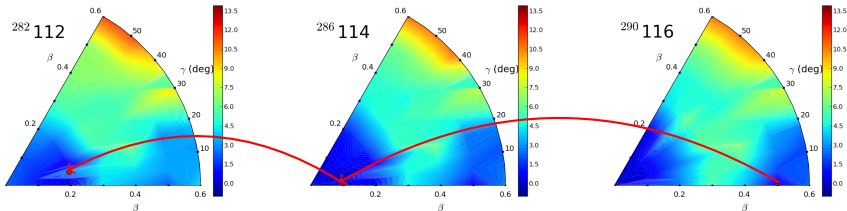
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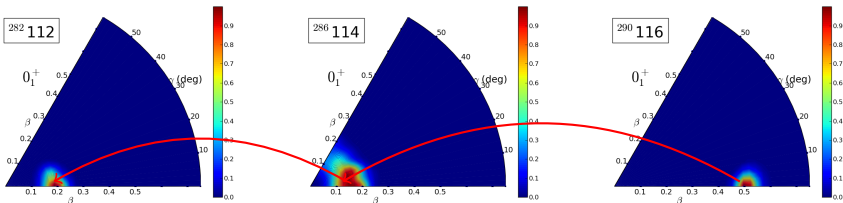
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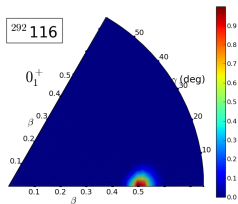
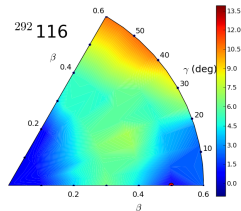


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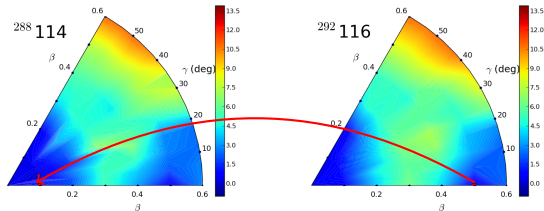


Decay chain of $^{292}\text{116}$
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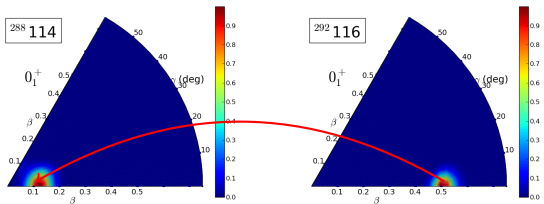
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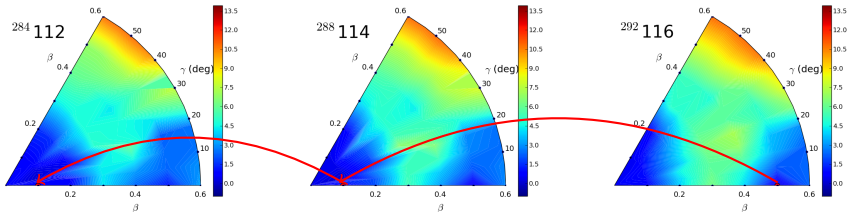
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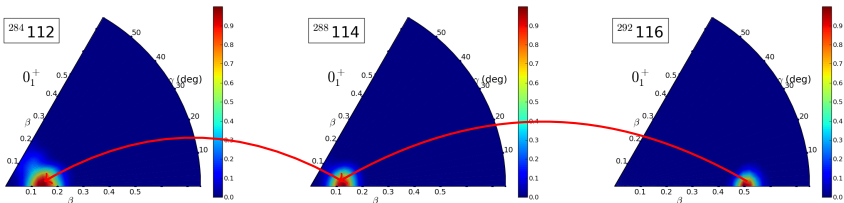
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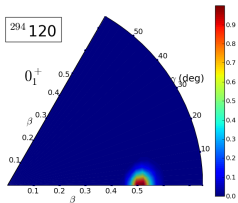
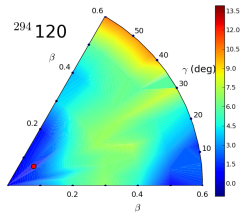


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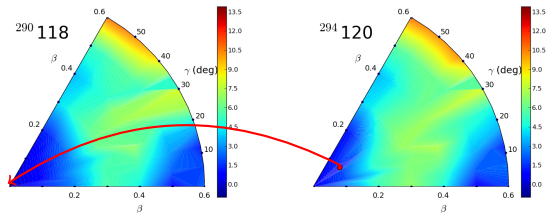


Decay chain of $^{294}\text{120}$
PES -3D MF

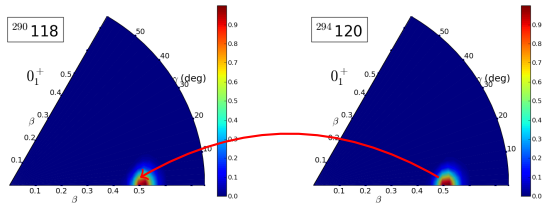
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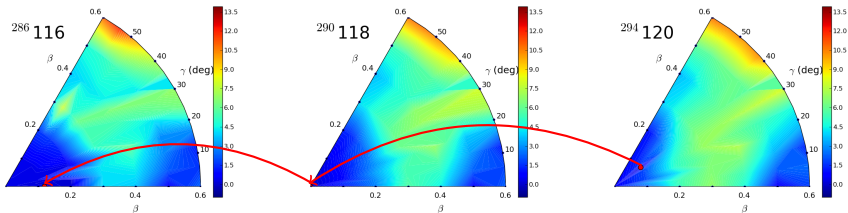
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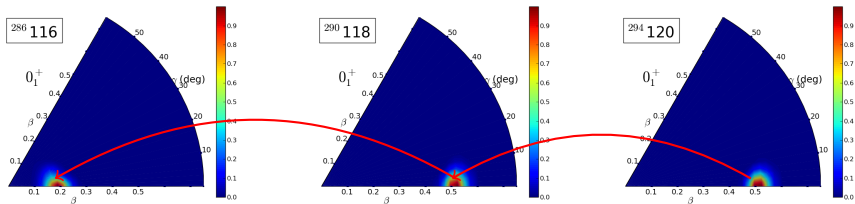
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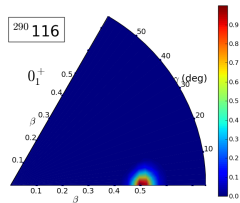


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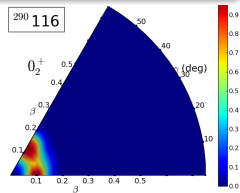


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0_2^+ states

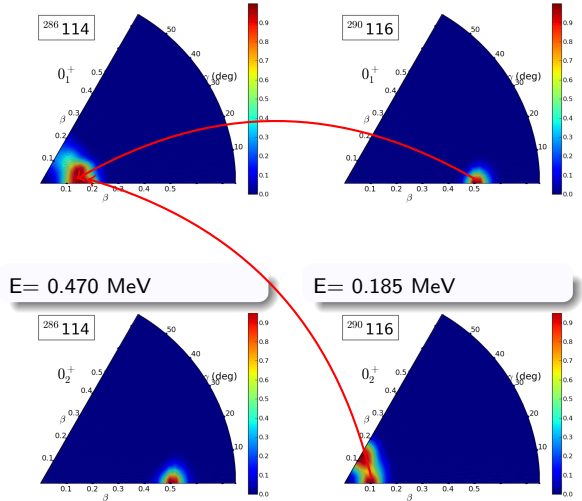


E = 0.185 MeV

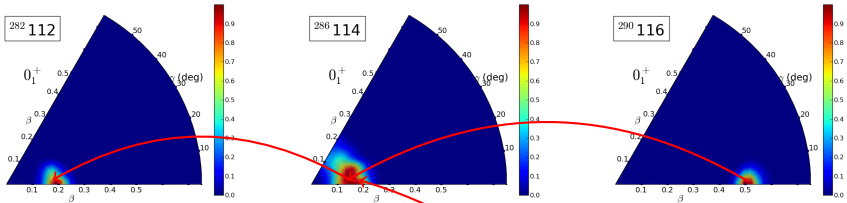


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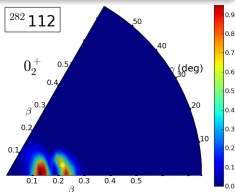


Decay chain of $^{290}\text{116}$
 0_1^+ states

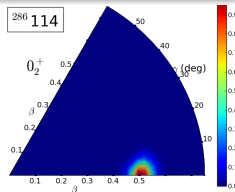


0_2^+ states

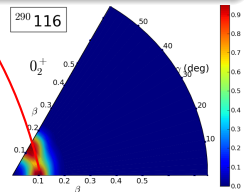
$E = 1.093$ MeV



$E = 0.470$ MeV

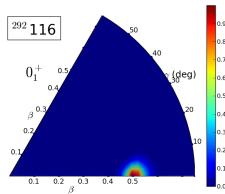


$E = 0.185$ MeV

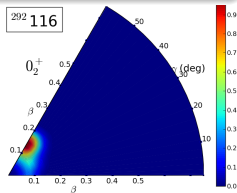


Decay chain of $^{292}\text{116}$
 0_1^+ states

0_2^+ states

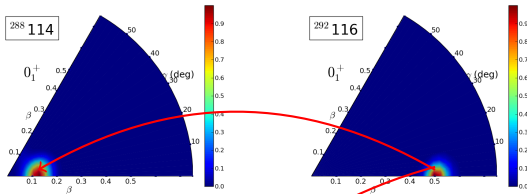


$E = 0.503$ MeV



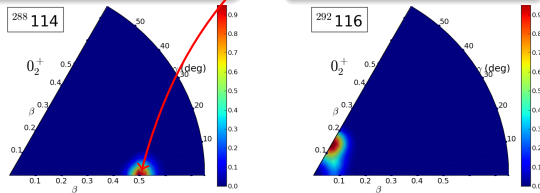
Decay chain of $^{292}\text{116}$
 0_1^+ states

0_2^+ states

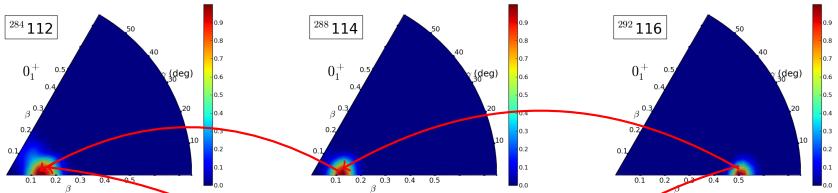


$E = 0.102 \text{ MeV}$

$E = 0.503 \text{ MeV}$

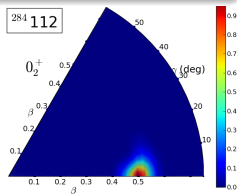


Decay chain of $^{292}\text{116}$
 0_1^+ states

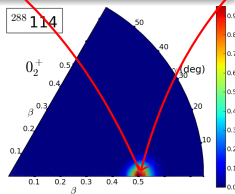


0_2^+ states

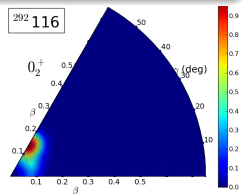
$E = 0.501$ MeV



$E = 0.102$ MeV

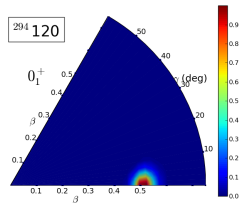


$E = 0.503$ MeV

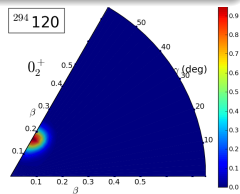


Decay chain of $^{294}\text{120}$
 0_1^+ states

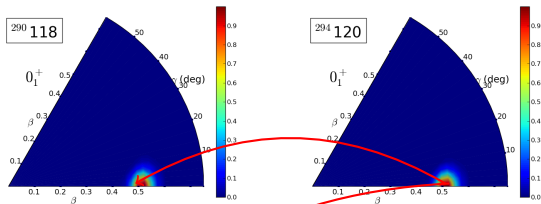
0_2^+ states



E=1.028 MeV

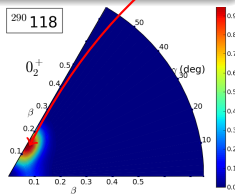


Decay chain of $^{294}_{120}$
 0_1^+ states

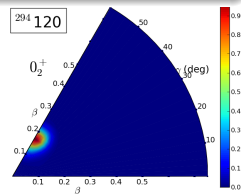


0_2^+ states

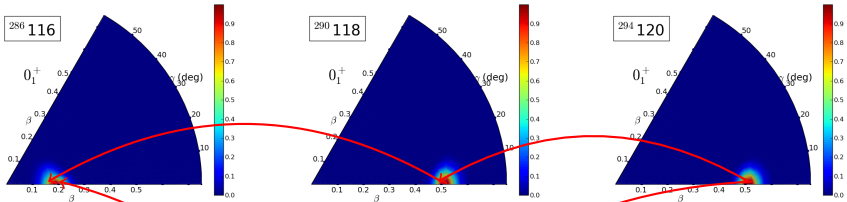
$E = 0.094$ MeV



$E = 1.028$ MeV



Decay chain of $^{294}_{120}$
 0_1^+ states

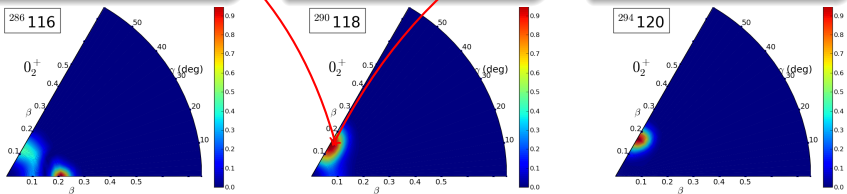


0_2^+ states

$E=0.8089$ MeV

$E= 0.094$ MeV

$E=1.028$ MeV



We presented theoretical results for SHE obtained with a self-consistent formalism based on NEDF.

The calculations involved even-even, odd-even and odd-odd nuclei with $104 < Z < 120$, in axial and triaxial schemes of RHB model and with a collective Bohr Hamiltonian, beyond the MF approximation.

Conclusions:

- **Deformation affects** play an important role in the description of SHE. Strongly deformed minima appear in the cases of isotopes $^{290}116$, $^{292}116$ and $^{294}120$.
- The inclusion of **triaxiality** gives soft or triaxial structures of those SHE.
- The calculated **Q_α -values** agree very well with the exp.data with a max discrepancy of 1MeV.
- The α -decay half-lives ($T_{1/2}$) were calculated with Viola-Seaborg-Sobiczewski formula. Good agreement with the exp.data, however they appear to be very sensitive to small changes of the Q_α -values.
- **Beyond MF calculations are necessary for the description of SHE.**



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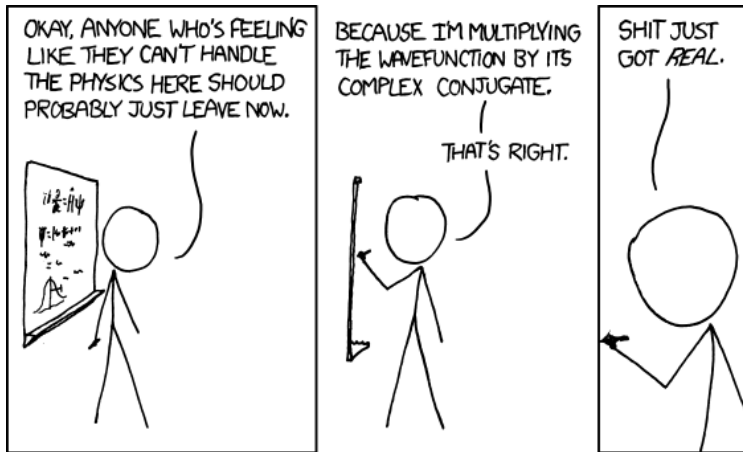
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THANK YOU FOR YOUR ATTENTION



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