

## How to determine Skyrme tensor interactions

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devoted to Peter Ring 70<sup>th</sup> anniversary  
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1. Introduction
2. Isotope and Isotone dependence of single particle energies
3. Spin and Spin-Isospin excitations
  - a) charge-exchange excitations
  - b) M1 excitations
4. Collective low-energy excitations
5. Summary

## Collaborations

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Gianluca Colo, Milano, Italy

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F. R. Xu, Peking University, China

PHYSICAL REVIEW C **83**, 034324 (2011)

### **Effects of tensor correlations on low-lying collective states in finite nuclei**

Li-Gang Cao (曹李刚),<sup>1,2,3</sup> H. Sagawa,<sup>2</sup> and G. Colò<sup>4,5</sup>

PHYSICAL REVIEW C **83**, 054316 (2011)

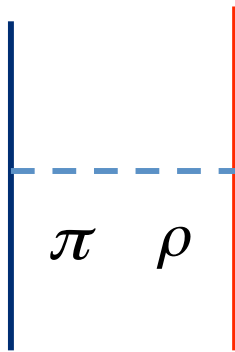
### **Spin-isospin excitations as quantitative constraints for the tensor force**

C. L. Bai,<sup>1,2</sup> H. Q. Zhang,<sup>2</sup> H. Sagawa,<sup>3</sup> X. Z. Zhang,<sup>2</sup> G. Colò,<sup>4</sup> and F. R. Xu<sup>5</sup>

## Nuclear Forces (short range, strong interaction)

### Meson exchange interactions

H. Yukawa, Prog. Theor. Phys. 1935



$$V_{\text{central}}(r) = V_0(r) + V_s(r)\sigma_1 \cdot \sigma_2 + V_t(r)\tau_1 \cdot \tau_2 + V_{st}(r)\sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2$$

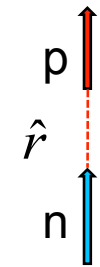
$$V_{\text{tensor}} = f(r) \left[ (\sigma_1 \times \sigma_2)^{(2)} Y_{l=2}(\hat{r}) \right]^{(0)} \tau_1 \cdot \tau_2$$

## Deformation of deuteron and Tensor Interaction



Rugby ball

prolate



$$S_{12} = +2$$

$$S_{12} = \left[ (\vec{\sigma}_1 \times \vec{\sigma}_2)^{(2)} \times Y_2(\hat{r}) \right]^{(0)} \sim 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



oblate

$$S_{12} = -1$$



pancake

attractive

$$V_T = f(r)S_{12}$$

repulsive

Rarita-Schwinger, Phys.Rev.59, 436(1941)

Blatt-Weisskopf, Theoretical Nuclear Phys.(1952)

Theoretical Mean Field Models  
(Hartree model or Hartree-Fock model)

Skyrme HF model  
Gogny HF model

+tensor interactions

RMF model  
RHF model

+pion-coupling, rho-tensor coupling

Long, Meng, Nguyen Van Giai

## Skyrme-type tensor interactions

### Two Advantages

1. A simple formula for spin-orbit splitting
2. Analytic multipole expansion for spin-dependent excitations

$$V^T = \frac{T}{2} \left\{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k'^2] \delta(\mathbf{r}_1 - \mathbf{r}_2) \right. \\ \left. + \delta(\mathbf{r}_1 - \mathbf{r}_2) [(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2] \right\} \quad \text{:Triplet-even}$$
$$+ \frac{U}{2} \left\{ (\sigma_1 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_2 \cdot \mathbf{k}) + (\sigma_2 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_1 \cdot \mathbf{k}) \right. \\ \left. - \frac{2}{3} [(\sigma_1 \cdot \sigma_2) \mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}] \right\} \quad \text{:Triplet-odd}$$

T.H.R. Skyrme, Nucl.Phys. 9,615(1959).

F.L. Stancu, D. M. Brink and H. Flocard, PLB68,108 (1977).

T.Lesinski, M. Bender, K. Bennaceur, T. Duguet, J. Meyer, Phys. Rev.C76, 014312(2007).

G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.

B.A.Brown, T. Duguet, T. Otsuka, D. Abe and T. Suzuki, Phys. Rev. C74(2006) 061303(R)

Mean field -----Spin-orbit splitting-----

$$\delta H = \frac{1}{2}\alpha(J_n^2 + J_p^2) + \beta J_n J_p.$$

It plays an important role for the spin-orbit splittings.

The contribution of the tensor to the total energy is not very large but may improve mass systematics (may not?).

$$U_{s.o.}^{(q)} = \frac{W_0}{2r} \left( 2 \frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr} \right) + \left( \alpha \frac{J_q}{r} + \beta \frac{J_{q'}}{r} \right),$$

$q \ (n=0, p=1) \qquad q' = 1-q$

$$J_q(r) = \frac{1}{4\pi r^3} \sum_i v_i^2 (2j_i + 1) \left[ j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4} \right] R_i^2(r).$$

Sly5+T

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2) = 80.7 \text{ MeV} \cdot \text{fm}^5$$

$$\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2) = -48.9 \text{ MeV} \cdot \text{fm}^5$$

$$\alpha_T = \frac{5}{12}U = -170 \text{ MeV} \cdot \text{fm}^5$$

$$\beta_T = \frac{5}{24}(T + U) = 100 \text{ MeV} \cdot \text{fm}^5$$

*TIJ* family

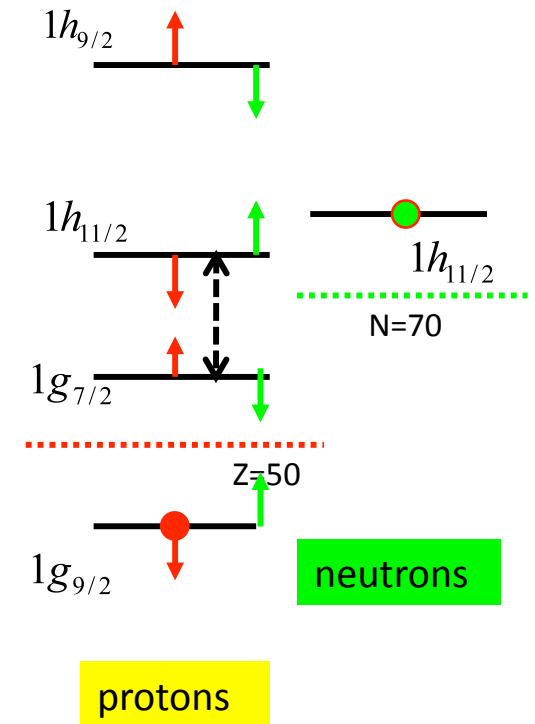
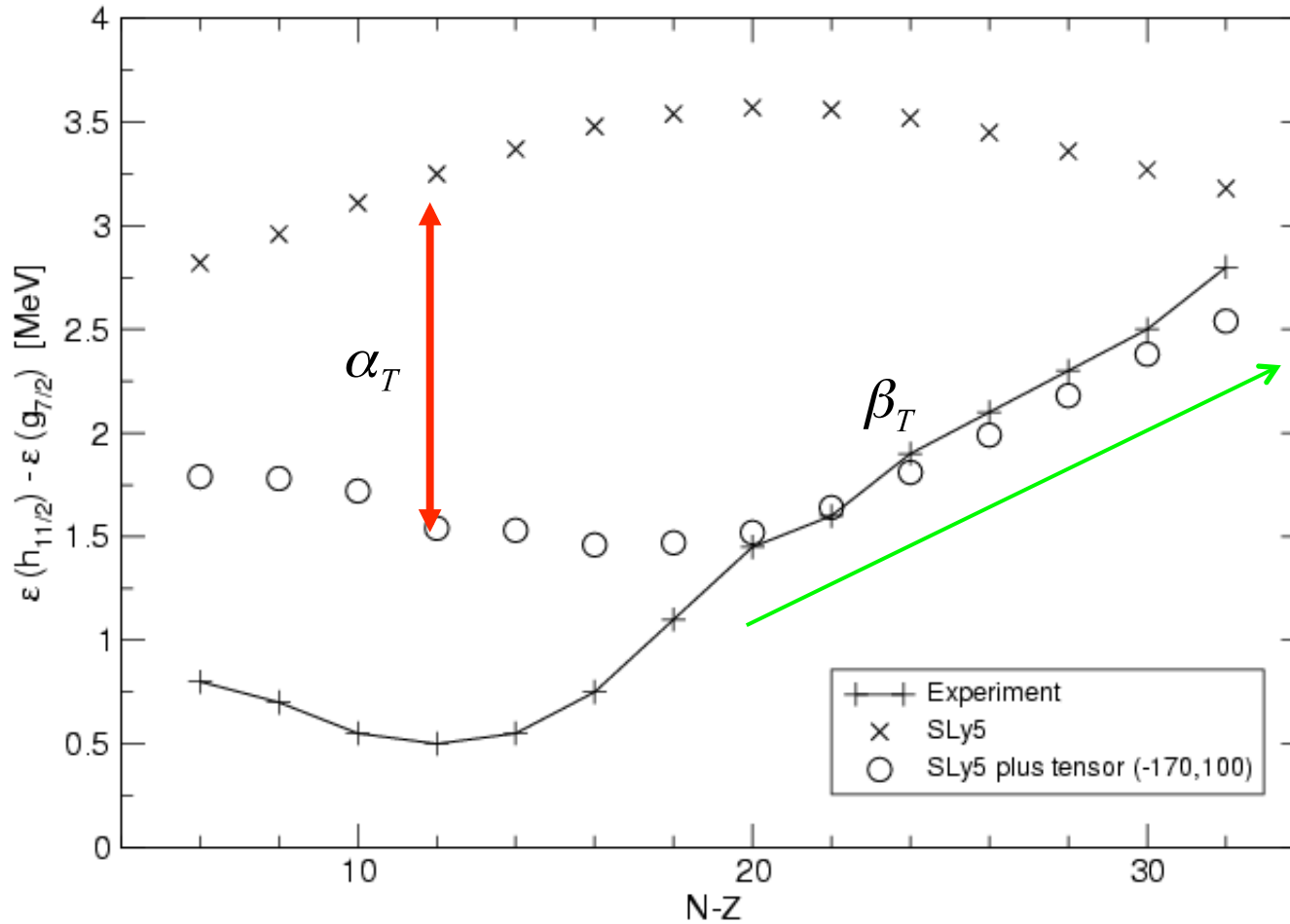
$$\alpha = \alpha_C + \alpha_T = 60(J - 2) \text{ MeV} \cdot \text{fm}^5$$

$$\beta = \beta_C + \beta_T = 60(I - 2) \text{ MeV} \cdot \text{fm}^5$$

	sign	spin - orbit splitting
$\alpha, \beta$	negative	larger
	positive	smaller

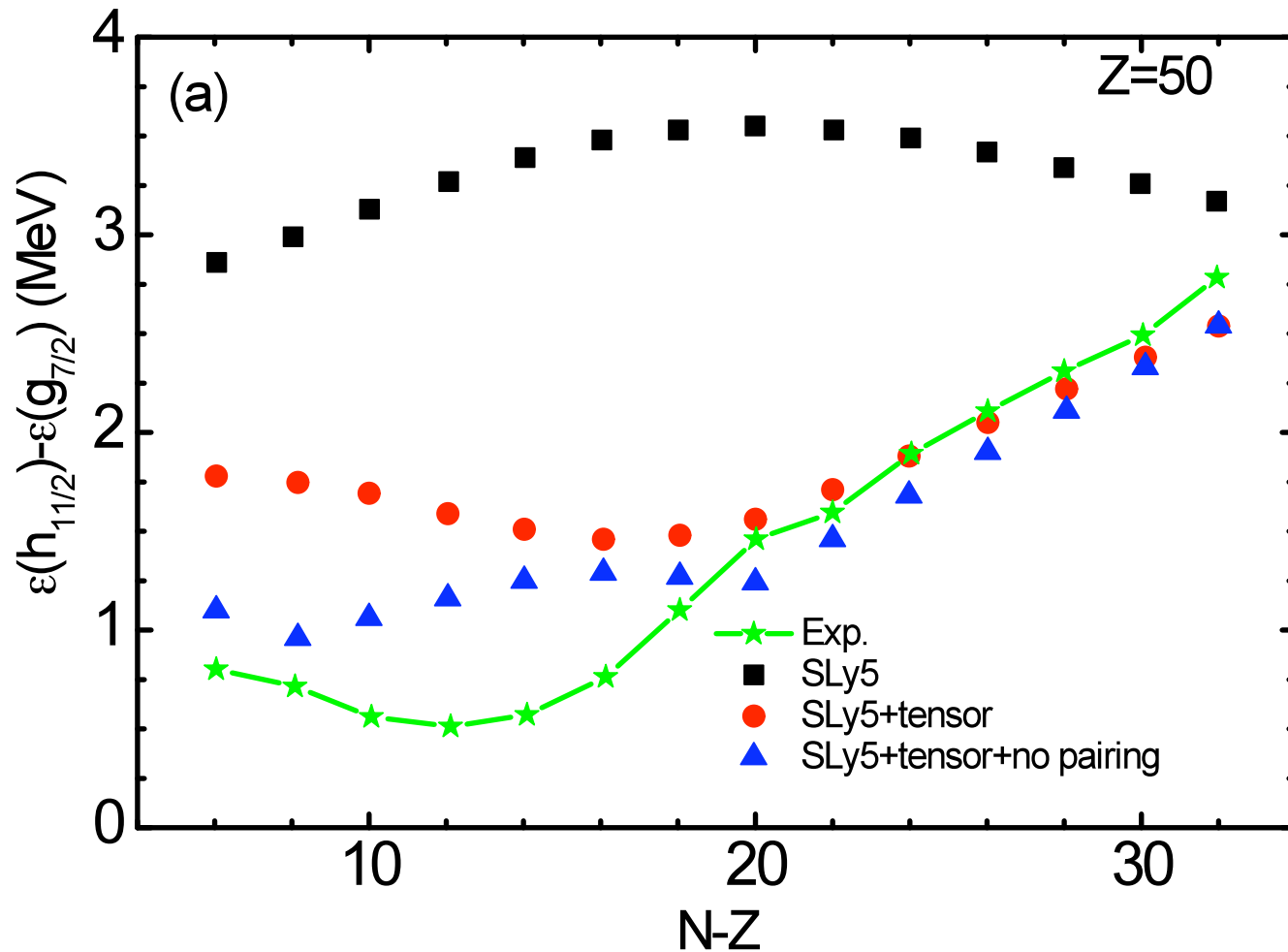
Exp. Data : J.P.Schiffer et al., P.R.L. 92, 162501(2004)

Protons on Z=50 core



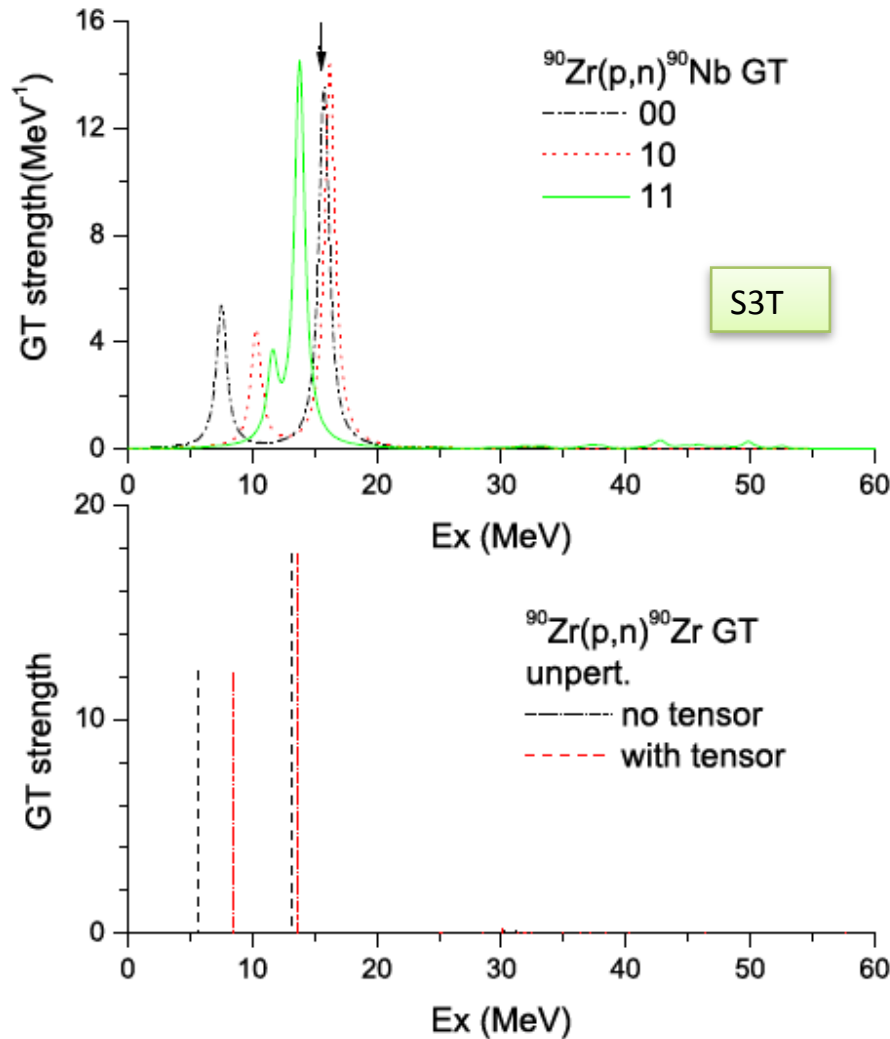
G.Colo, H. Sagawa, S. Fracasso, P.F. Bortignon, Phys. Lett. B 646 (2007) 227.



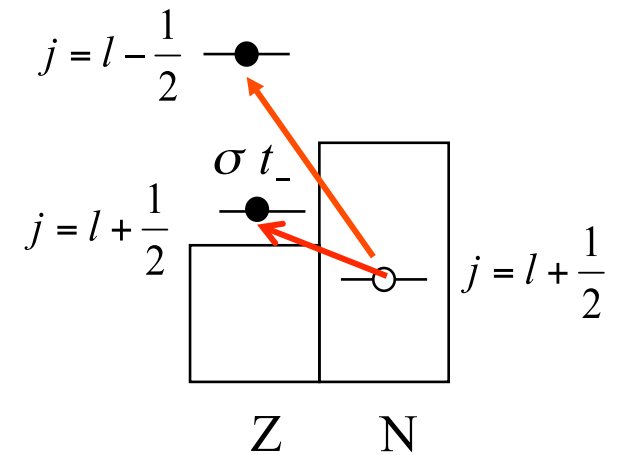


Not only tensor, but also pairing and particle-vibration coupling effects may play equally important roles. It is marginal just to look at s.p. states to find out the importance of tensor correlations!

# The tensor force and charge-exchange excitations



Gamow-Teller  $\lambda^\pi = 1^+$



The main peak is moved downward by the tensor force but the centroid is moved upwards!

C.L.Bai, HS, H.Q.Zhang, X.Z.Zhang, G.Colo and F.R.Xu, P.L.B675,28 (2009).

C.L.Bai, H.Q. Zhang, X.Z.Zhang, F,R,Xu, HS and G.Colo, PRC79, 041301(R) (2009).

	type of calculation	$m_{-}(0)$ 0-30MeV	$m_{-}(0)$ 30-60MeV	$m_{-}(1)$ 0-30 MeV	$m_{-}(1)$ 30-60 MeV	$m_{-}(1)$ total	$m_{+}(1)$ total
$^{90}\text{Zr}$	00	29.16	0.71	395	26.2	421.8	10.1
	10	29.16	0.79	444	22	466	11.1
	11	27.00	2.89	366.9	122	493.2	10.3
$^{208}\text{Pb}$	00	127.54	3.43	2080	124.5	2212.8	18.8
	10	127.38	3.68	2176	93	2269	21
	11	114.10	16.58	1658	694	2370	19.3

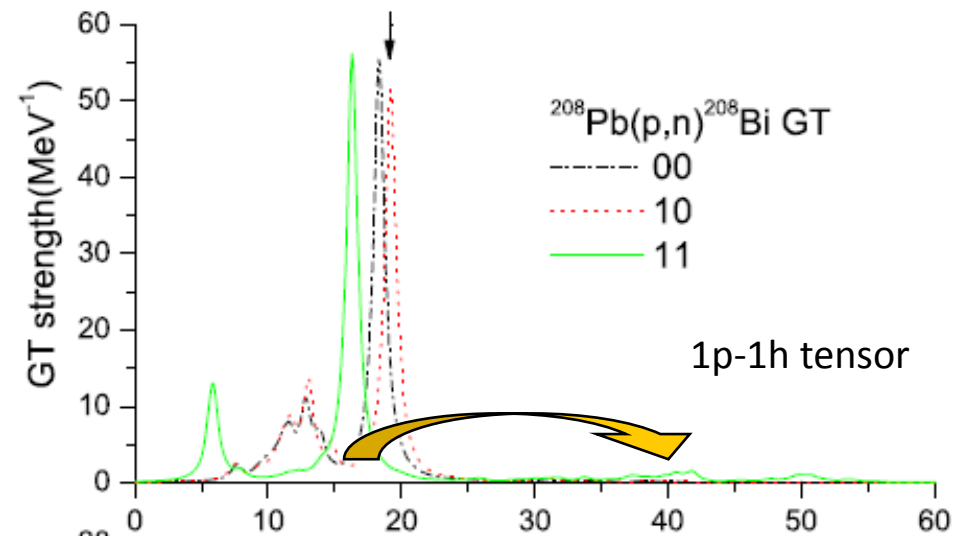
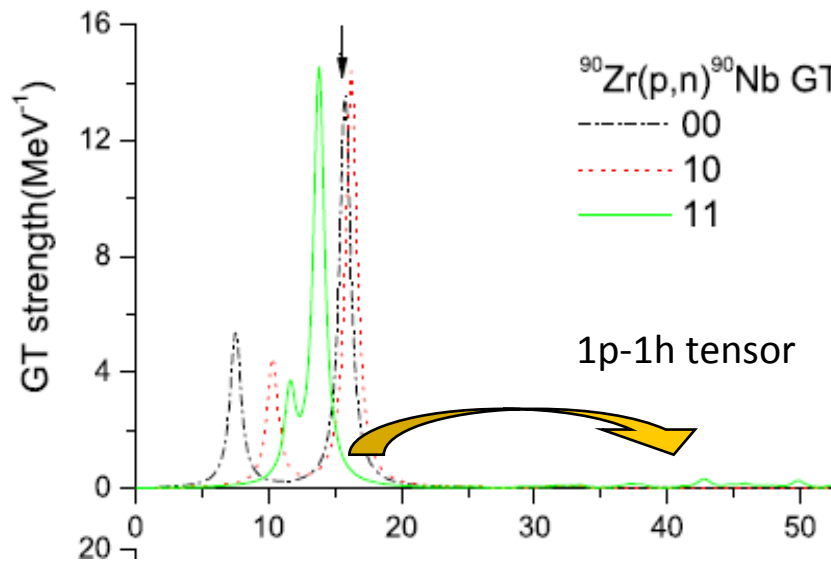
Energy-weighted sum rules

$$m(k) = \sum_i E_i^k \left| \langle i | \hat{O}_\lambda | 0 \rangle \right|^2$$

$$m(1) = \frac{1}{2} \langle 0 | [\hat{O}_\lambda, [H, \hat{O}_\lambda]] | 0 \rangle$$

About 10% of strength is moved by the tensor correlations to the energy region above 30 MeV.

Relevance for the GT quenching problem.



## Multipole Expansion of Tensor Interactions

$$\begin{aligned}
 V^T = & \frac{T}{2} \left\{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k'^2] \delta(\mathbf{r}_1 - \mathbf{r}_2) \right. \\
 & + \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[ (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k^2 \right] \left. \right\} \\
 & + \frac{U}{2} \left\{ (\sigma_1 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_2 \cdot \mathbf{k}) + (\sigma_2 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_1 \cdot \mathbf{k}) \right. \\
 & \left. - \frac{2}{3} [(\sigma_1 \cdot \sigma_2) \mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}] \right\}
 \end{aligned}$$

$$\delta(\vec{r}_1 - \vec{r}_2) = \sum_{lm} Y_{lm}(\hat{r}_1) Y_{lm}^*(\hat{r}_2) \frac{\delta(r_1 - r_2)}{r_1 r_2}$$

$$V^T \propto T_{(\lambda, \kappa)} \{ [\sigma_1 \times [\nabla_1 \times Y_{l=1}(\hat{r}_1)]^{(\lambda)} \}^{(\kappa)} [\sigma_2 \times [\nabla_2 \times Y_{l=1}(\hat{r}_2)]^{(\lambda')} \}^{(\kappa)} \}^{(0)} \delta(r_1 - r_2)$$

$$1^+ T_{(\lambda=\lambda'=2, \kappa=1)} \Rightarrow \textit{repulsive}$$

$$2^+ T_{(\lambda=\lambda'=2, \kappa=2)} \Rightarrow \textit{attractive}$$

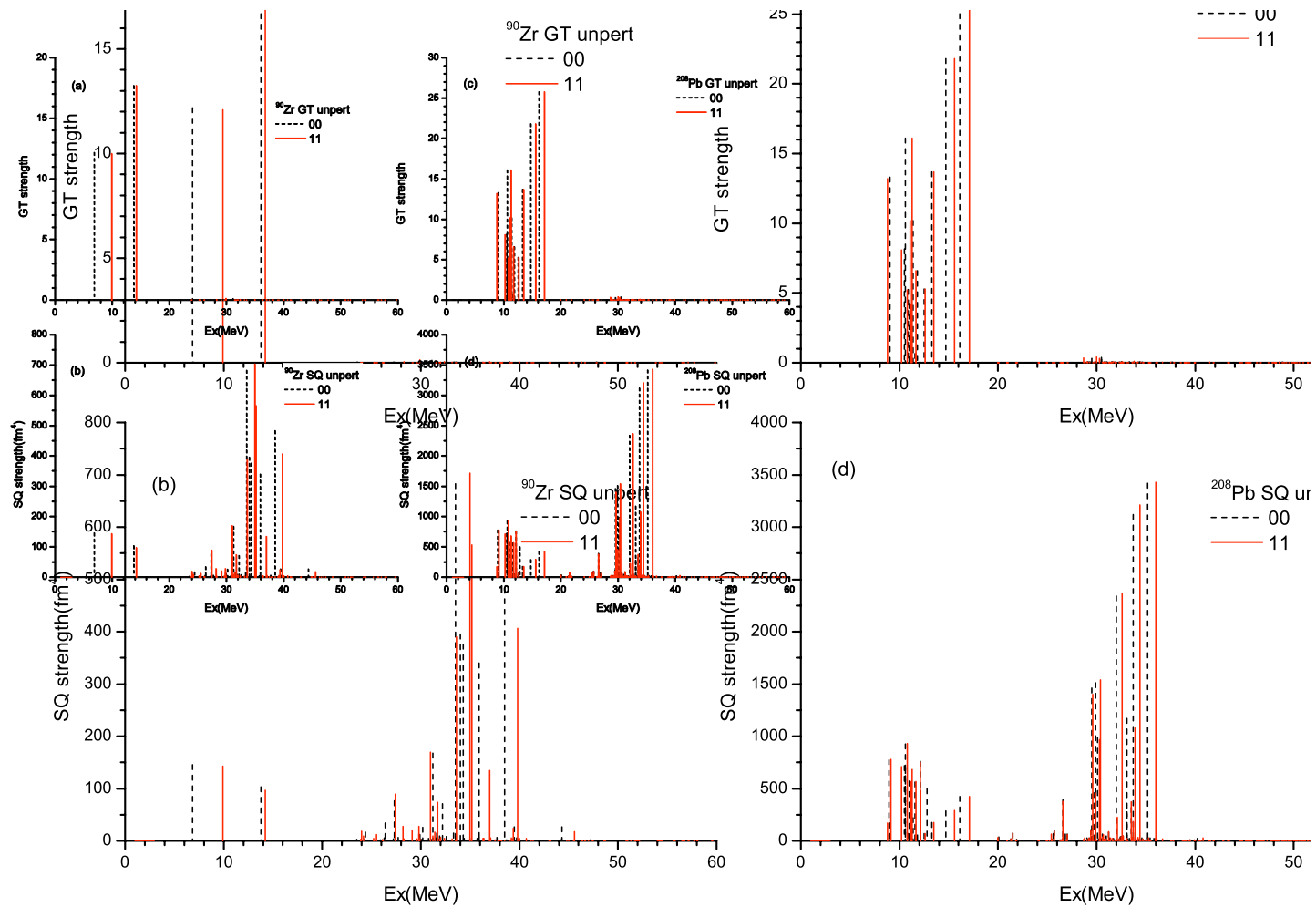
$$3^+ T_{(\lambda=\lambda'=2, \kappa=3)} \Rightarrow \textit{repulsive}$$

$$1^+ T_{(\lambda=2, \lambda'=0, \kappa=1)} \Rightarrow \text{strong mixing between Gamow - Teller and spin - quadrupole excitations!}$$

# Why does Tensor interaction decrease GT strength in peak region?

$1^+$  : Gamow - Teller excitation  $\sigma \cdot \tau$

Spin - Quadrupole excitation  $r^2 [\sigma \times Y_2]^{(\lambda)}$   $\lambda = 1^+, 2^+, 3^+$



## Tensor correlations on Spin-Dipole excitations

TIJ family

$$\alpha = 60(J - 2) \text{ MeV fm}^5,$$

$$\beta = 60(I - 2) \text{ MeV fm}^5.$$

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2),$$

$$\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2),$$

$$\alpha_T = \frac{5}{4}t_o = \frac{5}{12}U,$$

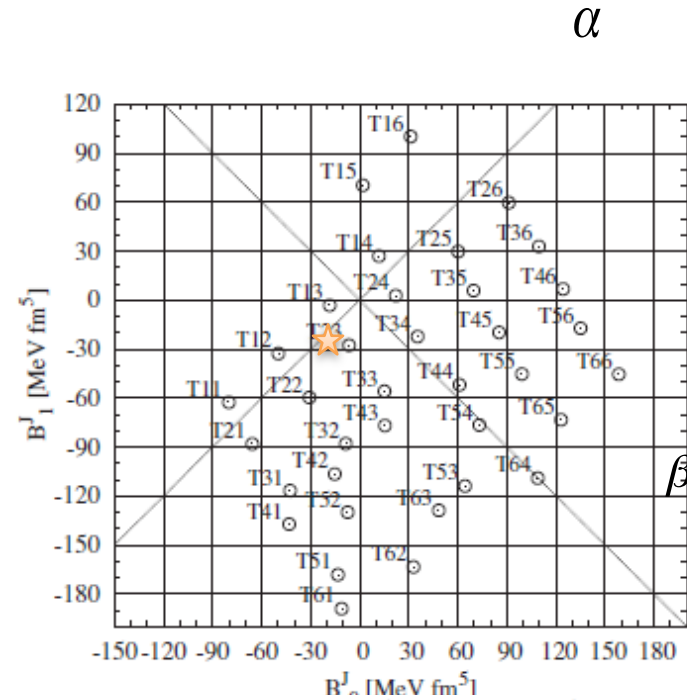
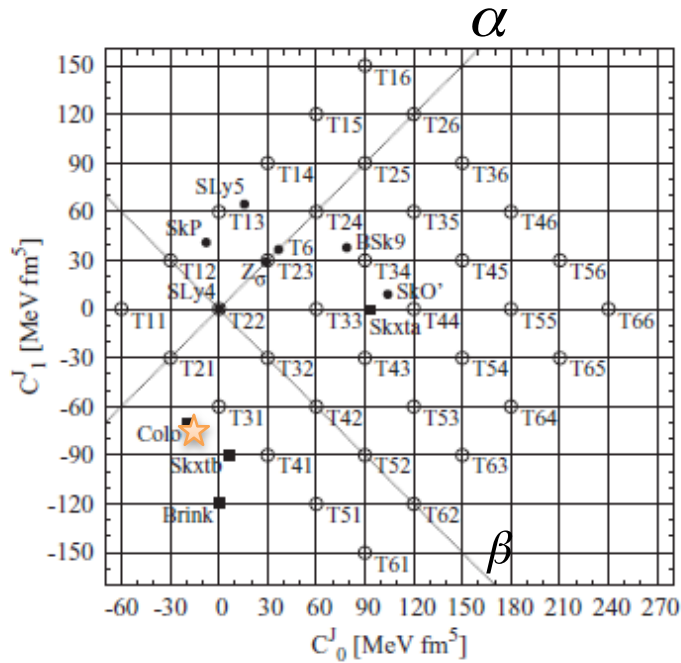
$$\beta_T = \frac{5}{8}(t_e + t_o) = \frac{5}{24}(T + U).$$

TABLE I. Parameters of the tensor terms in units of MeV-fm<sup>5</sup>. The  $T$  and  $U$  values are taken from Refs. [2,4-6], while the values  $\alpha$  and  $\beta$  are obtained by means of Eq. (5).

	$T$	$U$	$\alpha$	$\beta$
SLy5	888.0	-408.0	-89.8	51.1
SGII	1008.0	-432.0	-122.3	130.0
SIII	1008.0	-432.0	-118.7	120.0
SKXTA	384.0	144.0	93.6	94.2
SKXTB	811.2	-283.2	-83.9	96.1
T11	258.9	-342.8	-60.0	-60.0
T12	116.4	-198.2	0.0	-60.0
T13	-20.8	-51.7	60.0	-60.0
T14	-165.4	92.5	120.0	-60.0
T15	-500.9	173.3	180.0	-60.0
T16	-646.2	314.7	240.0	-60.0
T21	476.9	-369.4	-60.0	0.0
T22	356.1	-217.5	0.0	0.0
T23	183.9	-82.7	60.0	0.0
T24	33.7	59.2	120.0	0.0
T25	-69.4	216.0	180.0	0.0
T26	-209.7	362.1	240.0	0.0
T31	738.6	-382.5	-60.0	60.0
T32	613.1	-231.5	0.0	60.0
T33	439.3	-97.9	60.0	60.0
T34	246.6	30.8	120.0	60.0
T35	125.5	180.9	180.0	60.0
T36	27.2	341.8	240.0	60.0
T41	884.9	-433.6	-60.0	120.0
T42	730.7	-292.9	0.0	120.0
T43	590.6	-147.5	60.0	120.0
T44	520.9	21.5	120.0	120.0
T45	346.9	156.9	180.0	120.0
T46	249.6	314.6	240.0	120.0
T51	1179.9	-435.7	-60.0	180.0
T52	918.2	-329.9	0.0	180.0
T53	974.9	-119.1	60.0	180.0
T54	727.3	-8.4	120.0	180.0
T55	564.6	129.3	180.0	180.0
T56	448.3	282.9	240.0	180.0
T61	1335.5	-480.4	-60.0	240.0
T62	1256.5	-313.9	0.0	240.0
T63	1043.8	-193.3	60.0	240.0
T64	1046.8	-0.6	120.0	240.0
T65	823.2	119.7	180.0	240.0
T66	708.5	270.9	240.0	240.0

## Tensor part of the Skyrme energy density functional: Spherical nuclei

T. Lesinski,<sup>1,\*</sup> M. Bender,<sup>2,3,†</sup> K. Bennaceur,<sup>1,2</sup> T. Duguet,<sup>4</sup> and J. Meyer<sup>1</sup>



$$\alpha = C_0^J + C_1^J, \quad \beta = C_0^J - C_1^J,$$

$$C_0^J = \frac{1}{2}(\alpha + \beta), \quad C_1^J = \frac{1}{2}(\alpha - \beta).$$

TIJ family

$$\alpha = 60(J - 2) \text{ MeV fm}^5,$$

$$\beta = 60(I - 2) \text{ MeV fm}^5.$$

$$B_0^J = \frac{5}{16}(t_e + 3t_o) = \frac{5}{48}(T + 3U),$$

$$B_1^J = \frac{5}{16}(t_o - t_e) = \frac{5}{48}(U - T),$$

$$\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2),$$

$$\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2),$$

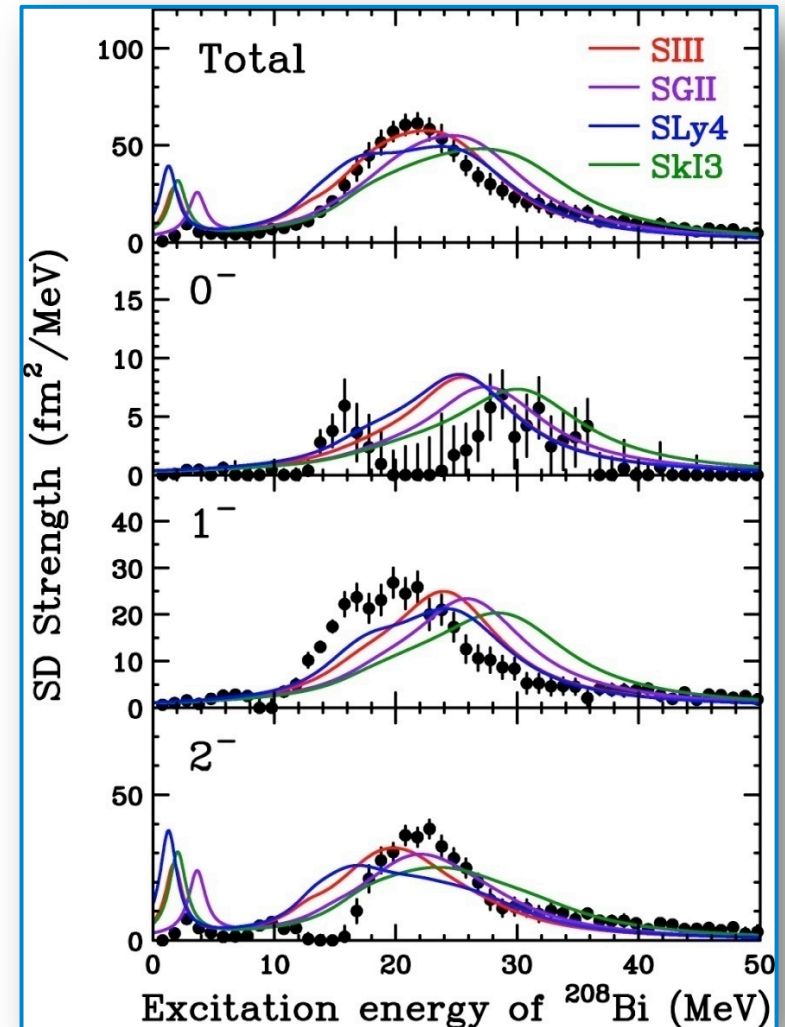
$$\alpha_T = \frac{5}{4}t_o = \frac{5}{12}U,$$

$$\beta_T = \frac{5}{8}(t_e + t_o) = \frac{5}{24}(T + U).$$

# SD Strength Distributions (Wakasa, SIR2010, 18-21 Feb., 2010)

H. Sagawa et al., PRC 76, 024301 (2007).

- Total strength
  - Asymmetric single bump
    - ☉ Extend up to  $\sim 50$  MeV
    - ☉ Same as  $^{90}\text{Zr}(p,n)$  results
  - SIII provides better description
- $0^-$  strength
  - Quenched
    - ☉ Seems to be fragmented
- $1^-$  strength
  - Softened compared with theory
    - ☉ Peak shift to lower  $E_x$
- $2^-$  strength
  - Hardened compared with theory
    - ☉ Peak shift to higher  $E_x$



- ☉ No Skyrme int. which reproduces both total and separated strengths
- ☉  $\Delta J^\pi$ -dependent correlation?  $\rightarrow$  Require further investigations



# A systematic study of tensor interactions on Spin-Isospin excitations

PHYSICAL REVIEW C 83, 054316 (2011)

## Spin-isospin excitations as quantitative constraints for the tensor force

C. L. Bai,<sup>1,2</sup> H. Q. Zhang,<sup>2</sup> H. Sagawa,<sup>3</sup> X. Z. Zhang,<sup>2</sup> G. Colò,<sup>4</sup> and F. R. Xu<sup>5</sup>

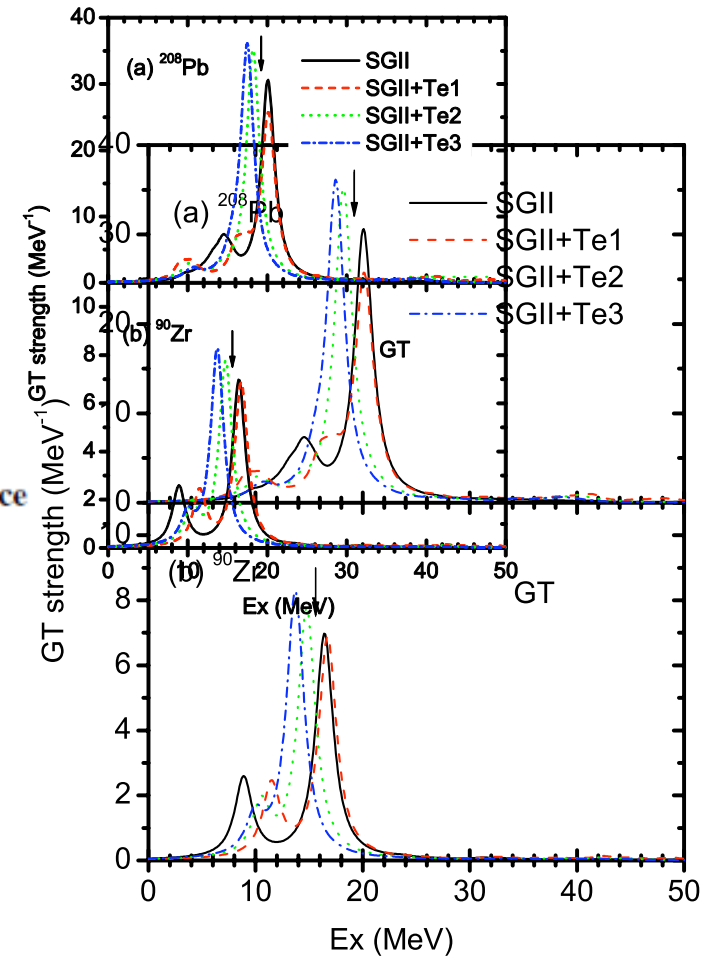
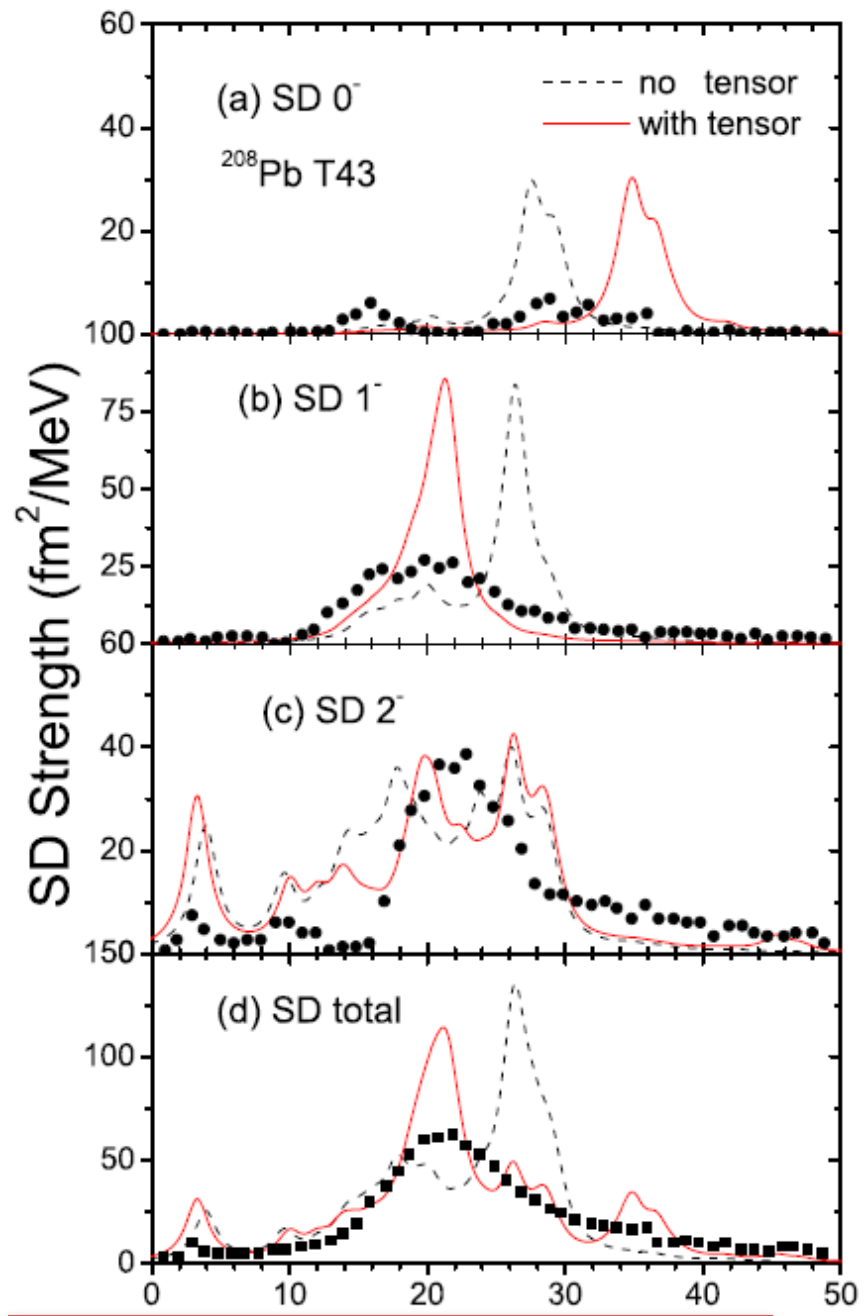
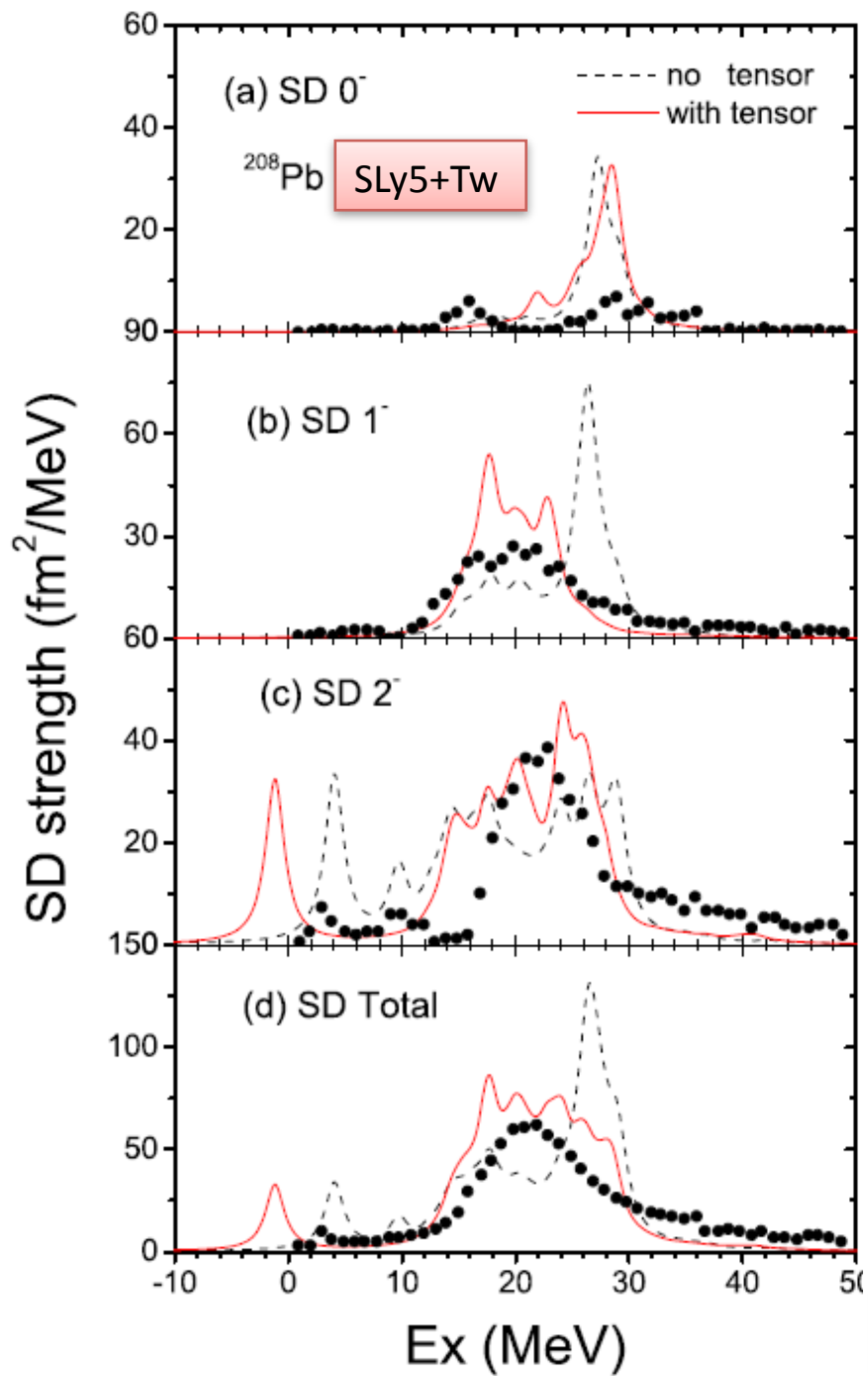


TABLE I. The calculated peak energies of the SD and GT strengths in  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  obtained by using the four interactions that reproduce the experimental data [14,18,19] within an accuracy of 2.5 MeV. See the text for a discussion.

	$^{90}\text{Zr}$					$^{208}\text{Pb}$				
	0 <sup>-</sup>	1 <sup>-</sup>	2 <sup>-</sup>	total SD	GT	0 <sup>-</sup>	1 <sup>-</sup>	2 <sup>-</sup>	total SD	GT
T21	39.3	23.3	25.3	23.5	15.9	40.8	24.1	25.0	23.3	18.0
T32	39.0	23.8	25.4	24.3	15.9	39.4	23.4	25.3	23.3	17.4
T43	38.6	24.3	25.3	24.9	16.2	37.7	24.0	25.4	23.6	17.2
T54	38.3	24.5	25.4	25.2	16.2	37.1	23.8	25.4	23.5	16.7
exp	...	...	...	26.0	15.6	34.5	22.8	25.8	25.2	19.2



C.L.Bai et al., PRL 105, 072501 (2010)

$$\mathbf{V}^{(\lambda)}_{\text{TE}} = \frac{-5}{12} T \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} \left| \langle p \| O_{1,\lambda} \| h \rangle \right|^2 \text{ for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

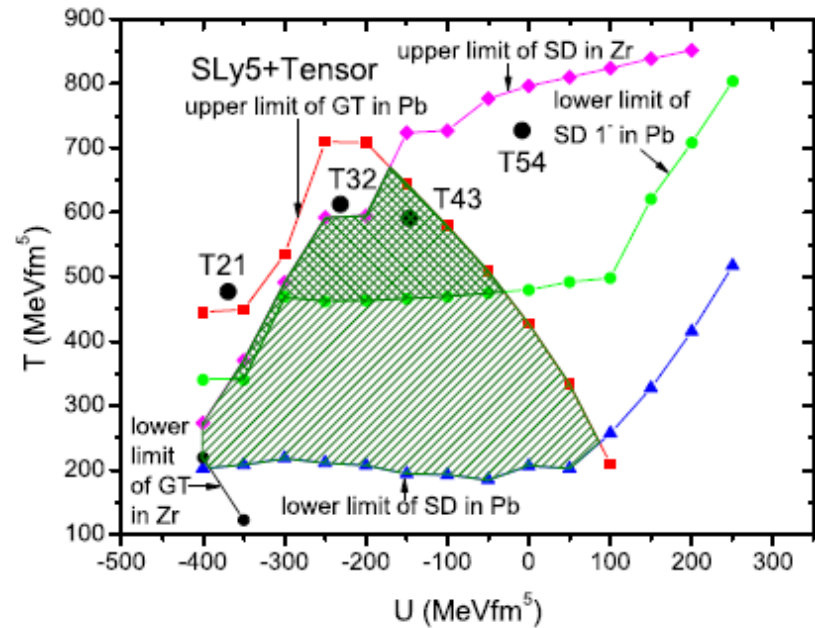
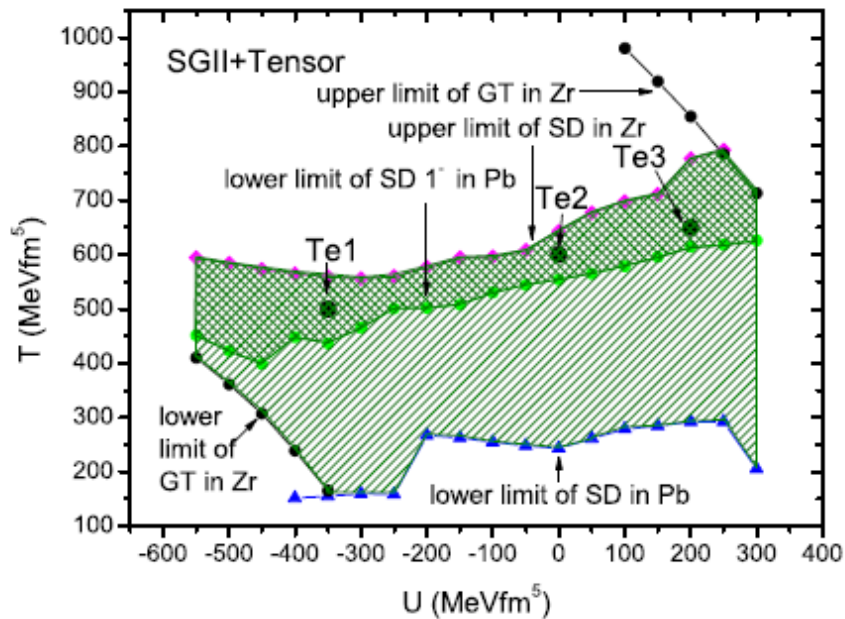
$$\mathbf{V}^{(\lambda)}_{\text{TO}} = \frac{5}{12} U \begin{Bmatrix} 1 \\ -1/6 \\ 1/50 \end{Bmatrix} \left| \langle p \| O_{1,\lambda} \| h \rangle \right|^2 \text{ for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

direct matrix  $\mathbf{V}^{(\lambda)}_{\text{T}} = \mathbf{V}^{(\lambda)}_{\text{TE}} + \mathbf{V}^{(\lambda)}_{\text{TO}} = a_\lambda T + b_\lambda U$

antisymmetric matrix  $\mathbf{V}^{(\lambda)}_{\text{T,AS}} = \left[ -\frac{1}{2} a_\lambda T + \frac{1}{2} b_\lambda U \right] \langle \tau, \tau \rangle$

$$\begin{Bmatrix} \text{repulsive} \\ \text{attractive} \\ \text{repulsive} \end{Bmatrix} \text{ for } \lambda = \begin{Bmatrix} 0^- \\ 1^- \\ 2^- \end{Bmatrix}$$

A systematic study of tensor interactions on Spin-Isospin excitations by HF+RPA



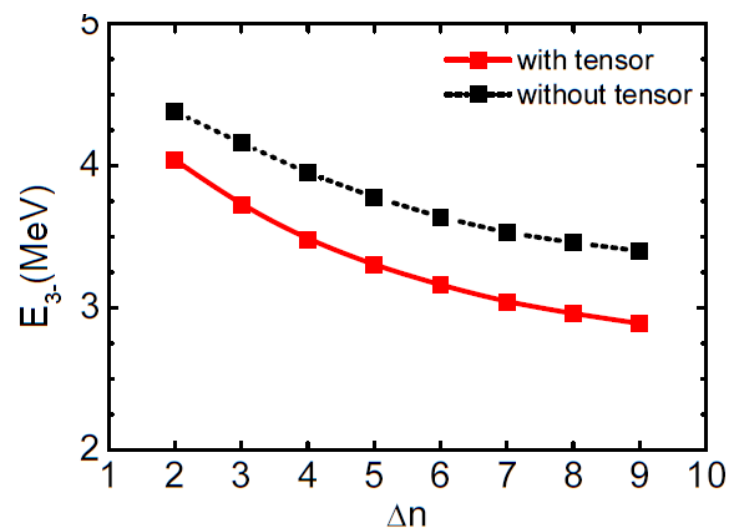
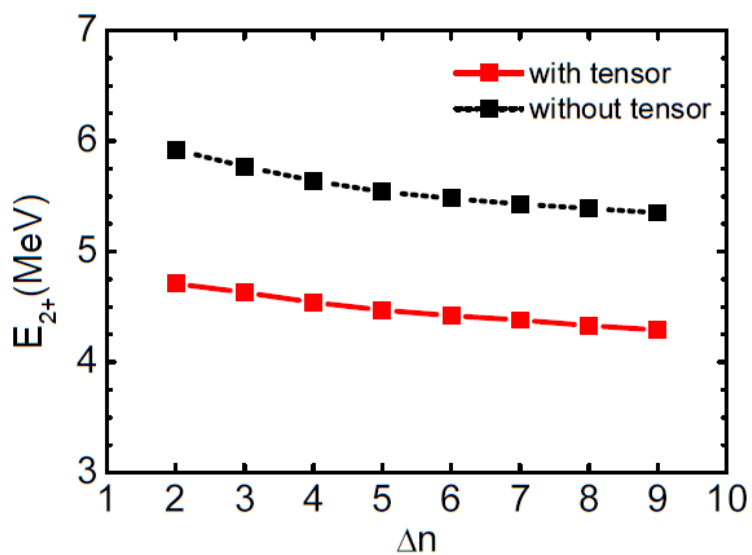
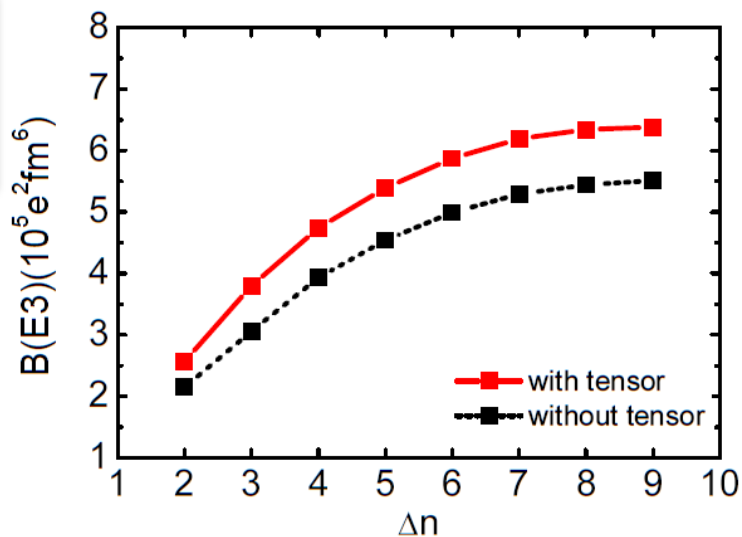
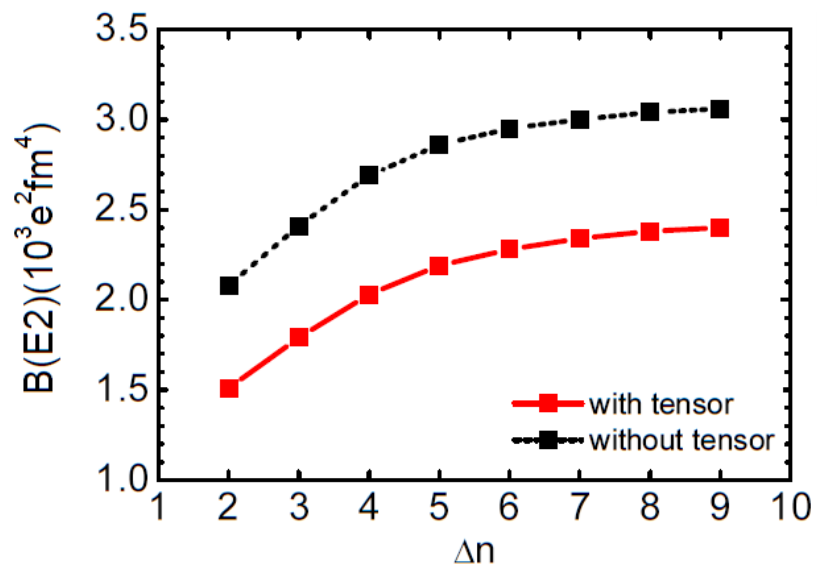
T(triplet-even tensor) is well constrained by spin-isospin excitations irrespective of central part of Skyrme forces.  $T=500\pm 100\text{MeVfm}^5$

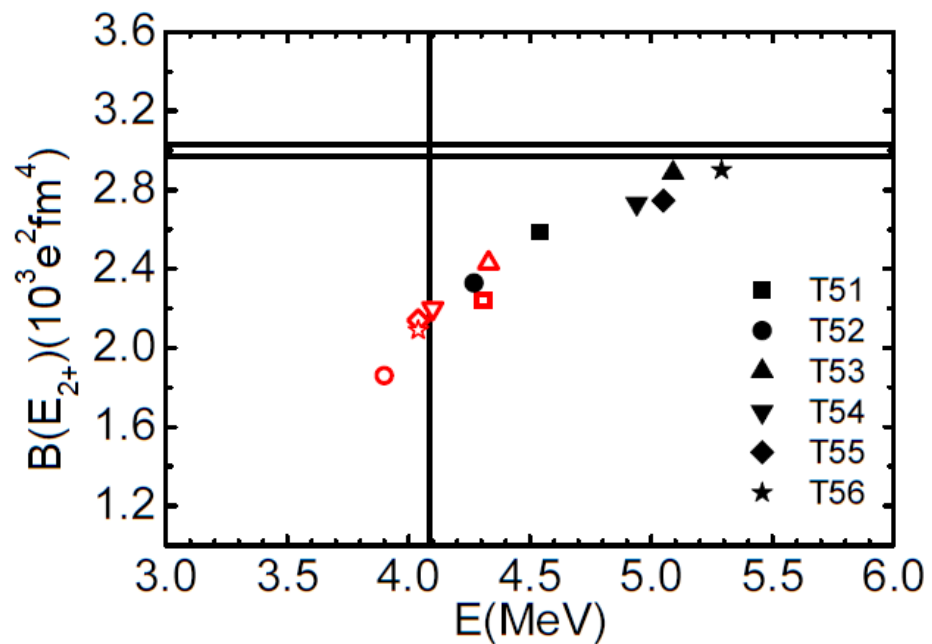
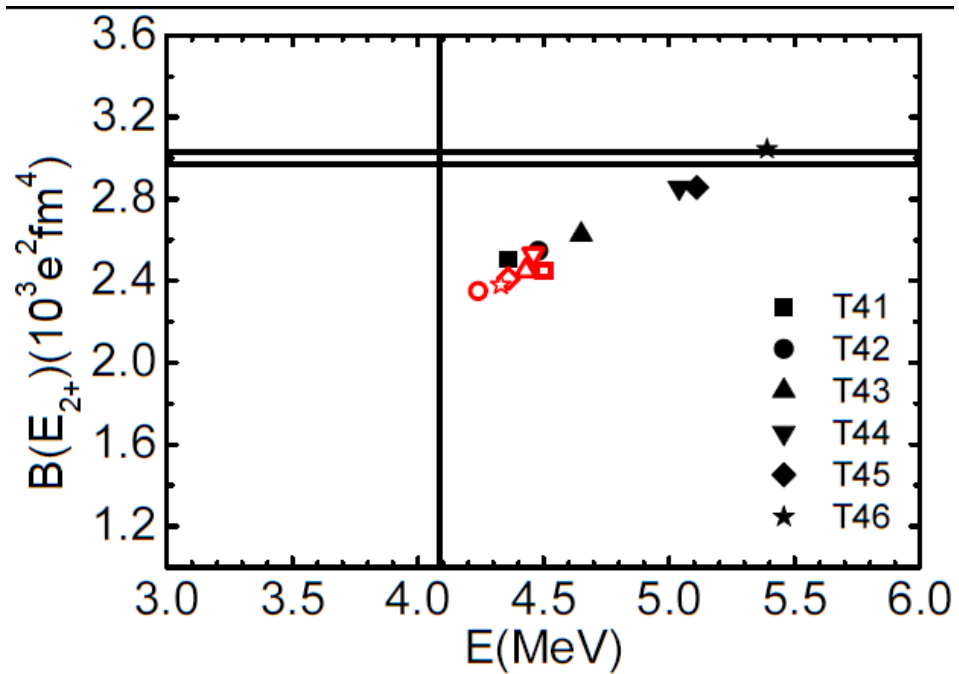
U(triplet-odd) is not well constrained by existing sets of experimental data.

### Effects of tensor correlations on low-lying collective states in finite nuclei

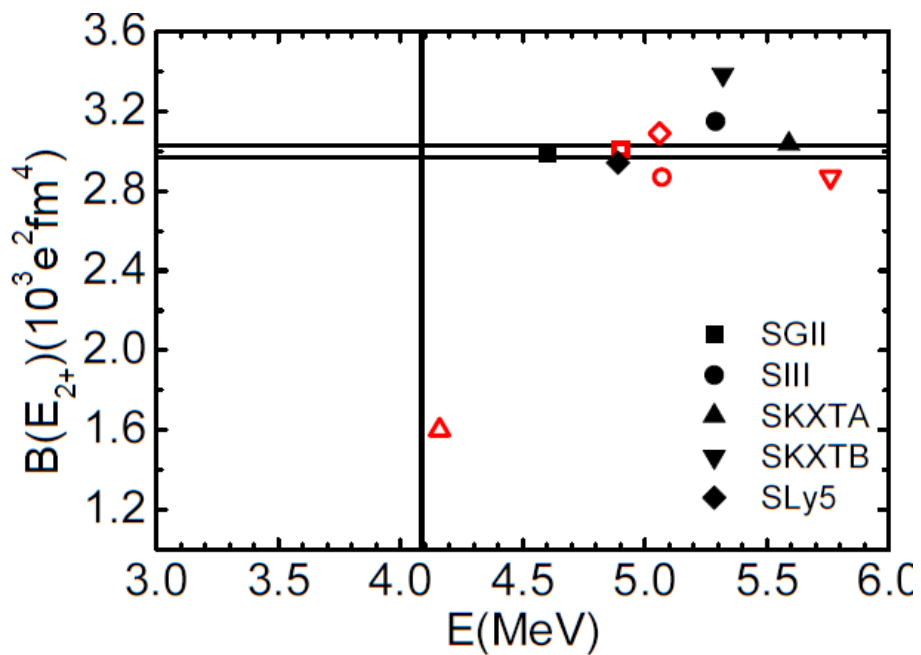
Li-Gang Cao (曹李刚),<sup>1,2,3</sup> H. Sagawa,<sup>2</sup> and G. Colò<sup>4,5</sup>

208Pb  
T46

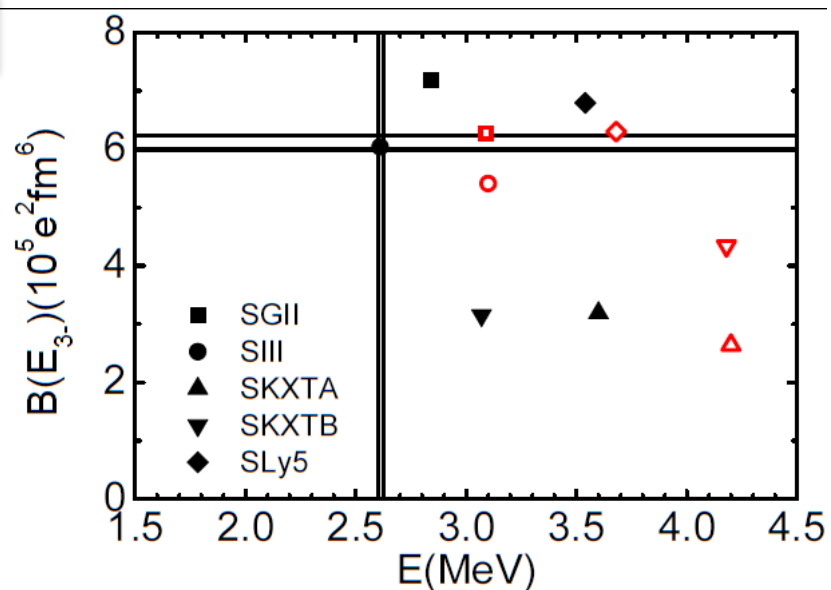
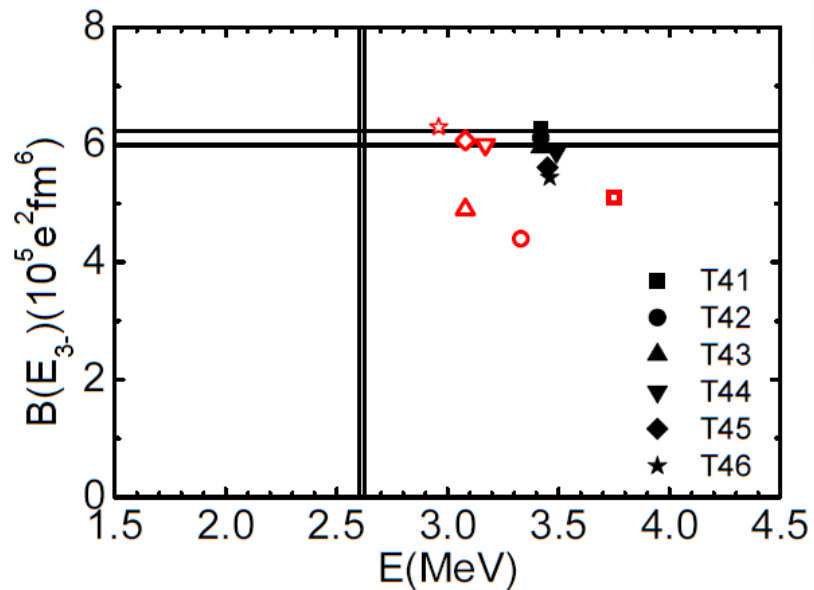




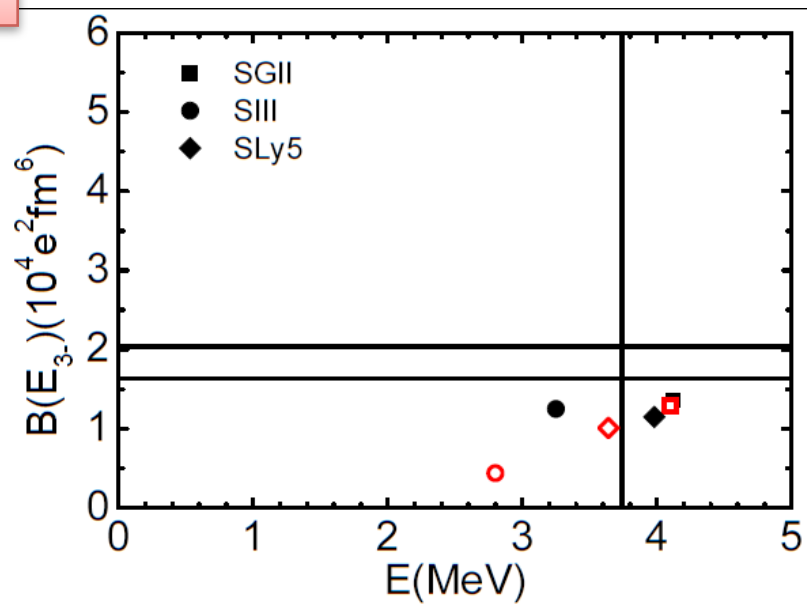
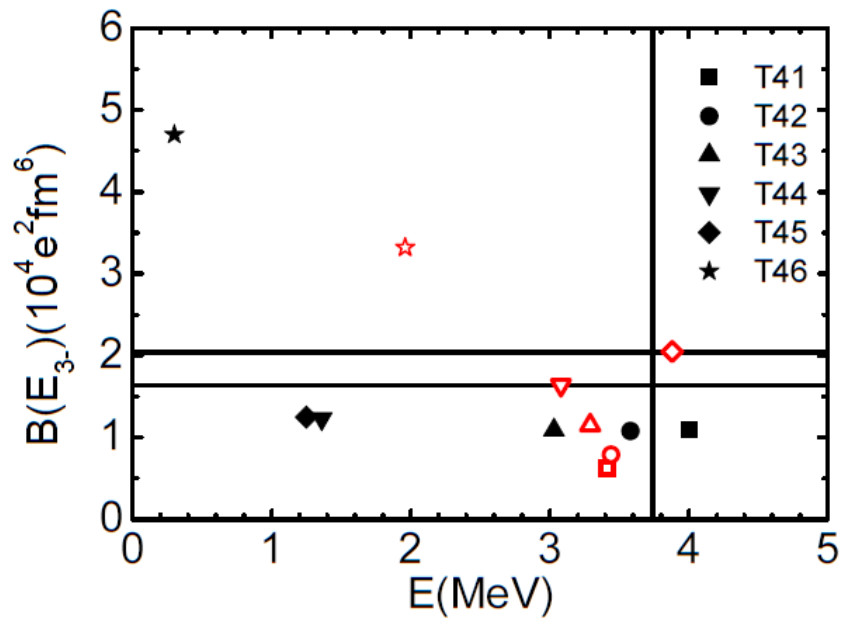
208Pb

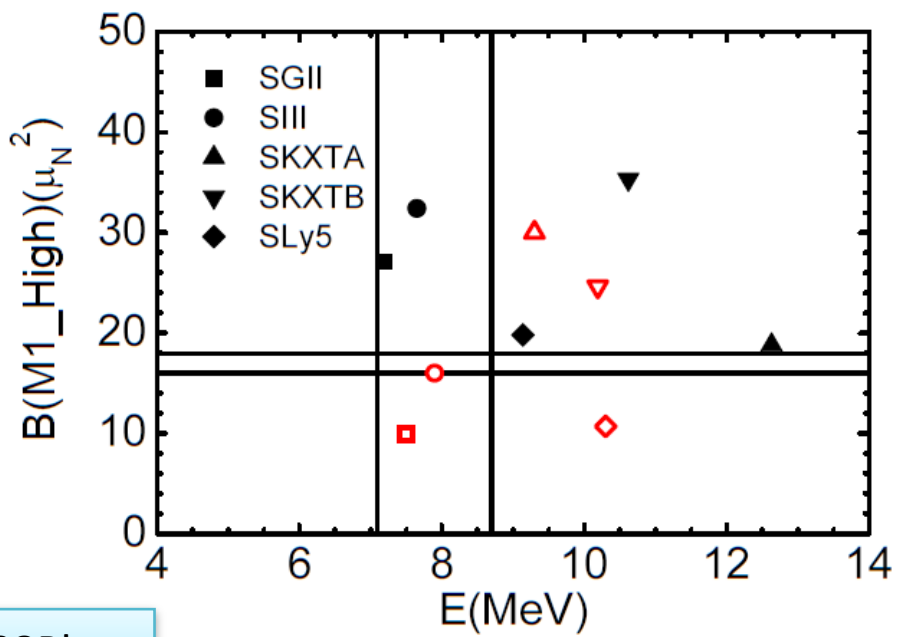
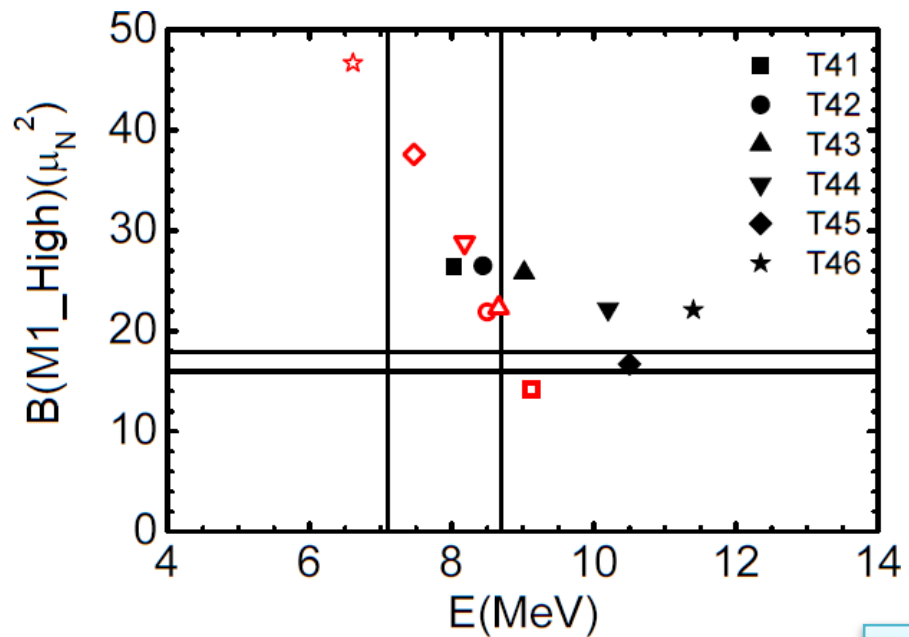


208Pb

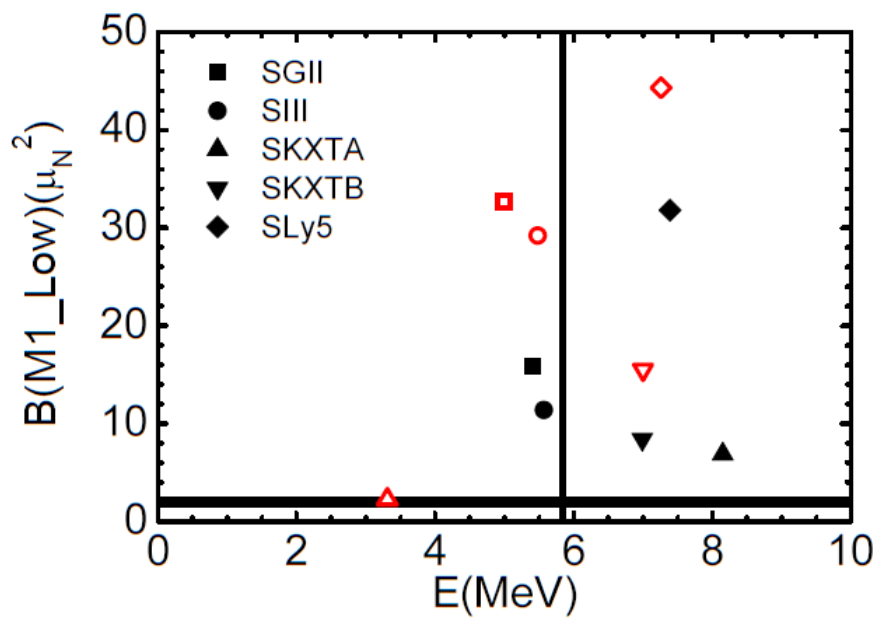
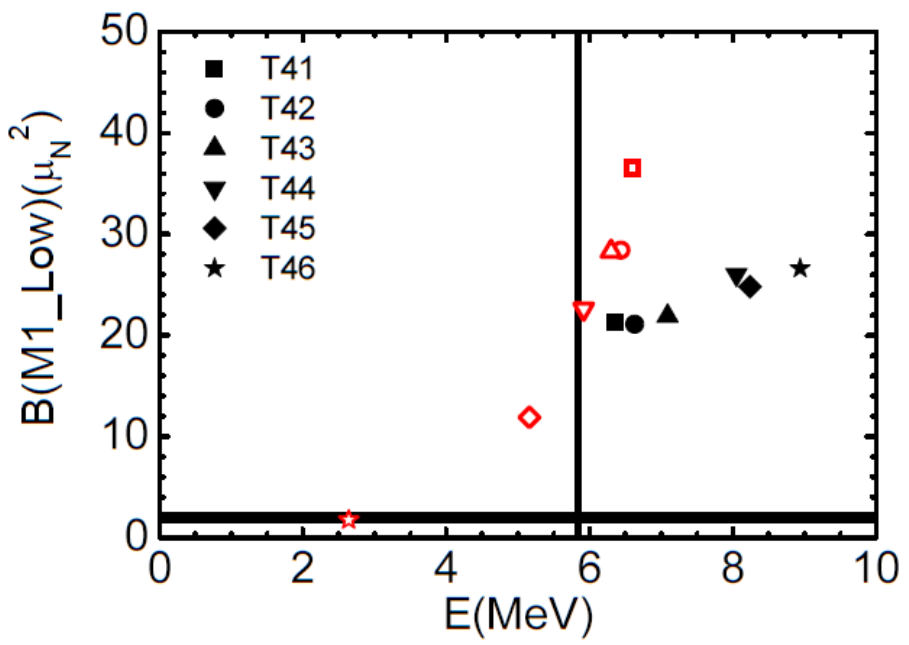


40Ca





208Pb





## Summary

1. Skyrme Tensor interactions are introduced in HF calculations. Triplet-Even and Triplet-Odd components
2. The isotope dependence of energy splitting ( $\epsilon(h11/2) - \epsilon(g7/2)$ ) of  $Z=50$  isotopes is well reproduced by a parameter set of tensor interactions. The same parameter set gives fairly good description of energy difference  $\epsilon(i13/2) - \epsilon(h9/2)$  of  $N=82$  isotones .
3. *HF+RPA calculations are performed for Gamow-Teller and Spin-multipole excitations in  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$ . We found that the sum rule strength of GT transitions is increased, while the main peak energy is slightly shifted to lower energy side. This is due to the coupling between GT and SQR with the tensor interactions.*
4. *10% of sum rule strength is removed from the main peak to higher energy region of SQR.*
5. *Softening and hardening of Spin-Dipole excitations are found in experimentally and RPA with tensor interactions reproduces well these experimental findings.*
6. T(triplet-even) interaction is well constrained by spin-isospin excitations irrespective of central part of Skyrme interactions.

7. U part (triplet-odd) is still not well constrained by the existing experimental data set.

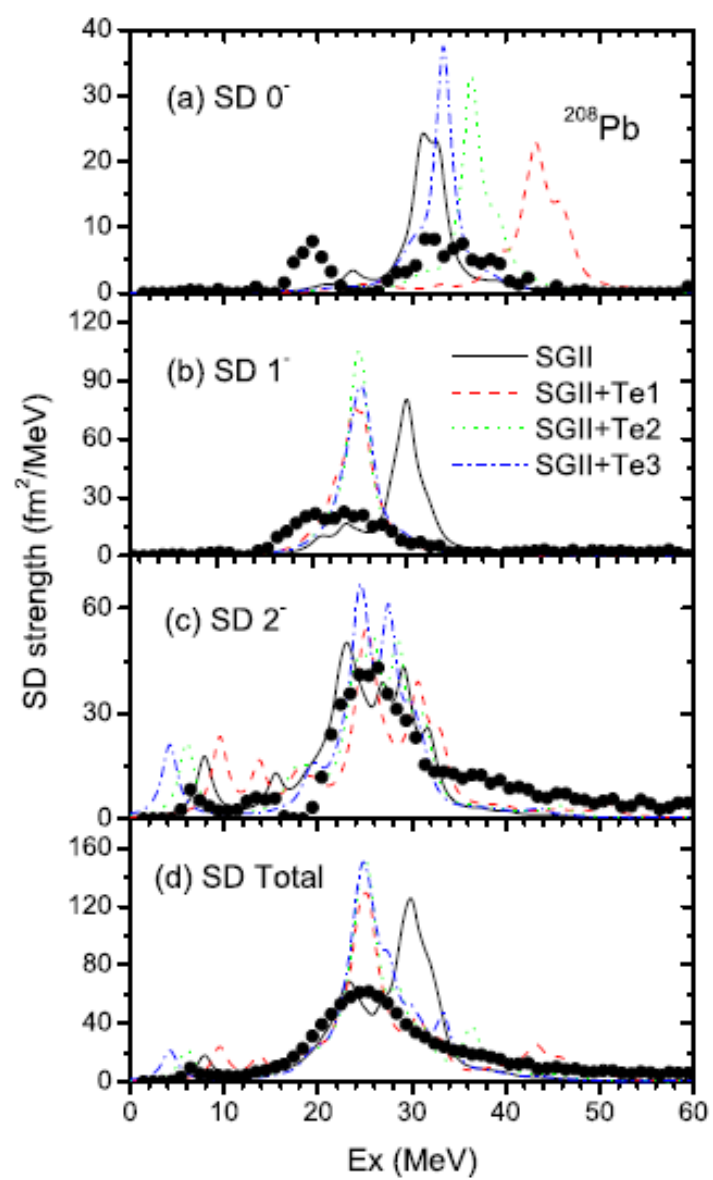
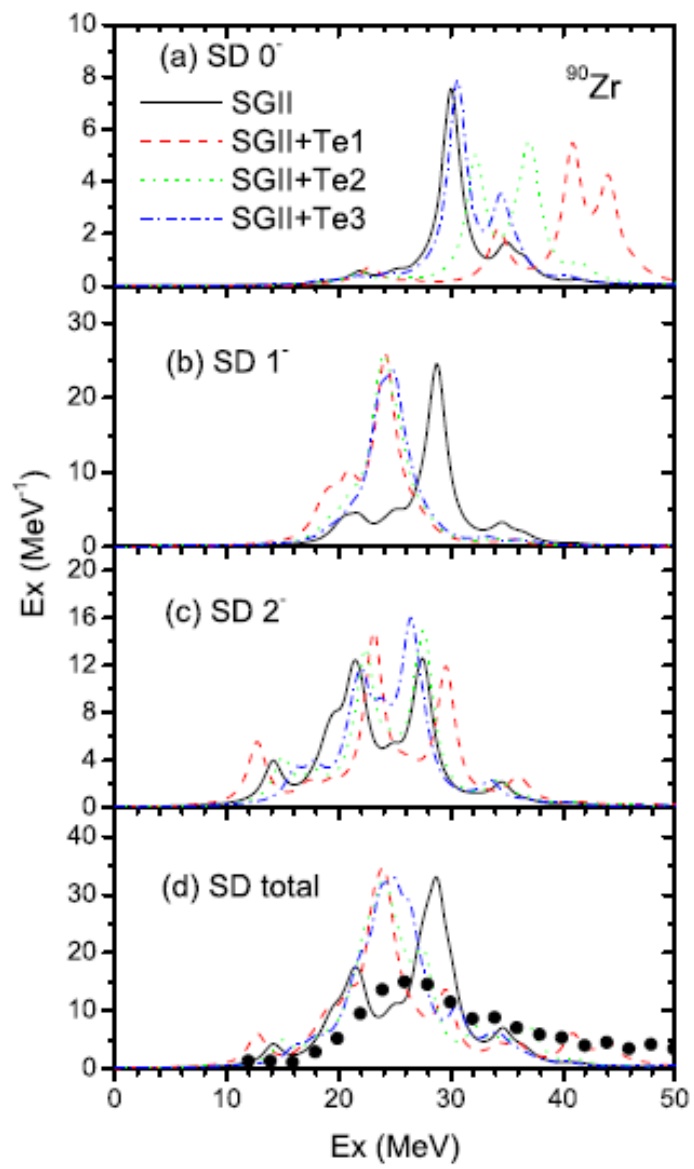
8. Recommended interactions:

Spin-Isospin excitations: T21, T32, T43, T54 and SGII+T

Low-energy collective excitations: T44, T45, T46, SGII+T

## Conclusion

Happy Birthday to Peter Ring!  
You may never retired from physics!  
I hope to see you many places all over the world  
again and again in future!



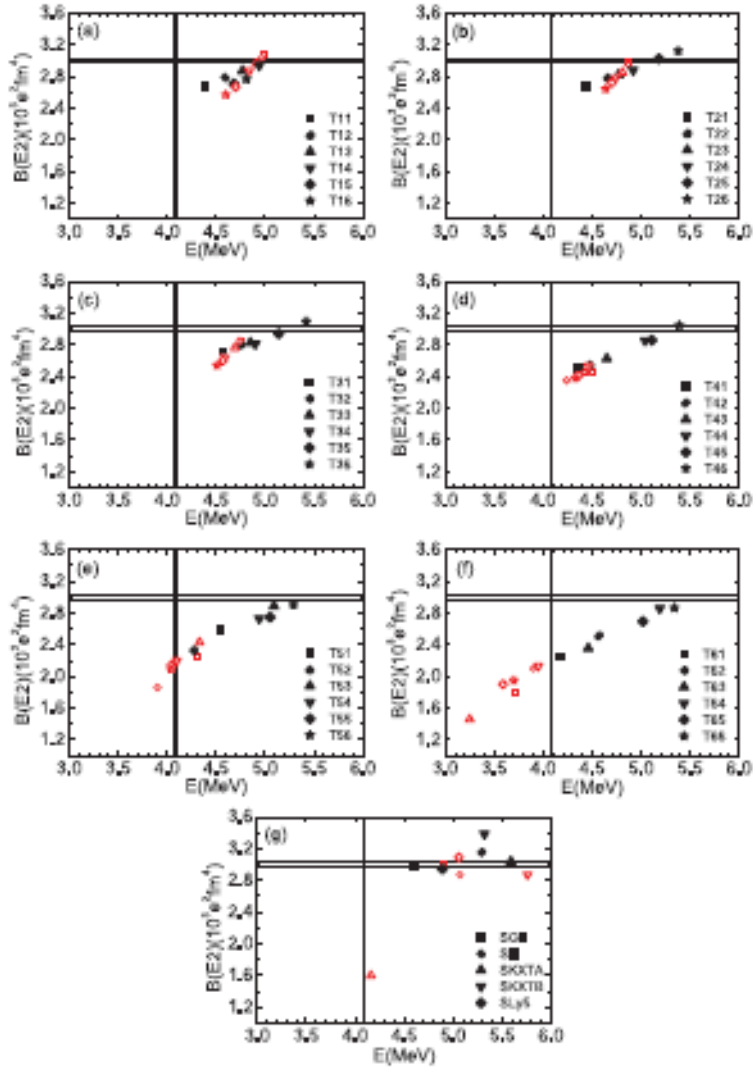


FIG. 2. (Color online) The HF + RPA results for the excitation energy and the  $B(E2)$  strength of the lowest quadrupole state in  $^{208}\text{Pb}$ . The calculations are performed with and without tensor terms, and the results are denoted by the open and the filled symbols, respectively. The vertical and horizontal lines mark the experimental values with their errors. These experimental data are taken from Ref. [17].

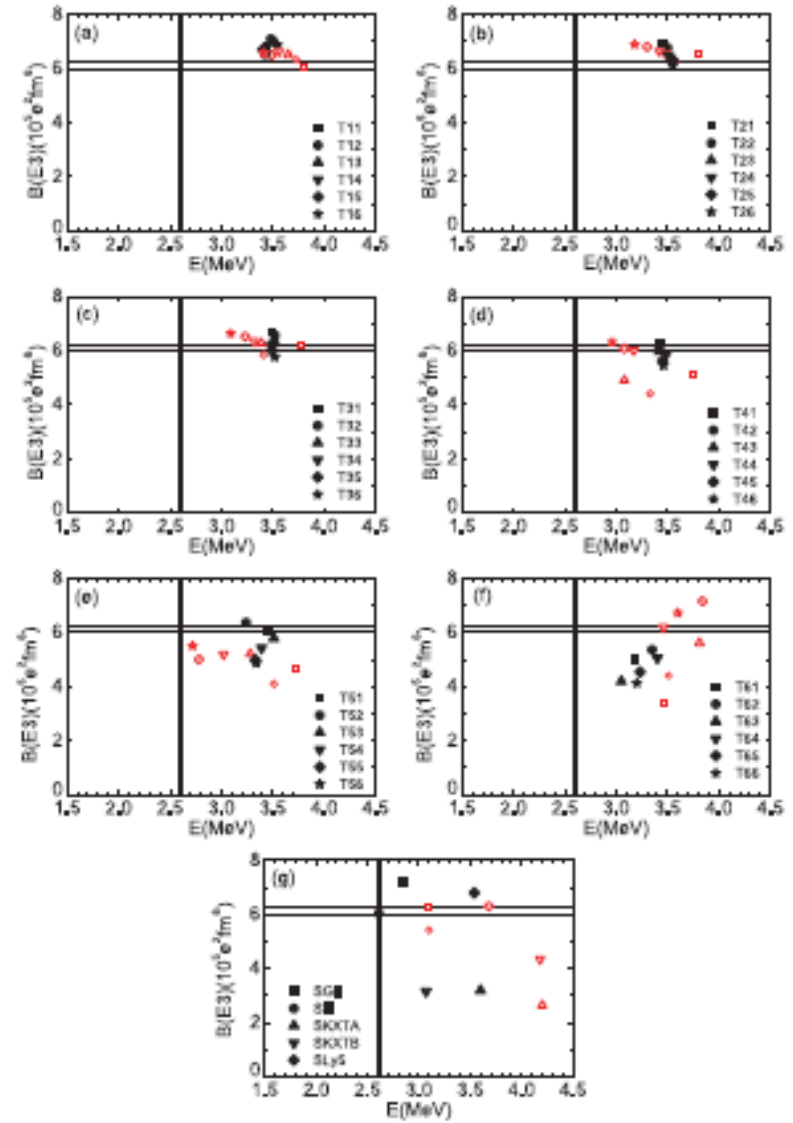


FIG. 3. (Color online) The HF + RPA results for the excitation energy and the  $B(E3)$  strength of the lowest octupole state in  $^{208}\text{Pb}$ . The calculations are performed with and without tensor terms, and the results are denoted by the open and the filled symbols, respectively. The vertical and horizontal lines mark the experimental values with their errors. These experimental data are taken from Ref. [18].

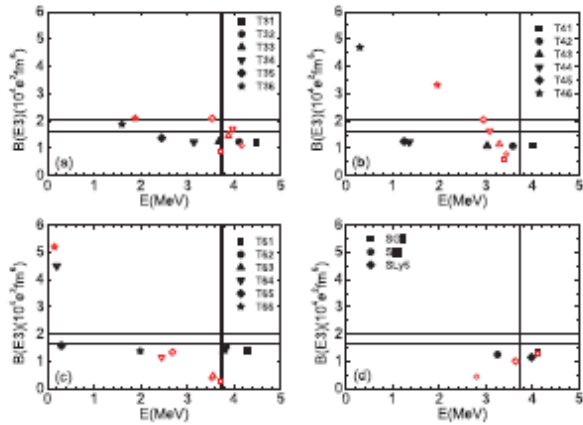


FIG. 4. (Color online) The same as Fig. 3 for the 3-state in  $^{40}\text{Ca}$ . Shown are selected results of T3J, T4J, T6J, SGII, SIII, and SLy5 interactions. Experimental data are taken from Ref. [18].

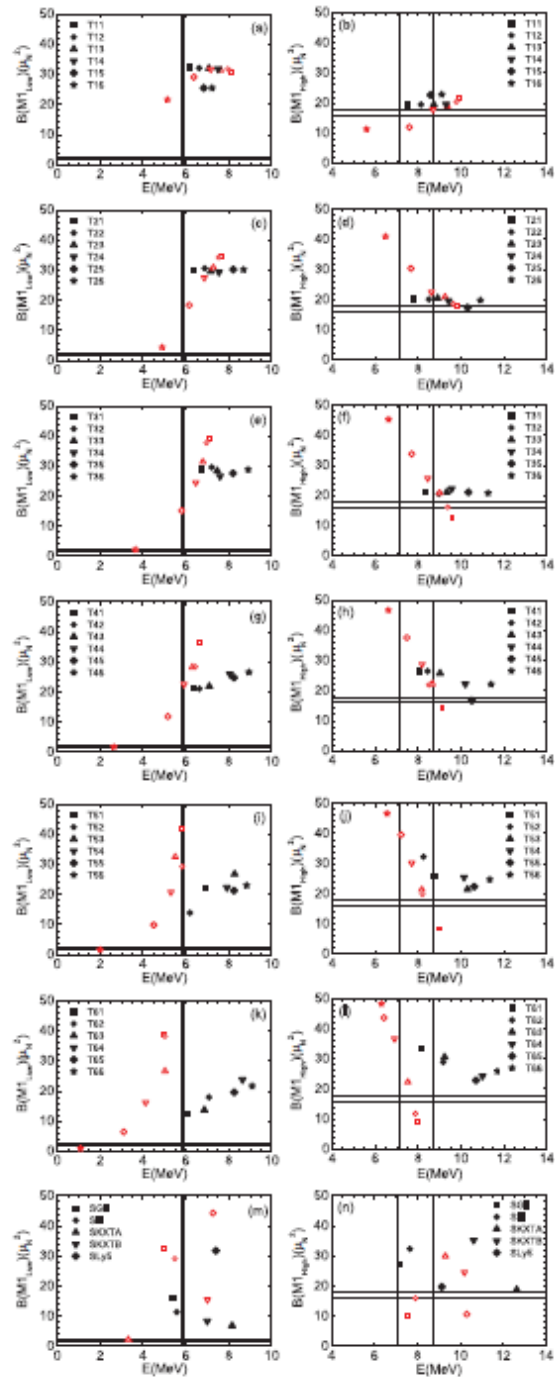


FIG. 5. (Color online) The HF+RPA results for the excitation energy and the B(M1) strength of the low (left panels) and high (right panels)  $1^+$  state in  $^{208}\text{Pb}$ . The calculations are performed with and without tensor terms, and the results are denoted by the open and the filled symbols, respectively. The vertical and horizontal lines mark the experimental values with their errors. These experimental data are taken from Ref. [22].

## Collaborators

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## Landau parameters and Stability condition with tensor interaction

Li-Gang Cao, G. Colo and H.S., PRC81,044302(2010)

$$\begin{aligned}
 V_{\text{ph}} = & \sum_{\ell} (F_{\ell} + F'_{\ell} \tau_1 \cdot \tau_2 + G_{\ell} \sigma_1 \cdot \sigma_2 \\
 & + G'_{\ell} (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2)) P_{\ell}(\cos\theta) , \\
 & + \frac{q^2}{k_F^2} H(\cos\vartheta) S_{12}(\hat{q}) + \frac{q^2}{k_F^2} H'(\cos\vartheta) S_{12}(\hat{q}) \tau \cdot \tau
 \end{aligned}$$

$$H_0 = N_0 k_F^2 \frac{1}{4} \left( \frac{1}{2} T + \frac{3}{2} U \right)$$

$$H'_0 = N_0 k_F^2 \frac{1}{4} \left( -\frac{1}{2} T + \frac{1}{2} U \right)$$

Stability conditions

no tensor

$$l=0 \quad \text{IS} \quad 1 + G_0 > 0 \quad \text{IV} \quad 1 + G'_0 > 0$$

$$l=1 \quad \text{IS} \quad 1 + \frac{1}{3} G_1 > 0 \quad \text{IS} \quad 1 + \frac{1}{3} G'_1 > 0$$

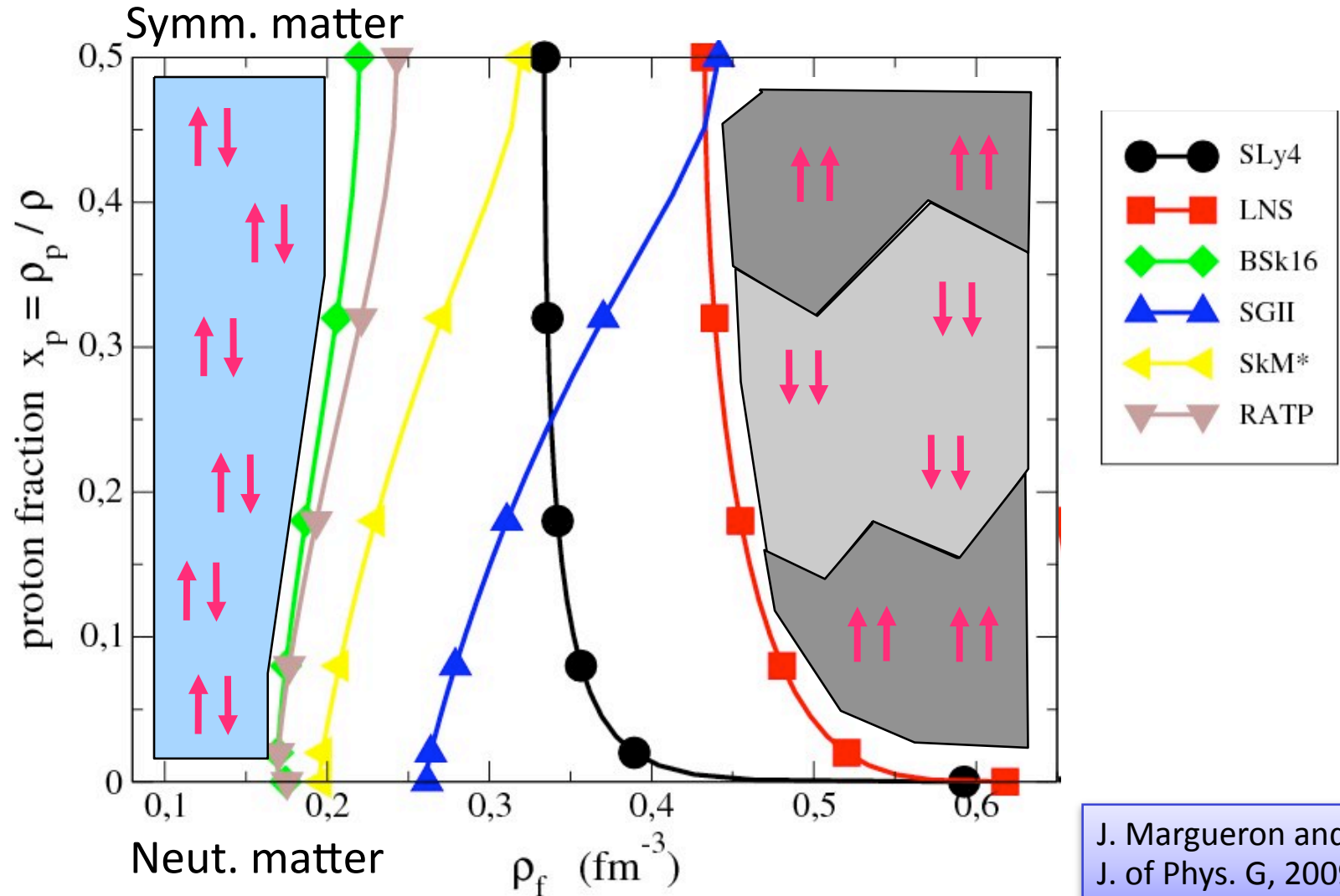
with tensor

$$l=1, J=0 \quad \text{IS} \quad 1 + \frac{1}{3} G_1 - \frac{10}{3} H_0 > 0 \quad \text{IV} \quad 1 + \frac{1}{3} G'_1 - \frac{10}{3} H'_0 > 0$$

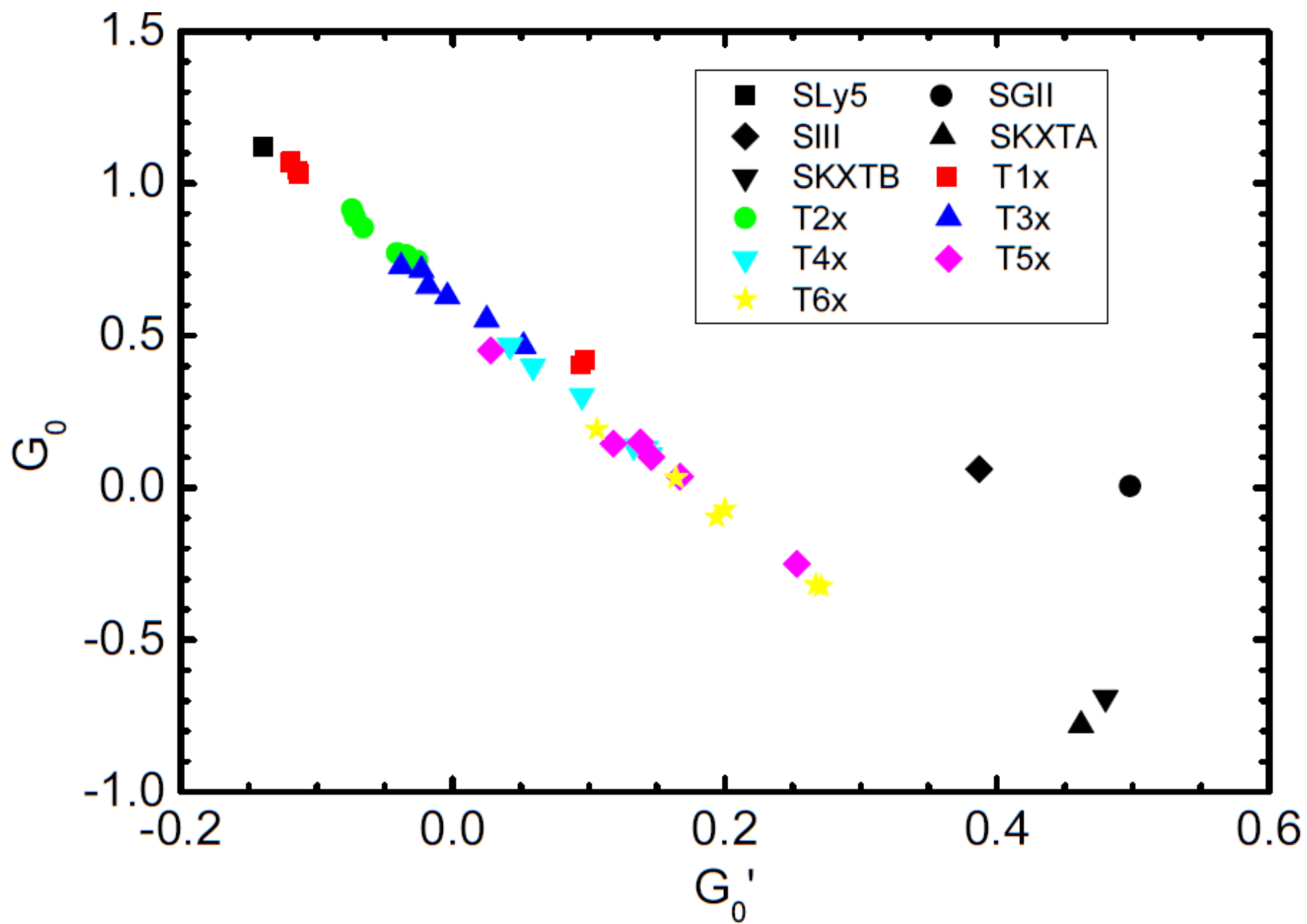
$$l=1, J=1 \quad \text{IS} \quad 1 + \frac{1}{3} G_1 + \frac{5}{3} H_0 > 0 \quad \text{IV} \quad 1 + \frac{1}{3} G'_1 + \frac{5}{3} H'_0 > 0$$

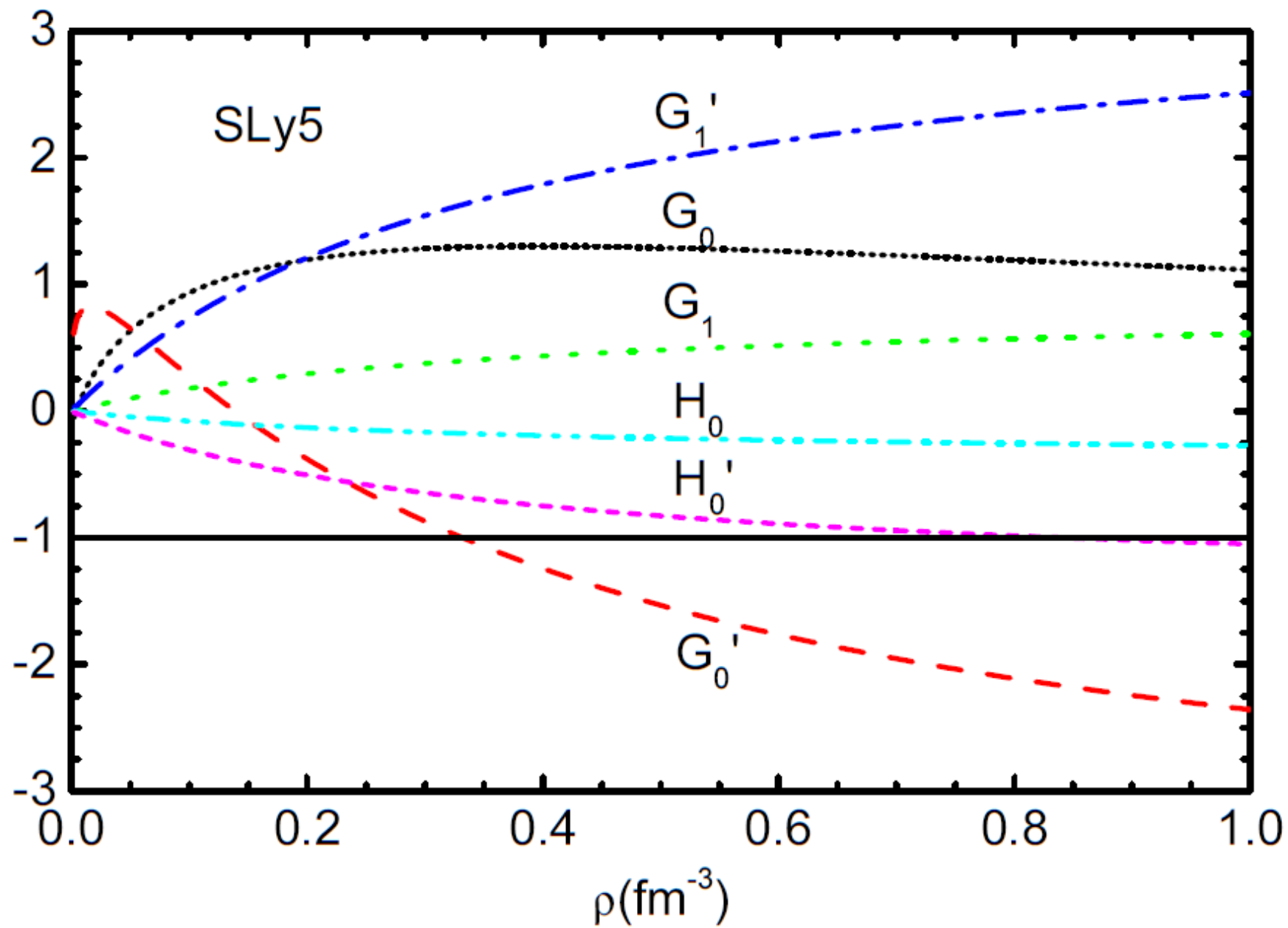


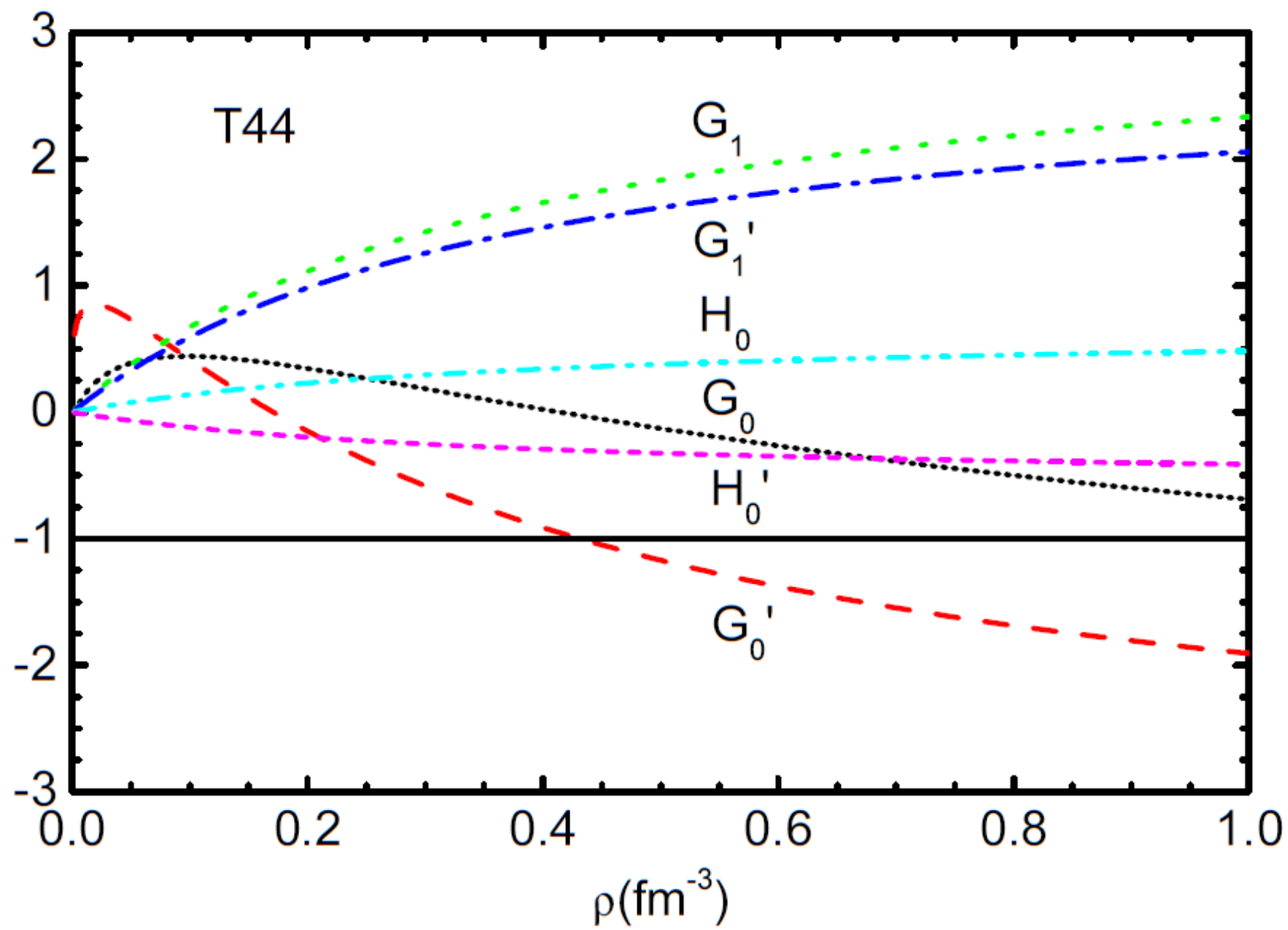
# Ferromagnetic phase diagram: G & G'



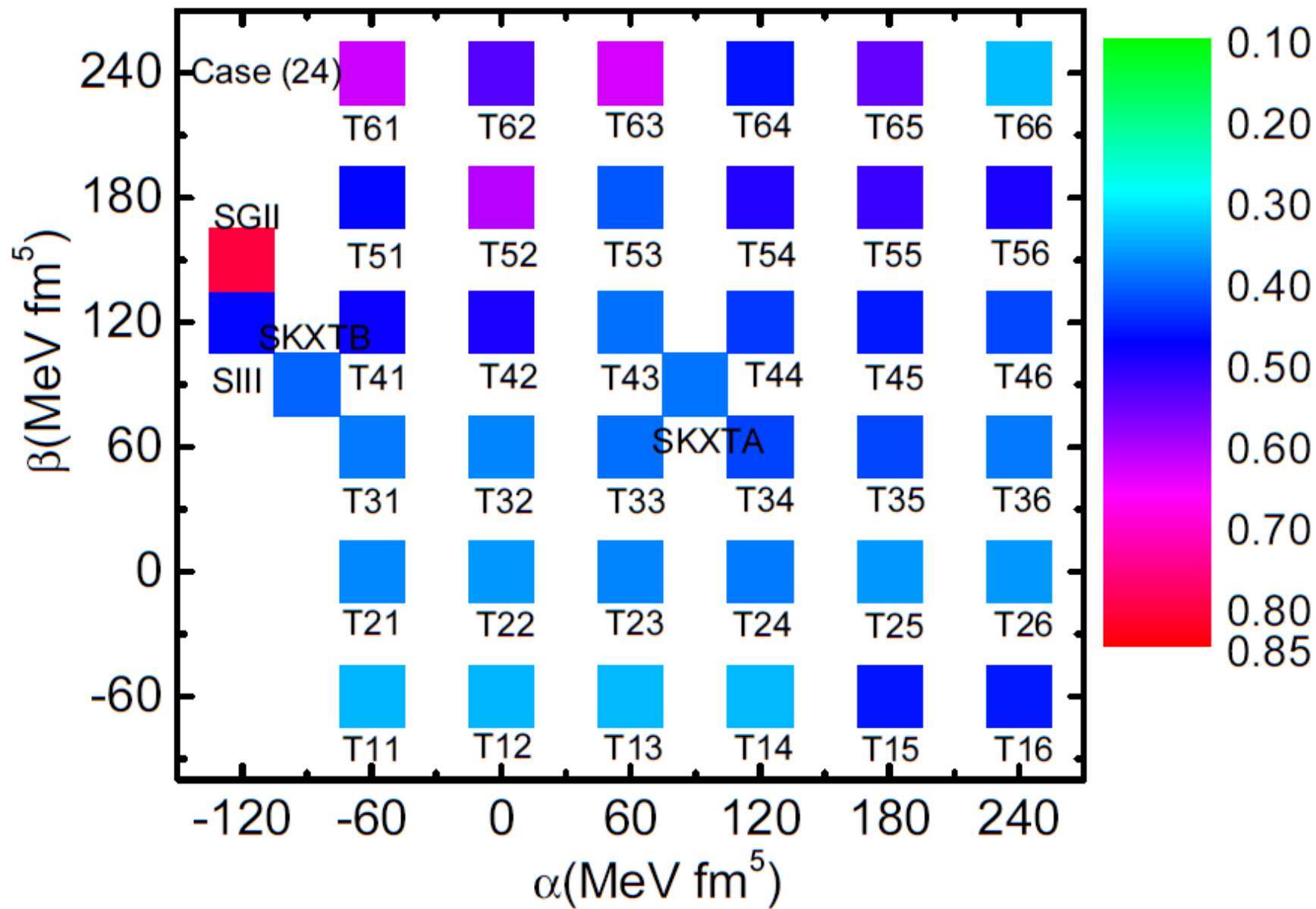
J. Margueron and HS,  
J. of Phys. G, 2009



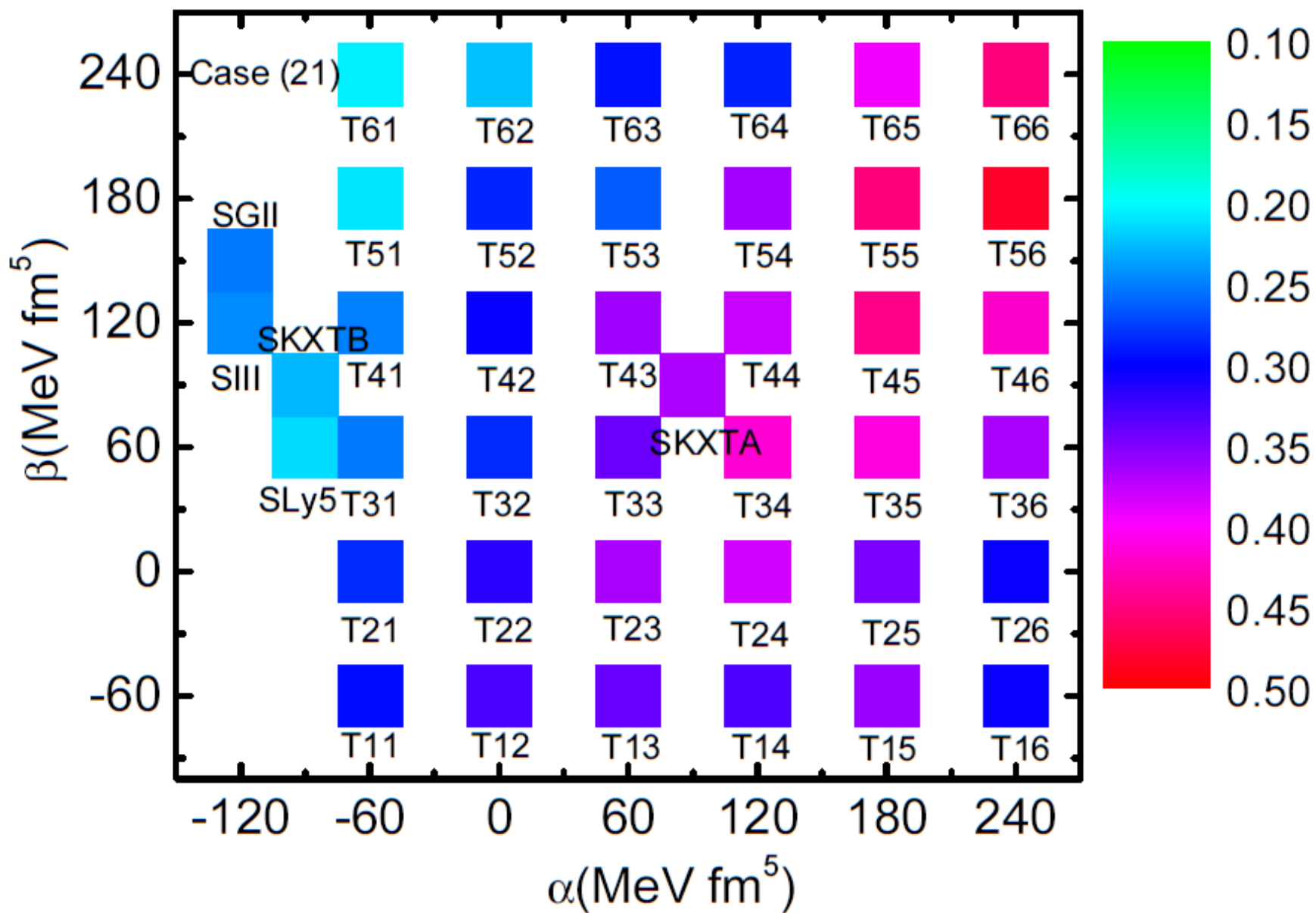




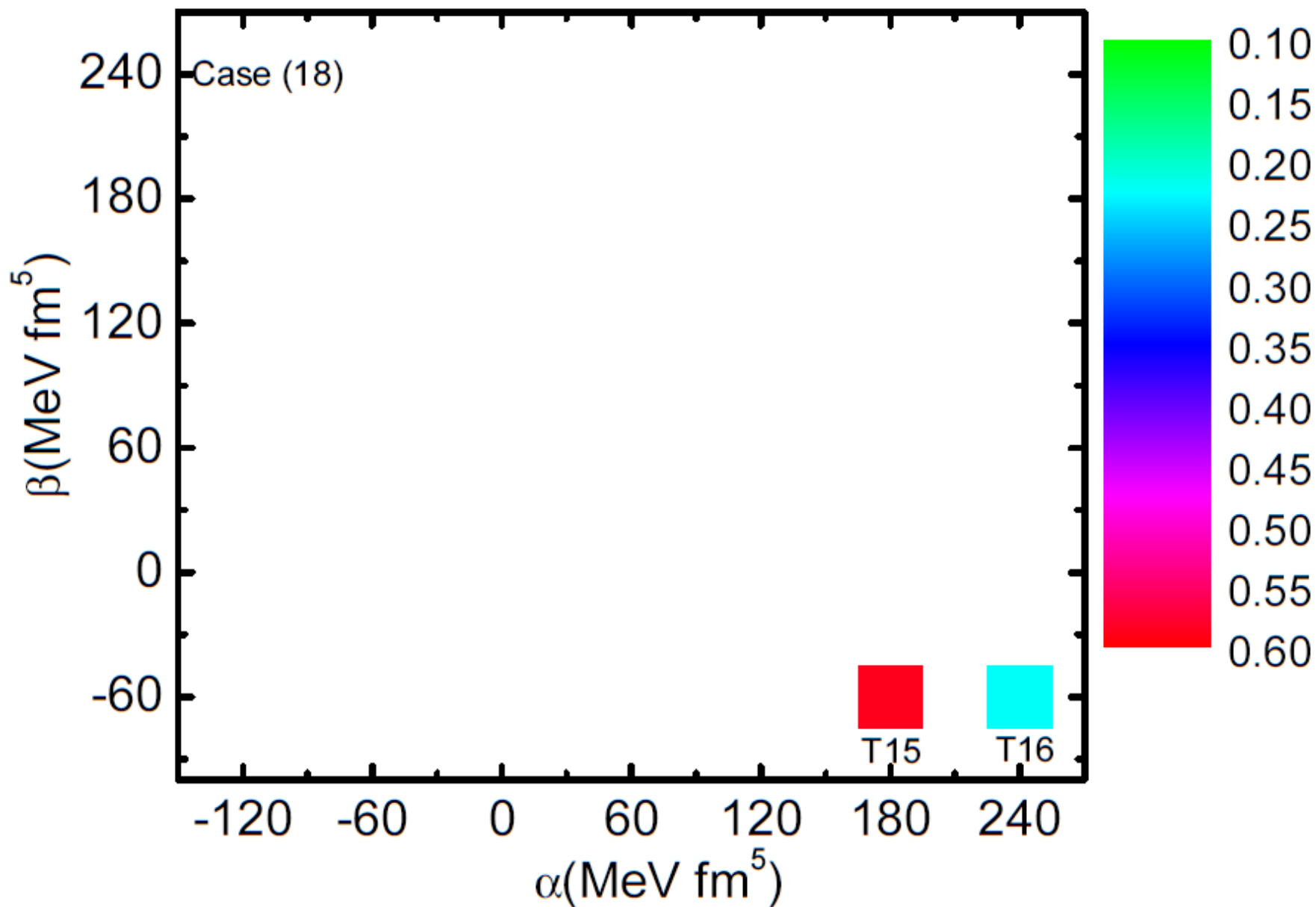
IV  $J^\pi = 1^+$   $1+G'_0 > 0$  (without tensor)



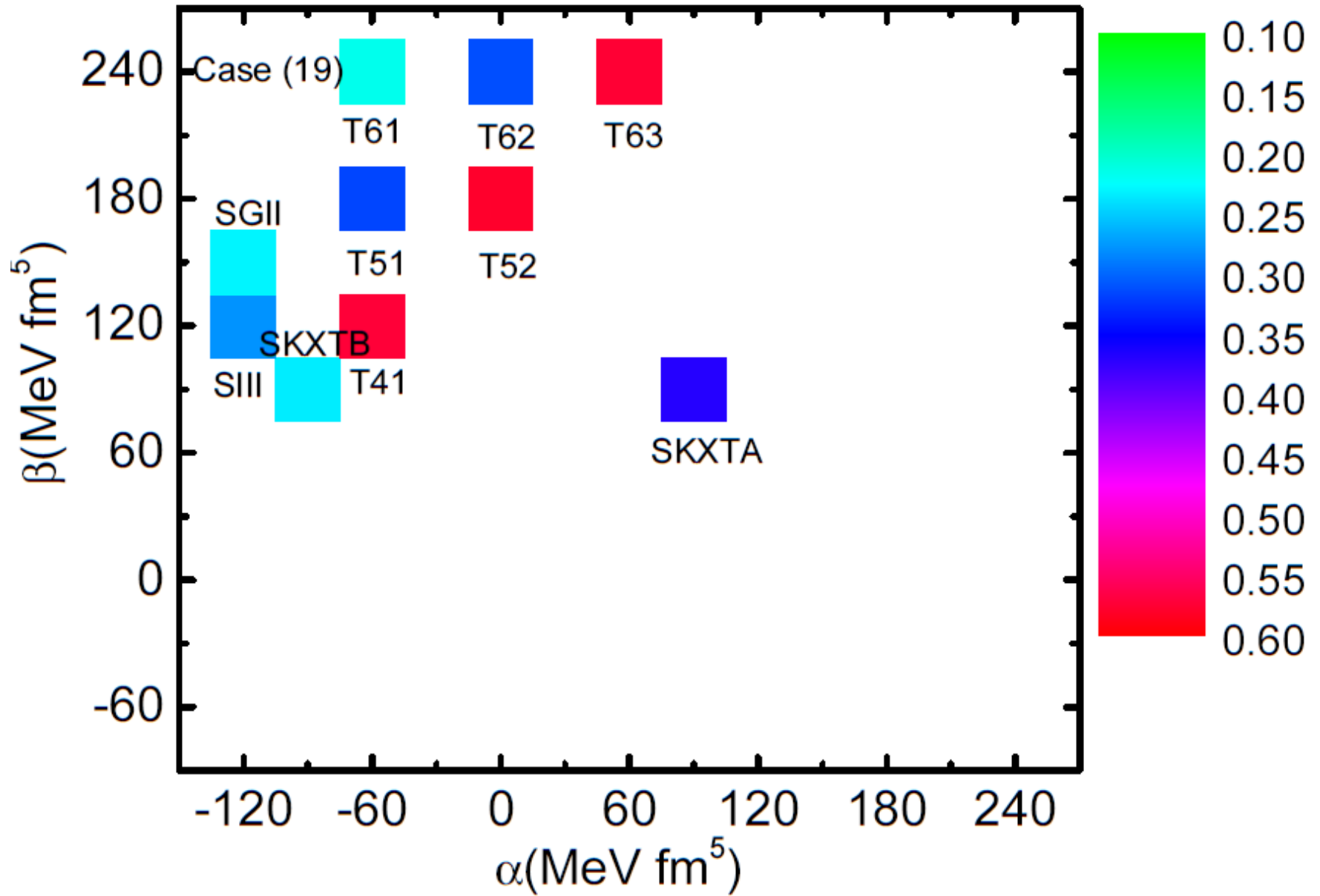
$$\text{IV } J = 1^+ \quad (2 + G'_0) - \sqrt{G'_0{}^2 + 8H'_0{}^2} > 0$$



$$\text{IV } J = 0^- \quad 1 + \frac{1}{3}G'_1 - \frac{10}{3}H'_0 > 0$$

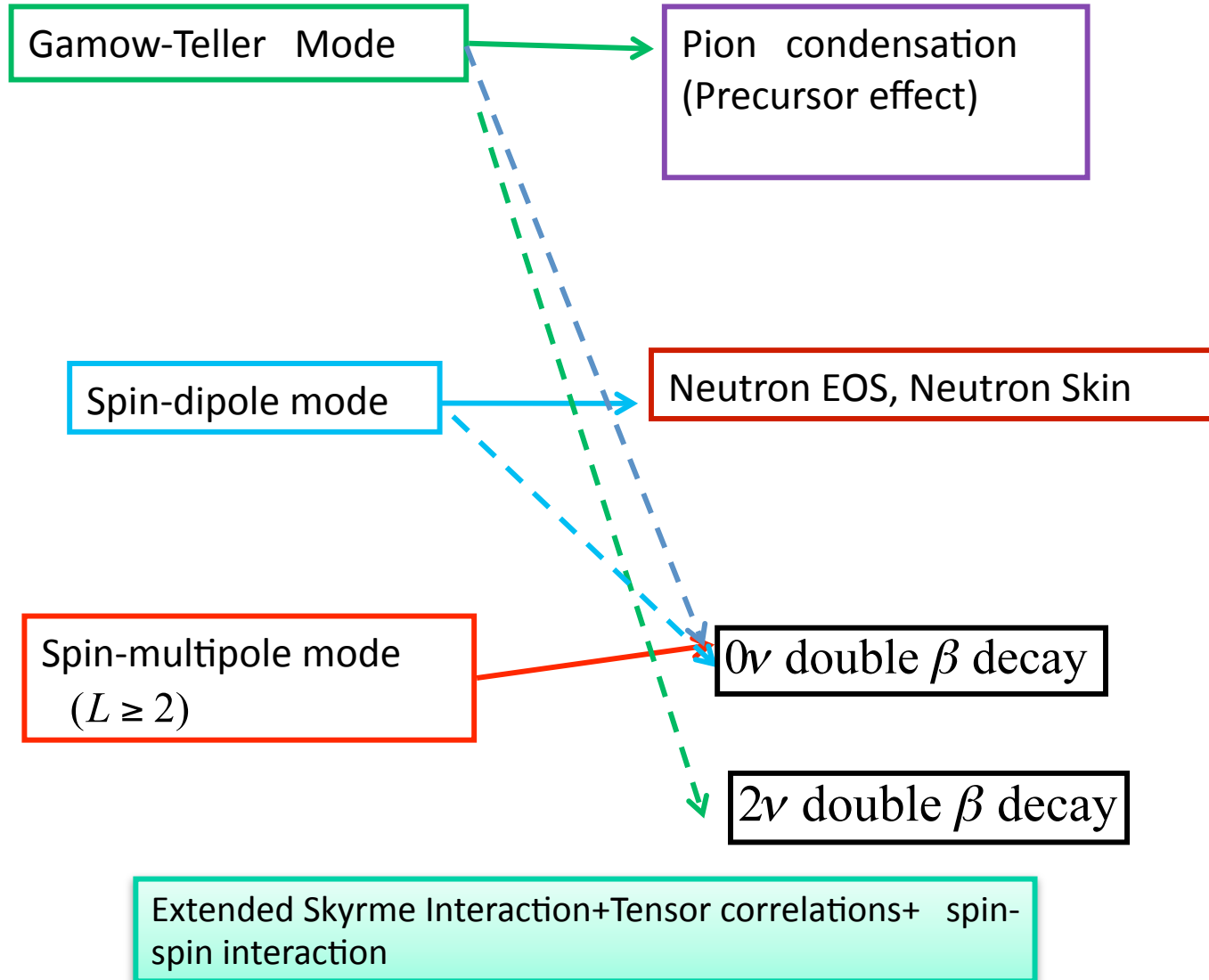


$$IV J = 1^- \quad 1 + \frac{1}{3}G'_1 + \frac{5}{3}H'_0 > 0$$

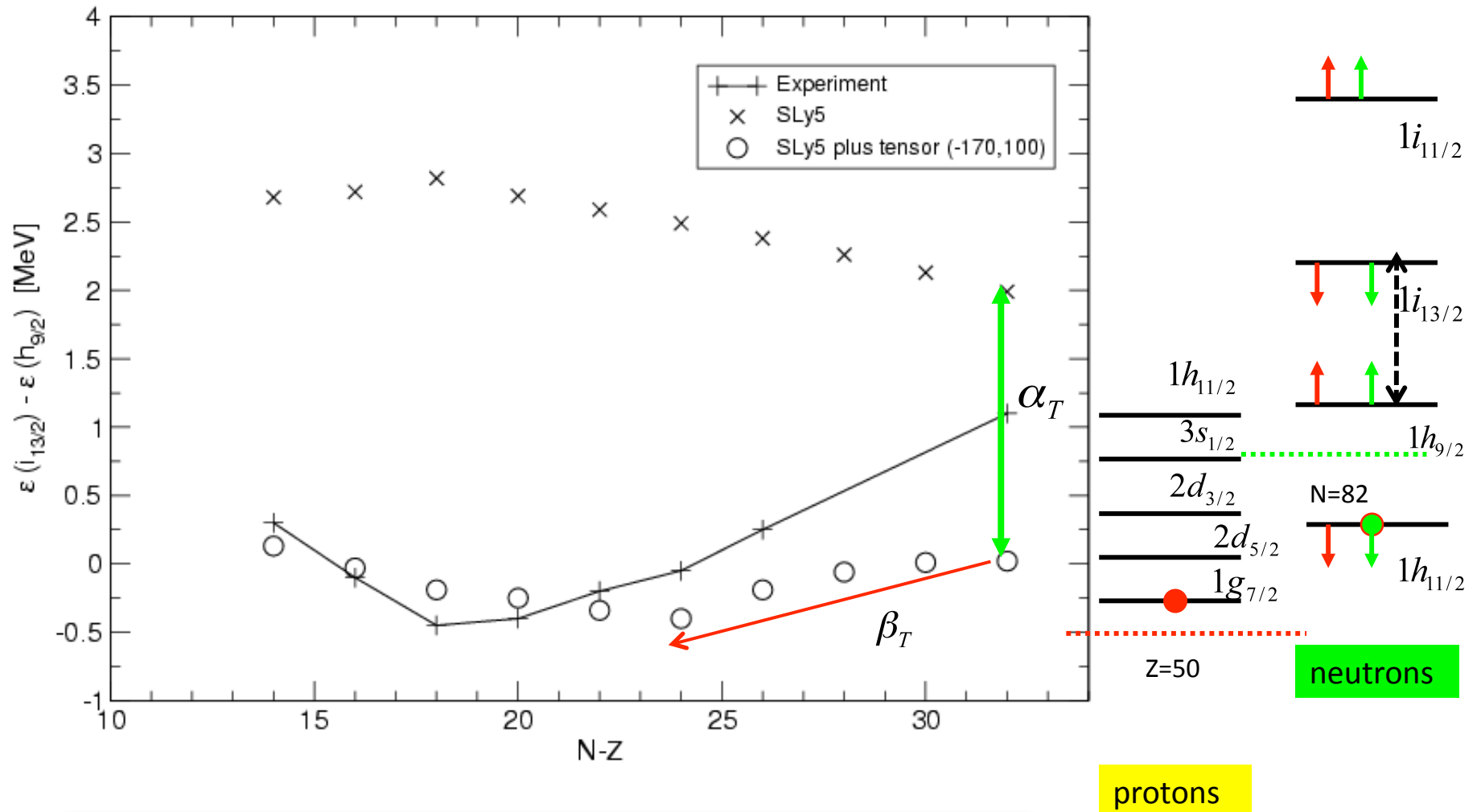




# Spin-Isospin mode Diagram

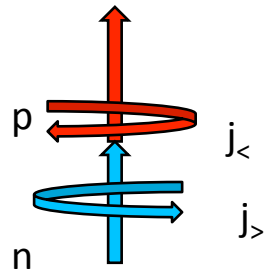


Neutrons on N=82 core

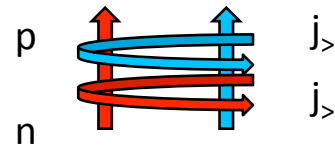


Effect of tensor correlations are shown on both p and n spin-orbit splittings.

# Tensor effect of pion and rho meson exchange potentials on Spin-orbit interaction



attractive

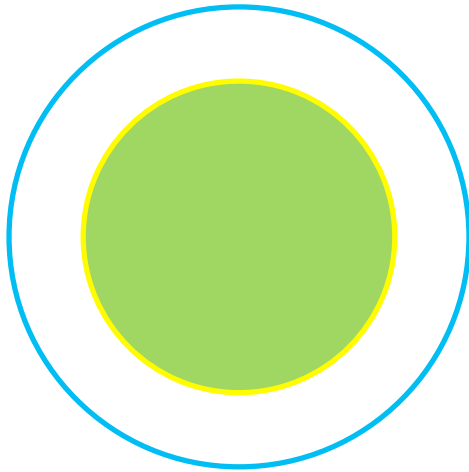


repulsive

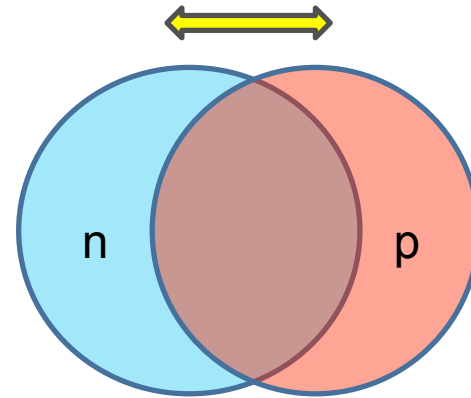
T. Otsuka et al., PRL95,232502 (2005)

Various excitation mode of finite nucleus (spin  $\times$  isospin  $\times$  multipolarity)

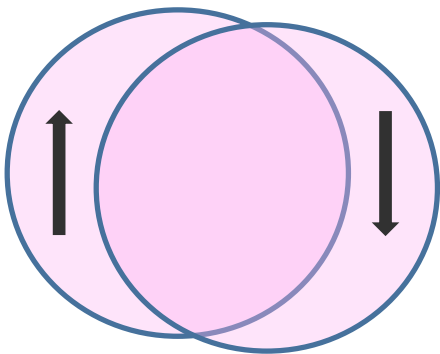
IS mode  $r^2 Y_0(\hat{r})$



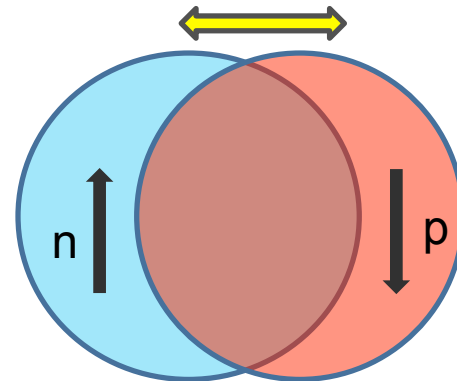
IV(Isospin) mode  $\tau$



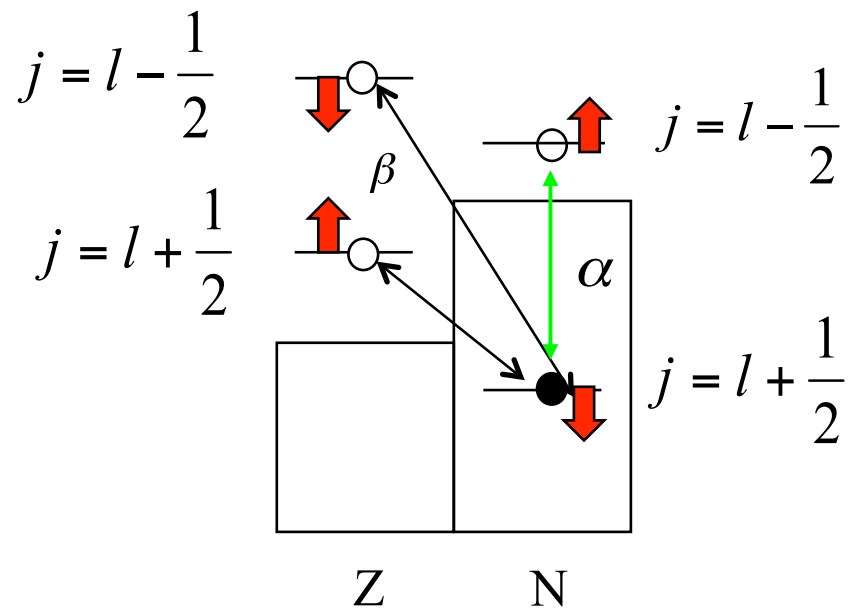
Spin mode  $\sigma$



Spin-Isospin mode  $\sigma\tau$



## Effect of tensor interaction on spin-orbit splitting



SLy5+T

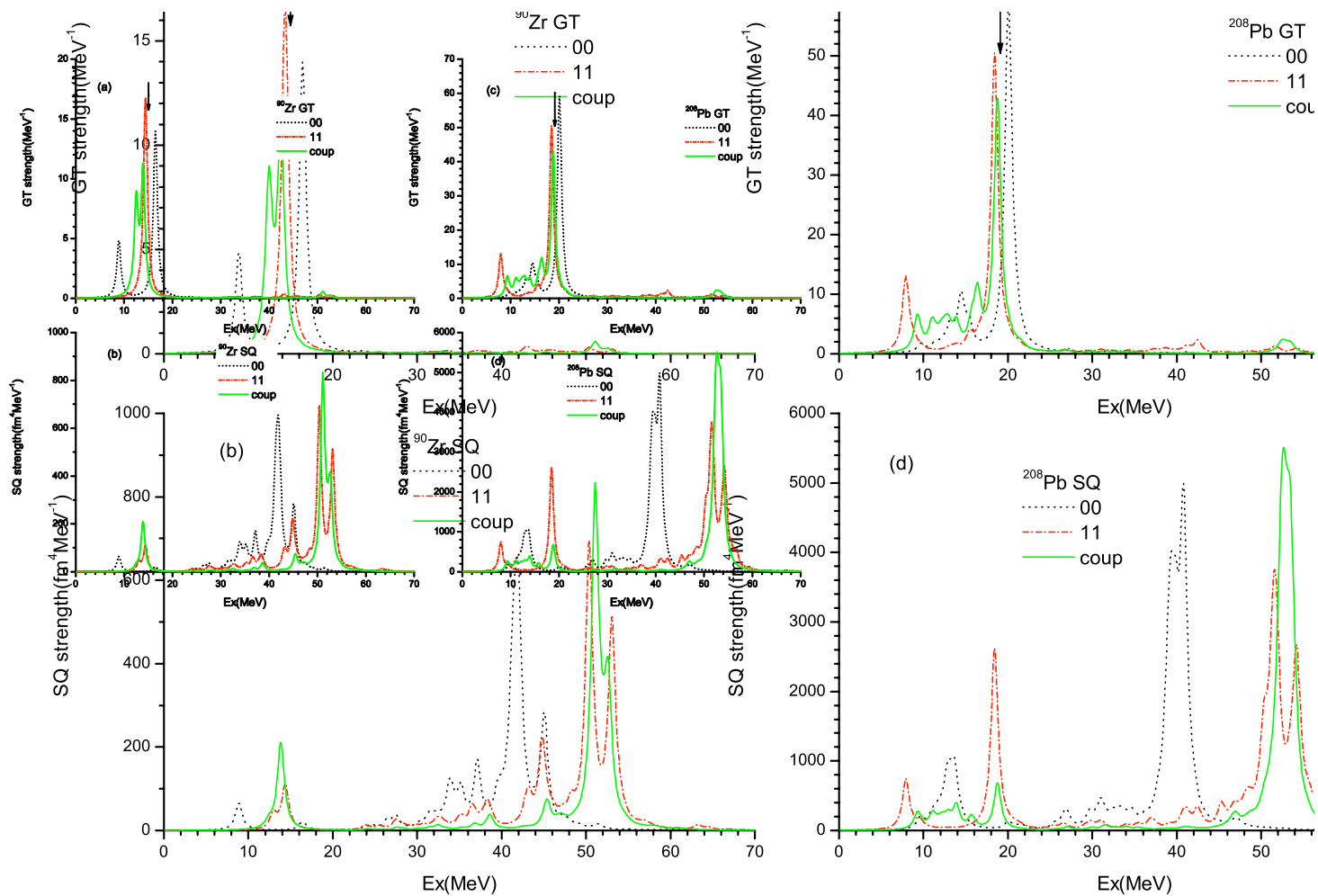
$$\alpha = \alpha_C + \alpha_T = -89.3 \text{ MeV} \cdot \text{fm}^5$$

$$\beta = \beta_C + \beta_T = 51.1 \text{ MeV} \cdot \text{fm}^5$$

T44

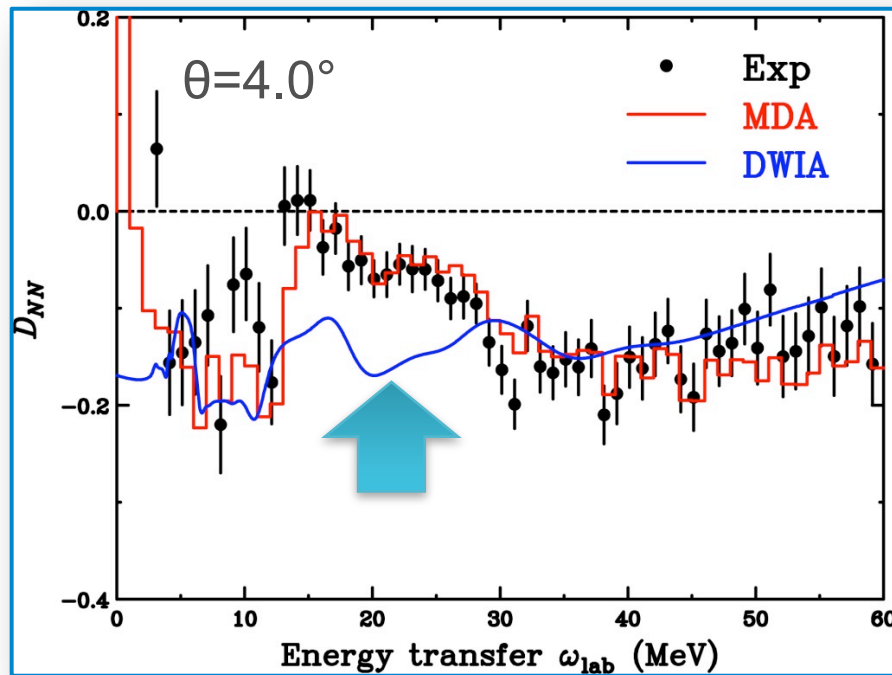
$$\alpha = \alpha_C + \alpha_T = 120 \text{ MeV} \cdot \text{fm}^5$$

$$\beta = \beta_C + \beta_T = 120 \text{ MeV} \cdot \text{fm}^5$$

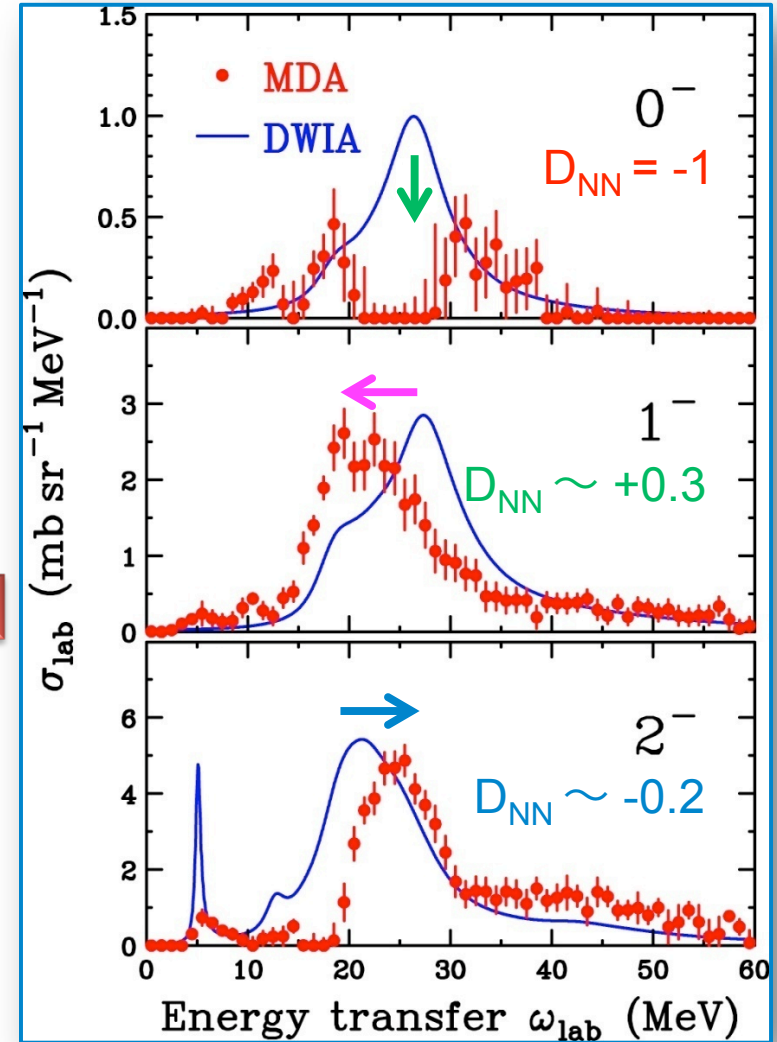


# Comparison between DWIA and MDA

$\Delta J^\pi$	MDA (compared with theory)
$0^-$	Quenching
$1^-$	Softening (shift to lower $\omega$ )
$2^-$	Hardening (shift to higher $\omega$ )



## SD Cross Sections



## Tensor correlations on Spin-Isospin mode

### Effect of Tensor Correlations on Gamow-Teller States in $^{90}\text{Zr}$ and $^{208}\text{Pb}$

C.L. Bai<sup>1,2)</sup>, H. Sagawa<sup>3)</sup>, H.Q. Zhang<sup>1,2)</sup>, X.Z. Zhang<sup>2)</sup>, G. Colò<sup>4)</sup> and F.R. Xu<sup>1)</sup>

$$O_- = \sigma \tau_-$$

$$O_+ = \sigma \tau_+$$

$$V^T = \frac{T}{2} \{ [(\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k'^2] \delta(\mathbf{r}_1 - \mathbf{r}_2) + \delta(\mathbf{r}_1 - \mathbf{r}_2) [(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)k^2] \} + \frac{U}{2} \{ (\sigma_1 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_2 \cdot \mathbf{k}) + (\sigma_2 \cdot \mathbf{k}') \delta(\mathbf{r}_1 - \mathbf{r}_2) (\sigma_1 \cdot \mathbf{k}) - \frac{2}{3} [(\sigma_1 \cdot \sigma_2) \mathbf{k}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \mathbf{k}] \}$$

$$m_-(0) - m_+(0) = \sum_{\nu} (|\langle \nu | O_- | 0 \rangle|^2 - |\langle \nu | O_+ | 0 \rangle|^2) = \langle 0 | [O_-, O_+] | 0 \rangle,$$

S3T

$$m_-(1) + m_+(1) = \sum_{\nu} (|\langle \nu | O_- | 0 \rangle| + |\langle \nu | O_+ | 0 \rangle|)^2 E_{\nu} = \langle 0 | [O_+, [H, O_-]] | 0 \rangle,$$

$$\Delta E_{GT} = \frac{m_-(1)}{m_-(0)} \sim \frac{m_-(1) + m_+(1)}{m_-(0) - m_+(0)} = \frac{4\pi}{3(N-Z)} \int dr r^2 [ -(\frac{5}{2}U + \frac{5}{2}T) J_n J_p - \frac{5}{3}U (J_n^2 + J_p^2) ]$$

	$m_-(1; \text{no tensor})$	$m_-(1; \text{with tensor})$	$\delta E_{RPA}$	$\delta E_{DC}$
	MeV	MeV	MeV	MeV
$^{90}\text{Zr}$	271.45	338.68	2.241	2.276
$^{208}\text{Pb}$	1854.12	2000.76	1.111	1.118



## Energy-weighted sum rules

$$m(k) = \sum_i E_i^k \left| \langle i | \hat{O}_\lambda | 0 \rangle \right|^2$$

$$m(1) = \frac{1}{2} \langle 0 | [\hat{O}_\lambda, [H, \hat{O}_\lambda]] | 0 \rangle$$

