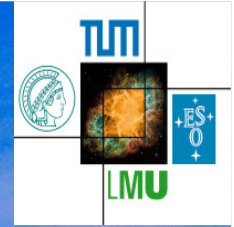


Puzzles with solutions and without solutions



Peter Ring

45 years in nuclear structure



Puzzles with solutions
and puzzles without solutions

Peter Ring

45 years in nuclear structure

Pimosten, June 9, 2011

Puzzles:

- peak in the moment of inertia (1968)
- attenuation of the Coriolis interaction (1974)
- sign change in pair transfer matrix element (1985)
- Egido poles (1983)
- divergence of self energy (1973)
- quenching of the tensor force (2008)

Three basic equations:

$$\begin{pmatrix} M^* + V & \sigma p \\ \sigma p & -M^* + V \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = \varepsilon \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{pmatrix} M^* + V & \sigma p \\ \sigma p & -M^* + V \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = \varepsilon \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\varepsilon = V \pm \sqrt{p^2 + M^{*2}}$$

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix}$$

$$E = \pm \sqrt{h^2 + \Delta^2}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\omega = \pm \sqrt{A^2 - B^2}$$

$$\begin{pmatrix} M^* + V & \sigma p \\ \sigma p & -M^* + V \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = \varepsilon \begin{pmatrix} F \\ G \end{pmatrix}$$

$$\Delta = \begin{pmatrix} & \Delta \\ -\Delta^T & \end{pmatrix}$$

Symplectic structure of Hartree-Bogoliubov canonical basis

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

Three basic equations:

$$\begin{pmatrix} M^* + V & \sigma p \\ \sigma p & -M^* + V \end{pmatrix} \begin{pmatrix} F & \bar{G} \\ G & \bar{F} \end{pmatrix} = \begin{pmatrix} F & \bar{G} \\ G & \bar{F} \end{pmatrix} \begin{pmatrix} \varepsilon & \\ & -\bar{\varepsilon} \end{pmatrix}$$

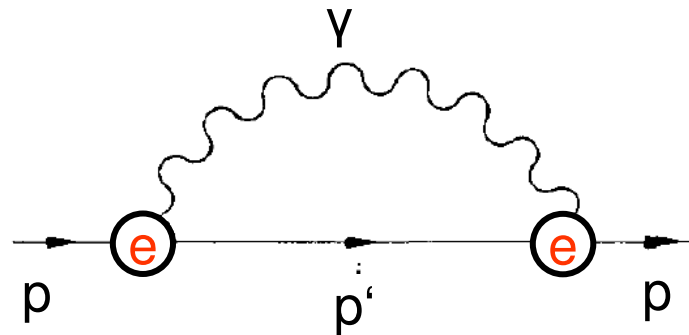
$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U & V^* \\ V & U^* \end{pmatrix} = \begin{pmatrix} U & V^* \\ V & U^* \end{pmatrix} \begin{pmatrix} E & \\ & -E \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X & Y^* \\ Y & X^* \end{pmatrix} = \begin{pmatrix} X & Y^* \\ Y & X^* \end{pmatrix} \begin{pmatrix} \omega & \\ & -\omega \end{pmatrix}$$

Diploma work in QCD:

$$\begin{pmatrix} m & \boldsymbol{\sigma p} \\ \boldsymbol{\sigma p} & -m \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = \varepsilon \begin{pmatrix} F \\ G \end{pmatrix}$$

Self-energy of the electron:



new anti-particle concept



F. Bopp

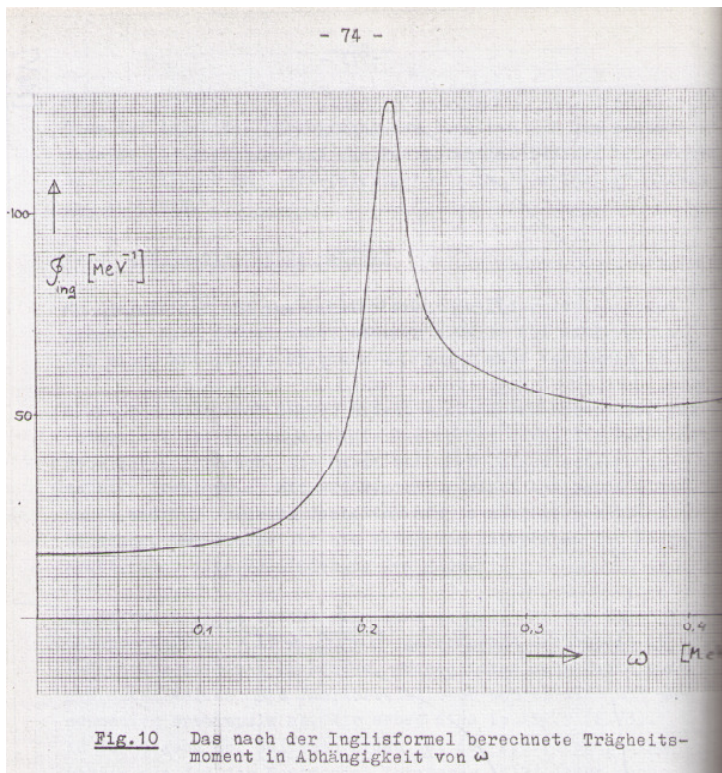


H.J. Mang

$$\langle P^I \hat{H} P^I \rangle$$

Variation after projection:

$$\langle P^I \hat{H} P^I \rangle = \langle \hat{H} \rangle + \Omega(I - \langle \hat{J}_x \rangle) - \frac{1}{2\mathcal{J}}(I - \langle \hat{J}_x \rangle)^2 - \frac{\langle \Delta \hat{J}^2 \rangle^2}{2\mathcal{J}} + \dots$$

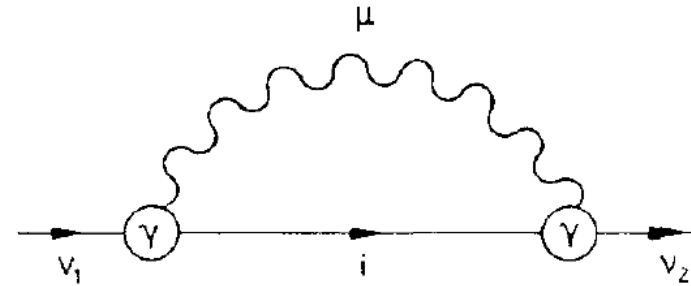


$$\langle \hat{H} \rangle \Rightarrow \langle \hat{H} - \Omega \hat{J}_x \rangle \quad \text{with} \quad \langle \hat{J}_x \rangle = I$$

P.Ring, PhD thesis TUM Munich (1969) unpublished

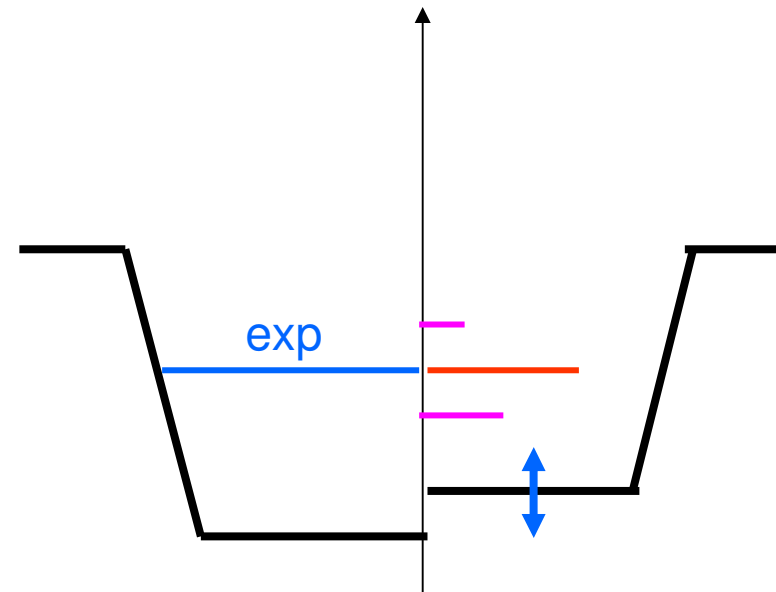
Particle-vibrational coupling:

P. R and E. Werner, NPA 211 (1973)



Adding more and more configurations μ leads to diverging results

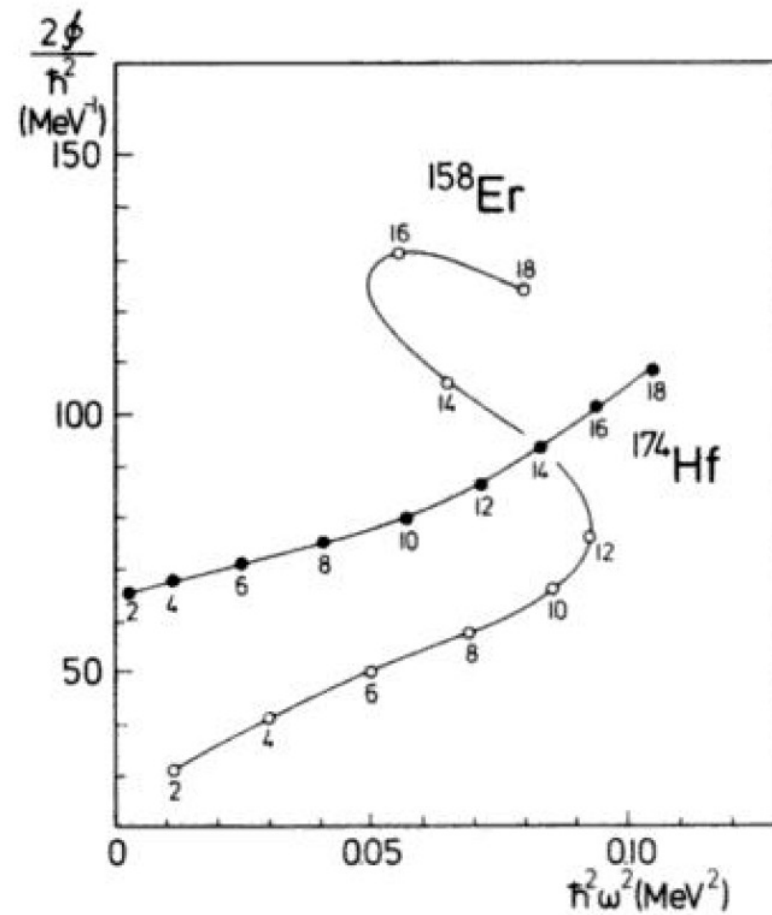
Hamamoto, Siemens (1976)
Perazzo, Reich, Sofia (1980)
Bortignon et al (1980)
Bernard, Giai (1980)
Platonov (1981)
Kamerdzhiev, Tselyaev (1986)
Litvinova, Colo,



PVC only adds energy dependence without changing the ground state

Backbending:

Johnson et al, PLB (1971):

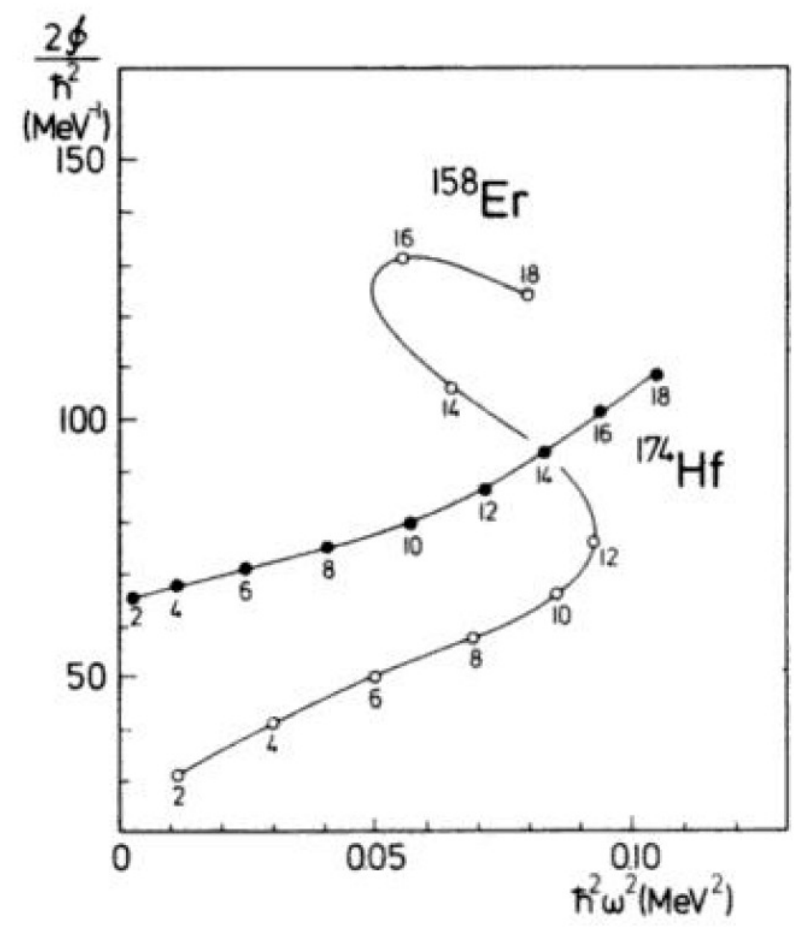
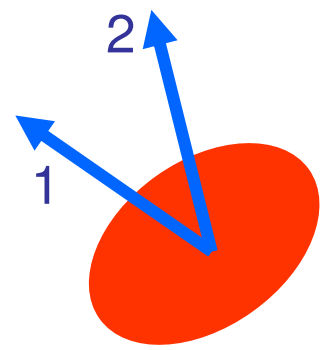


Backbending:

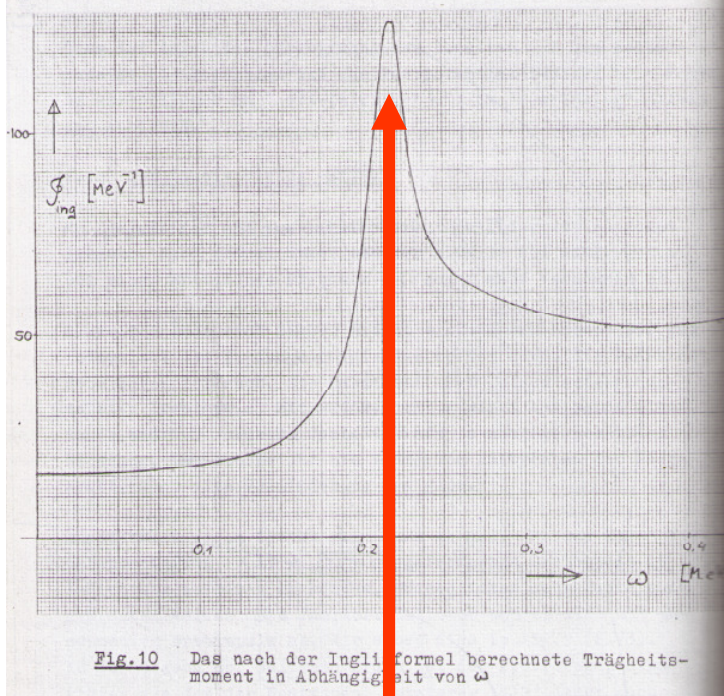
Pairing collapse ?

Shape change ?

Stephens, Simon, NPA (1972):
rotational alignment ?

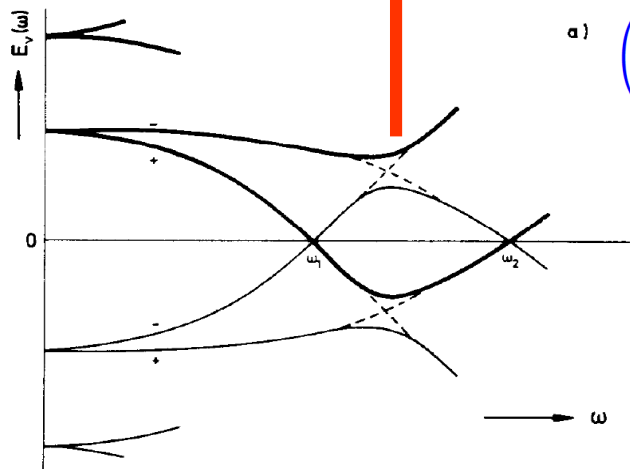


$$|\Psi^I\rangle = c_0 | \text{red ellipsoid} \rangle_I + \sum_{12} c_{12} \left[a_1^\dagger a_2^\dagger | \text{red ellipsoid} \rangle \right]_I$$



Backbending:

Nucl. Phys. A215 (1973)



$$a) \begin{pmatrix} h - \Omega J_x & \Delta \\ \Delta^\dagger & -h^* - \Omega J_x \end{pmatrix} \begin{pmatrix} U & \bar{V}^* \\ V & \bar{U}^* \end{pmatrix} = \begin{pmatrix} U & \bar{V}^* \\ V & \bar{U}^* \end{pmatrix} \begin{pmatrix} E & \\ & -\bar{E} \end{pmatrix}$$

gap-less superconductivity
 negative quasiparticle energies
 crossing frequency

Backbending

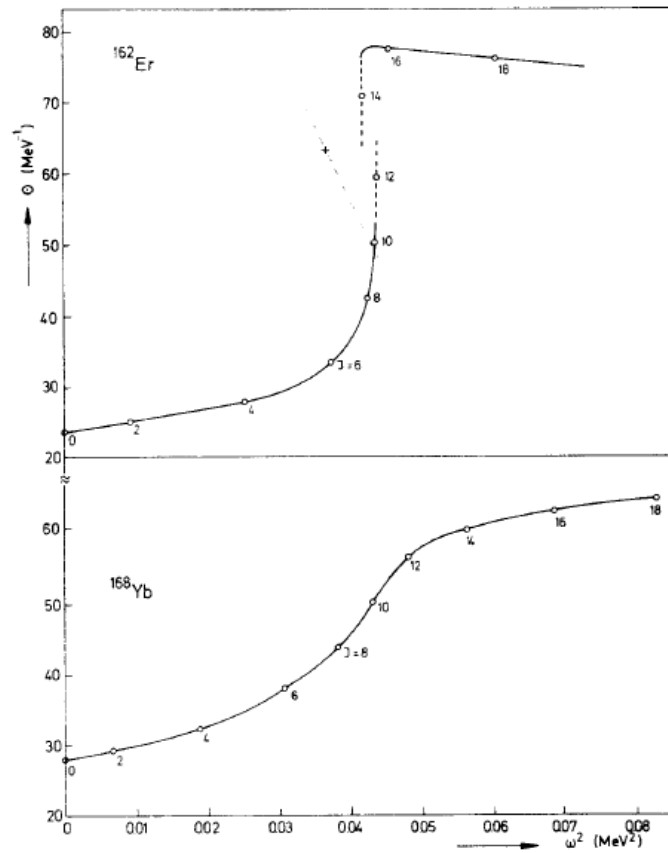
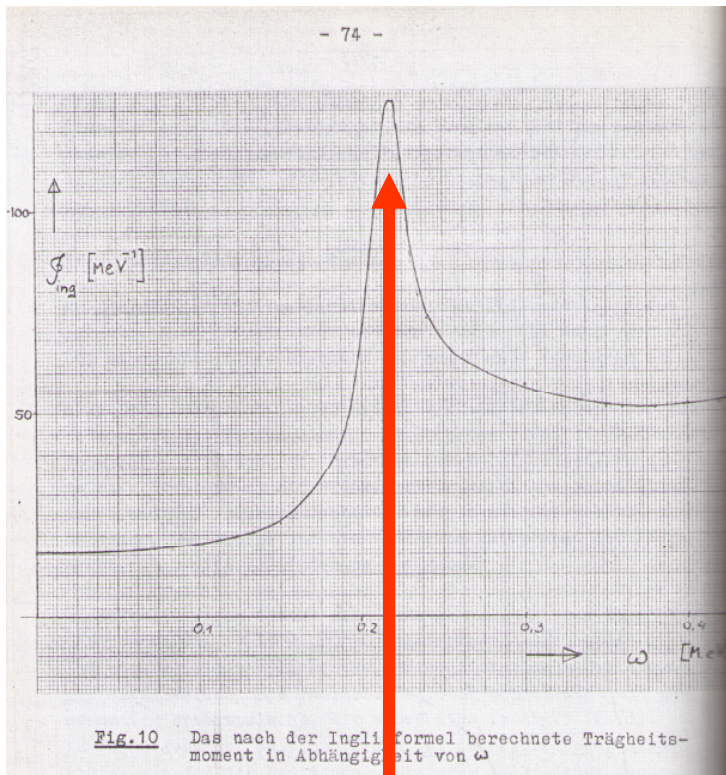
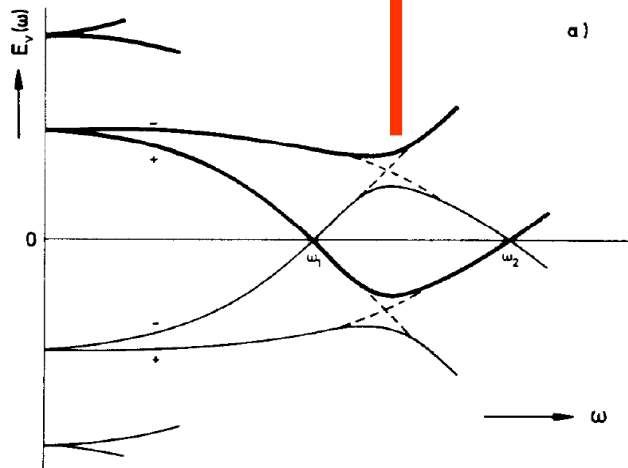
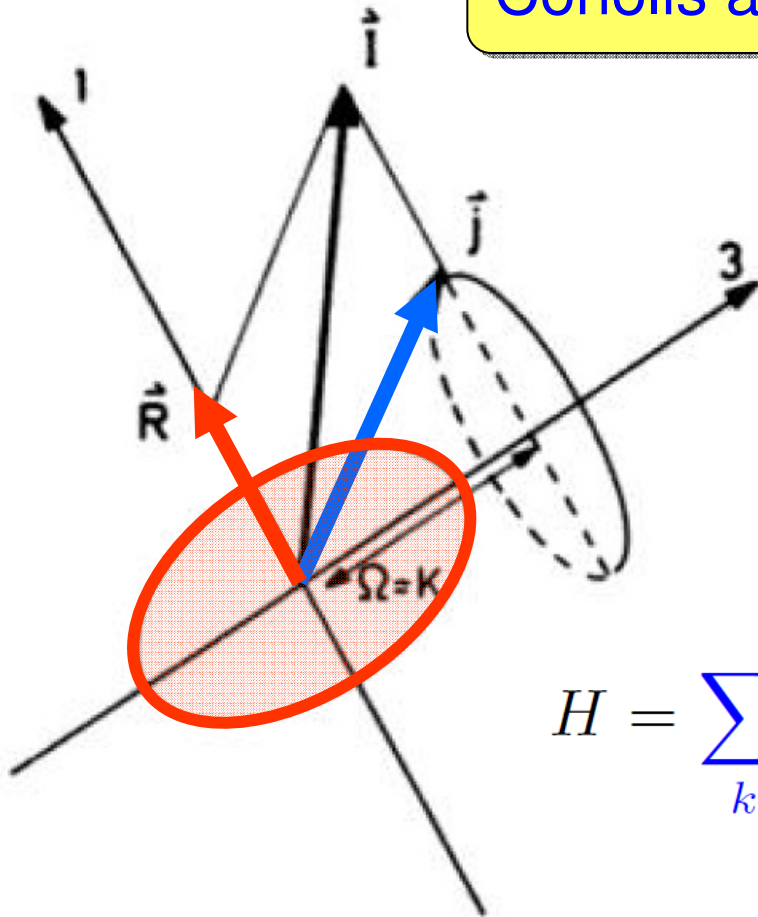


Fig. 2. Moment of inertia as a function of ω^2 : (a) ^{162}Er ; (b) ^{168}Yb .



gap-less superconductivity
 negative quasiparticle energies
 crossing frequency

Coriolis attenuation in the particle-rotor model:

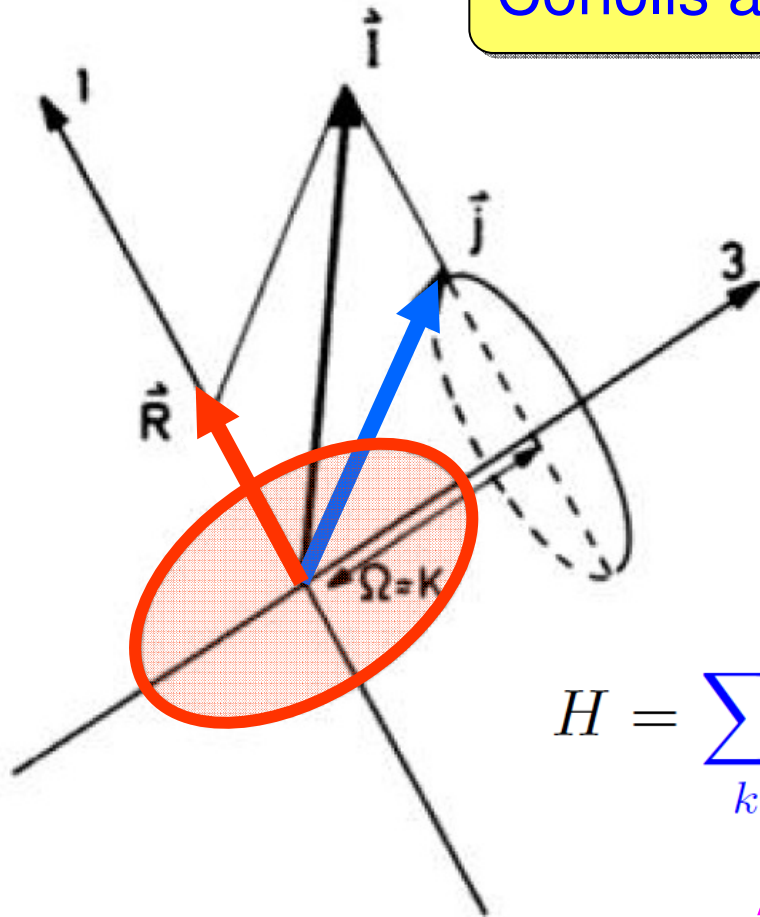


$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{R^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{(I - j)^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k - \frac{\mathbf{I} \cdot \mathbf{j}}{\mathcal{J}} + \frac{I(I + 1)}{2\mathcal{J}} + \frac{j^2}{2\mathcal{J}}$$

Coriolis attenuation in the particle-rotor model:



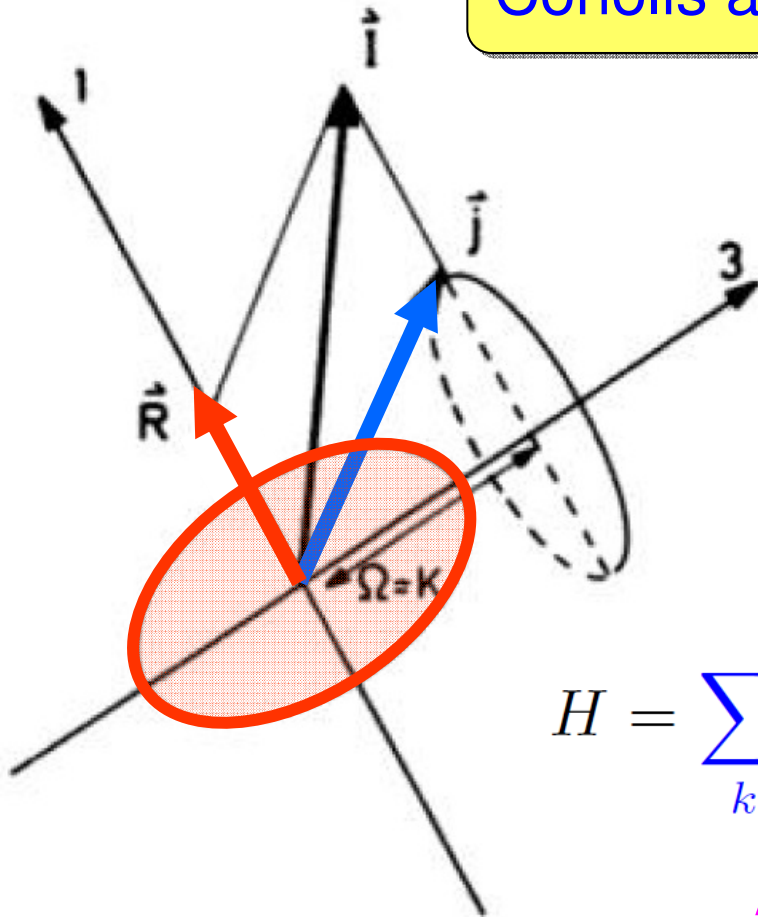
$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{R^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{(I - j)^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k - \mathbf{x} \frac{I \cdot j}{\mathcal{J}} + \frac{I(I + 1)}{2\mathcal{J}} + \frac{j^2}{2\mathcal{J}}$$

Attenuation: $\mathbf{x} \approx 0.4 - 0.5$

Coriolis attenuation in the particle-rotor model:



$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{R^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{(I - j)^2}{2\mathcal{J}}$$

$$H = \sum_k \epsilon_k a_k^\dagger a_k - \mathbf{x} \frac{I \cdot j}{\mathcal{J}} + \frac{I(I + 1)}{2\mathcal{J}} + \frac{j^2}{2\mathcal{J}}$$

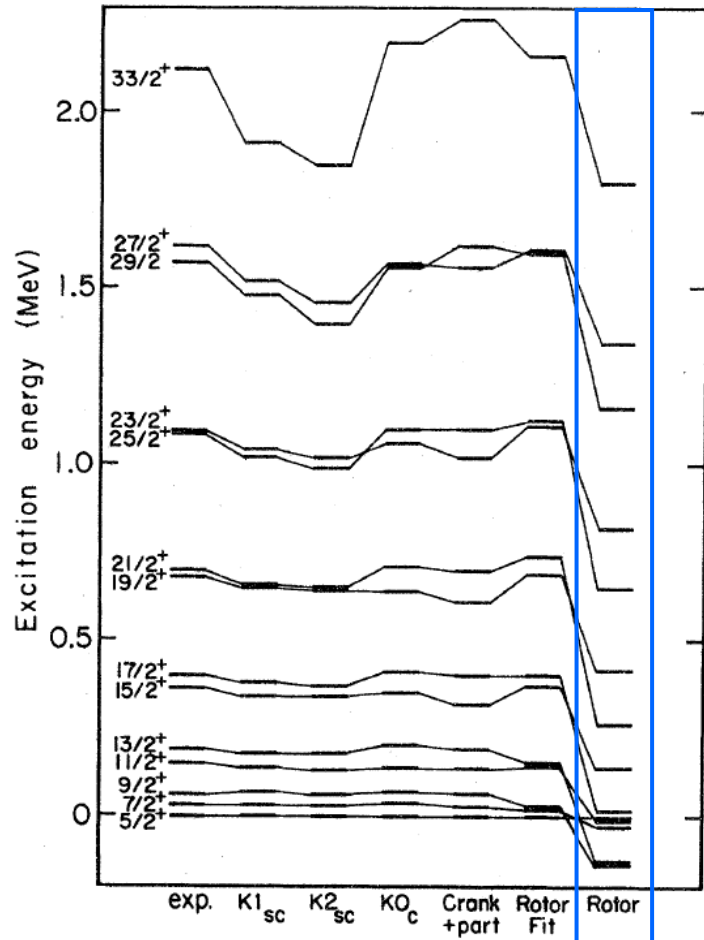
Attenuation: $\mathbf{x} \approx 0.4 - 0.5$

Cranking model:

$$H = \sum_k \epsilon_k a_k^\dagger a_k - \Omega j_x + \frac{I(I + 1)}{2\mathcal{J}}$$

^{159}Dy

Coriolis-attenuation ?

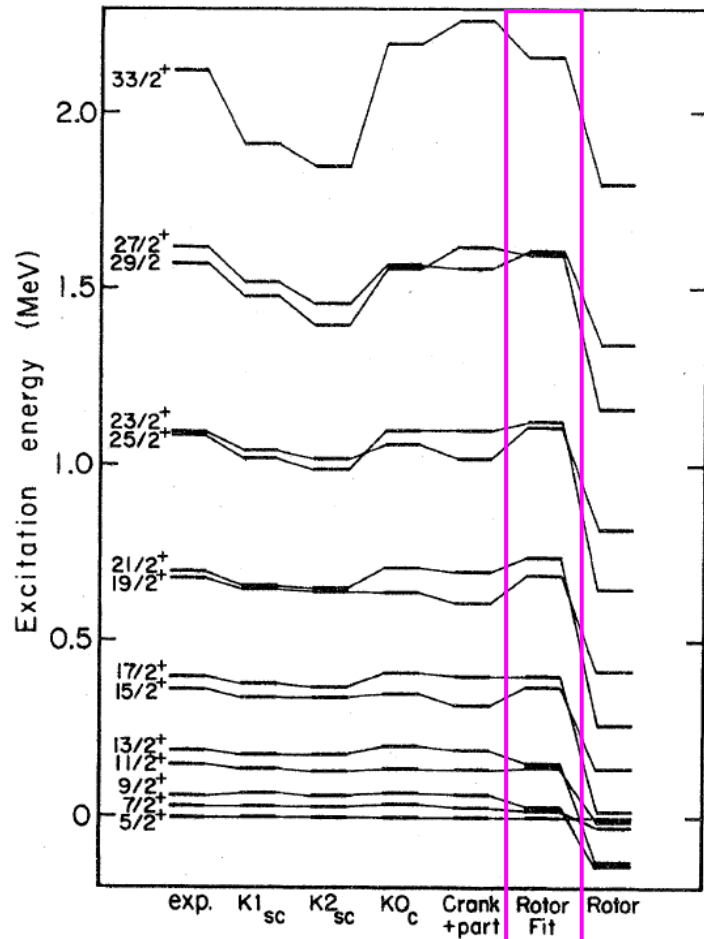


P.R., Mang, PRL. 33 (1974)

Particle+Rotor (PRM)

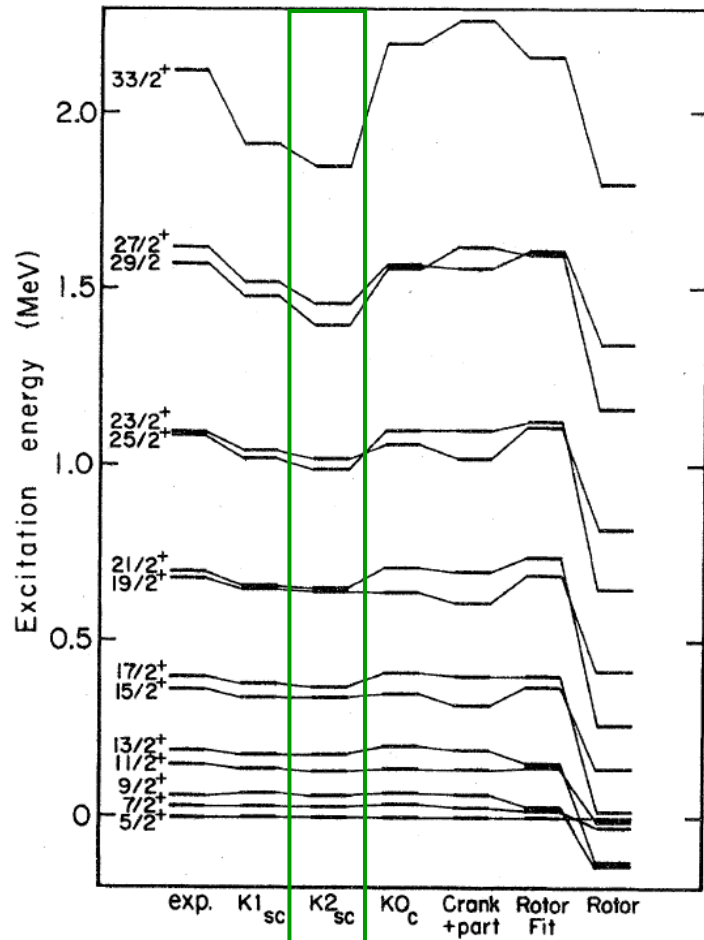
^{159}Dy

Coriolis-attenuation ?



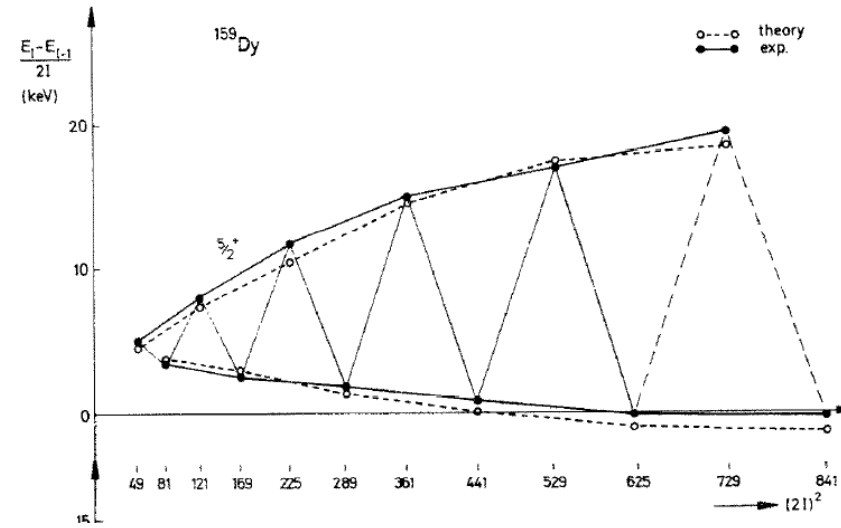
P.R., Mang, PRL. 33 (1974)

Particle+Rotor (PRM) x

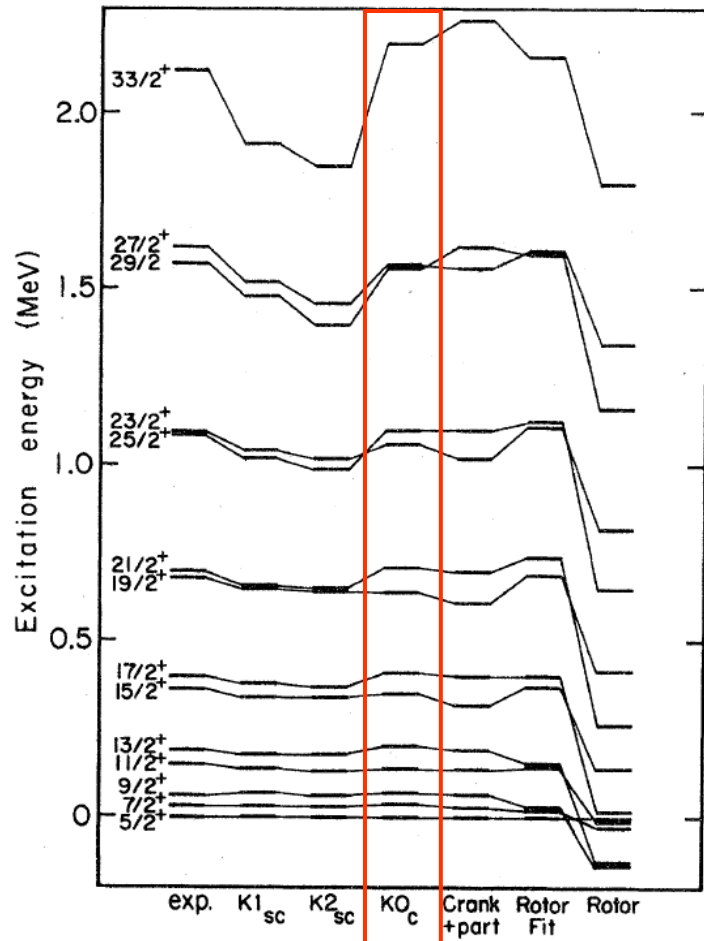


¹⁵⁹Dy

Coriolis-attenuation ?



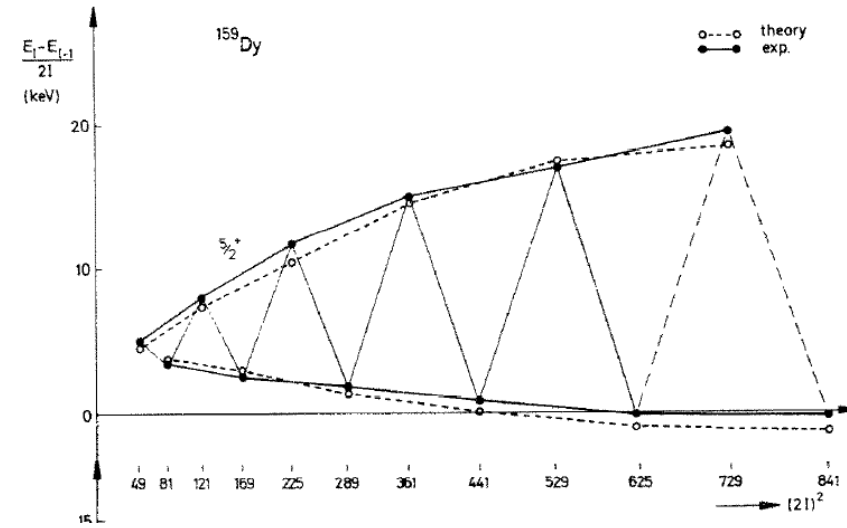
P.R., Mang, PRL. 33 (1974)



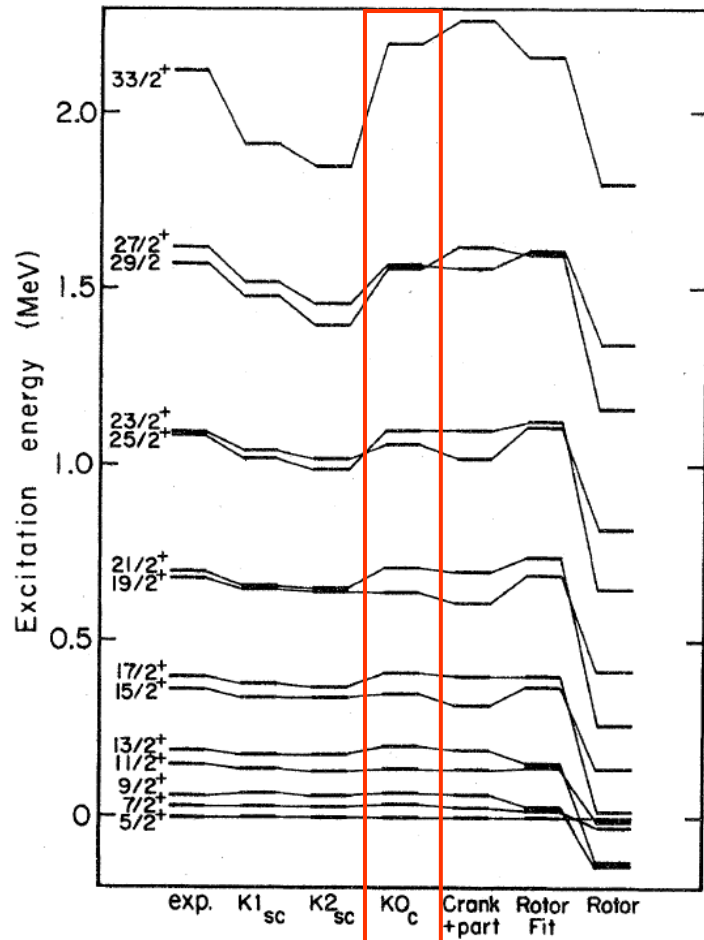
Fixed potentials

^{159}Dy

Coriolis-attenuation ?



P.R., Mang, PRL. 33 (1974)

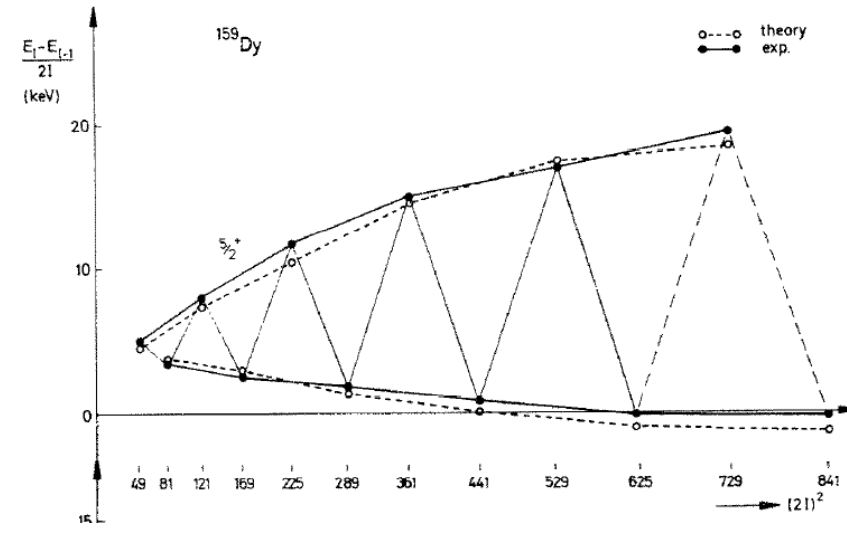


Fixed potentials

Cranked Shell Model (CSM) Bengtsson + Frauendorf, NPA (1979)

^{159}Dy

Coriolis-attenuation ?



P.R., Mang, PRL. 33 (1974)

$$H = \sum_k \varepsilon_k a_k^\dagger a_k - \mathbf{X} \frac{I \cdot \mathbf{j}}{\mathcal{J}} + \frac{I(I+1)}{2\mathcal{J}} + \frac{j^2}{2\mathcal{J}}$$

Attenuation: $\mathbf{X} \approx 0.4 - 0.5$

$$H = \sum_k \varepsilon_k a_k^\dagger a_k - \Omega \mathbf{j}_x + \frac{I(I+1)}{2\mathcal{J}}$$

The question remains:

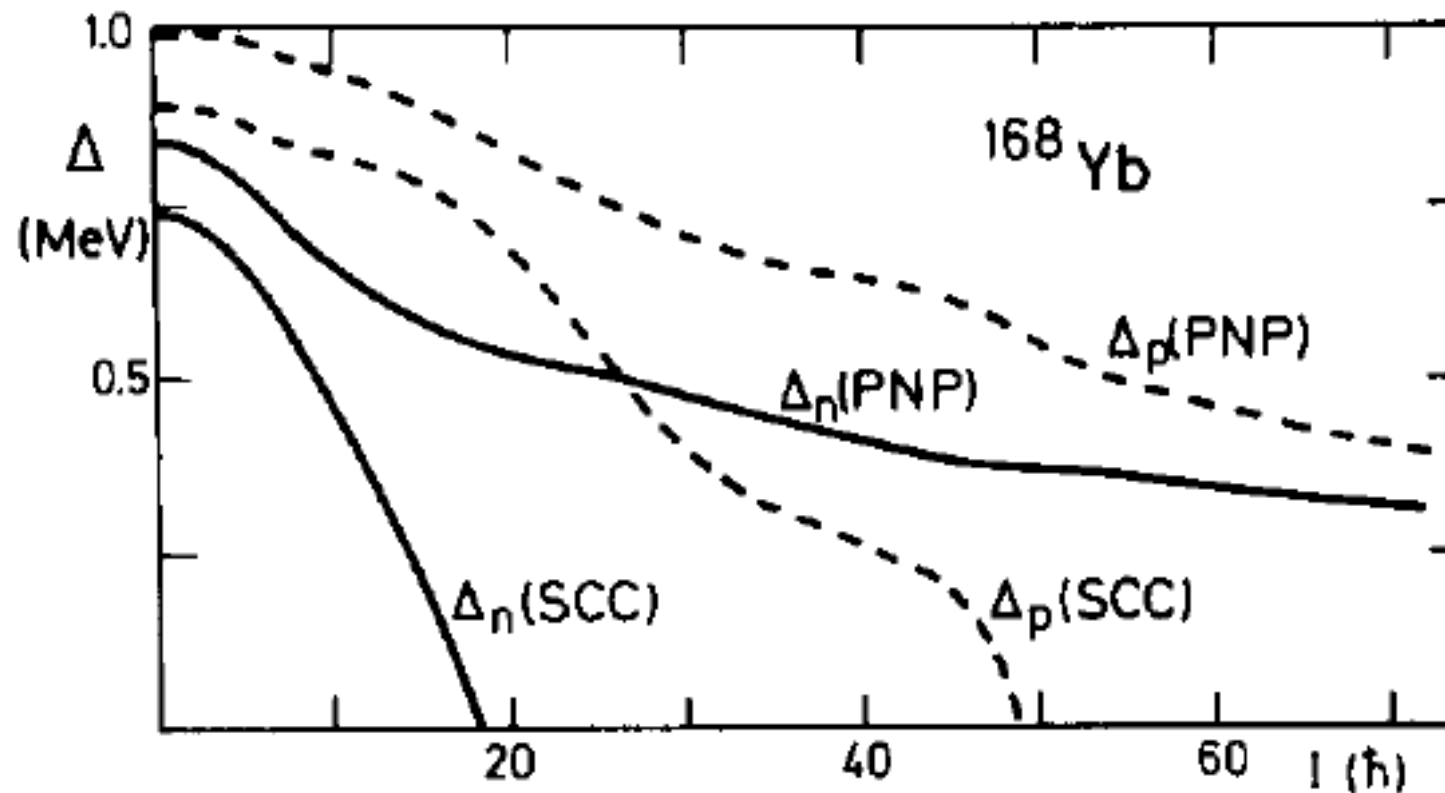
Why do we need Coriolis attenuation \mathbf{X} ?

or: How to treat the recoil term ?

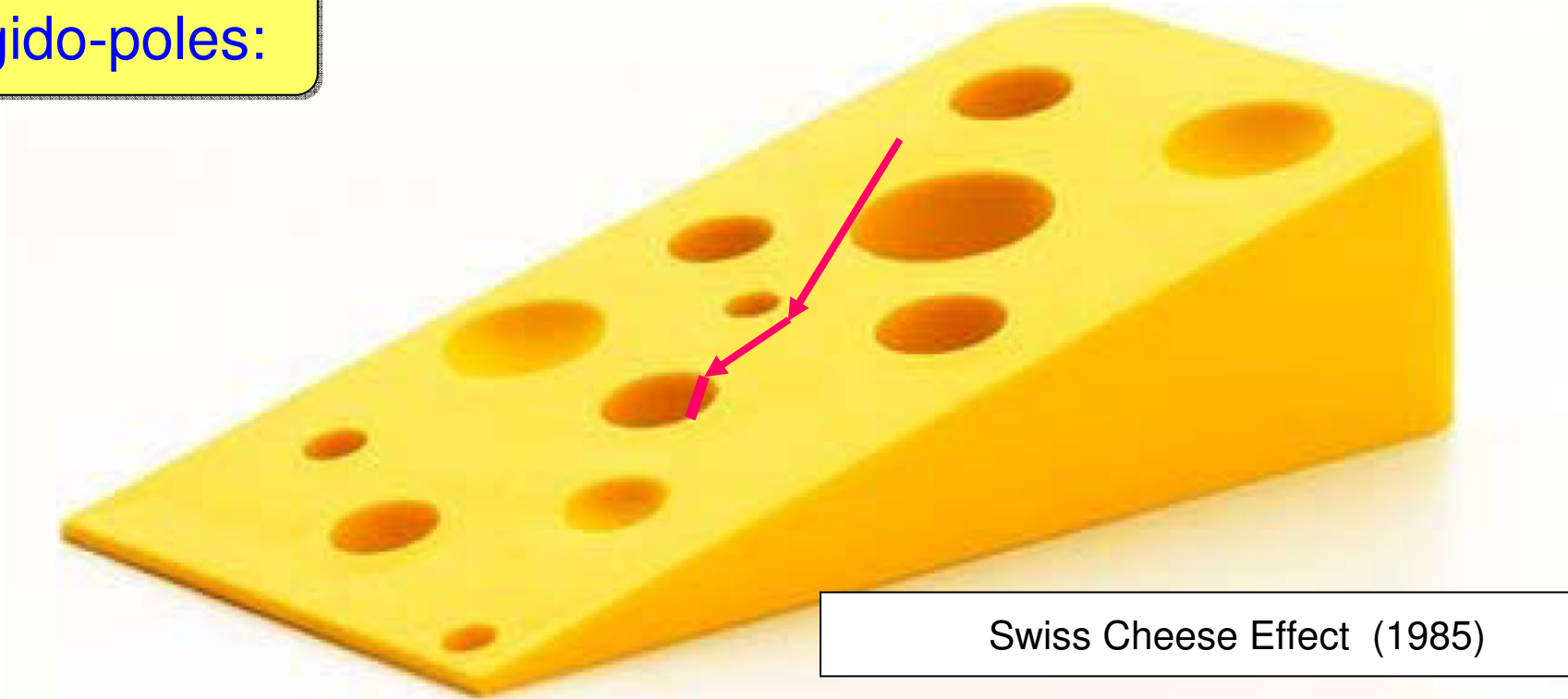
or: Microscopic derivation of the PRM? (by GCM?)

Coriolis anti-pairing and pairing collapse:

Why it has not been observed experiment ?



Egido-poles:



Swiss Cheese Effect (1985)

15 years later Egido understood the problem:

$$E^N = \langle 0 | H P^N | 0 \rangle = \int d\varphi e^{-iN\varphi} \langle 0 | H | \varphi \rangle \quad \text{where: } |\varphi\rangle = e^{i\hat{N}\varphi} |0\rangle$$

$$\langle 0 | a_1^\dagger a_2 a_3^\dagger a_4 | \varphi \rangle = \langle 0 | \varphi \rangle \frac{\langle 0 | a_1^\dagger a_2 | \varphi \rangle}{\langle 0 | \varphi \rangle} \frac{\langle 0 | a_3^\dagger a_4 | \varphi \rangle}{\langle 0 | \varphi \rangle} + \dots$$

can vanish

Angiano, Egido, Robledo, NPA (2001)

Two totally different concepts

Wave function and Hamiltonian

$$\langle 0 | H P^N | 0 \rangle$$

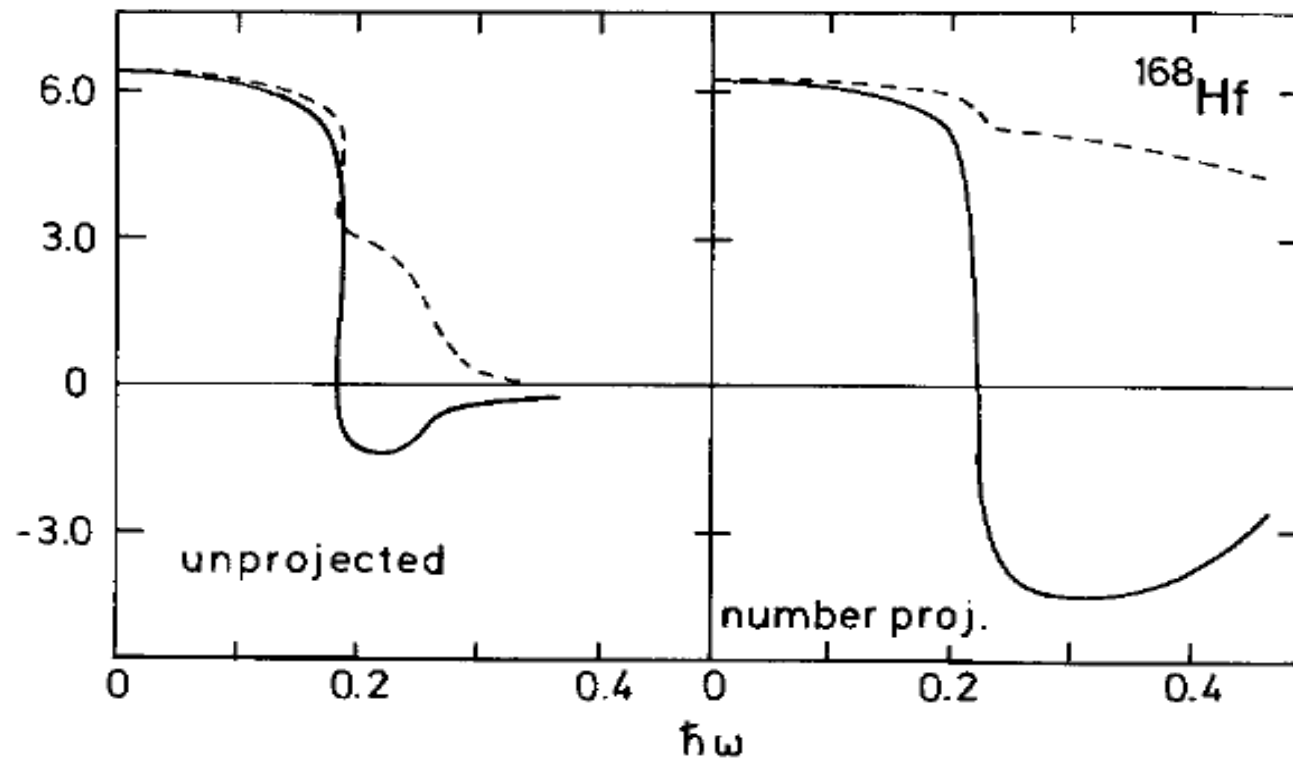
Functional of the local density

$$E[\rho(r)]$$

How to find a solution ?

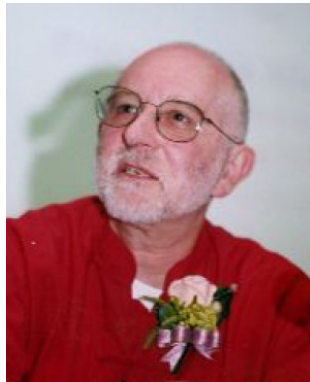
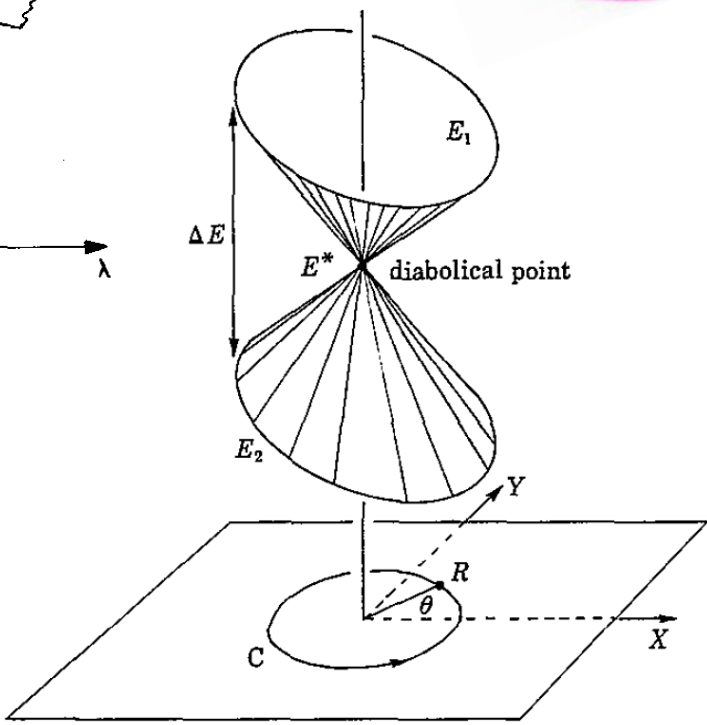
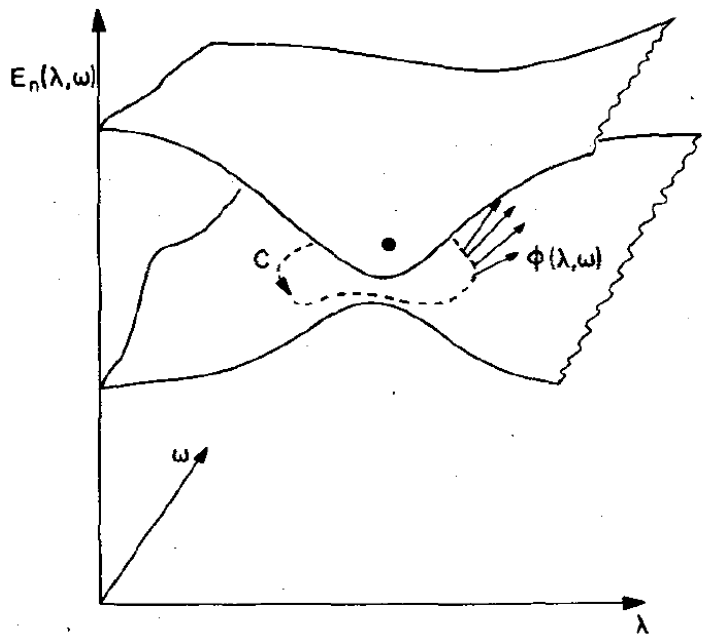
Diaboloic pair transfer:

- Pair transfer matrix element $\langle A + 2, I | [a^\dagger a^\dagger]_0 | A, I \rangle$
- - - Gap parameter $\Delta(\Omega)$



R.S.Nikam, P.R., PRL 58 (1987)

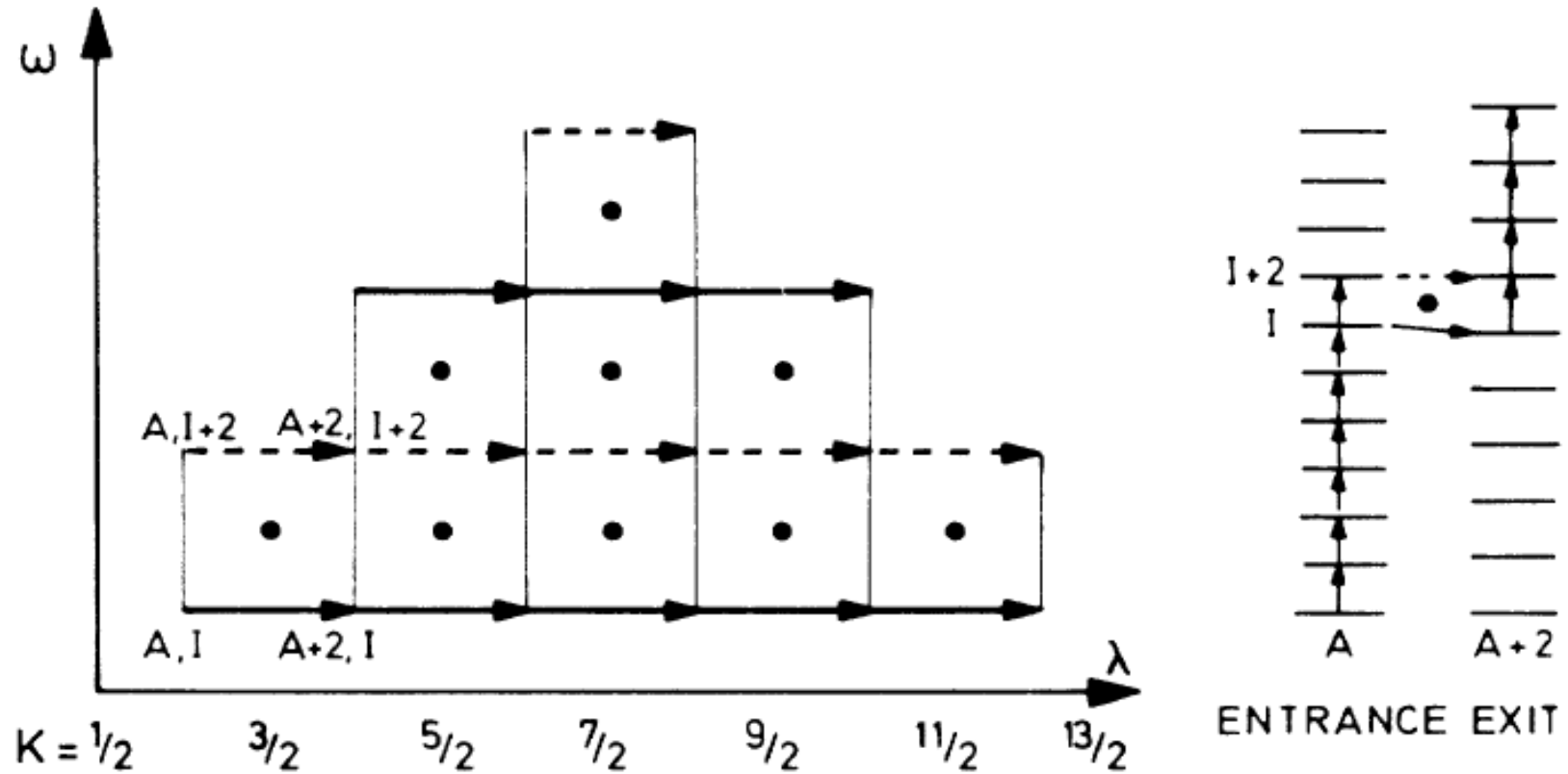
Diabological point:



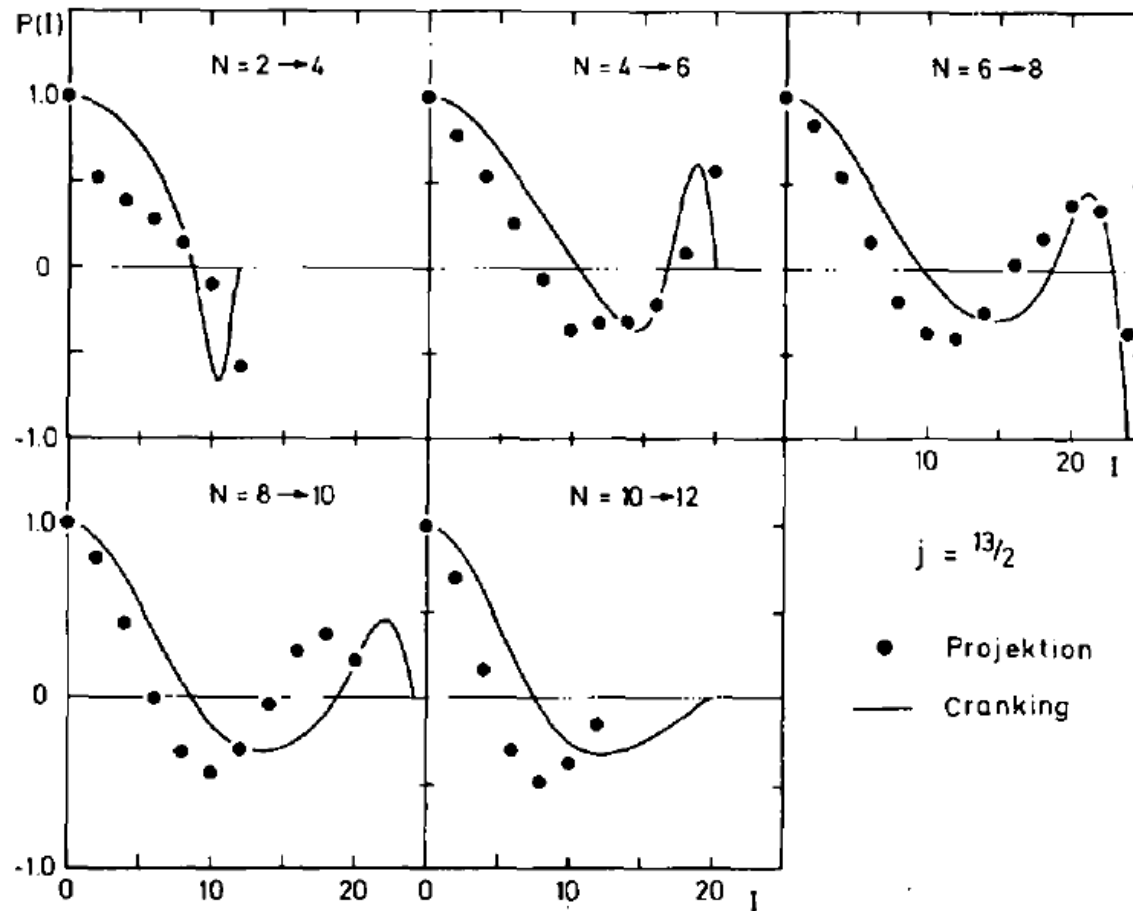
M. Berry (1983)

FIGURE 8. Geometry and notation near a diabolical point.

Pattern of diabolical points in the (λ, ω) -plane



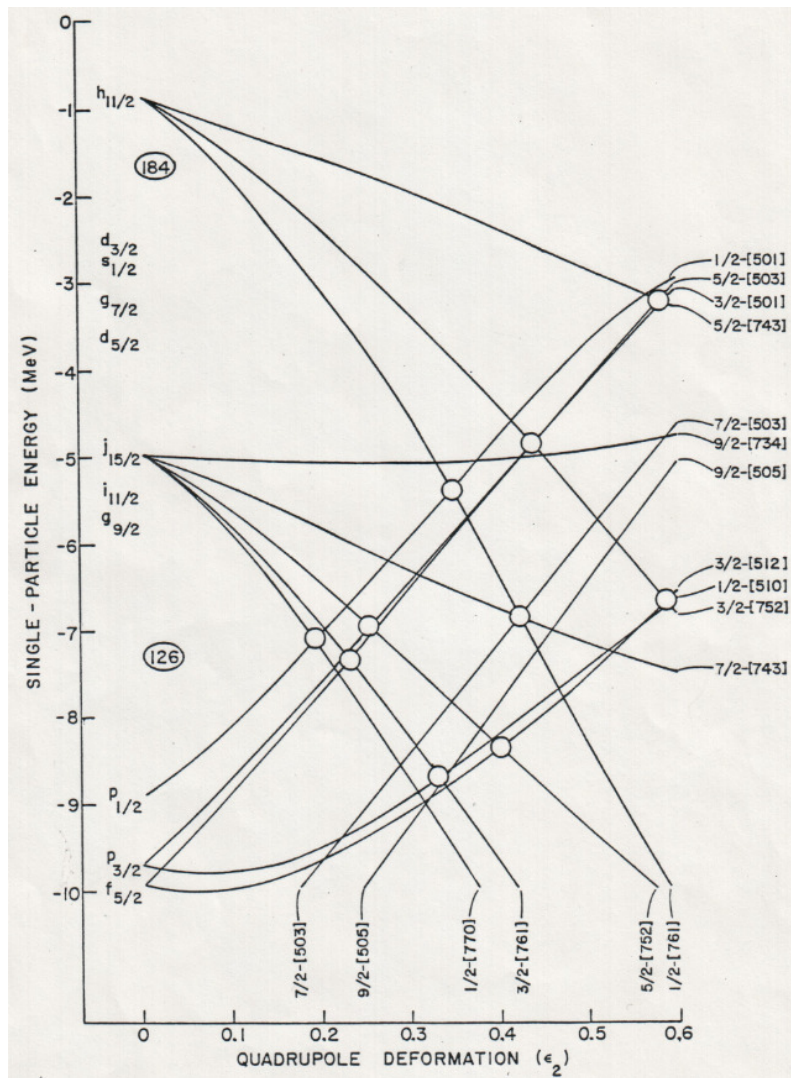
Oscillating behaviour of the pair transfer matrix element



...Y. Sun, P.R., ... PLB (1990)

Puzzle: Why it has not been seen in experiment ?

Diabological points in the (β_2, β_4) -plane of the Nilsson diagram:



R. Chasman, P.R., PLB (1990)

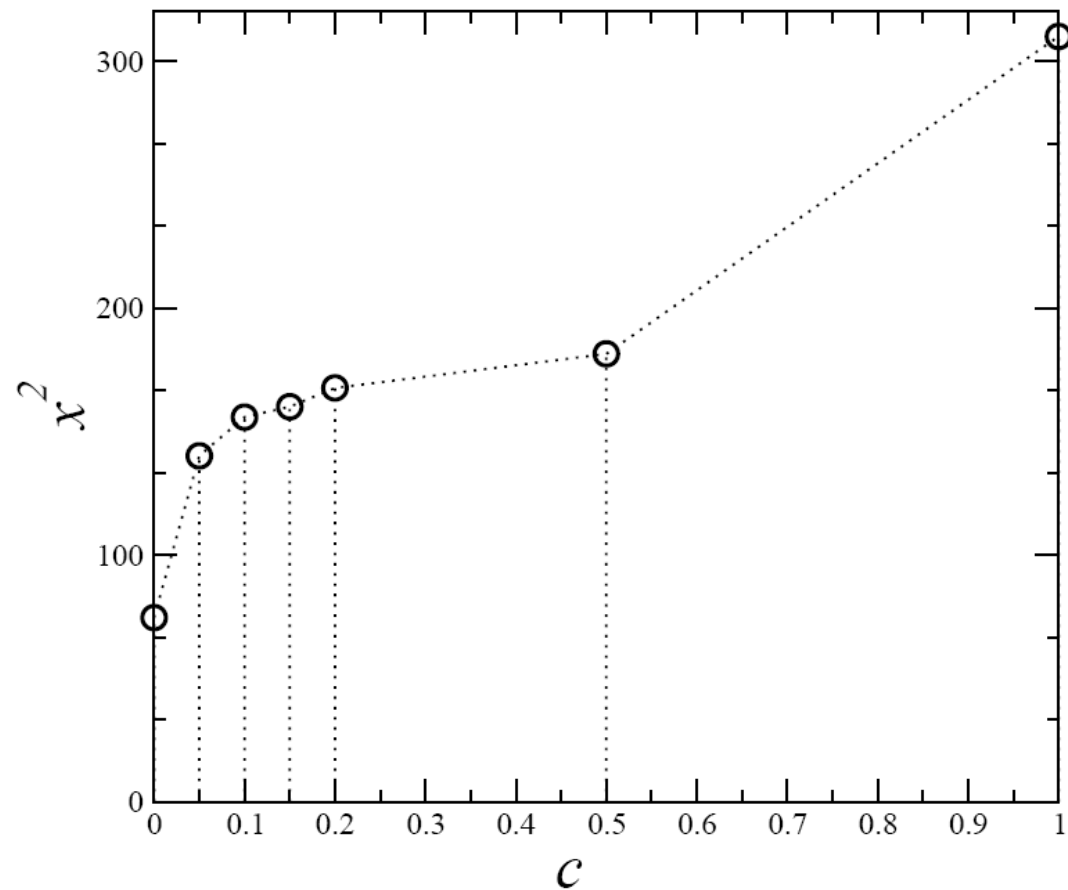
Puzzles in the relativistic description:

$$\begin{pmatrix} M^* + V & \boldsymbol{\sigma} \boldsymbol{p} \\ \boldsymbol{\sigma} \boldsymbol{p} & -M^* + V \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix} = \varepsilon \begin{pmatrix} F \\ G \end{pmatrix}$$

Spurious states in the oscillator expansion

Proper treatment of the Dirac-see in RRPA

Tensor force:



no pion

full pion

Lalazissis, Karatzikos, Serra, Otsuka, P.R., PRC 80 , 041301(R) (2009)

Puzzles, waiting for solution:

Microscopic derivation of the particle + rotor model (Coriolis attenuation)

Projected Shell model derived from CDFT (more than quadrupole degr.)

Are the Egido poles really important (or only a technical problem) ?

How to include tensors in CDFT in a simple way ? (single particle states)

Why the fits do not like the tensor ? (rel. BHF for finite nuclei)

To what extent can we derive CDFT *ab initio* ?

.....

Second edition of Ring+Schuck ???

My thanks goes to:

Dario Vretenar, T. Niksic, N. Paar, T. Marketin

Georgios Lalazissis

Jie Meng

all my collaborators since many years

all of you

Gabi