

# The Core-Particle-Hole Coupling model of odd-odd nuclei with microscopic cores

L. Próchniak<sup>1</sup>, Ch. Droste<sup>2</sup>, S. G. Rohoziński<sup>2</sup> & K. Starosta<sup>3</sup>

<sup>1</sup> Maria Curie-Skłodowska University, Lublin

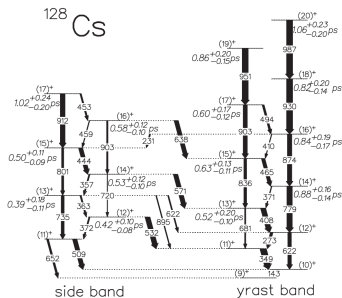
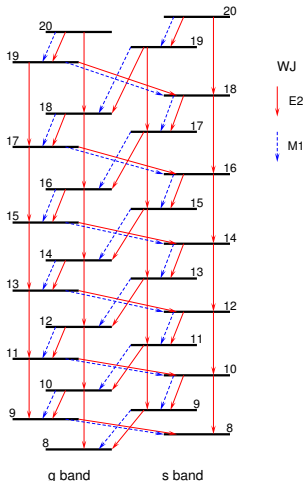
<sup>2</sup> University of Warsaw

<sup>3</sup> Simon Fraser University, Vancouver

## Outline

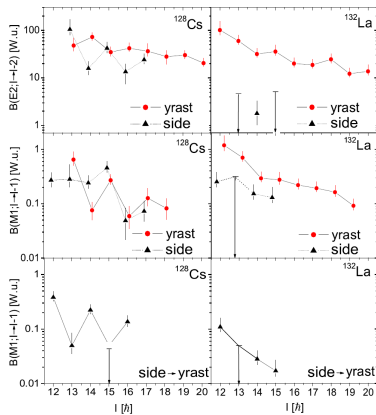
1. Properties of partner (twin, doublet) bands in odd-odd nuclei
2. The Core-Particle-Hole Coupling model
3. The  $S$  symmetry and its consequences
4. Some examples of microscopically calculated cores

## Some properties of partner bands in odd-odd nuclei



- ▶ Almost degenerate g and s bands
- ▶ Similar EM properties of g and s bands
- ▶ Staggering of  $\Delta I = 1$  transitions

## Properties of partner bands cont.



E. Grodner *et al*, Phys. Rev. Lett **97** (2006) 172501

## Core-Particle-Hole Coupling model

$$|\text{odd} - \text{odd}, i\rangle = \sum_{j,k,m} U_{jkm}^i |\text{core}, j\rangle |p, k\rangle |n, m\rangle$$

$$H_{\text{o-o}} = H_{\text{core}} - \chi Q q_p - \chi Q q_n - \chi' q_p q_n + h_p + h_n$$

$Q, q_{p,n}$  — quadrupole operators

Cores:

- ▶ Bohr Hamiltonian (FQ, full quadrupole),  $(\beta, \gamma, \Omega)$ ;  
inertial functions and potential energy:
  - given by analytic formulas
  - calculated microscopically
- ▶ rigid rotor (DF, Davydov-Filipov);  $(\Omega)$ ;  $[\tilde{\beta}, \tilde{\gamma} \text{ — fixed}]$

One-particle part:

$$p h_{11/2} \otimes n h_{11/2}^{-1}, \quad A \sim 130$$

## The Bohr Hamiltonian

$$H_{\text{GBH}}(\beta, \gamma, \Omega) = T_{\text{vib}} + T_{\text{rot}} + V$$

$$T_{\text{vib}}(\beta, \gamma) = -\frac{1}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[ \partial_{\beta} \left( \beta^4 \sqrt{\frac{r}{w}} B_{\gamma\gamma} \right) \partial_{\beta} - \partial_{\beta} \left( \beta^3 \sqrt{\frac{r}{w}} B_{\beta\gamma} \right) \partial_{\gamma} \right] + \right. \\ \left. + \frac{1}{\beta \sin 3\gamma} \left[ -\partial_{\gamma} \left( \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \right) \partial_{\beta} + \frac{1}{\beta} \partial_{\gamma} \left( \sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\beta} \right) \partial_{\gamma} \right] \right\}$$

$$T_{\text{rot}}(\beta, \gamma, \Omega) = \frac{1}{2} \sum_{k=1}^3 I_k^2(\Omega) / J_k; \quad J_k(\beta, \gamma) = 4B_k(\beta, \gamma) \beta^2 \sin^2(\gamma - 2\pi k/3)$$

$$w = B_{\beta\beta} B_{\gamma\gamma} - B_{\beta\gamma}^2; \quad r = B_x B_y B_z$$

Simplest form of the kinetic energy

$$T_{\text{kin}} = \frac{1}{2} B |\dot{\alpha}|^2 \quad \leftrightarrow \quad B_{\beta\beta} = B_{\gamma\gamma} = B_k = B, \quad B_{\beta\gamma} = 0$$

## Symmetry $S$ and its consequences

$$S = P_5 C_{pn}$$

Core part:  $P_5$  ( $\alpha$ -parity of the core)

FQ core

$$P_5 \alpha_\mu = -\alpha_\mu \quad (\text{laboratory coordinates})$$

$$P_5(\beta, \gamma, \Omega) = (\beta, \gamma \pm \pi, \Omega)$$

$$P_5(\beta, \gamma, \Omega) = (\beta, \pi/3 - \gamma, R_x(\pi/2)\Omega)$$

DF core

$$P_5 \rightarrow R_x(\pi/2)$$

Proton-neutron part:  $C_{pn}$

$$C_{pn}|(p, i)(n, k)\rangle = |(p, k)(n, i)\rangle$$

$$C_{pn}|((p, j)(n, j))^L\rangle = (-1)^{L+1}|((p, j)(n, j))^L\rangle$$

$$S^2 = (P_5 C_{pn})^2 = I$$

## Inertial functions and potential energy of the $P_5$ symmetric Hamiltonian

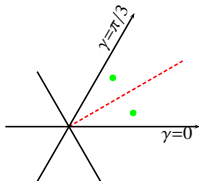
Conditions imposed by the  $P_5$  symmetry

$$f(\beta, \pi/3 - \gamma) = f(\beta, \gamma) \quad \text{for } f = B_{\beta\beta}, B_{\gamma\gamma}, B_x$$

$$B_{\beta\gamma}(\beta, \pi/3 - \gamma) = -B_{\beta\gamma}(\beta, \gamma)$$

$$B_y(\beta, \pi/3 - \gamma) = B_z(\beta, \gamma)$$

$$V(\beta, \pi/3 - \gamma) = V(\beta, \gamma)$$





## Symmetry $S$ . Quadrupole operators, angular momenta. $H_{0-0}$ Hamiltonian

Quadrupole operators and angular momenta

$$P_5 Q P_5^+ = -Q$$

$$P_5 R P_5^+ = R$$

$$C_{pn} q_{p,n} C_{pn}^+ = -q_{n,p}$$

$$C_{pn} j_{p,n} C_{pn}^+ = j_{n,p}$$

Odd-odd nucleus Hamiltonian  $H_{0-0} = H_{\text{core}} - \chi Q q_p - \chi Q q_n - \chi' q_p q_n$

$$H_{\text{core}} \text{ is } P_5\text{-symmetric} \quad \longrightarrow \quad S H_{0-0} S^+ = H_{0-0}$$

New additional label  $s = \pm 1$  for eigenstates of  $H_{0-0}$

## E2 & M1 transition operators

E2

$$T(E2) = \kappa Q + e_p q_p + e_n q_n$$

$$SQS^+ = -Q$$

$$\longrightarrow \langle J, s | T(E2) | J', s \rangle \approx 0$$

$$e_p q_p, e_n q_n \text{ — small}$$

M1

$$T(M1) = \sqrt{\frac{3}{4\pi}} \mu_N (g_R R_{\text{core}} + g_p j_p + g_n j_n)$$

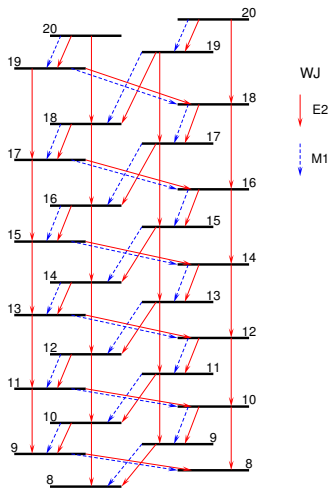
$$P_5 R_{\text{core}} P_5^+ = R_{\text{core}}$$

$$\left. \begin{array}{l} g_R - (g_p + g_n)/2 = 0 \\ \text{some other properties} \end{array} \right\}$$

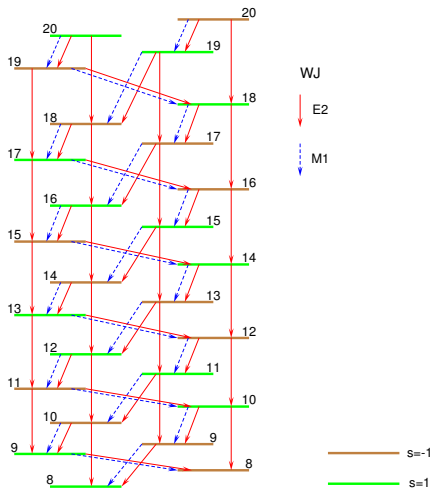
$$\longrightarrow \langle J, s | T(M1) | J', s \rangle = 0, \quad J \neq J'$$

Example (reasonable values):  $g_R = 0.44$ ,  $g_p = 1.22$ ,  $g_n = -0.21$

## Selection rules



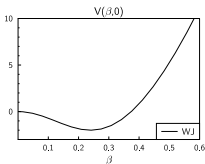
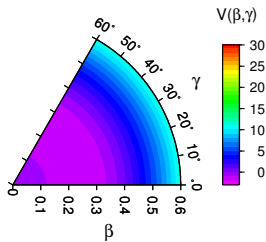
## Selection rules



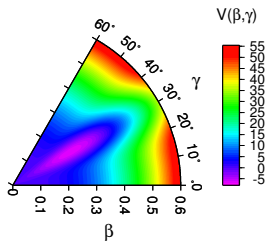
## Examples of the $S$ symmetric Hamiltonians

Wilets-Jean, Potential Well, Potential Barrier. Simple kinetic energy

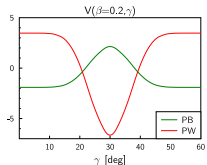
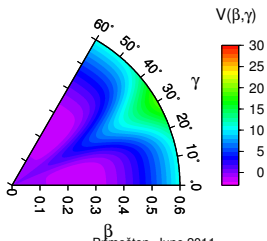
WJ



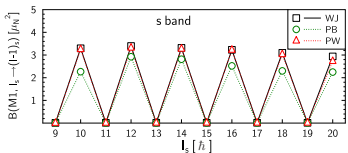
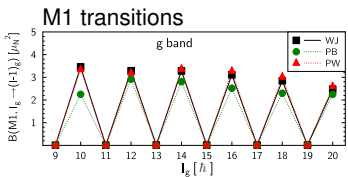
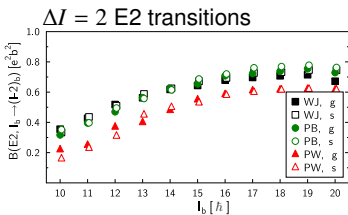
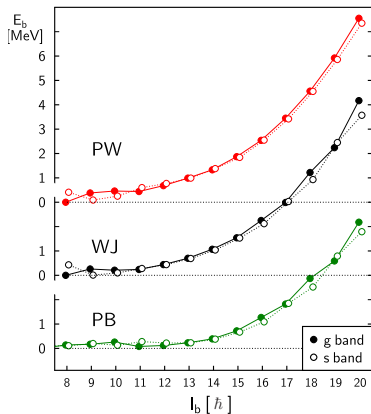
PW



PB



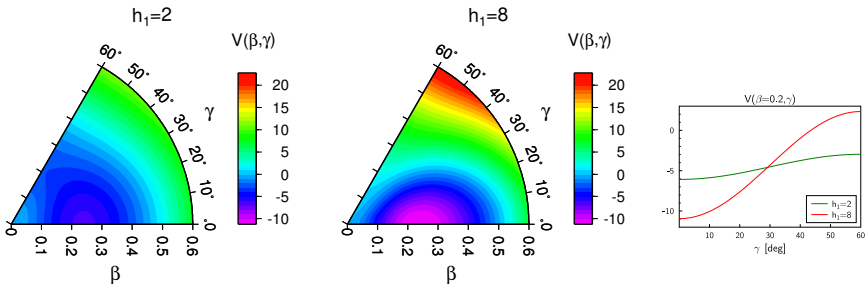
## Results for the $S$ symmetric cases



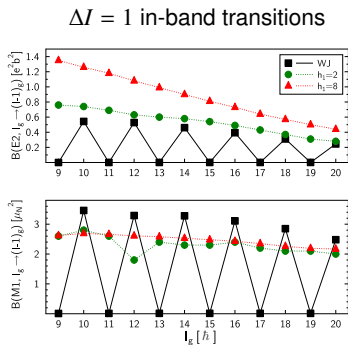
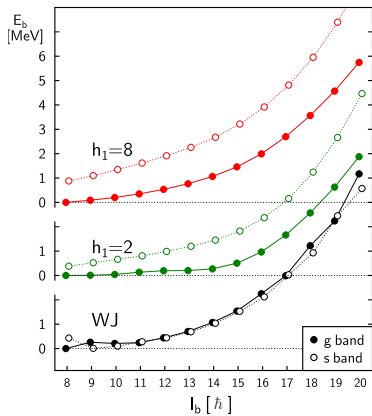
## Deviations from the $S$ symmetry

Non-symmetric core (potential or/and kinetic energy)  
Different one-particle spaces for proton and neutron

Example: non-symmetric potential energy



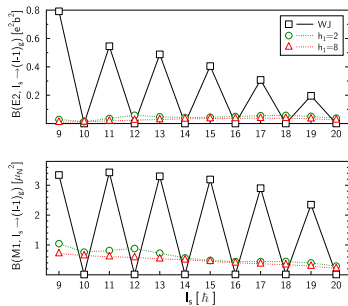
## Deviations from the $S$ symmetry cont.



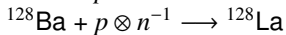
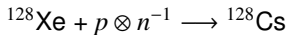


## Deviations from the $S$ symmetry cont.

### Inter-band transitions



## Microscopic cores



Mean field:

Skyrme SIII and SLy4

RMF NL3

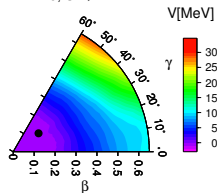
Pairing: seniority force

Kinetic energy — ATDHFB type formulas

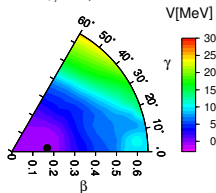
## Microscopic cores, potential energy

$^{128}\text{Xe}$

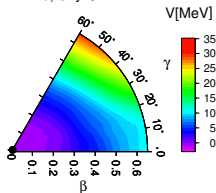
$^{128}\text{Xe}$ , SIII, sen



$^{128}\text{Xe}$ , NL3, sen

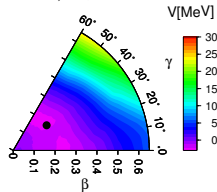


$^{128}\text{Xe}$ , SLy4, sen

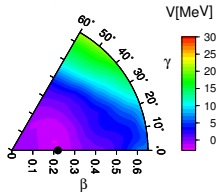


$^{128}\text{Ba}$

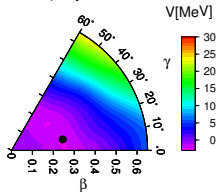
$^{128}\text{Ba}$ , SIII, sen



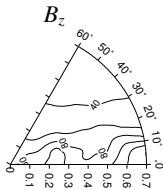
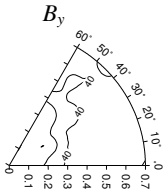
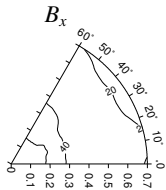
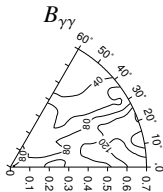
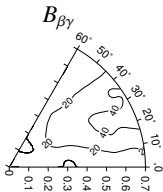
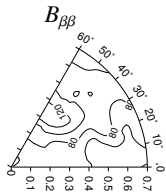
$^{128}\text{Ba}$ , NL3, sen



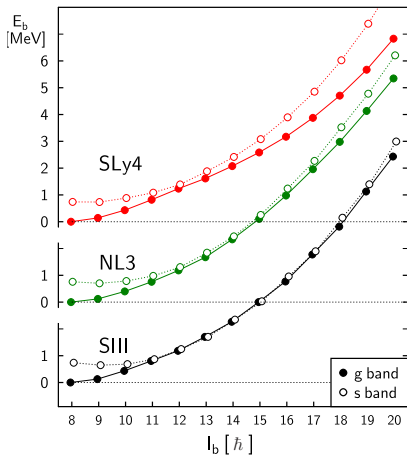
$^{128}\text{Ba}$ , SLy4, sen



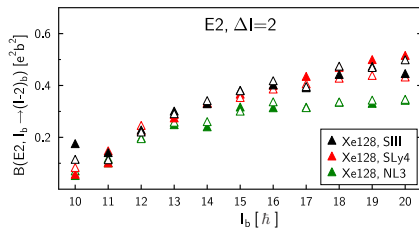
# Inertial functions, $^{128}\text{Xe}$ , Skyrme III



# Odd-odd nucleus with $^{128}\text{Xe}$ core

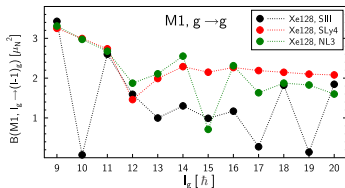


## E2 in-band transitions

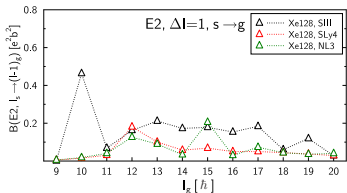
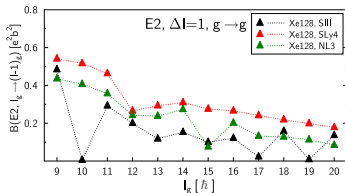
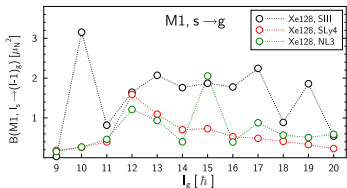


$\Delta I = 1$  M1 and E2 transitions

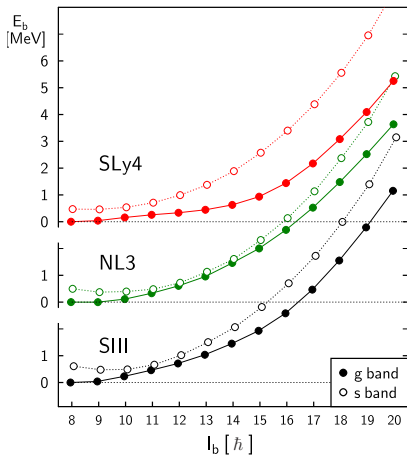
In-band



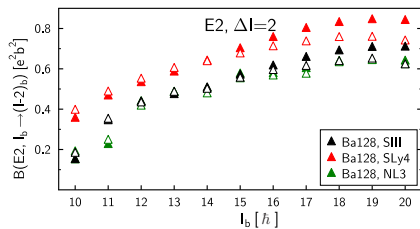
Inter-band



# Odd-odd nucleus with $^{128}\text{Ba}$ core



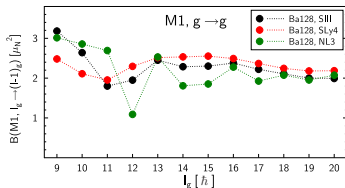
## E2 in-band transitions



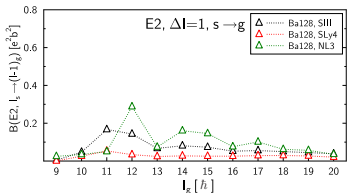
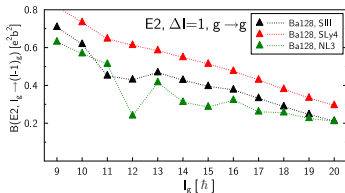
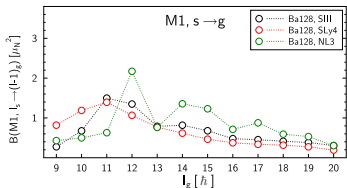
# Odd-odd nucleus with $^{128}\text{Ba}$ core cont.

$\Delta I = 1$  M1 and E2 transitions

In-band



Inter-band





## Conclusions

- ▶ Existence of the partner bands do not require rigid deformation of the core
- ▶ The  $S$  symmetry — useful in model studies, not much probable to exist in real nuclei
- ▶ It is possible to obtain partner bands with microscopic cores