Superfluidity-kill at overflow of trapped fermions. Quantal and semiclassical studies

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Outline

1. The physical context
2. General considerations about semiclassics for pairing
3. The slab model. Quantal and Thomas-Fermi approach
4. Wigner-Seitz cells in neutron stars with BCP functional
4. Conclusions

Physical context

Overflow situations of superfluid fermions in finite mean field potential $\rightarrow$

nuclei (drip line),

nuclei in Wigner Seitz cells in crust of neutron stars,

Cold atoms,

metallic clusters,

etc

Semiclassics for pairing

In weak coupling, we have BCS:

$$\Delta_n = \sum_{n'} \langle n\bar{n}|v|n'\bar{n}'\rangle \frac{\Delta_{n'}}{2\sqrt{(\varepsilon_{n'} - \mu)^2 + \Delta_{n'}}} \tag{1}$$

In LDA we have

$$\Delta(R,p) = \int \frac{d^3p'}{(2\pi\hbar)^3} V_{p,p'} \frac{\Delta(R,p')}{2\sqrt{(\varepsilon_{p'} - \mu(R))^2 + \Delta^2(R,p')}} \tag{2}$$

where

$$\mu(R) = [\mu - V(R)] \ \Theta(\mu - V(R))$$

is the local Fermi energy. The condition for validity of LDA is that

coherence length is $<<$ oscillator length.

hbar corrections

$$\mathcal{C}_0{}^{\beta} = e^{-\mathcal{H}_W} + O(\hbar^2) \tag{3}$$

$$\mathcal{H}_W = \begin{pmatrix} h(\mathbf{R},\mathbf{p}) & \Delta(\mathbf{R},\mathbf{p}) \\ \Delta(\mathbf{R},\mathbf{p}) & -h(\mathbf{R},\mathbf{p}) \end{pmatrix} \tag{4}$$

O($\hbar^2$) gradient correction extremely complicated!

In novel Thomas-Fermi approximation, we take $\hbar \to 0$ of gap equation in configuration space

$$\langle r1r2|n\bar{n}\rangle = \langle r1|n\rangle\langle n|r2\rangle \tag{5}$$

Then for $\hbar \to 0$, we have

$$\{|n\rangle\langle n|\}_{Wigner} \to f_{E_n}(\mathbf{R},\mathbf{p}) = \frac{1}{g^{TF}(E)}\delta(E_n - H_{cl.}) \tag{6}$$

with $H_{cl.} = \frac{p^2}{2m^*} + V(R)$ and the level density

$$g^{TF}(E) = \int d^3R \int \frac{d^3p}{(2\pi\hbar)^3}\delta(E - H_{cl.}) \tag{7}$$

With this we can calculate pairing matrix element semiclassically and obtain for gap eq.:

$$\Delta(E) = \int dE' g^{TF}(E')V(E,E')\frac{\Delta(E')}{2\sqrt{(E'-\mu)^2 + \Delta^2(E')}} \tag{8}$$

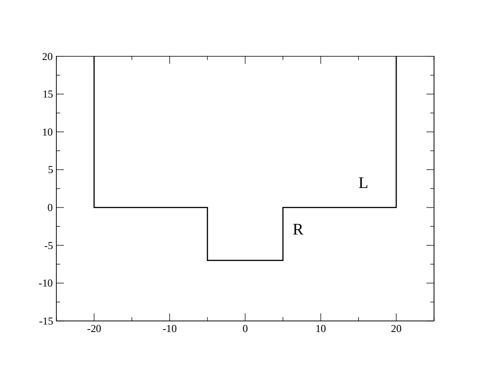
$$V(E,E') = \int d^3R \int\int \frac{d^3p}{(2\pi\hbar)^3}\frac{d^3p'}{(2\pi\hbar)^3} f_E(\mathbf{R},\mathbf{p}) v(\mathbf{p}-\mathbf{p}') f_{E'}(\mathbf{R},\mathbf{p}') \quad (9)$$

delta force $\rightarrow$

$$V(E,E') \sim -\frac{g}{g^{TF}(E)g^{TF}(E')} \int d^3R \sqrt{E - V(\mathbf{R})}\sqrt{E' - V(\mathbf{R})} \quad (10)$$

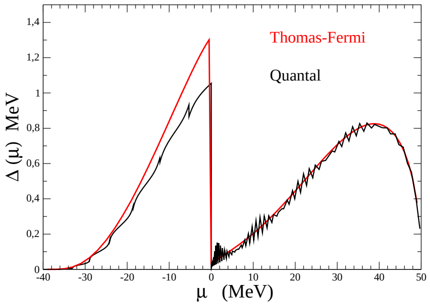
very simple!

For a check, we use a SLAB with following transverse profile
L=100fm, R=10fm

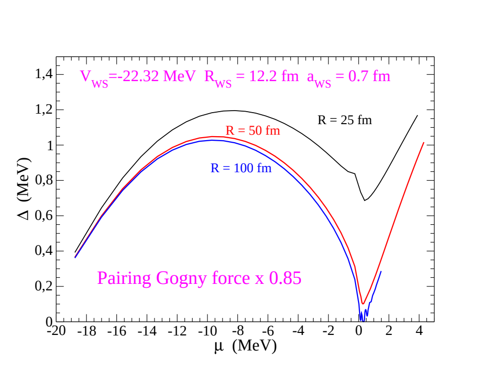


We will solve gap equation as a fct of filling, i.e. fct of μ

red: TF; black: quantal



Gap for various Wigner-Seitz cell radii R



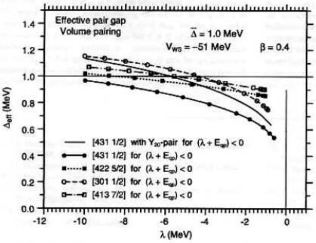


FIG. 3. Effective pair gap [Eq. (12)] of several deformed one-particle orbits as a function of the Fermi level λ, taking the quadrupole deformation $\beta = 0.4$ and $\bar{\Delta} = 1$ MeV. The curves with symbols show the Δ_{eff} values estimated by including the Y_{00} pairing only, whereas the solid curve without filled circles is calculated for the [431 1/2] orbit, including the Y_{20} pairing on top of the Y_{00} pairing without renormalizing the total strengths. See the text for details.

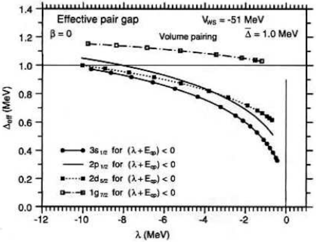
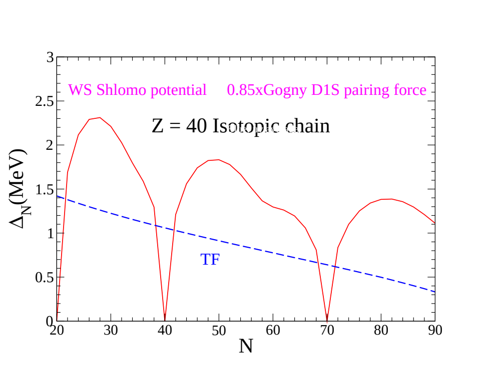


FIG. 4. Effective pair gap in the case of spherical nuclei as a function the Fermi level λ. Δ_{eff} is defined as the minimum value of $E_{qp}(\ell j)$, analogous to Eq. (12) in the case of deformed nuclei: For a given potential strength where $V_{WS} = -51$ MeV is fixed while R is varied, the minimum value of $E_{qp}(\ell j)$ is looked for and plotted as a function of λ.

HFB: I. Hamamoto, PRC 71;
also HFB in WS cells by Grasso, Khan, Margueron, N.v. Giai, NPA 807

In nuclei shell fluctuations are very strong but tendency can clearly be seen

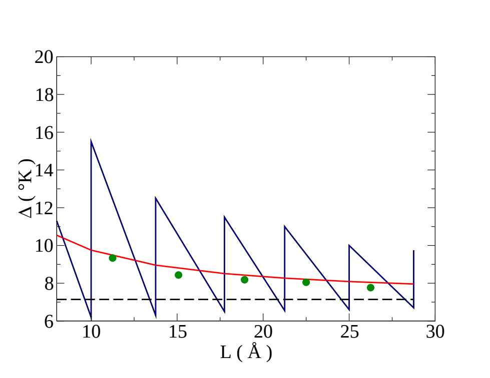


Size dependence of gap in finite Fermi systems:

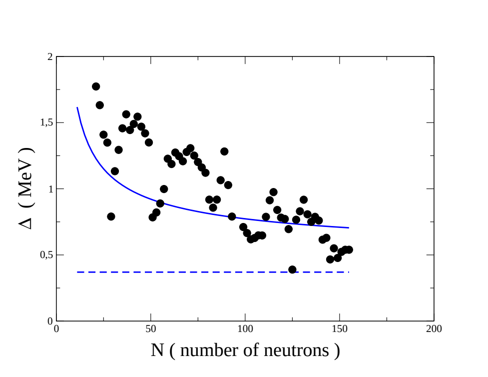
$$\Delta = \Delta_B e^{-C\frac{S}{V}} \sim \Delta_B [1 - C\frac{S}{V}] \quad (11)$$

$$C = \frac{1}{v_F^B g_F^B} \frac{8}{\pi} \frac{1}{k_F^B} \quad (12)$$

a) metallic films and grains



b) Size dependence of gaps in nuclei



1/R = $A^{-1/3}$ dependence!

Digression: new BCP functional (together with L Robledo)
What is it?
Baldo: nuclear matter, neutron matter:

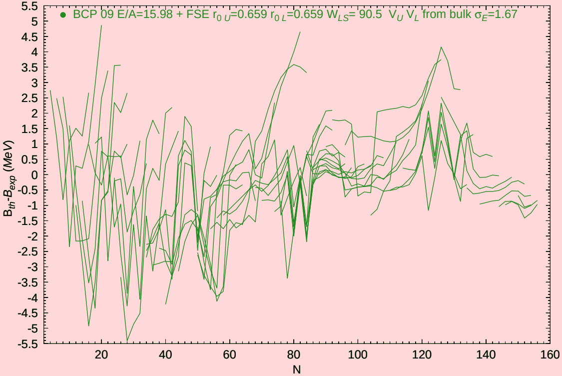
$$E_{pot}^{\infty} = a_1\rho + a_2\rho^2 + ... \tag{13}$$

finite nuclei $\rightarrow$

$$a_2\rho^2 \rightarrow \quad \int d^3r \int d^3r' \rho(\mathbf{r}) v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}') \tag{14}$$

$$v(r) = v_0 e^{(r/r_0)^2} \tag{15}$$

r_0 the only finite range parameter! Surface energy!



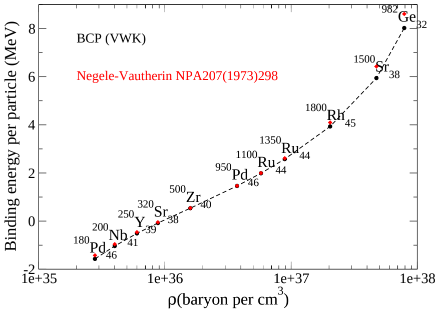
rms masses $\sim$ 1.7

rms radii: $\sim$ 0.03

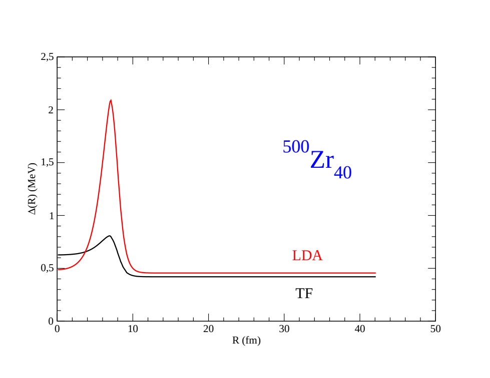
Fine tuning of Baldo EOS: E/A = 16.02 MeV. Only two parameters!
Bulk+ surface.

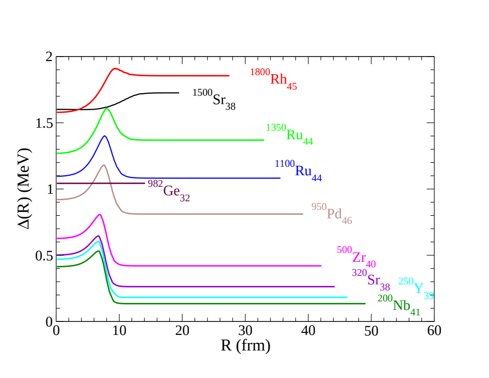
Wigner-Seitz cells fully selfconsistently with BCP functional with ETF + TF pairing

comparison with Negele Vautherin:



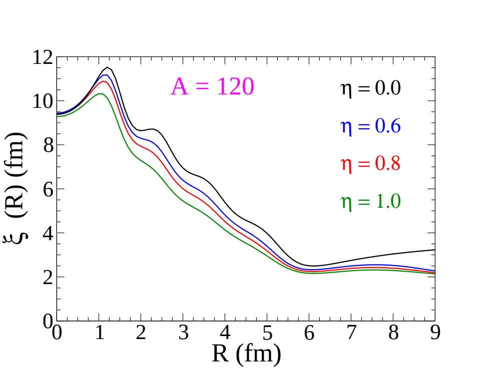
Knowing that TF works well, we compare with LDA in Wigner-Seitz cells





Gap also disappears inside cluster at drip!

Finally some remarks on size of Cooper pairs in nuclei



Conclusions

Novel TF approach for pairing performs very well.

Drip and overflow situations reduce or kill pairing.

new size dependence of nuclear gaps

new BCPM functional with only two adjustable parameters

semiclassical treatment of WS cells with BCPM

DEAR PETER,

I WISH YOU GOOD HEALTH AND THAT NEXT DECADE BE AS FRUITFUL SCIENTIFICALLY AS PAST ONE!