Pairing correlations tested in heavy-ion induced reactions: two-particle transfer and two-particle break-up reactions

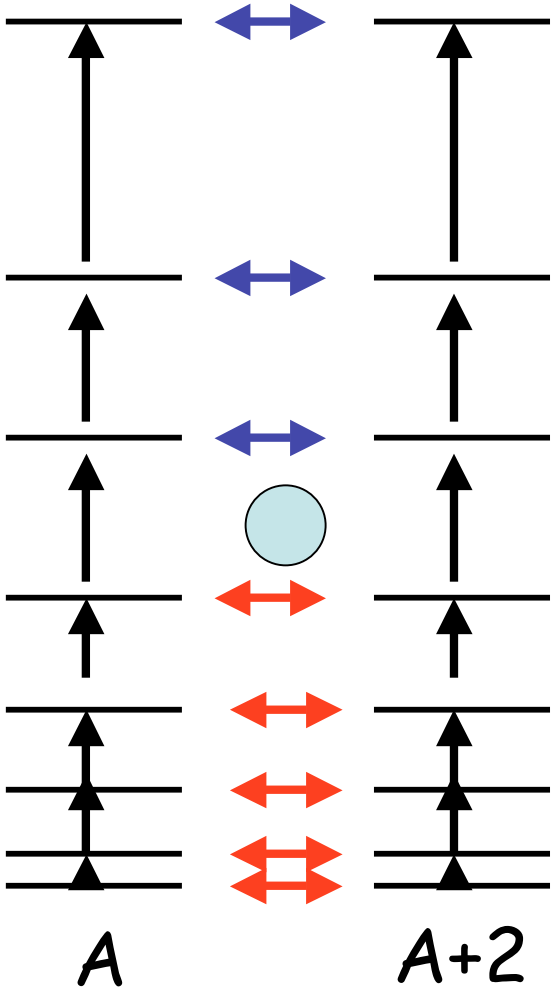
Outline

- pairing correlation and correlations in space
- systems at the drip lines
- role and treatment of continuum states
- reaction models for two-particle break-up and two-particle transfer reactions

A nuclear squid: Diabolic pair transfer in rotating nuclei

Nikam, R. S.; Ring, P.; Canto, L. F.

Physics Letters B, Volume 185, p. 269-274, 1987



Coulomb (and nuclear) excitation

Two-nucleon transfer

How to use dynamics to study pairing correlations?

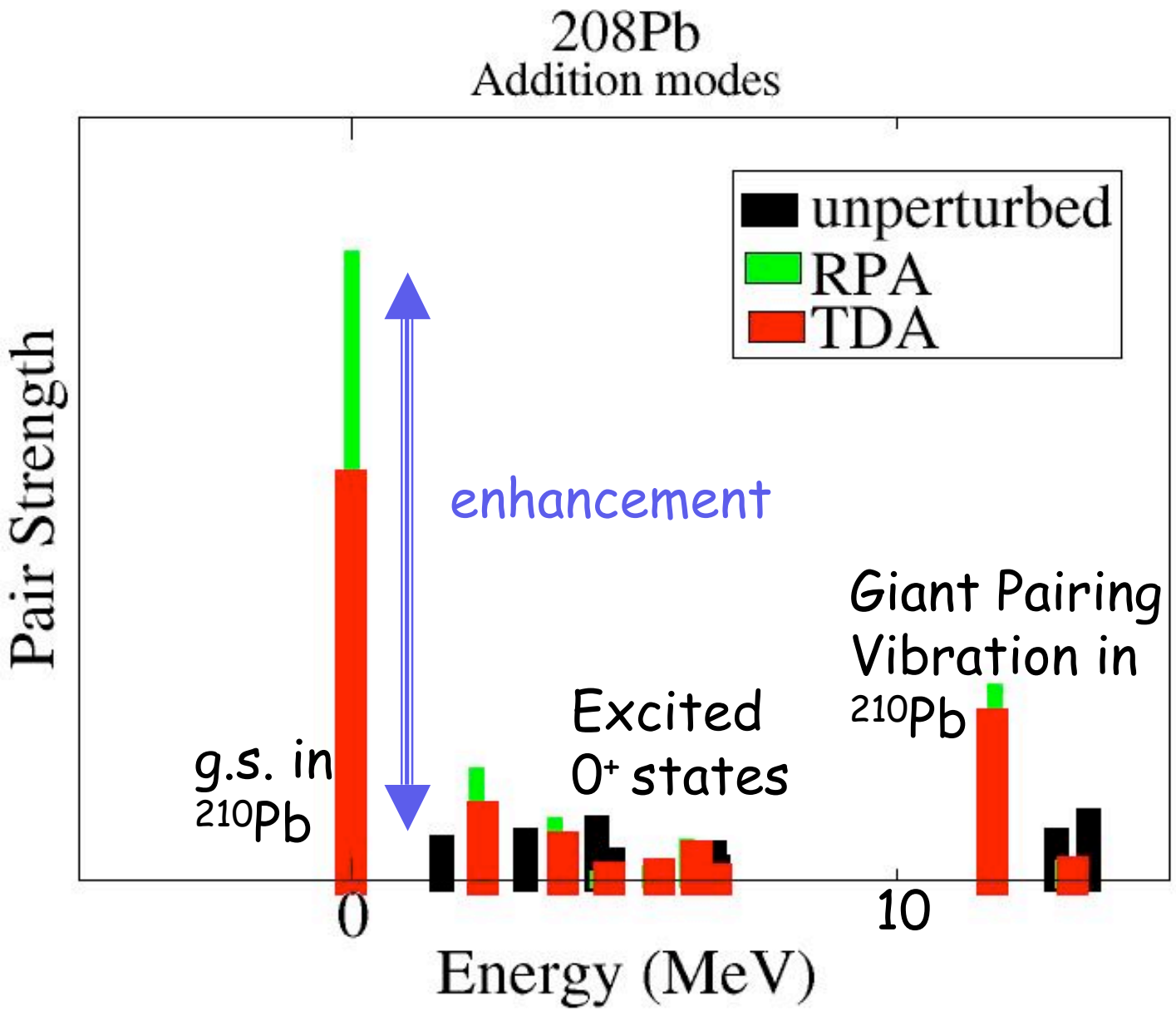
The main road is clearly provided by the study of those processes where a pair of particles in involved, e.g. transferred from/to another nucleus (two-particle transfer) or ejected onto the continuum (two-particle break-up).

Unfortunately, the situation is different, for example, from low-energy one-step Coulomb excitation, where the excitation probability is directly proportional to the B(Eλ) values. Here the reaction mechanism is much more complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious. The situation is actually more complicated even with respect to other processes (as inelastic nuclear excitation) that may need to be treated microscopically, but where the reaction mechanism is somehow well established.

It is often assumed that the cross section for two-particle transfer just scale with the square of the matrix element of the pair creation (or removal) operator

$$P^{+} = \Sigma_{j} [a^{+}a^{+}]_{00}$$

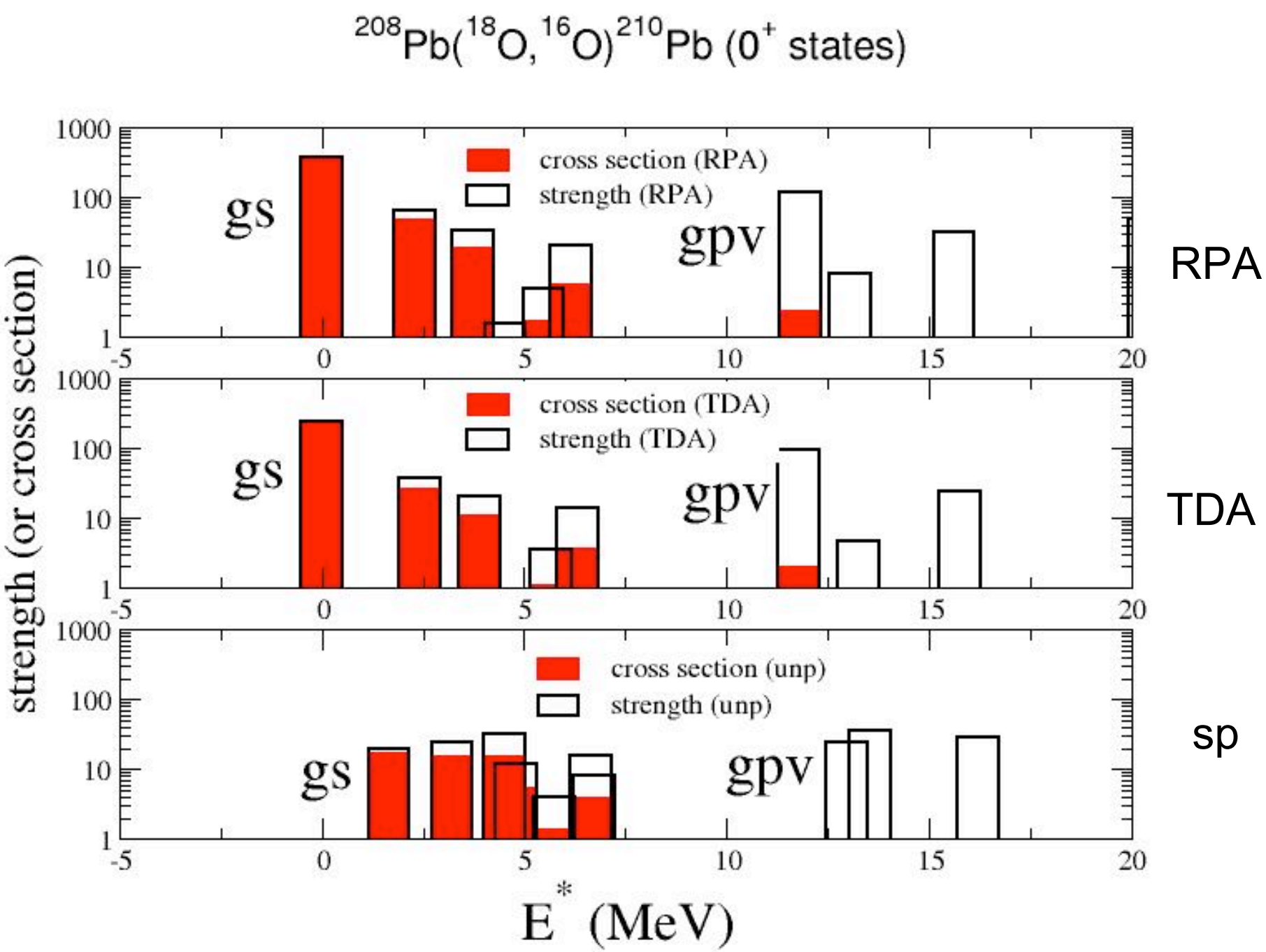
For this reason the easiest way to define and measure the collectivity of pairing modes is to compare with single-particle pair transition densities and matrix elements to define some "pairing" single-particle units and therefore "pairing" enhancement factors.

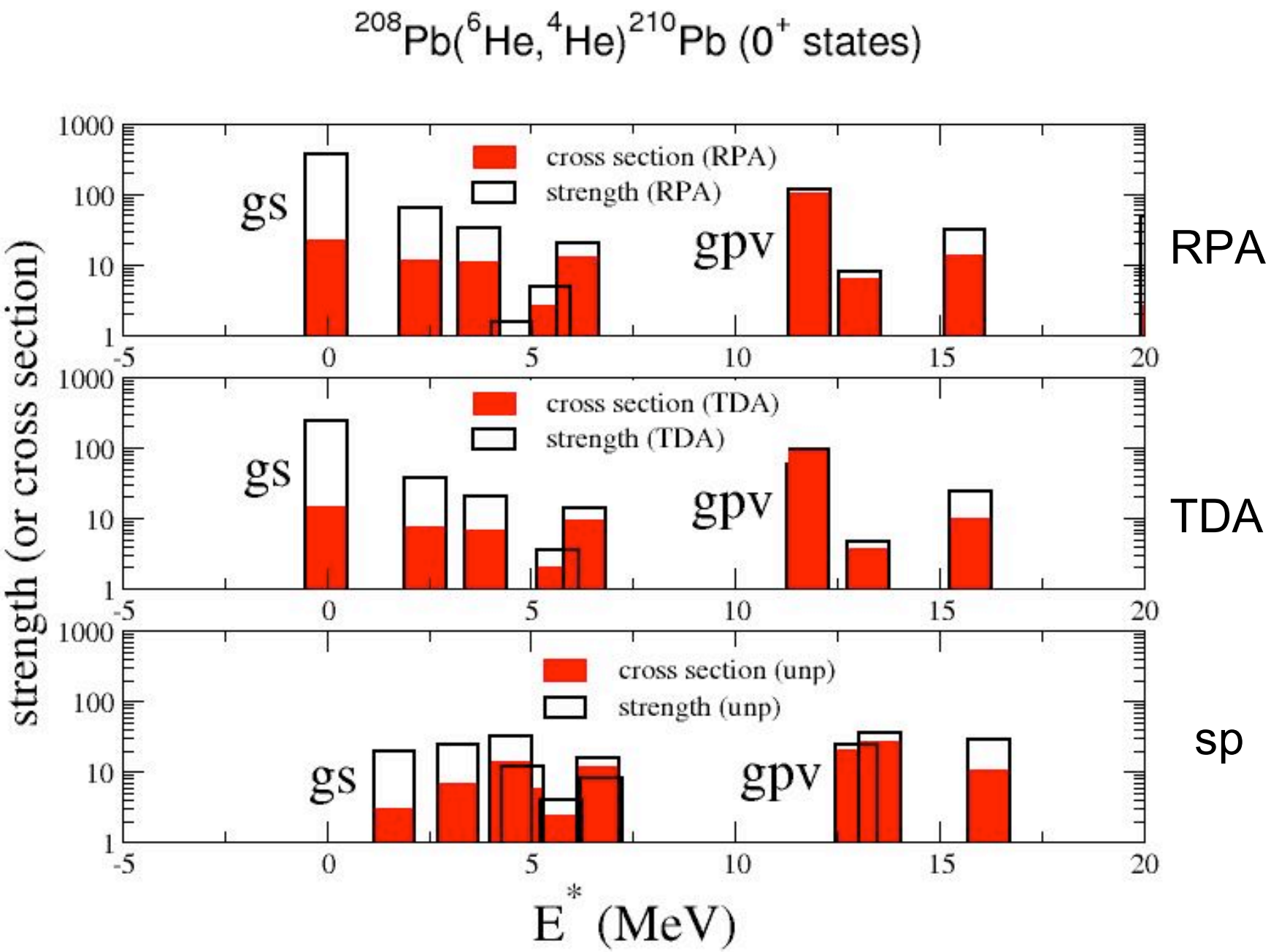

208Pb
Addition modes
unperturbed
RPA
TDA
Pair Strength
enhancement
g.s. in
210Pb
Excited
0+ states
Giant Pairing
Vibration in
210Pb
0
10
Energy (MeV)

Even without entering into the details of the reaction mechanism, one should at least take into account the Q-value effect

Keeping fixed any other parameter, the probability for populating a definite final channel depends on the Q-value of the reaction. The dependence is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

In the specific case of L=0 two-neutron transfer, the optimal Q-value is zero. This can modify the cross-section distribution with respect to the strength distribution


^{208}Pb(^{18}O,^{16}O)^{210}Pb (0^{+} states)
strength (or cross section)
cross section (RPA)
strength (RPA)
gs
gpv
RPA
cross section (TDA)
strength (TDA)
gs
gpv
TDA
cross section (unp)
strength (unp)
gs
gpv
sp
1000
100
10
1
-5
0
5
10
15
20
E* (MeV)

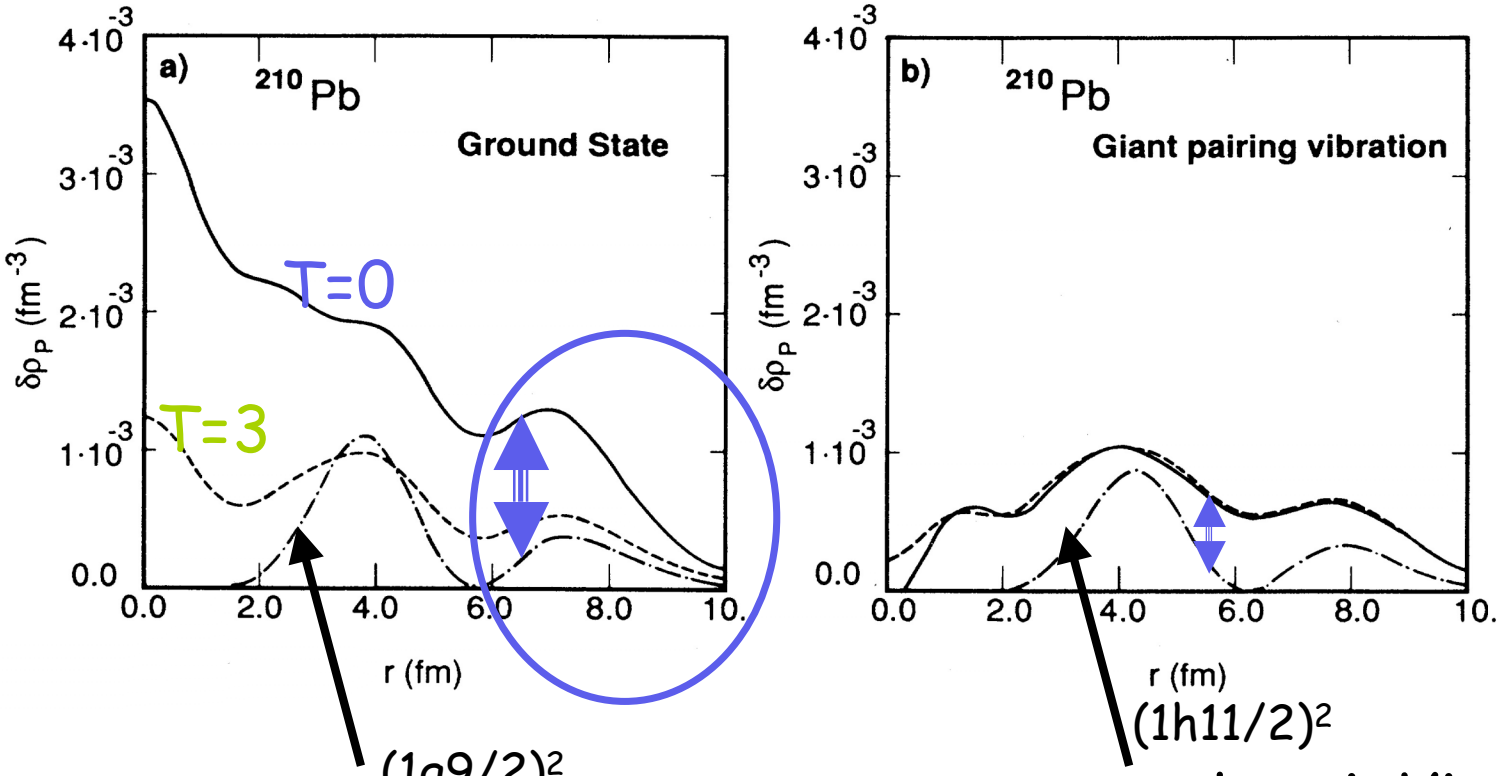

^{208}Pb(^{6}He,^{4}He)^{210}Pb (0^{+} states)
strength (or cross section)
cross section (RPA)
strength (RPA)
gs
gpv
RPA
cross section (TDA)
strength (TDA)
gs
gpv
TDA
cross section (unp)
strength (unp)
gs
gpv
sp
1000
100
10
1
-5
0
5
10
15
20
E* (MeV)

But the two-particle transfer process in not sensitive to just the pair matrix element. We have to look at the radial dependence, which is relevant for the reaction mechanism associated with pair transfer processes.

Comparison with pure single-particle configurations

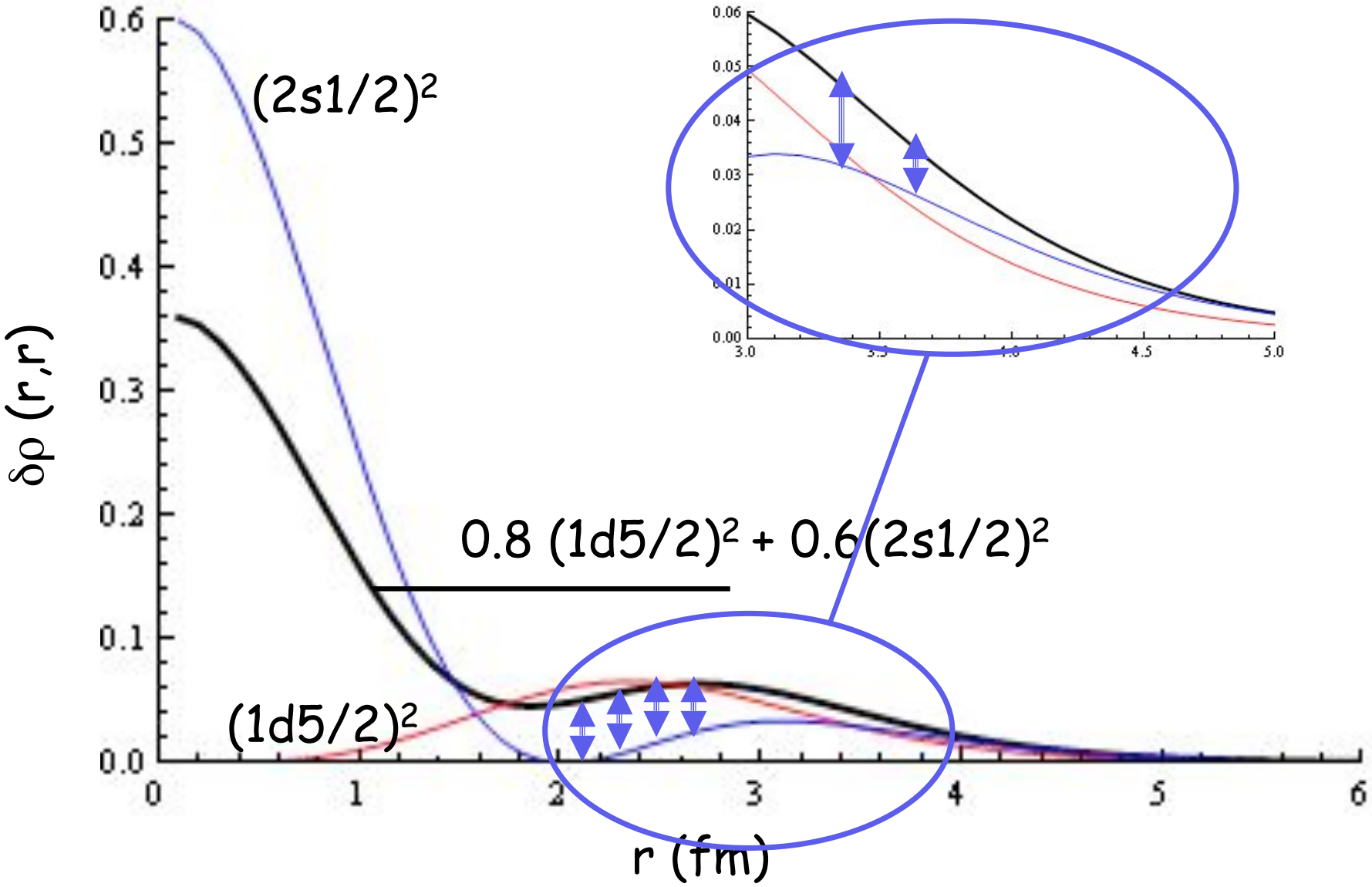
pair transition density

$$\rho^{\nu}_{p}(r,r)=\kappa^{\nu}(r\sigma)=\langle 0|c(r\bar{\sigma})c(r\sigma)|\nu\rangle$$



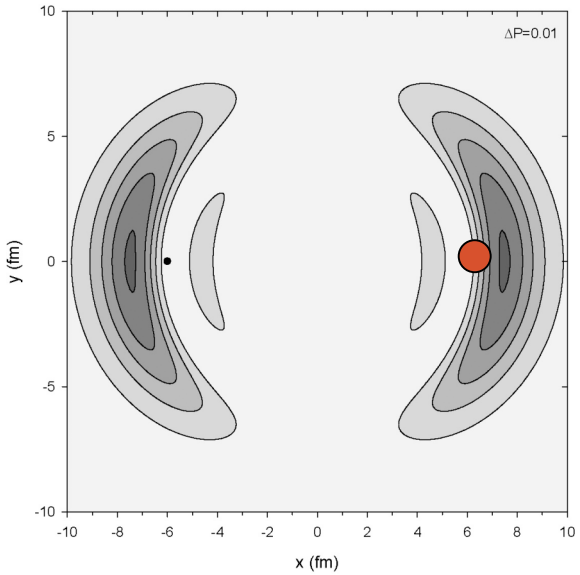
Lotti, Vitturi etal

^{18}O

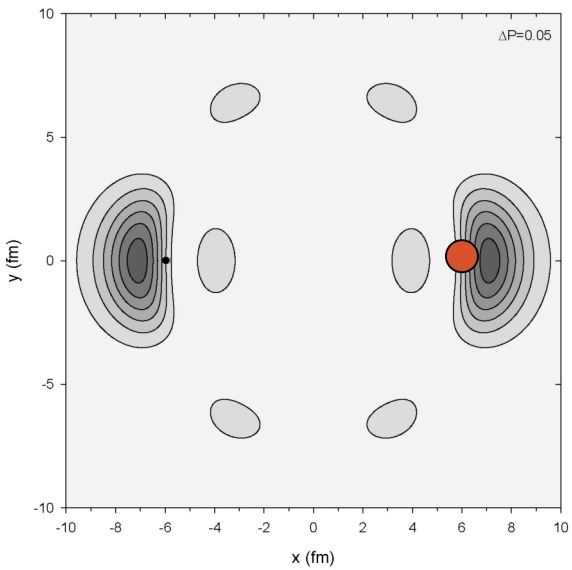

(2s1/2)²
0.8 (1d5/2)² + 0.6 (2s1/2)²
(1d5/2)²
δρ (r,r)
r (fm)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
0.00
0.01
0.02
0.03
0.04
0.05
0.06
3.0
4.5
5.0

^{206}Pb

$(3p1/2)^2$

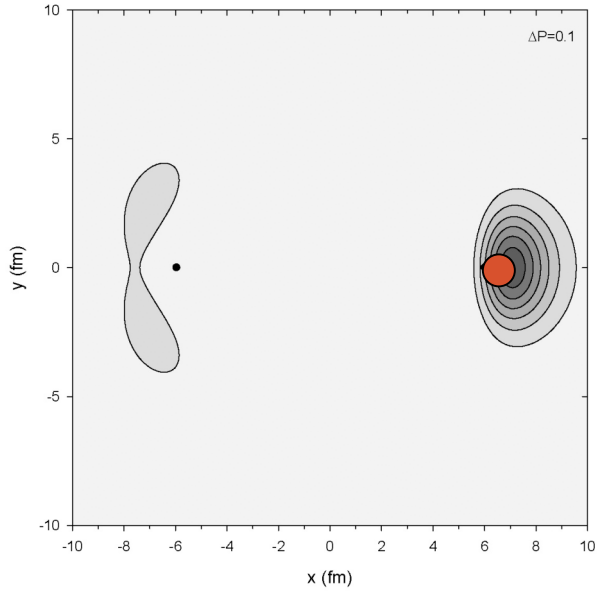


$(2f5/2)^2$



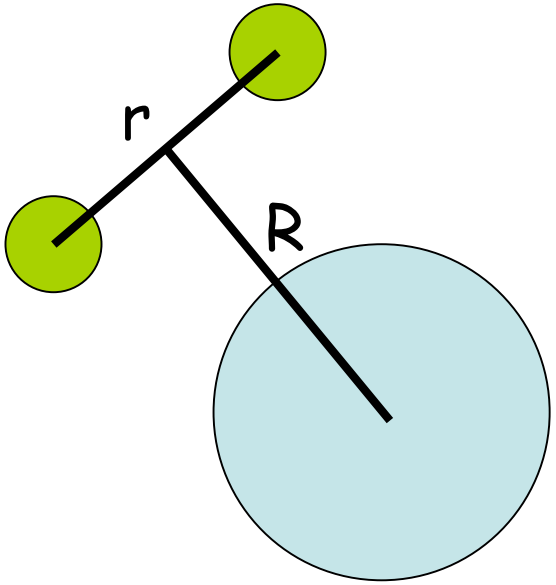
Correlated ground state

$|\Psi(r_1,r_2)|^2$ as a function of r_2, for fixed r_1



OBS: mixing of configurations with opposite parity

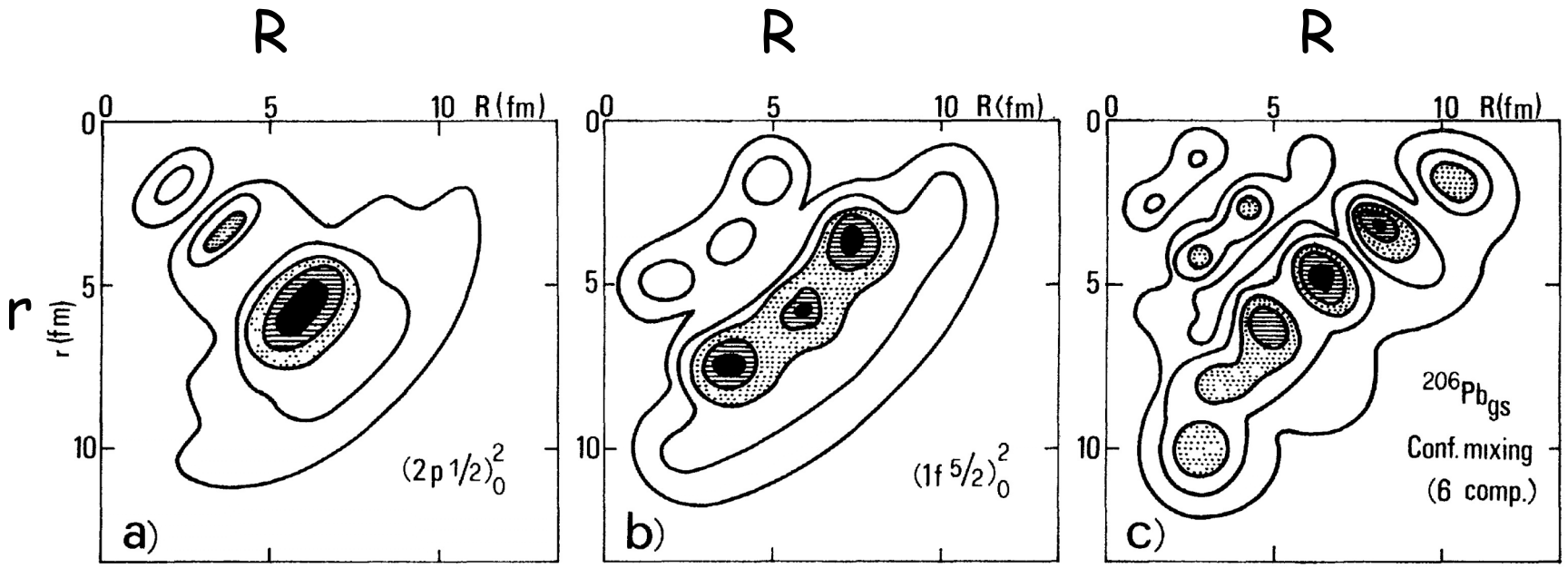
● position of particle 1



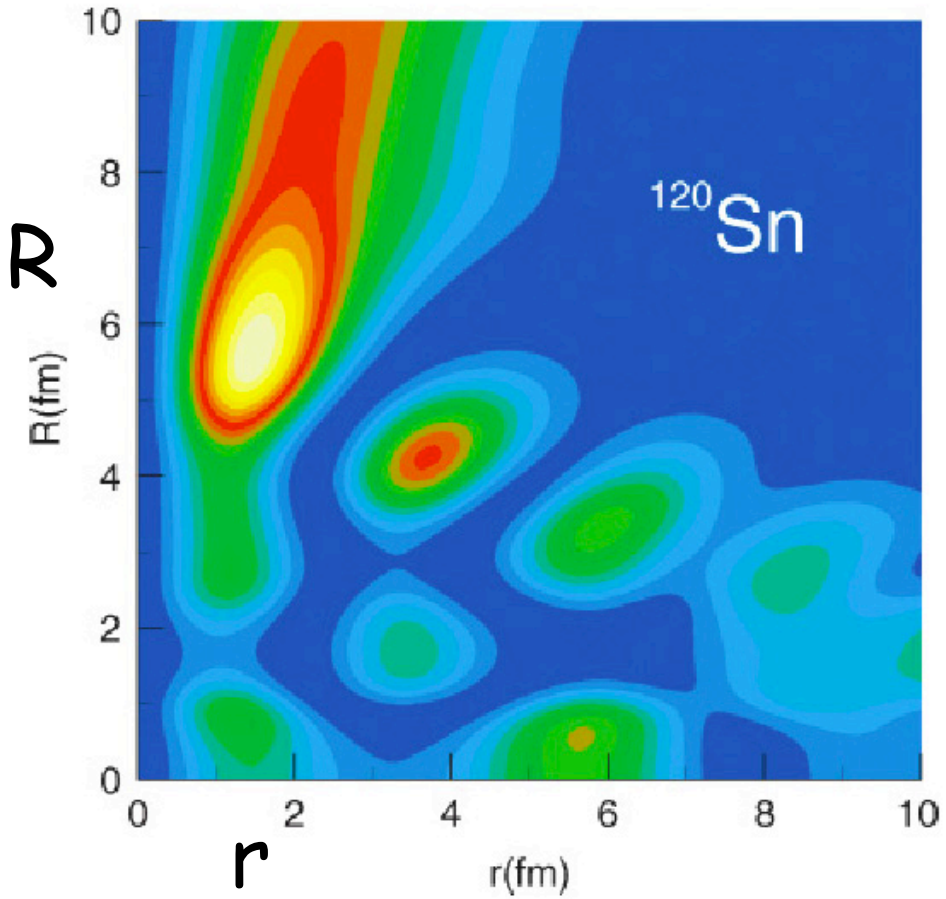
$\delta\rho_p(R,r)$

Catara etal, 1984

^{206}Pb



$(3p1/2)^2$ $(2f5/2)^2$ Correlated g.s.

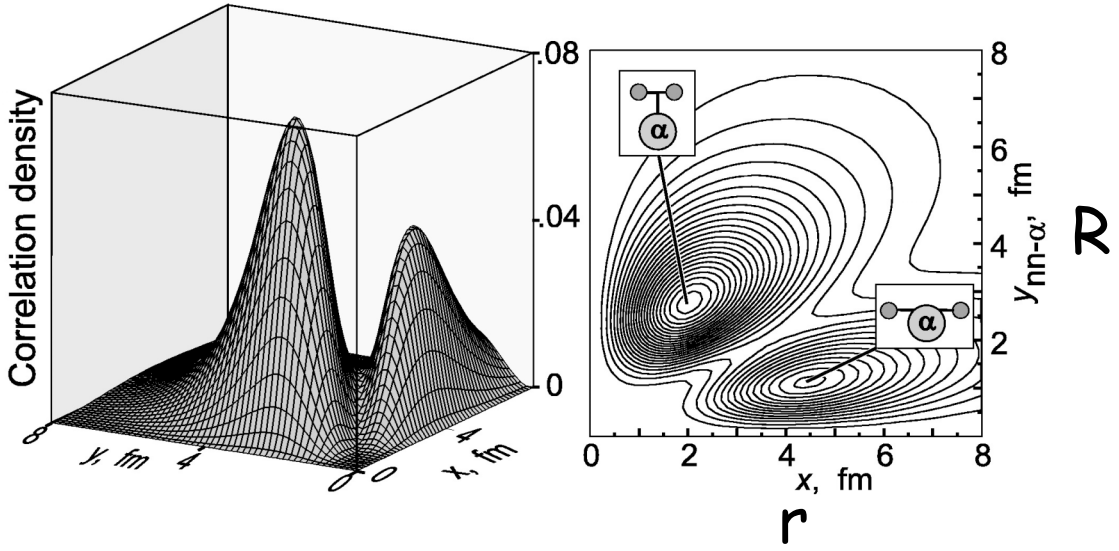


$\delta\rho_p(R,r)$

Pillet, Sandulescu, Schuck

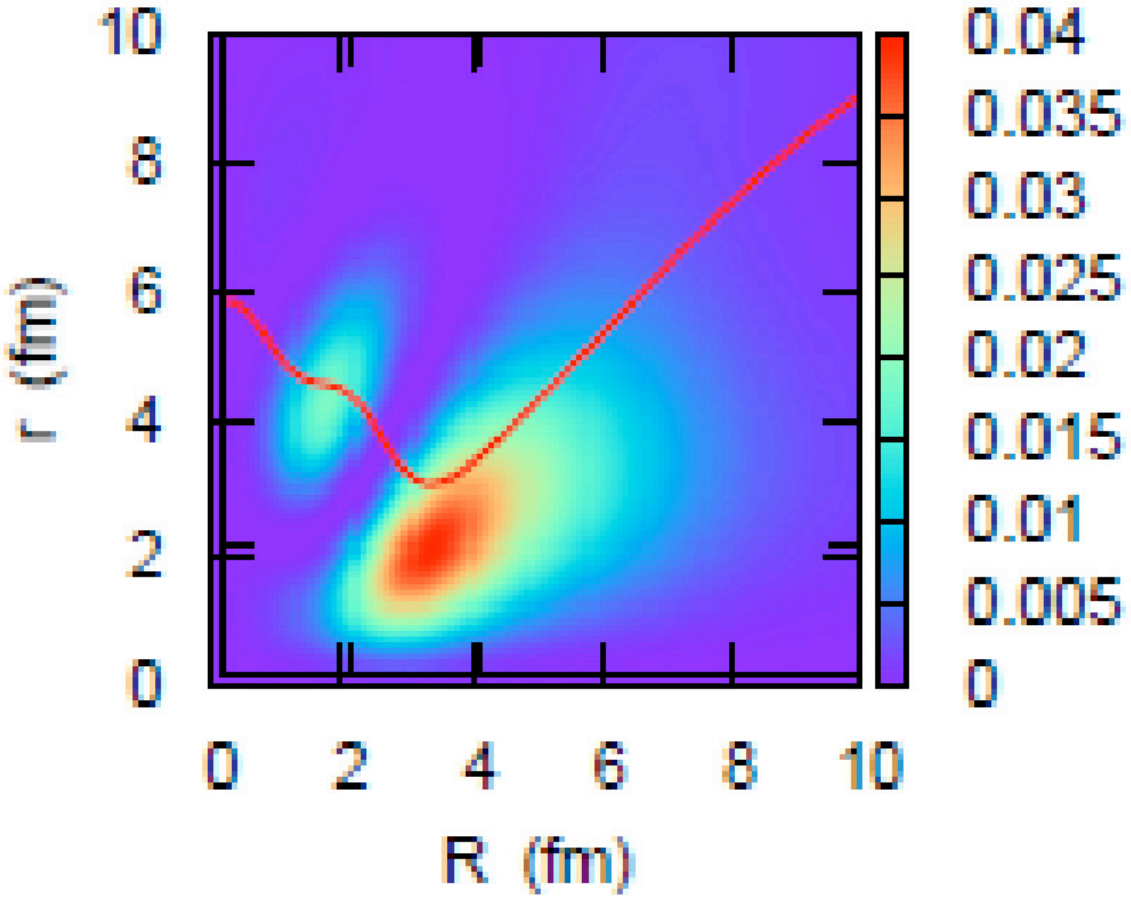
Interesting problem:
how is changed the picture as we move closer or even beyond the drip lines?

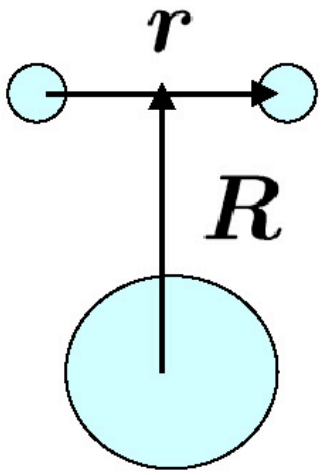
Example:
the case of
^{6}He



Oganessian, Zagrebaev, Vaagen, 1999

Other example: the case of ^{11}Li





K.Hagino, H. Sagawa, and P. Schuck,
J. of Phys. G37(‘10) 064040.

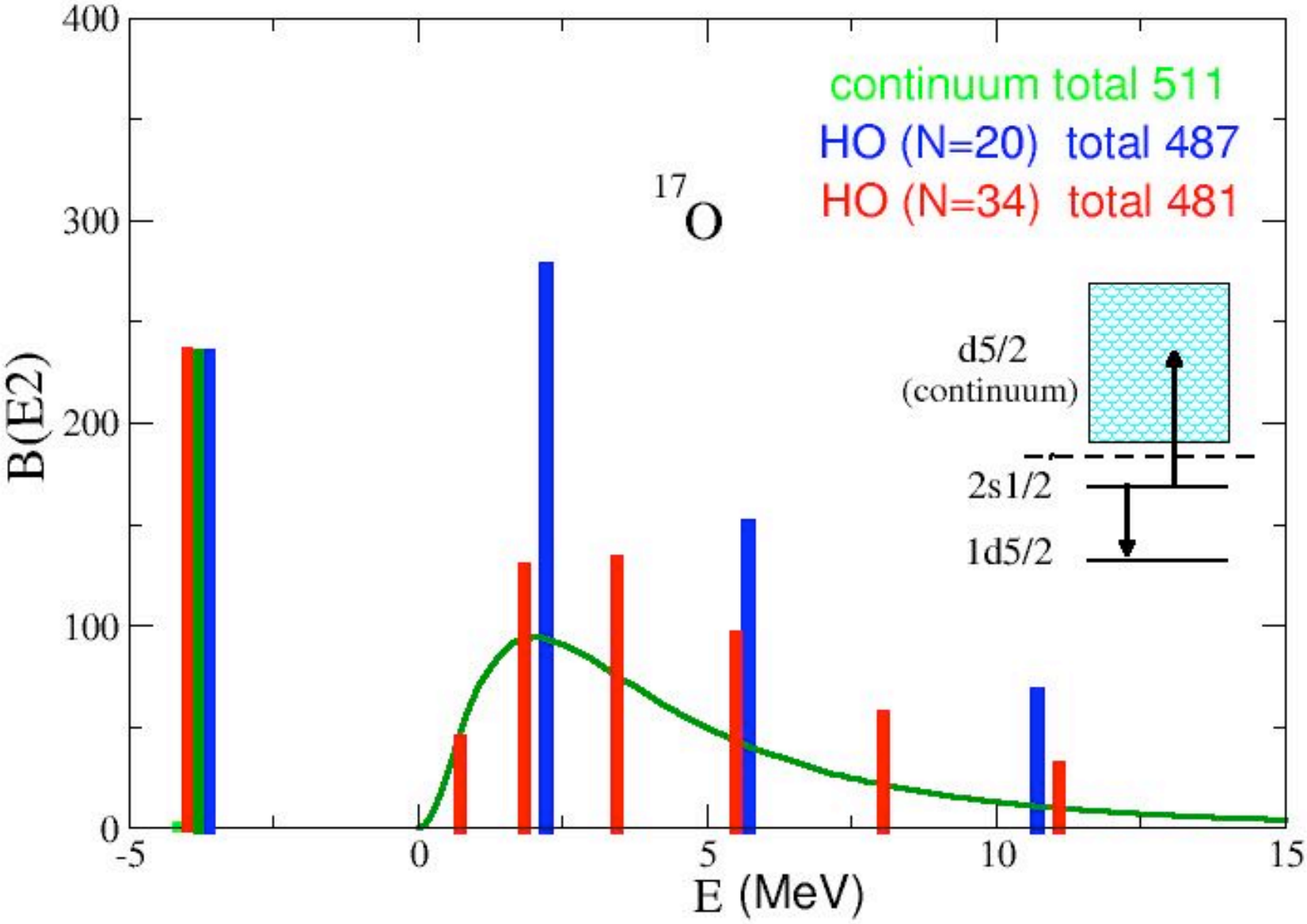
For weakly-bound systems at the drip lines it is mandatory to include in the models the positive energy part of the spectrum. If one wants to still use the same machinary used with bound states, the most popular approach is the discretization of the continuum. But the discretization MUST go in parallel in a consistent way both in the structure and in reaction parts.

All discretization procedures are equivalent as long as a full complete basis is used. In practice all procedudes contain a number of parameters and criteria, that make not all procedures equally applicable in practical calculations. Computational constraints may in fact become a severe problem.

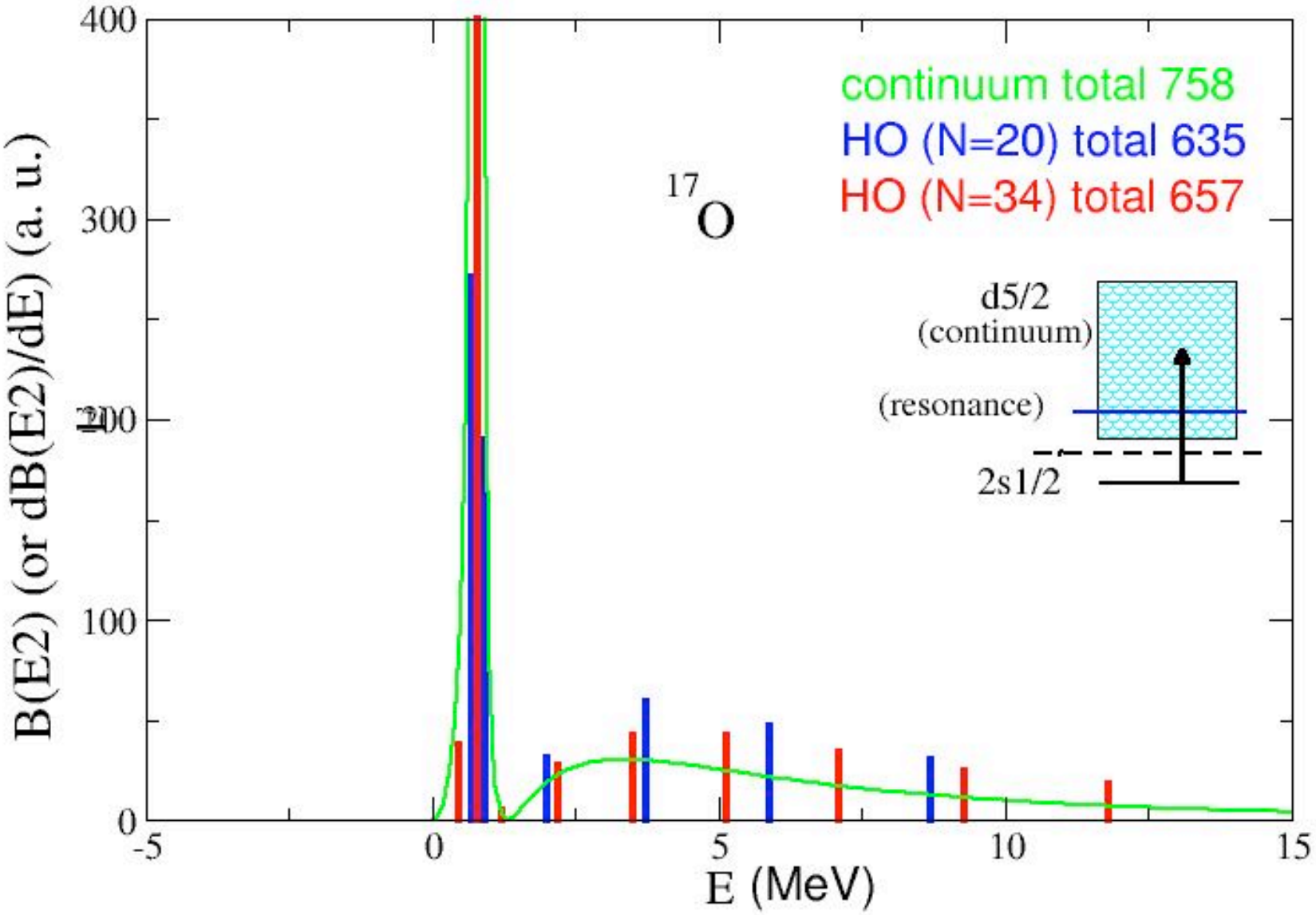
As possibilities we can consider

- diagonalization in a basis given by HO wave functions
- impose boundary conditions in a BOX
- the case of discretized wave functions with scattering boundary conditions (CDCC)
- Gamow states (complex energies)

Case of non-resonant continuum
(Woods-Saxon single-particle potential in a HO basis)

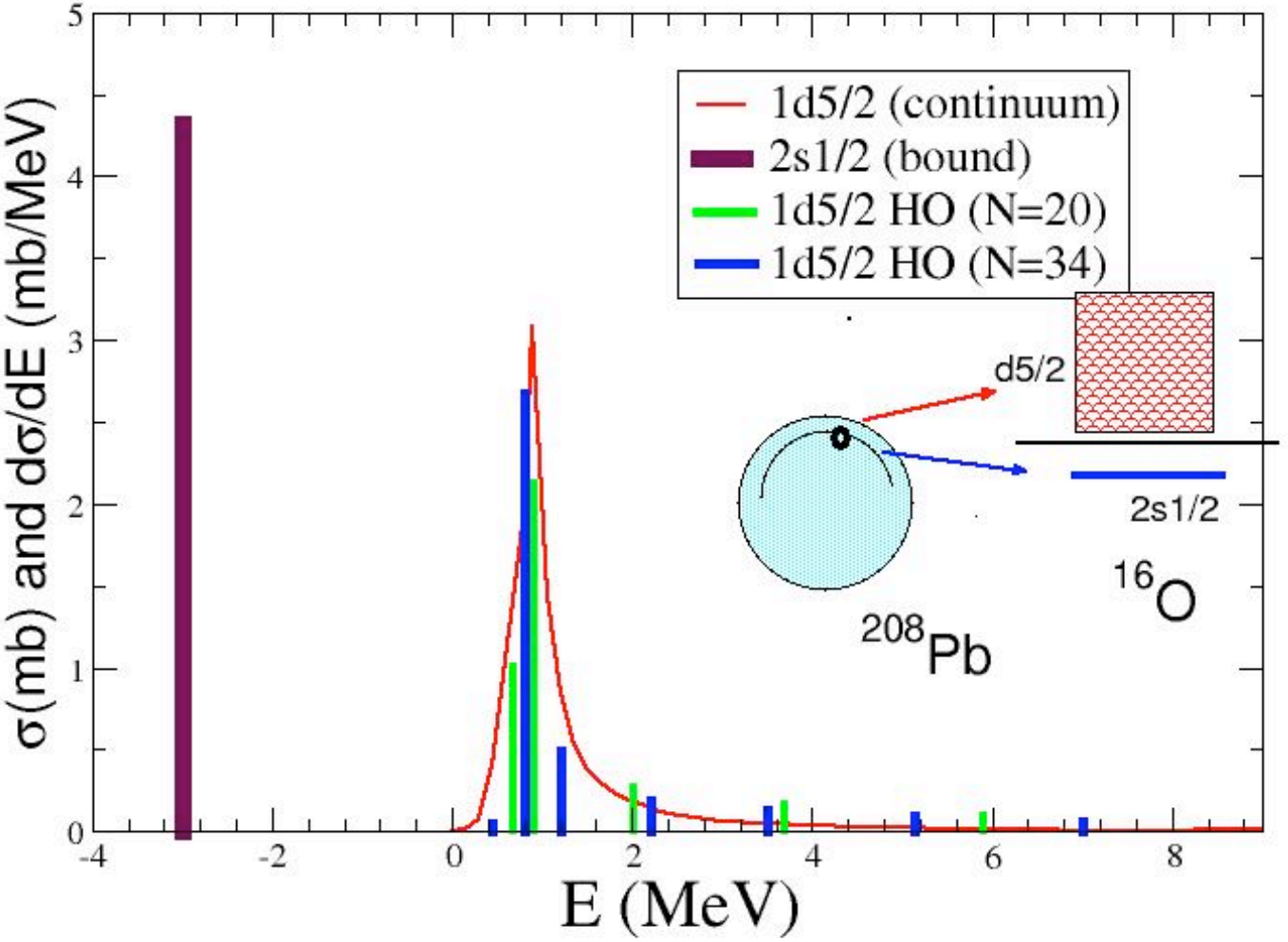


Case of resonant + non-resonant continuum



One-particle transfer (in DWBA)
Case of resonant + non-resonant continuum

$^{208}Pb(^{16}O,^{17}O^{*})^{207}Pb(gs,(3p1/2)^{-1})$

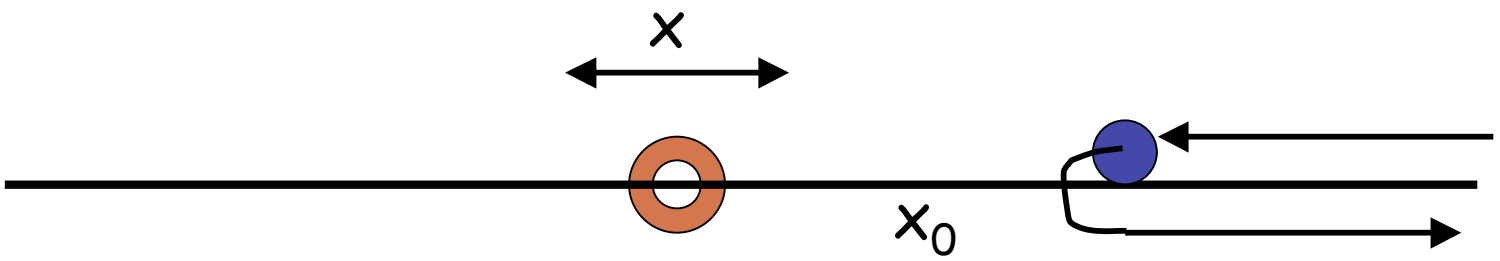


How does the continuum discretization work in break-up processes involving one particle in the continuum in the final stage?
Couplings are strong and first-order perturbation may not be sufficient

Simple modelling of one-particle halo break-up

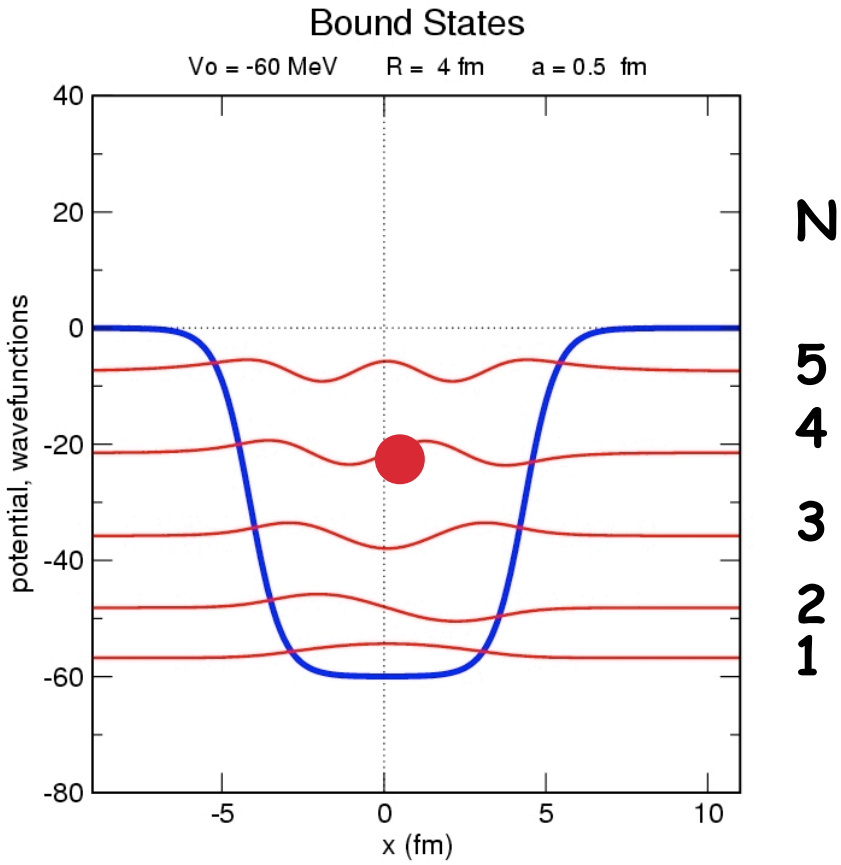
Single particle, initially moving in a one-dimensional Woods-Saxon potential V_0, perturbed by a time-dependent interaction V(x,t), assumed to be of gaussiam shape

$$V(x,t)=V \exp(-t^2/\sigma_t) \exp(-(x-x_0)^2/\sigma_x)$$



Obs: simulation of the nuclear field generated in a collision with a heavy partner

The particle is assumed to be initially in one of the bound states $\Phi_N(x)$ of V_0



Exact full evolution of the system obtained by solving the time-dependent Schroedinger equation

$$ih\,\partial\,\Psi(x,t)/\,\partial\,t = [H_0 + V(x,t)]\,\Psi(x,t)$$

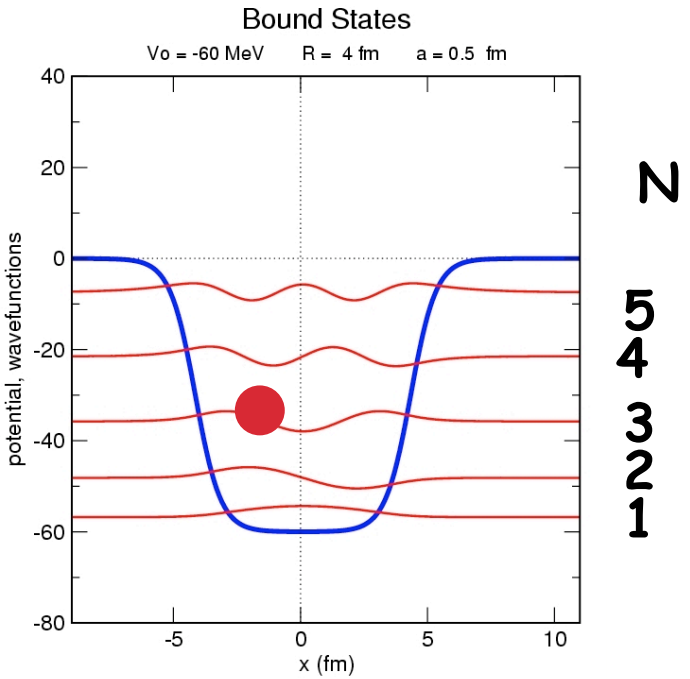
with

$$H_0 = -(h^2/2\mu)\,d^2/dx^2 + V_0(x)$$

Coupling with excitation to the continuum

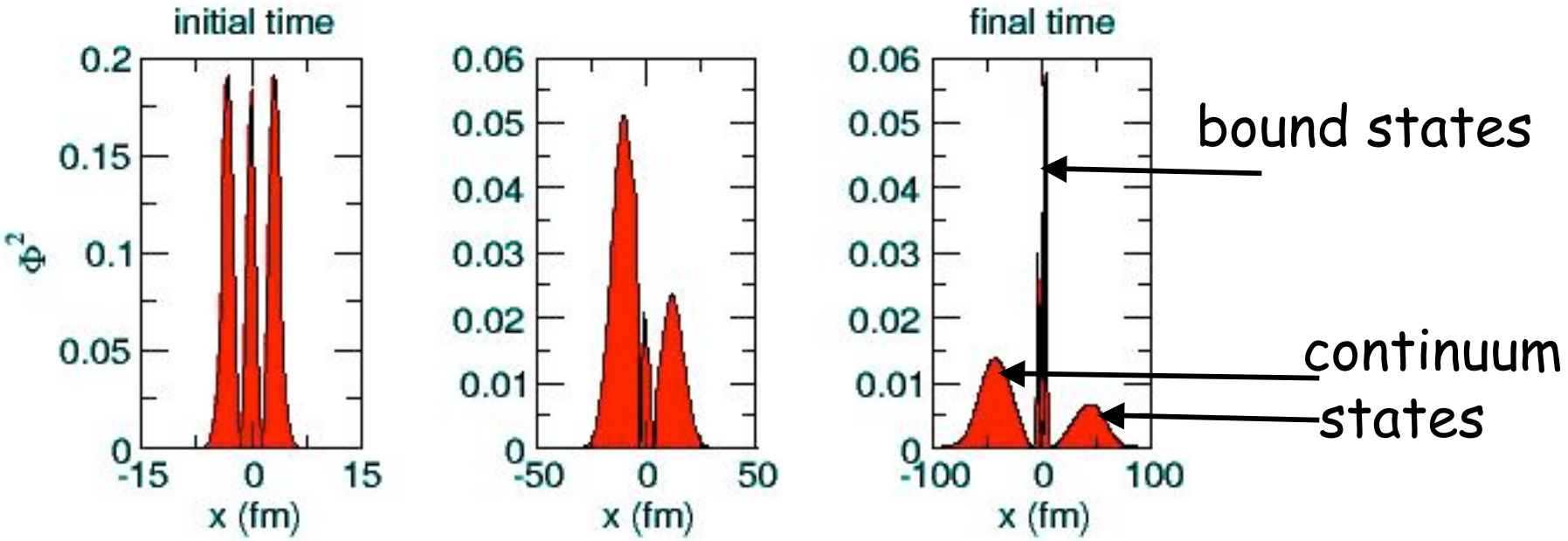
(still with some final probability of being bound):

partial break-up



initial bound state

N=3

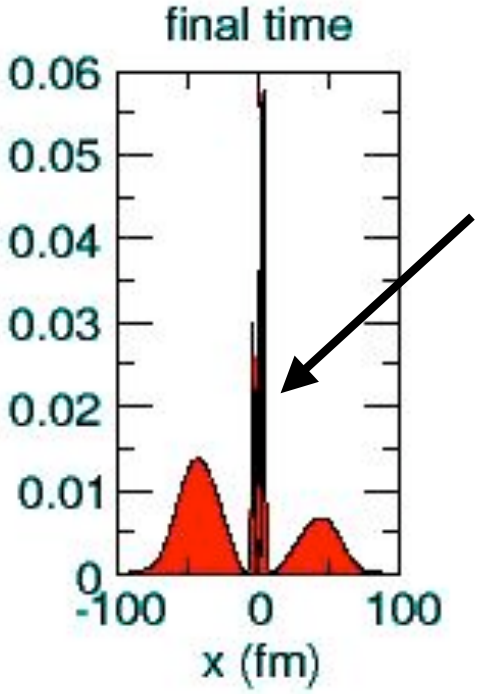


Obs: change of scale

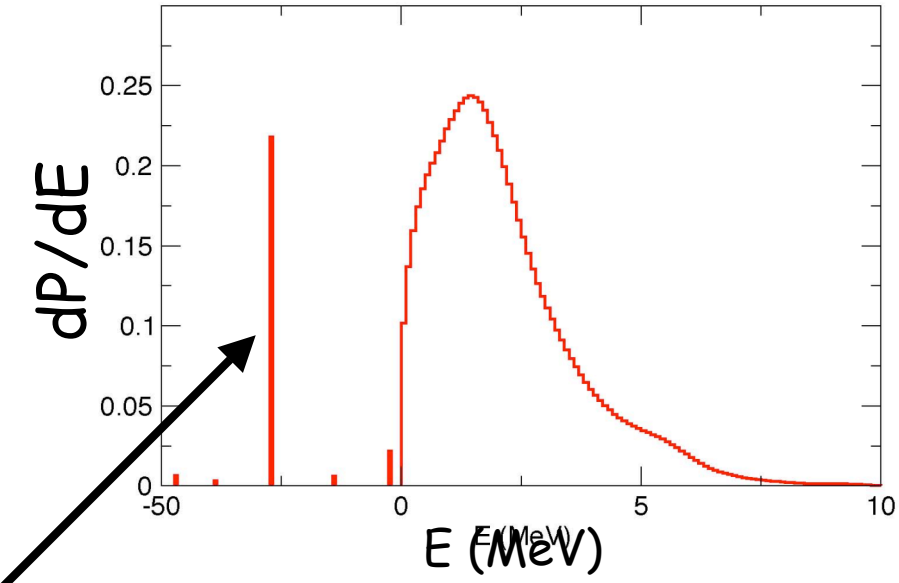
Final wave function
NOT confined in
potential well

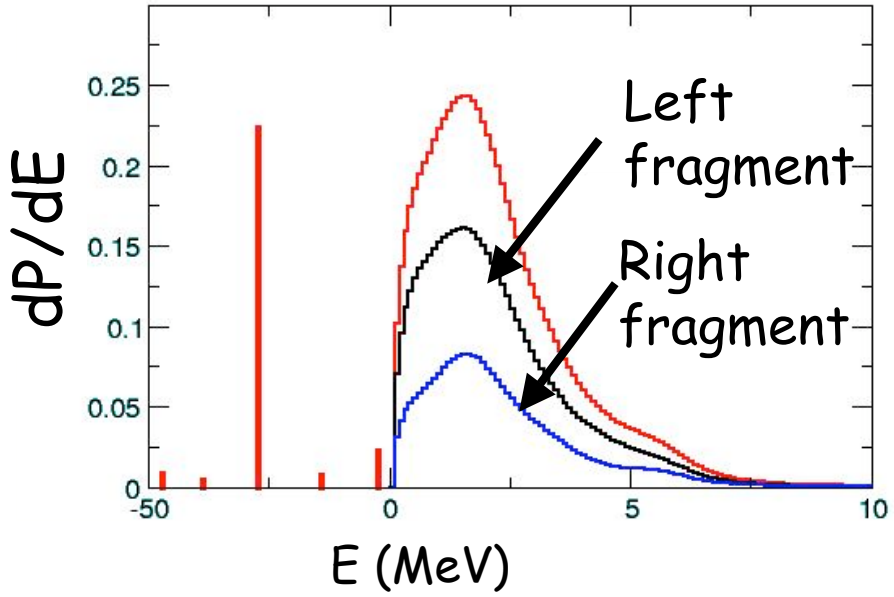
Q-value final distribution

Asymmetric partial breakup



bound





The same problem can be approached in the ''standard'' coupled-channel formalism where the Schrödinger equation is solved by expanding the total wave function into a stationary basis

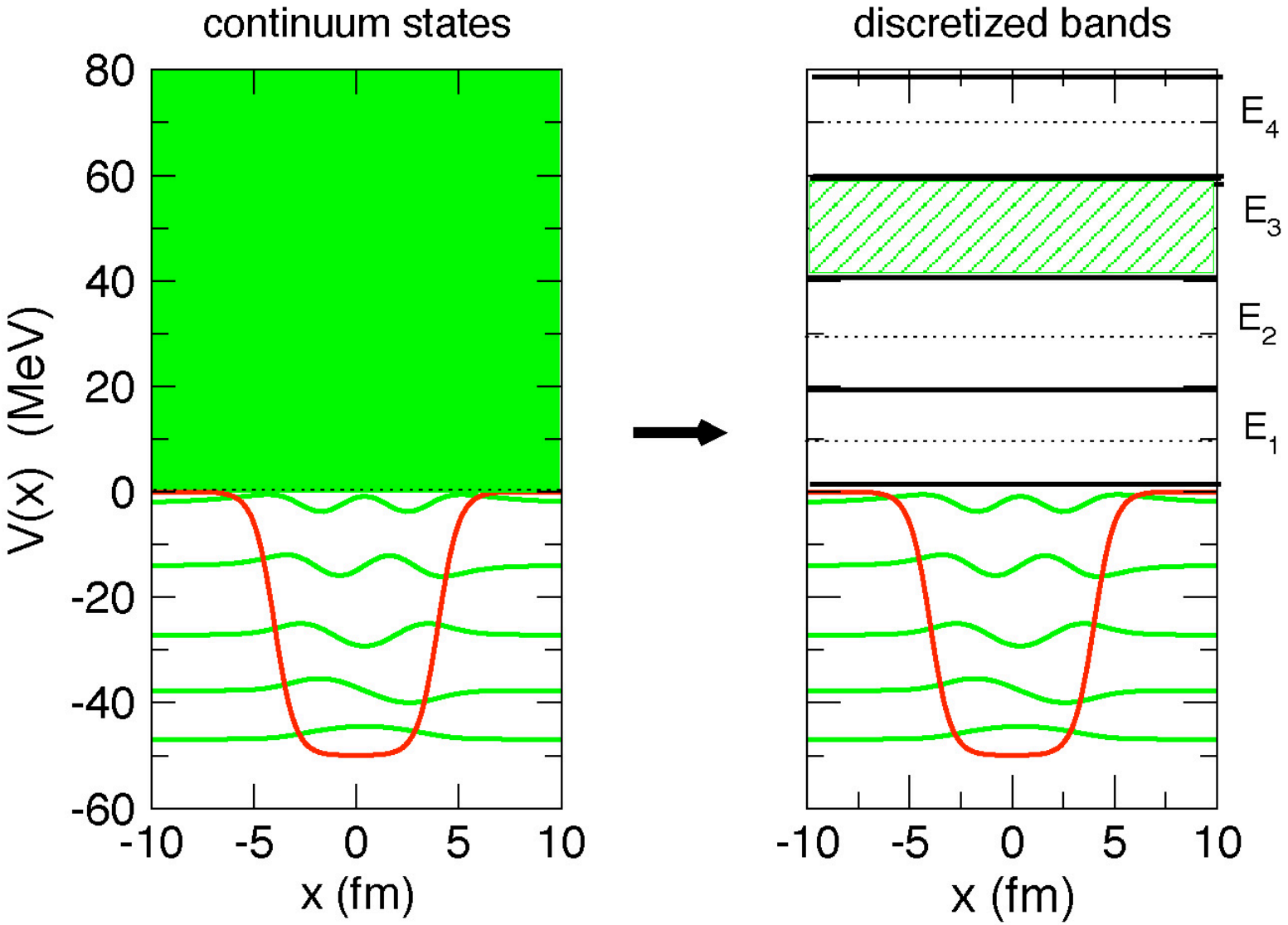
$$\Psi(x,t)=\Sigma_N\, a_N(t)\, \Phi_N(x)$$

and solving the coupled system of equations for the amplitudes $a_N(t)$

$$ihda_N(t)/dt=\Sigma_M \exp(-i(E_N-E_M)t) \langle\Phi_N|\, V\,(x,t)|\Phi_M\rangle\, a_M(t)$$

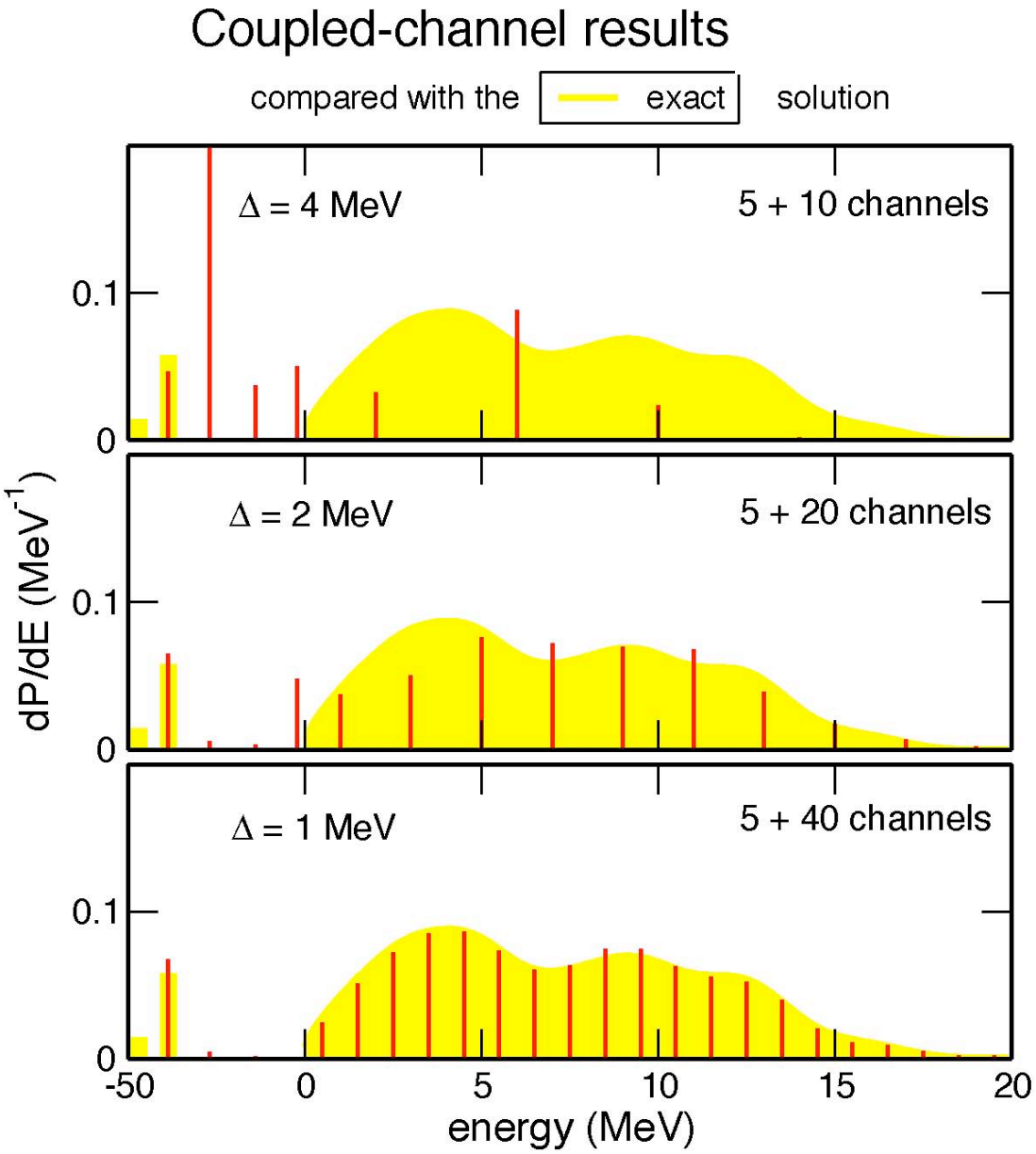
In cases where the inclusion of continuum states
are essential in the proper description of the
evolution of the system, one is naturally led
to the procedure of energy discretization:
we will now slice the continuum
and compare the different approximations
to the full exact solution

Slicing the continuum (in steps of ΔE) and averaging within each band (CDCC)



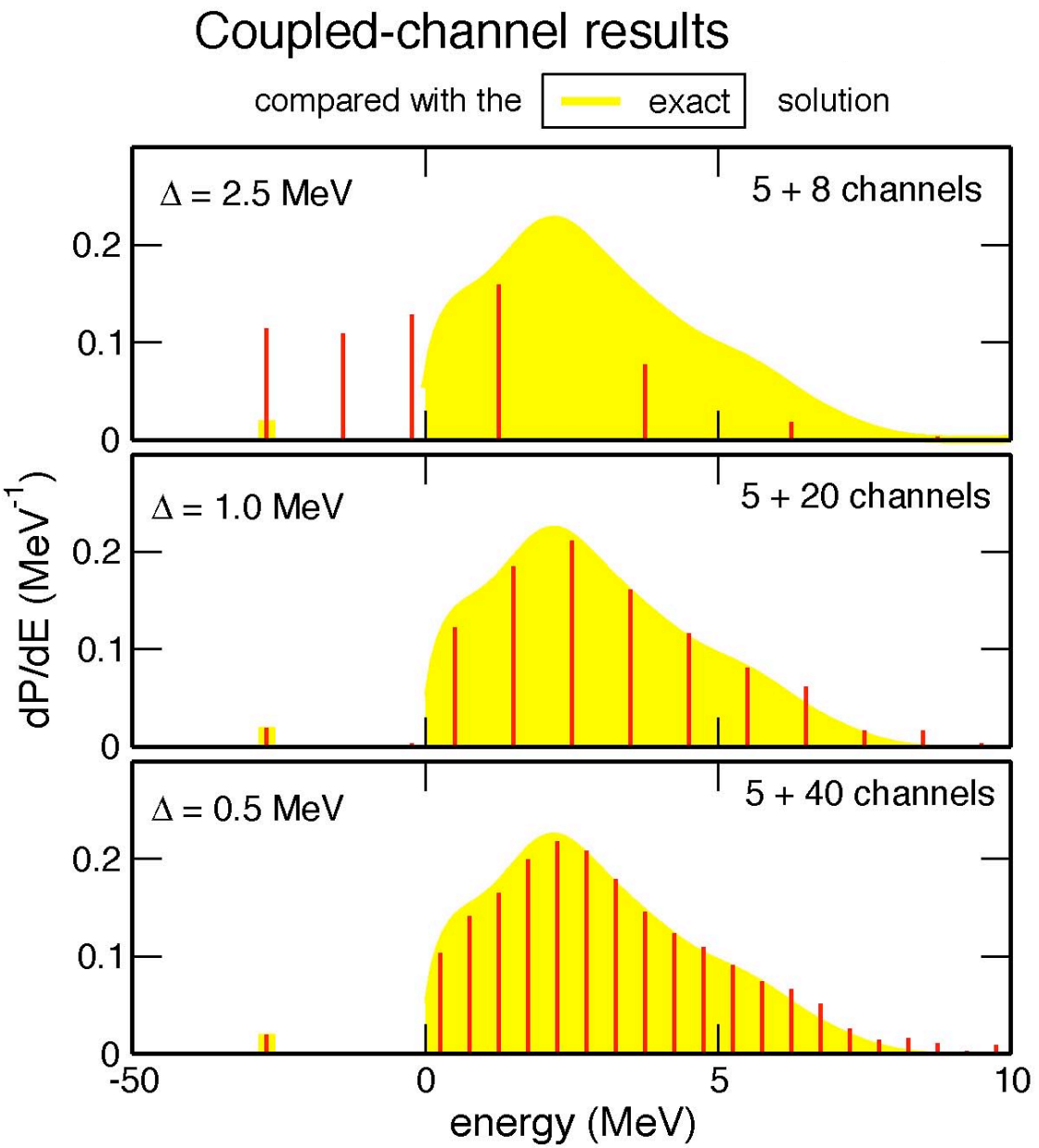
Case of partial break-up starting from a deeply-bound orbital (N=3)

Final Q-value distribution



Case of partial break-up starting from a weakly-bound orbital (N=5)

Final Q-value distribution



Moving from the case of just one particle in the continuum to cases with more particles in the continuum

Simple test cases in structure

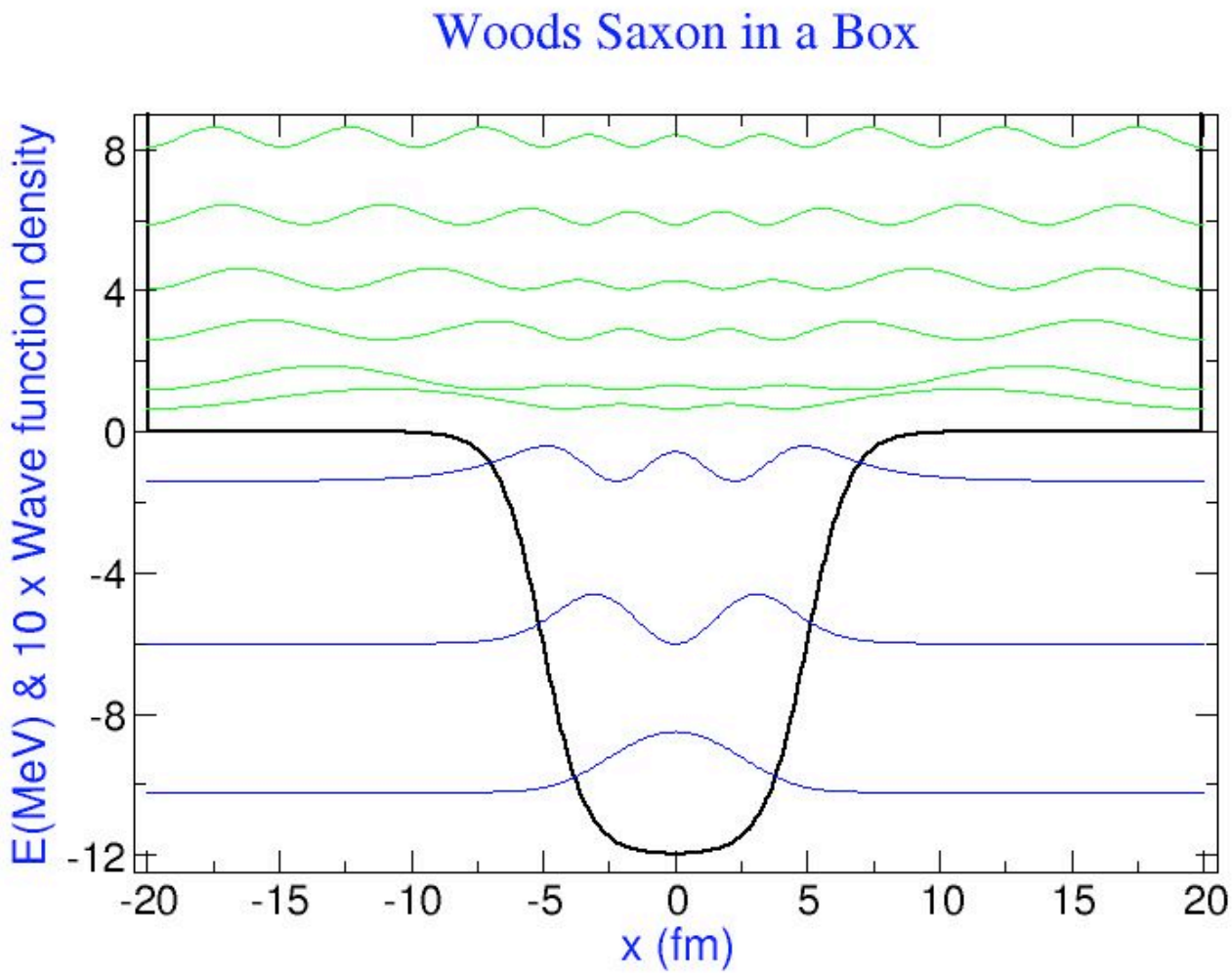
Two valence particles, moving in a one-dimensional Woods-Saxon potential V_0, interacting via a residual density-dependent short-range attractive interaction. Modelling a drip-line system, one can choose the Fermi surface in such a way that there are no available bound states, and the two unperturbed particles must be in the continuum. The residual interaction

$$V(x_1,x_2) = V_0\, \delta(x_1-x_2)\, \rho((x_1+x_2)/2)/\rho_0$$

can be chosen in such a way that the final correlated wave function is however bound. Such a system is normally called "Borromean"

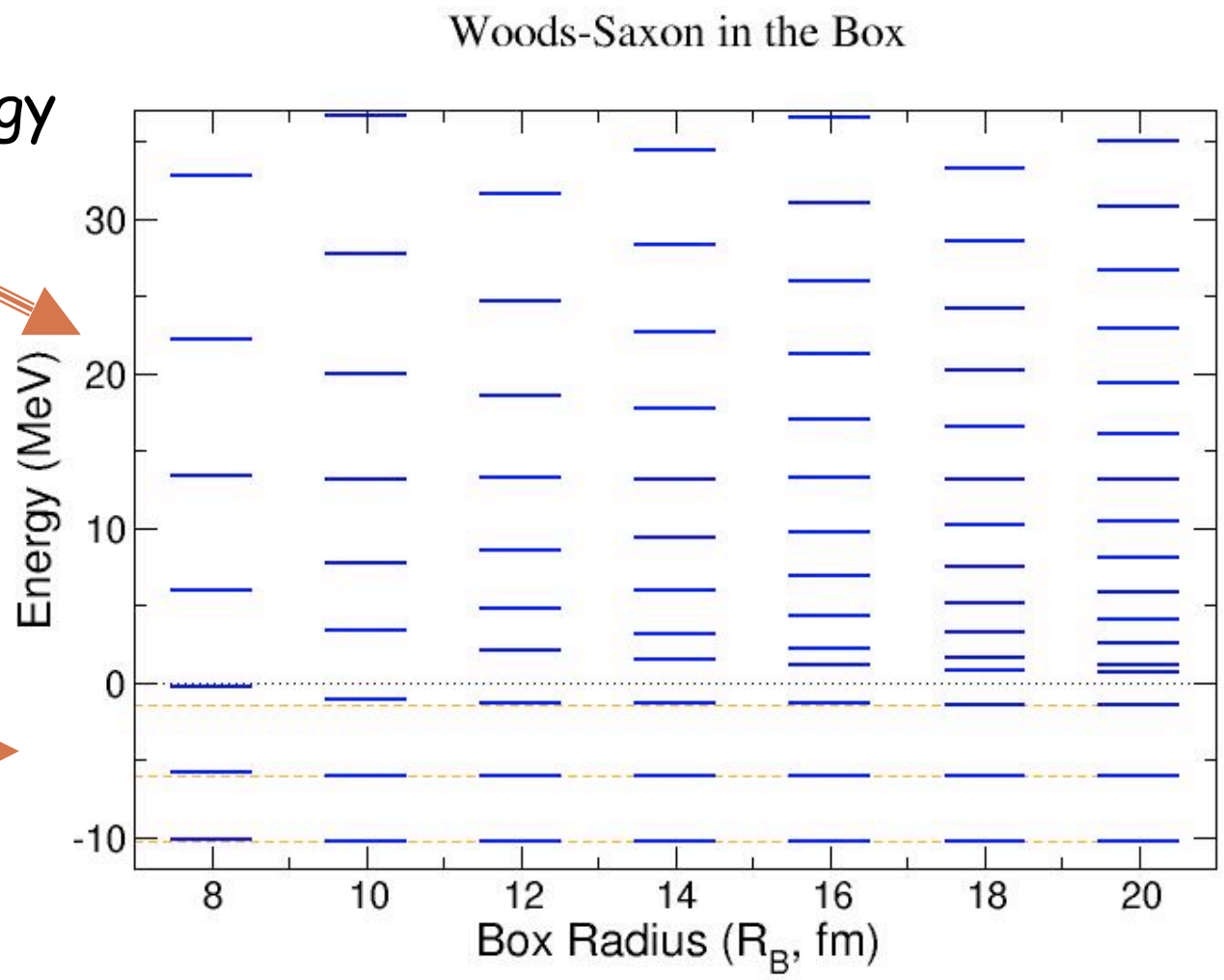
Diagonalization in a box

WS single-particle states obtained imposing boundary conditions at a box (R=20 fm)

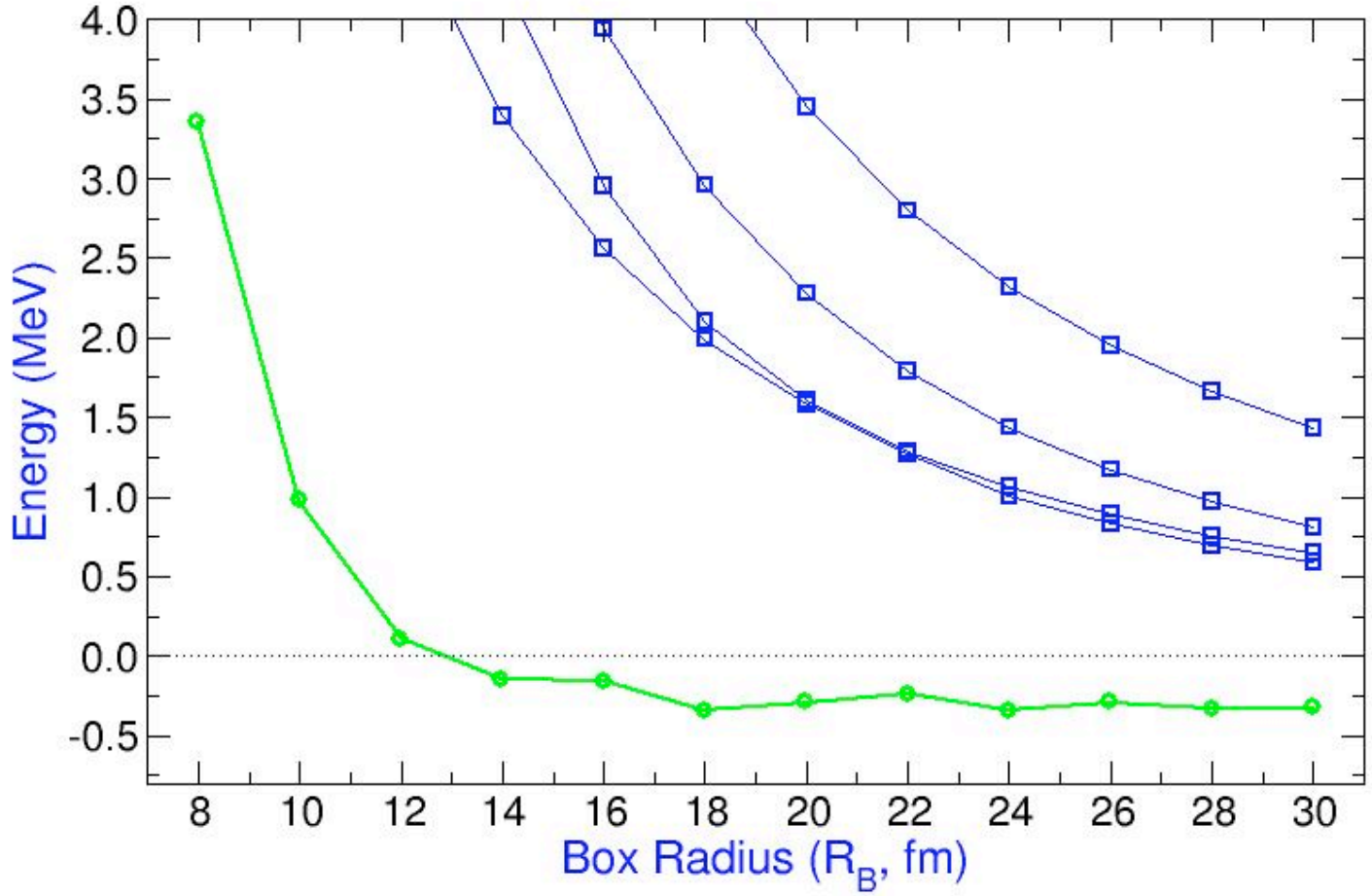


positive energy states

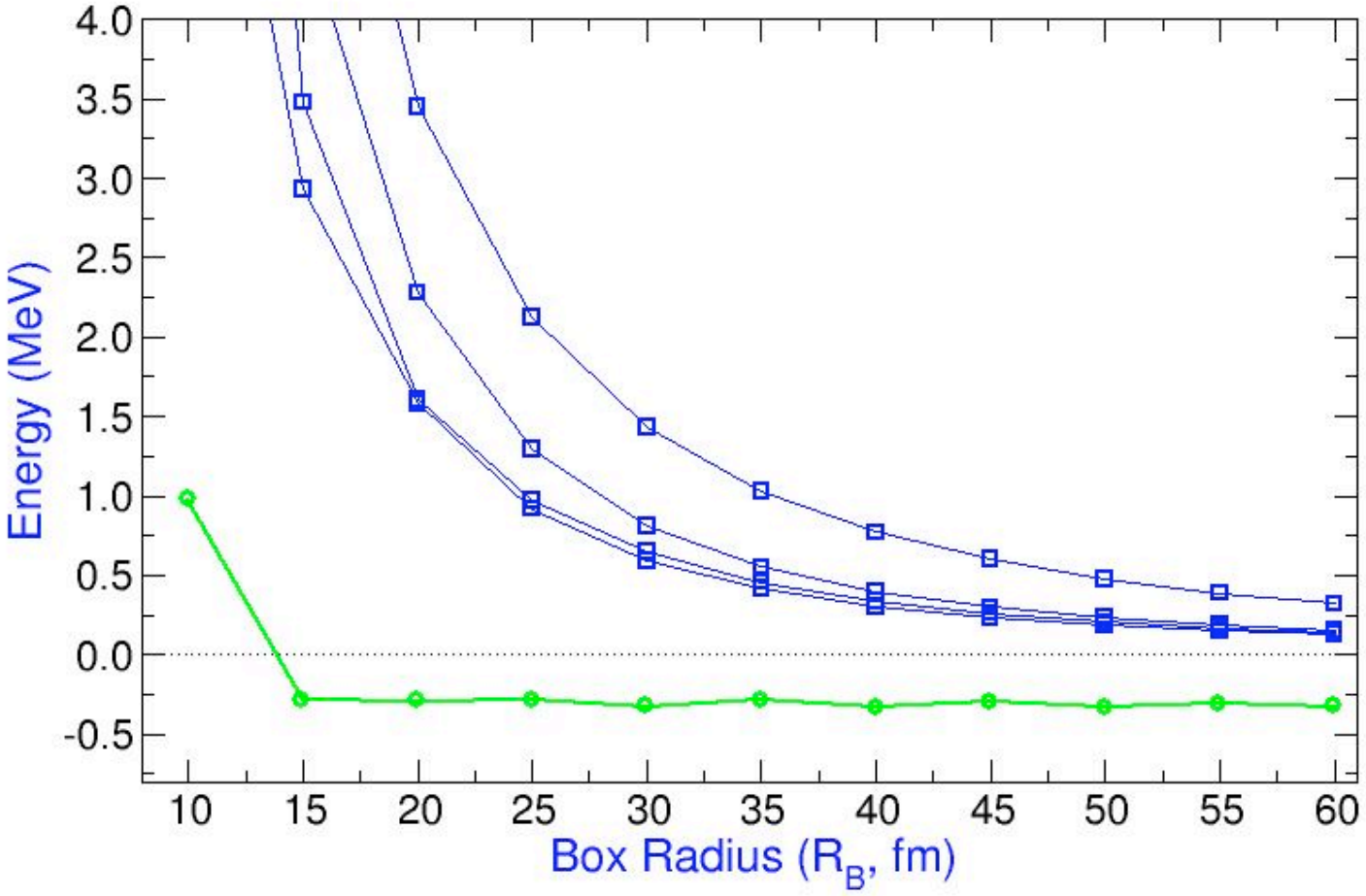
bound states



Correlated energy of the two-particle system (as a function of the box radius)

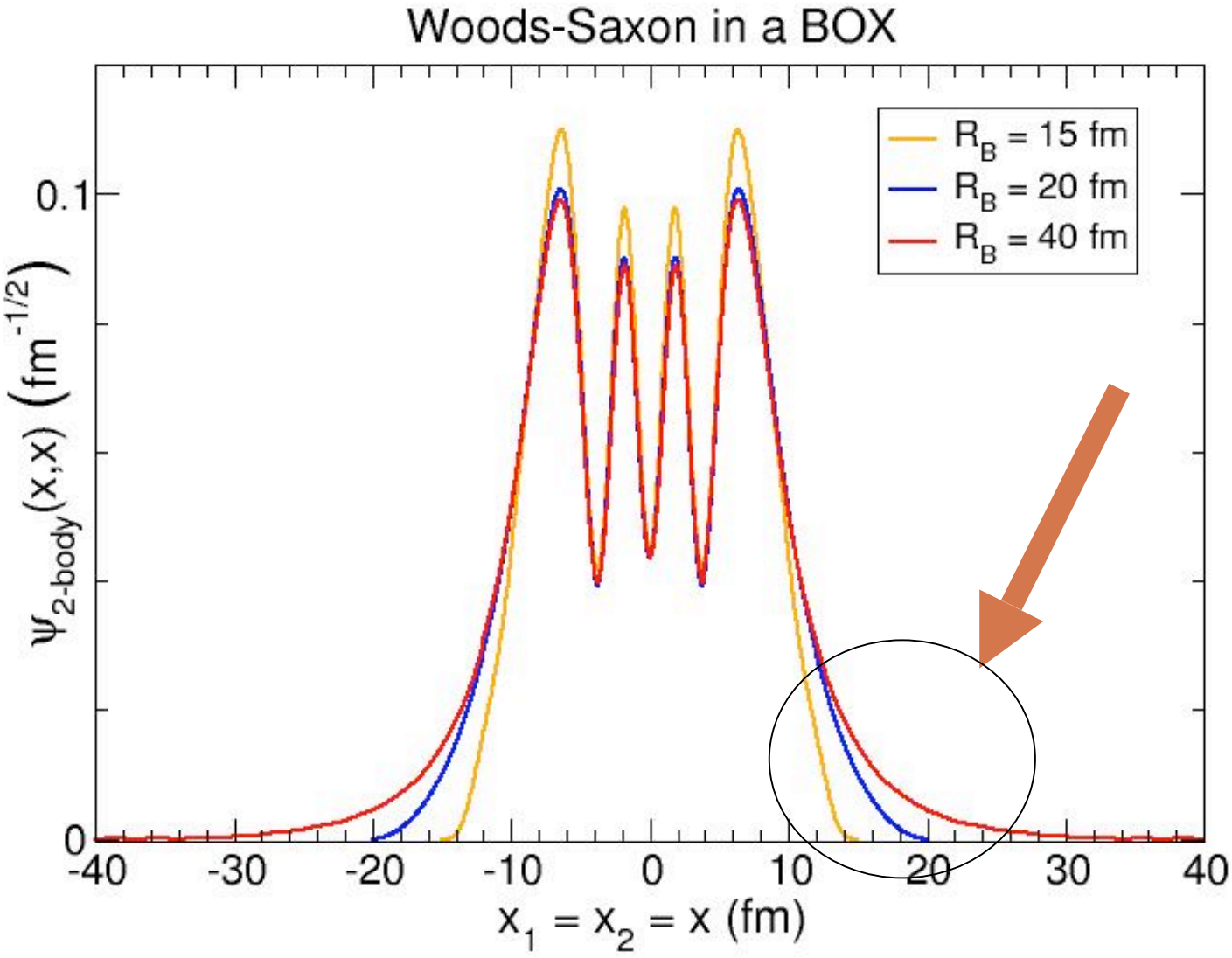


The value of the binding energy is converging (with some oscillations) to the final value

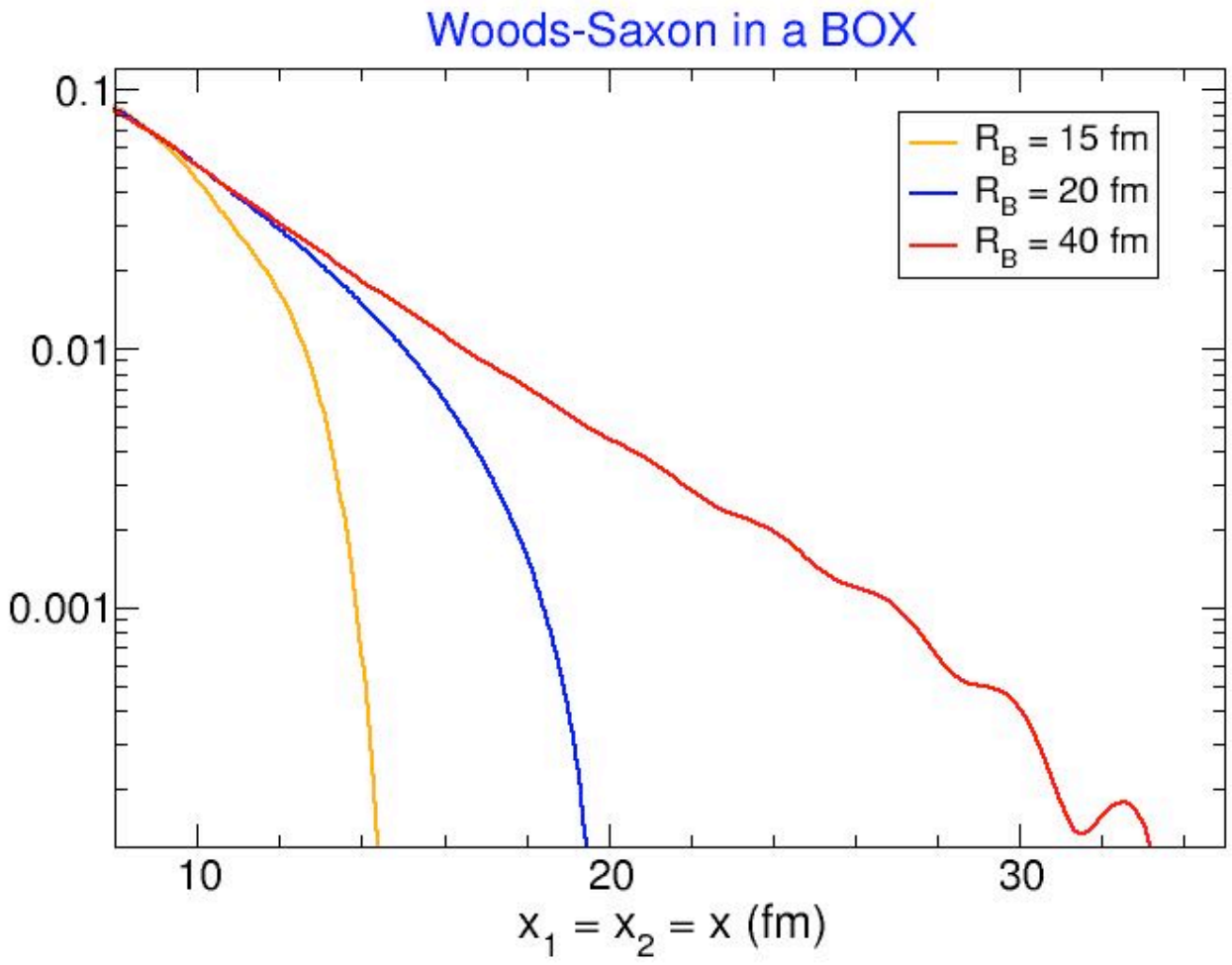


Energy already practically correct with a box of 15 fm, but what about the wave function? In particular, how does it behave in the tail?

Radial dependence $\delta\rho(x,x)$

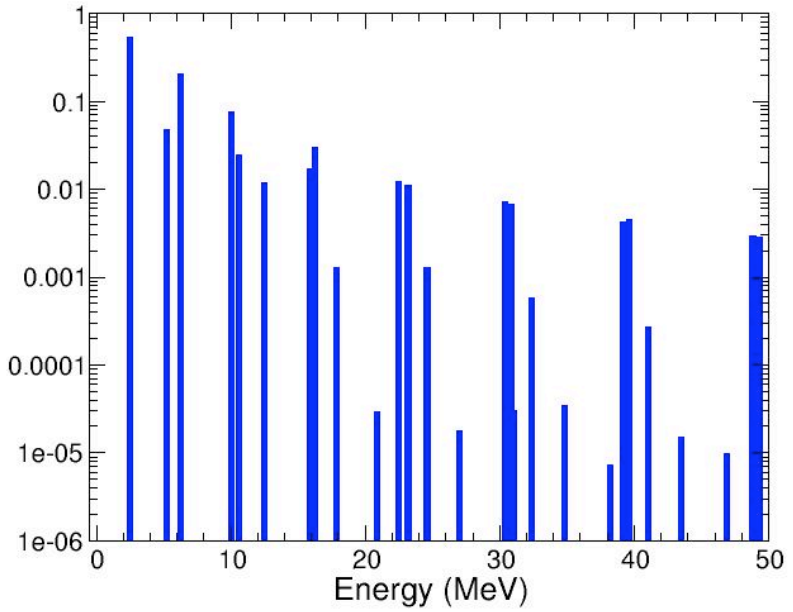


Logaritmic scale



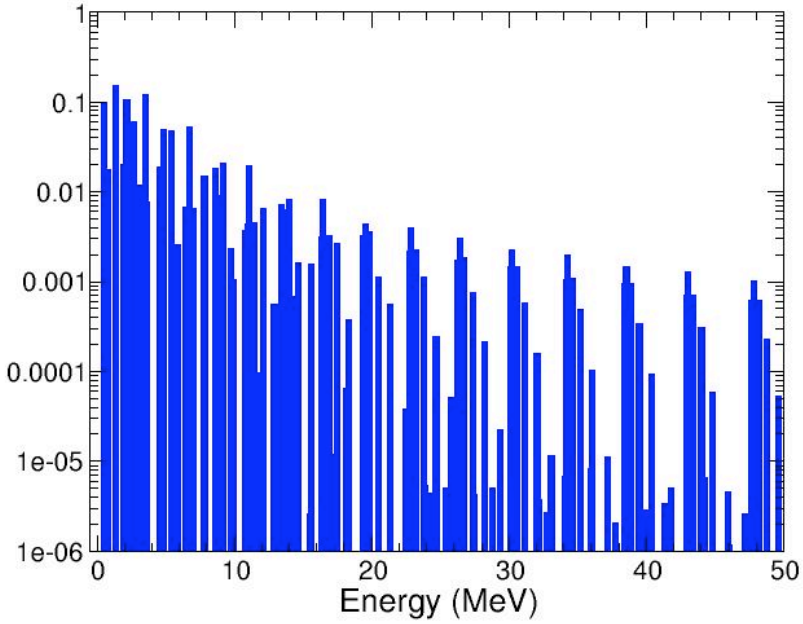
Correlated two-particle wave-function expanded over discretized two-particle positive energy states

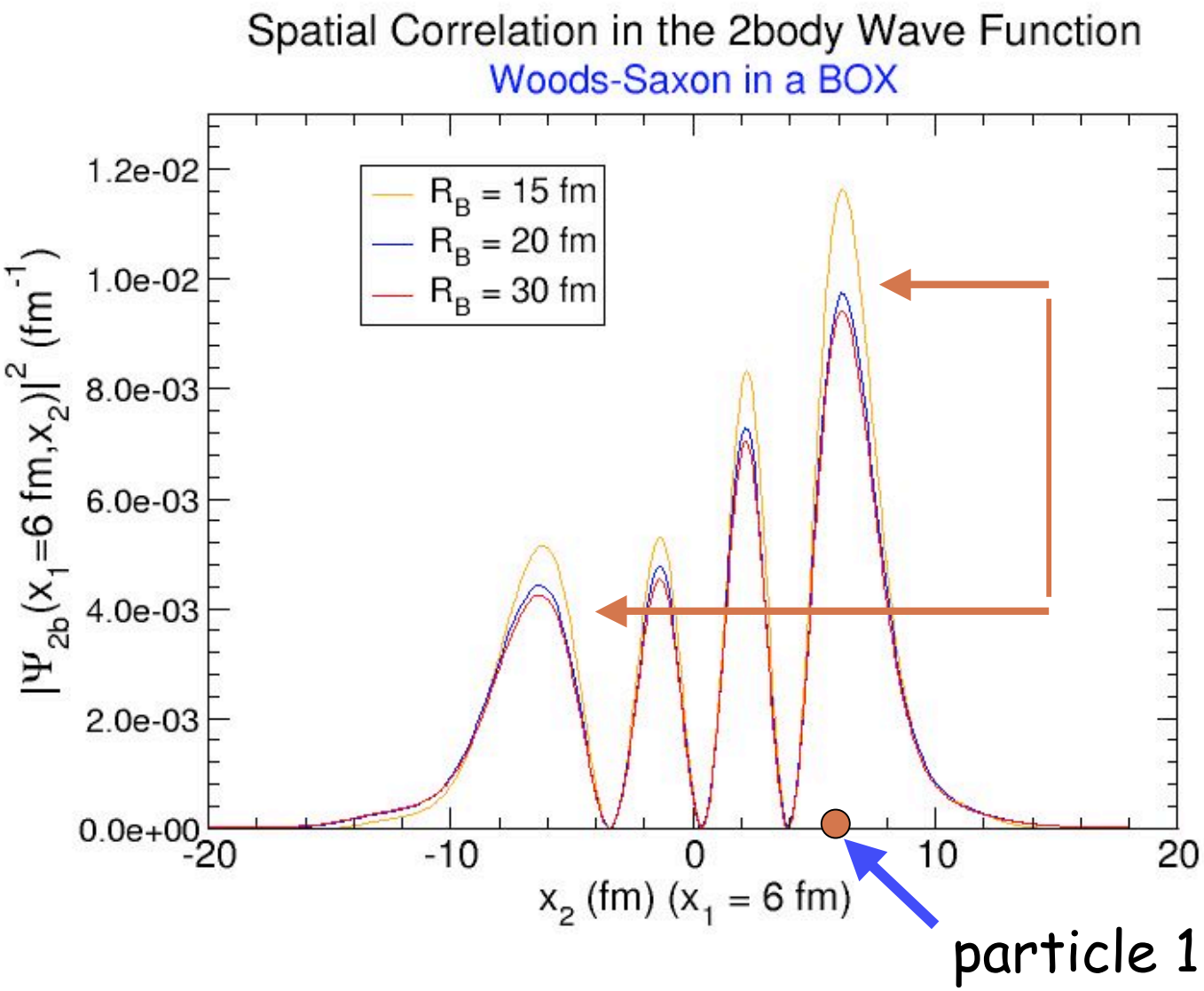
R=15fm



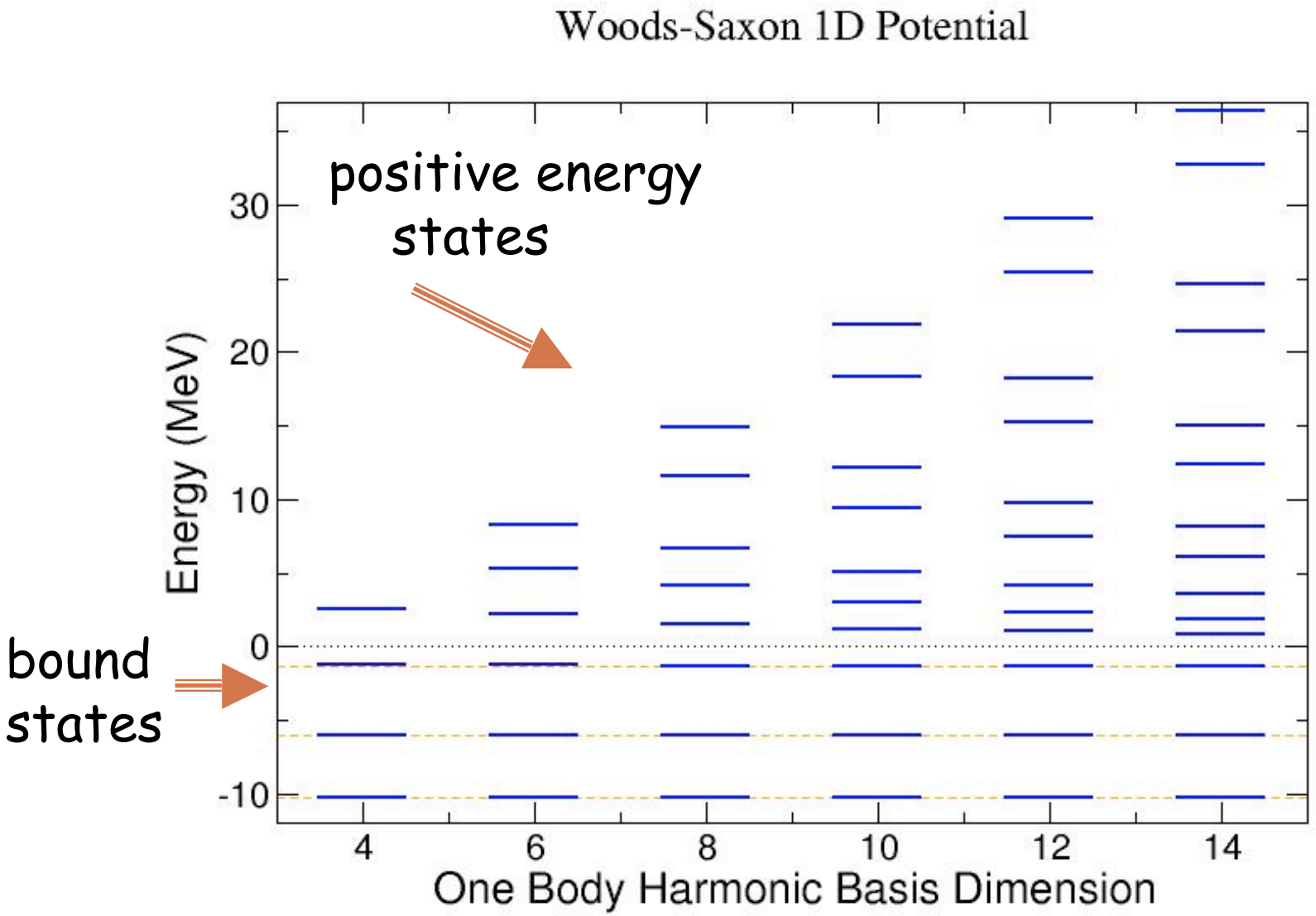
OBS Enormous number of components

R=40fm

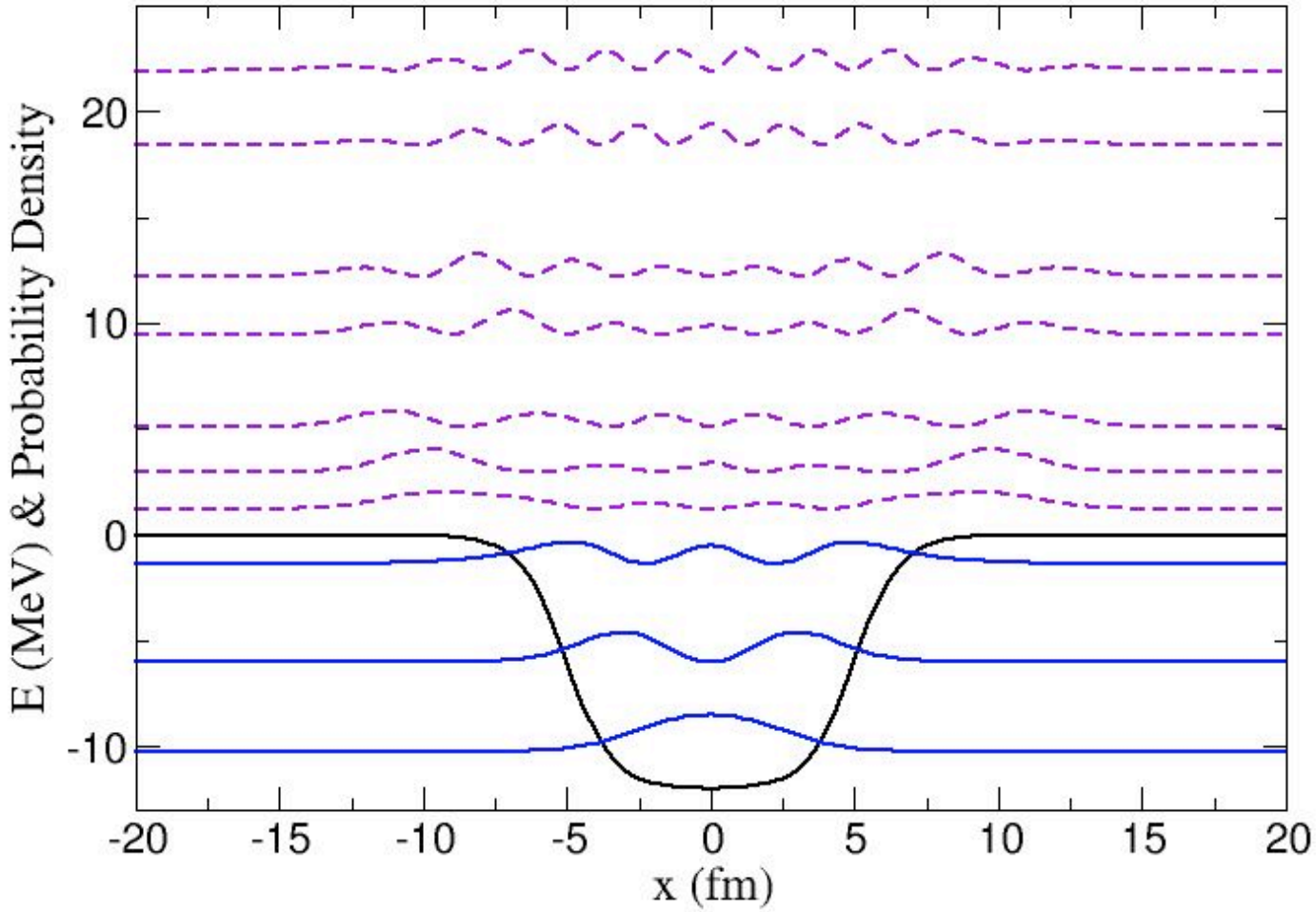



Spatial Correlation in the 2body Wave Function
Woods-Saxon in a BOX
R_B = 15 fm
R_B = 20 fm
R_B = 30 fm
$|\Psi_{2b}(x_1=6\ \mathrm{fm},x_2)|^2\ (\mathrm{fm}^{-1})$
1.2e-02
1.0e-02
8.0e-03
6.0e-03
4.0e-03
2.0e-03
0.0e+00
-20
-10
0
10
20
x_2 (fm) (x_1 = 6 fm)
particle 1

Other option: diagonalization in a harmonic oscillator basis

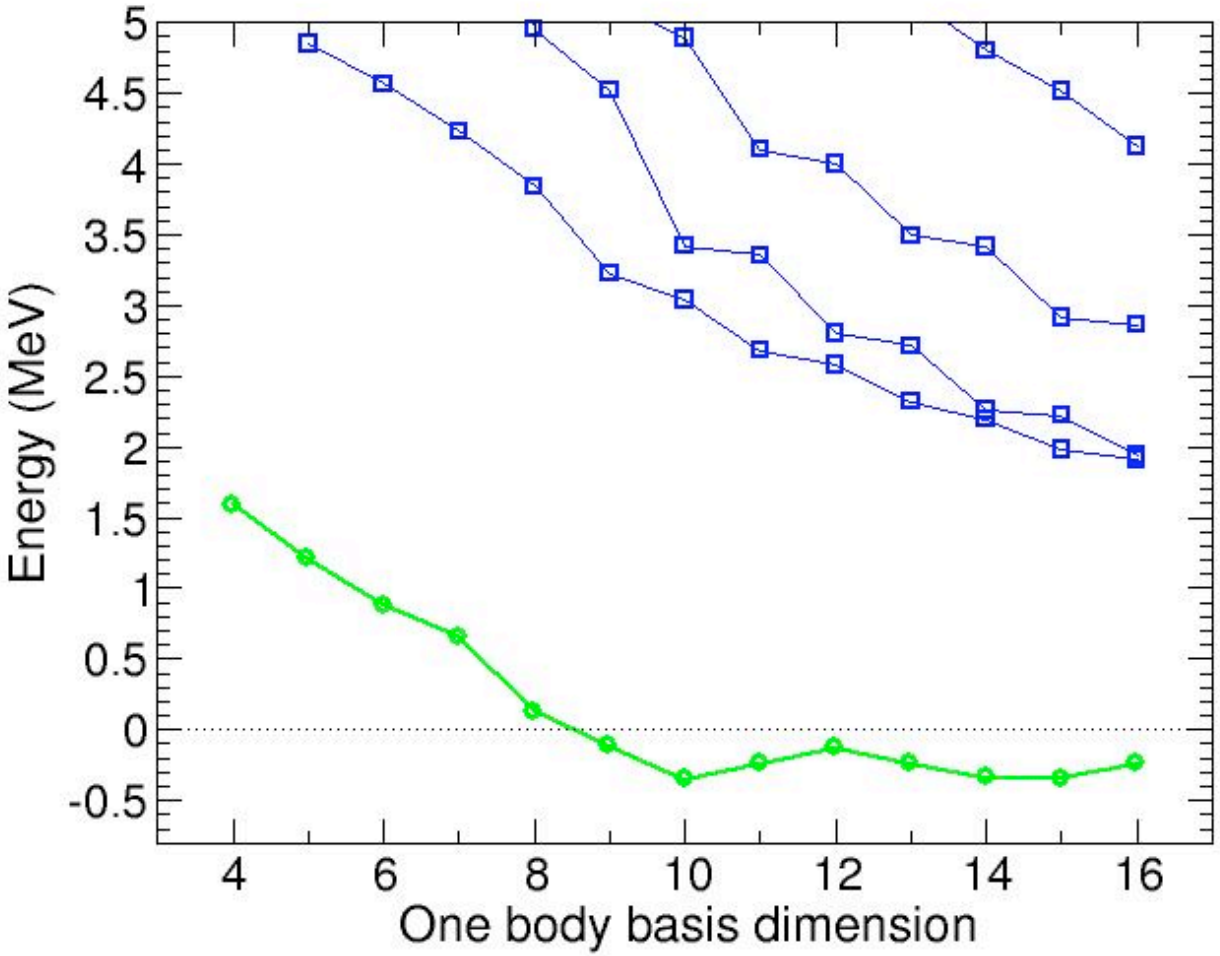


WS single-particle states obtained from Harmonic Oscillator basis (N=10)



Mon Apr 27 15:39:35 2009

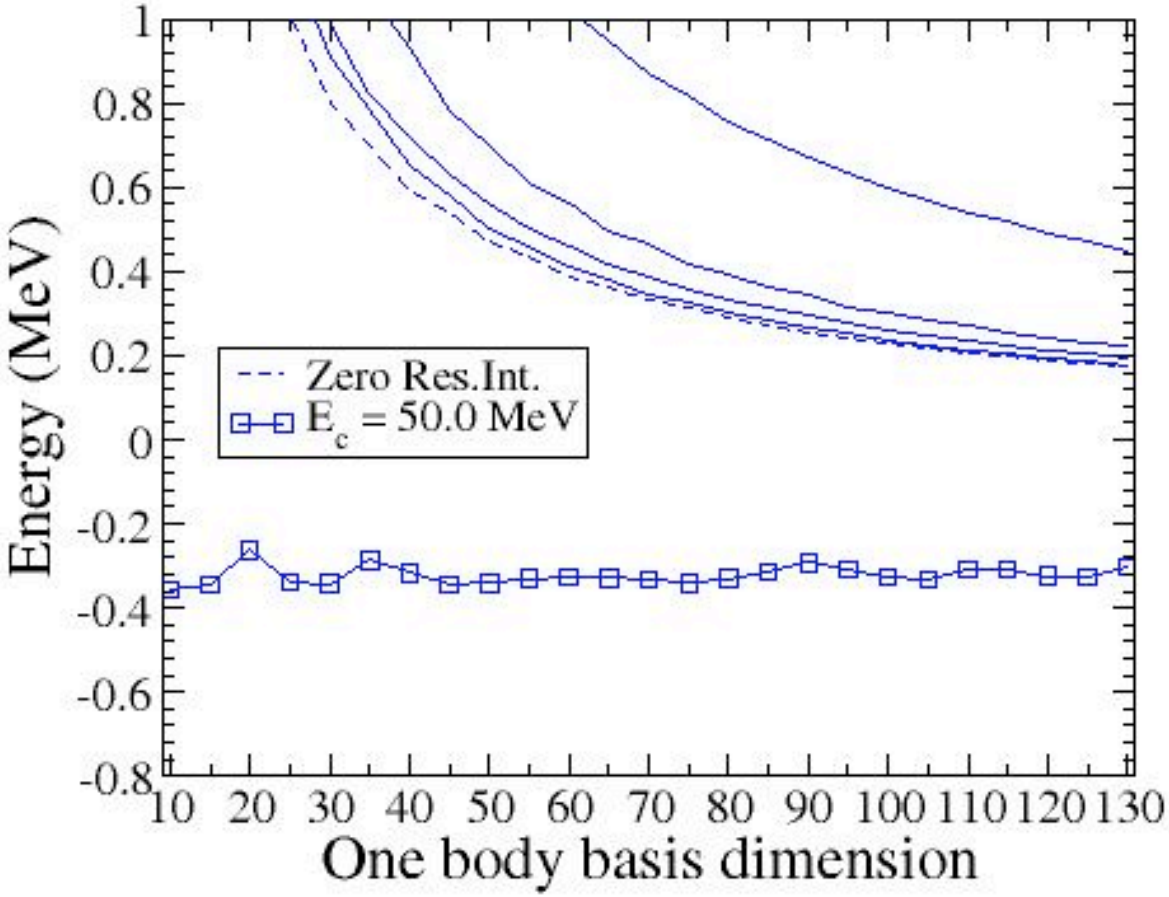
Two-body correlated energies: Harmonic Oscillator basis



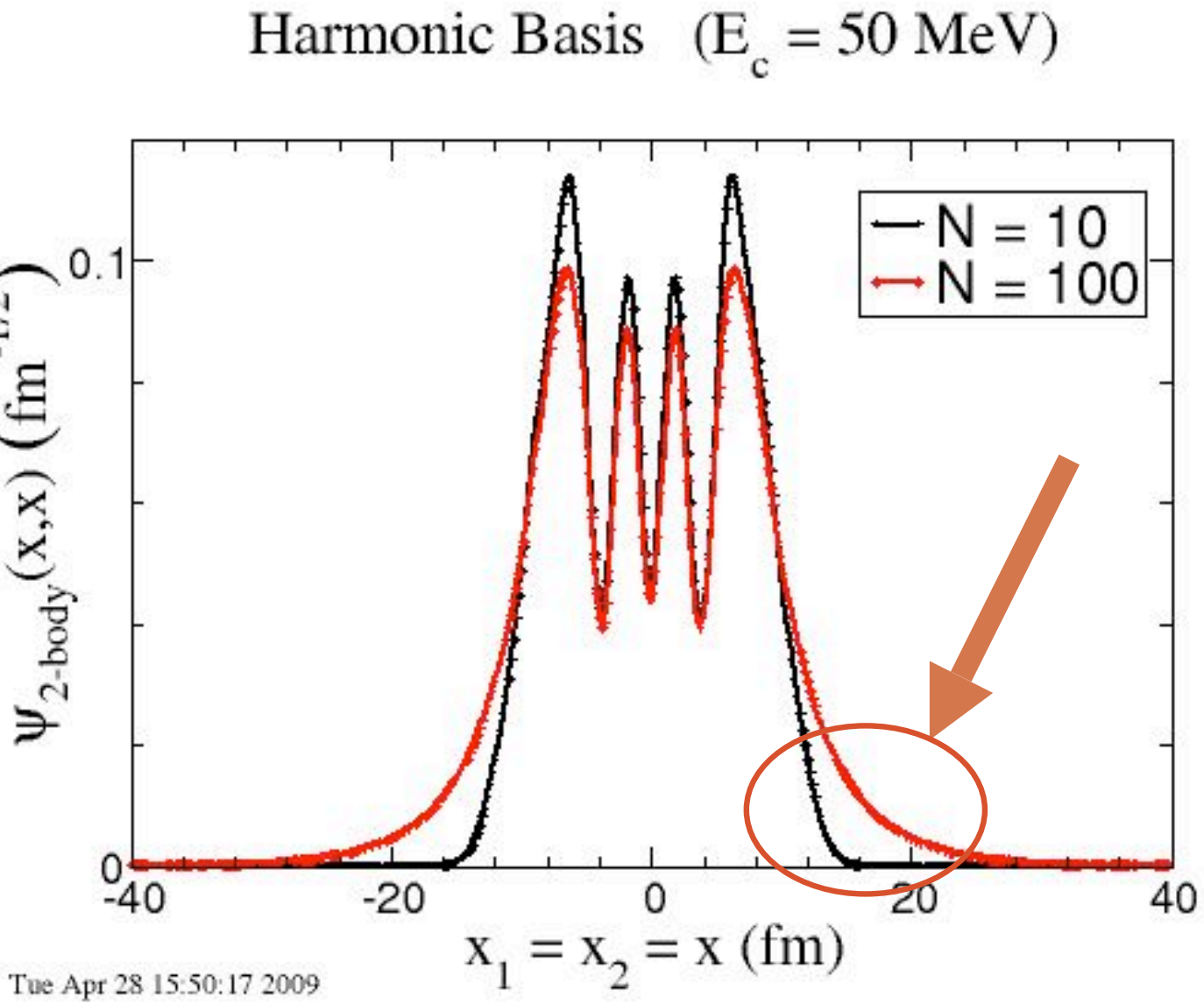
unphysical two-particle states (basis dependent)

physical two-particle bound state

The value of the binding energy is converging (with some oscillations) to the final value

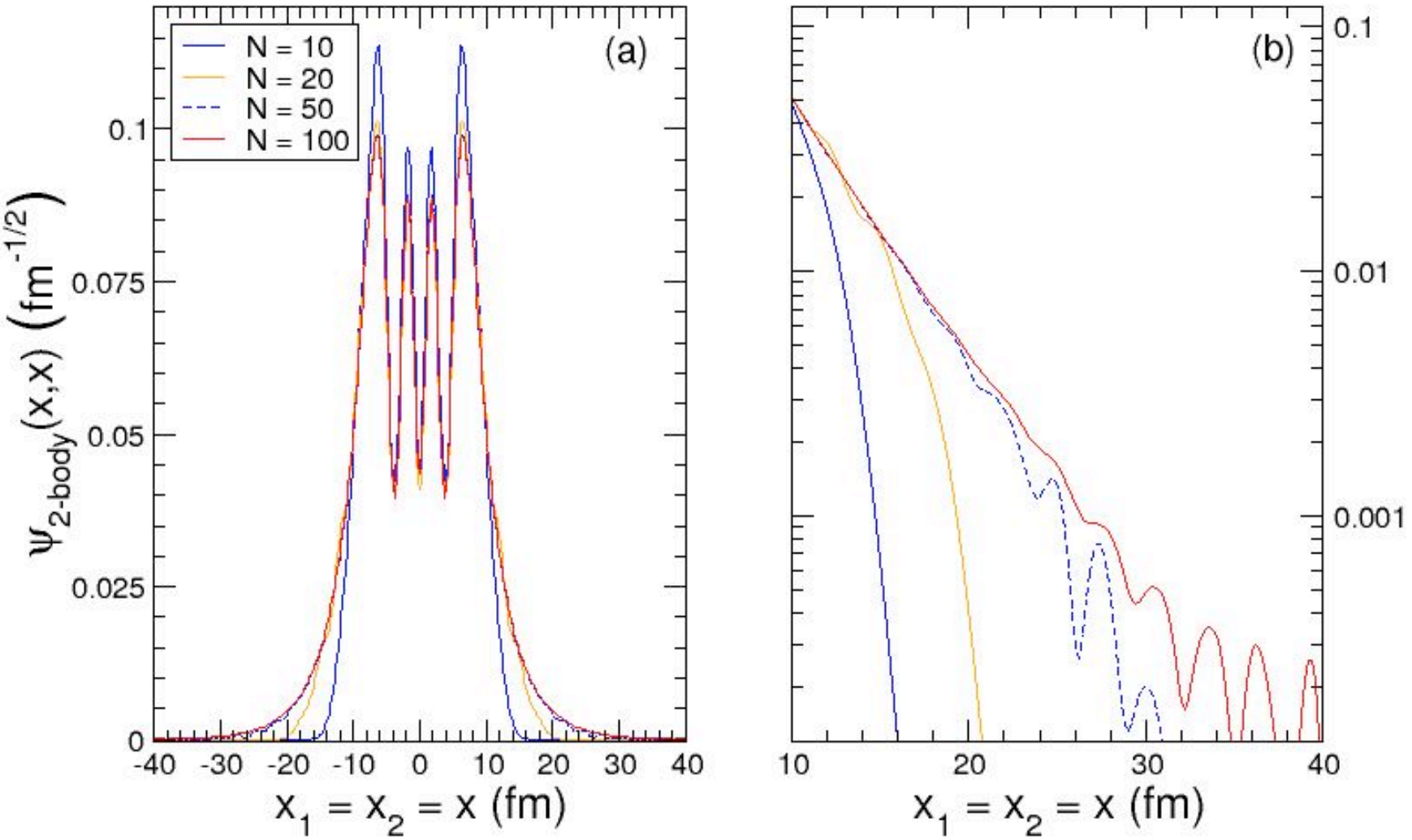


The radial dependence, however …. $\Psi(x,x)$



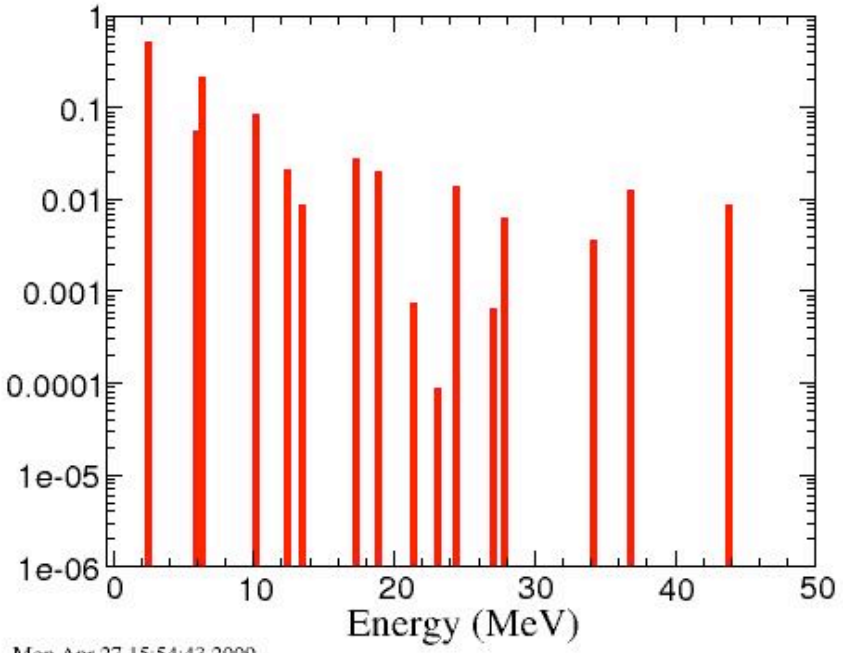
linear

logaritmic



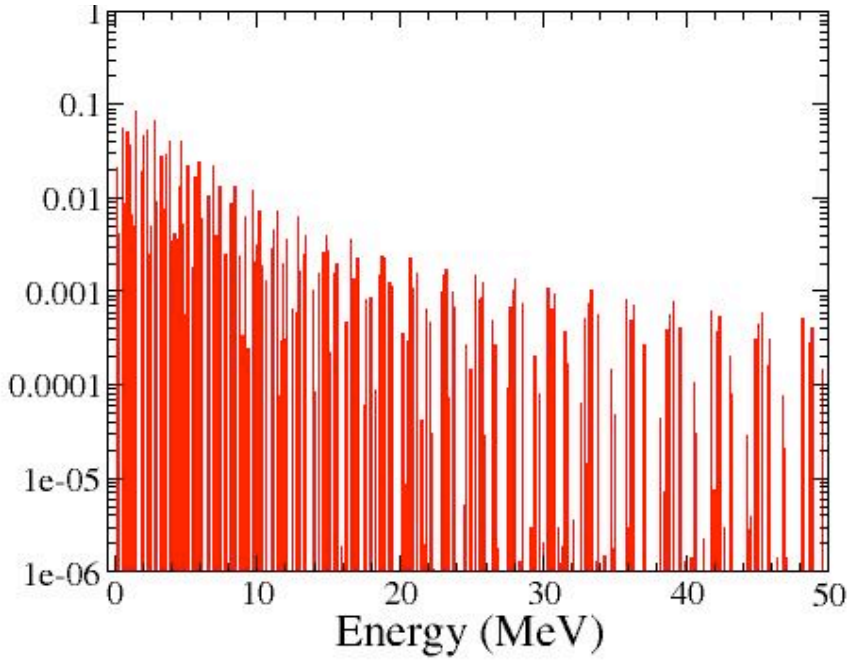
Correlated two-particle wave-function expanded over discretized two-particle positive energy states (amplitudes **2)

N=10



Mon Apr 27 15:54:43 2009

N=100



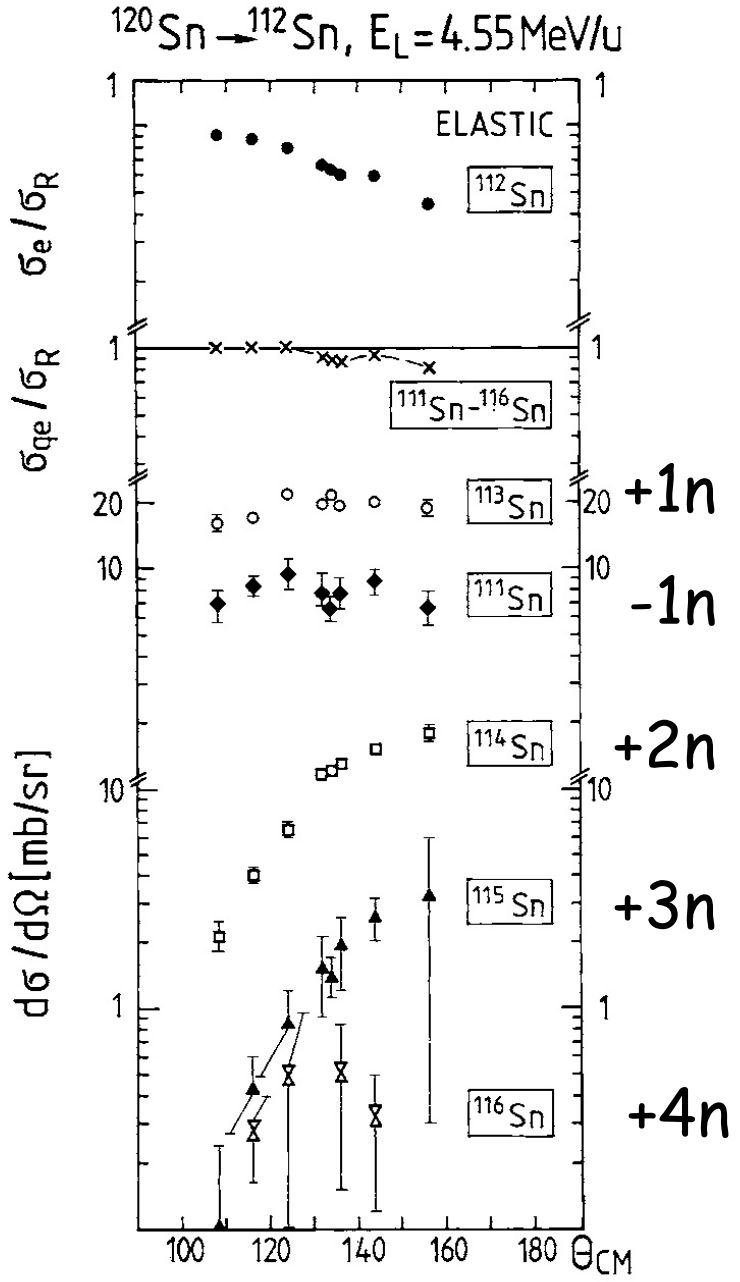
I move now to considering two classes of reactions where pairing correlations play a dominant role and the continuum affects (directly or indirectly) not simply one, but two particles:

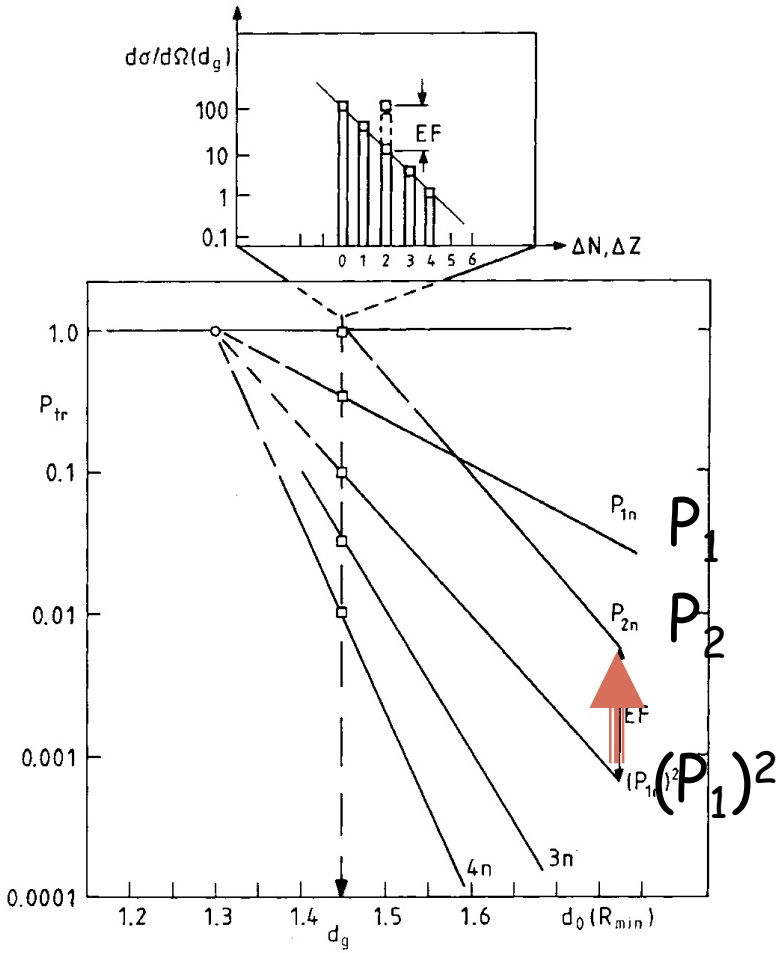
- Two-particle transfer reaction

- Break-up of a two-particle (Borromean) halo system

Two-particle transfer reactions

The classical example:
Sn+Sn
(superfluid on superfluid)





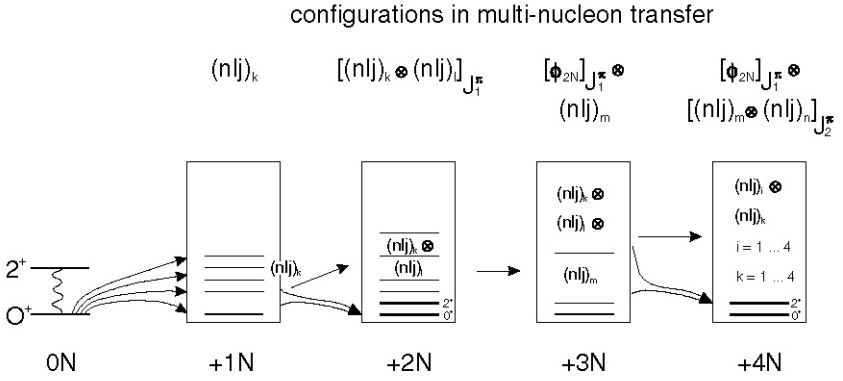
distance of closest approach

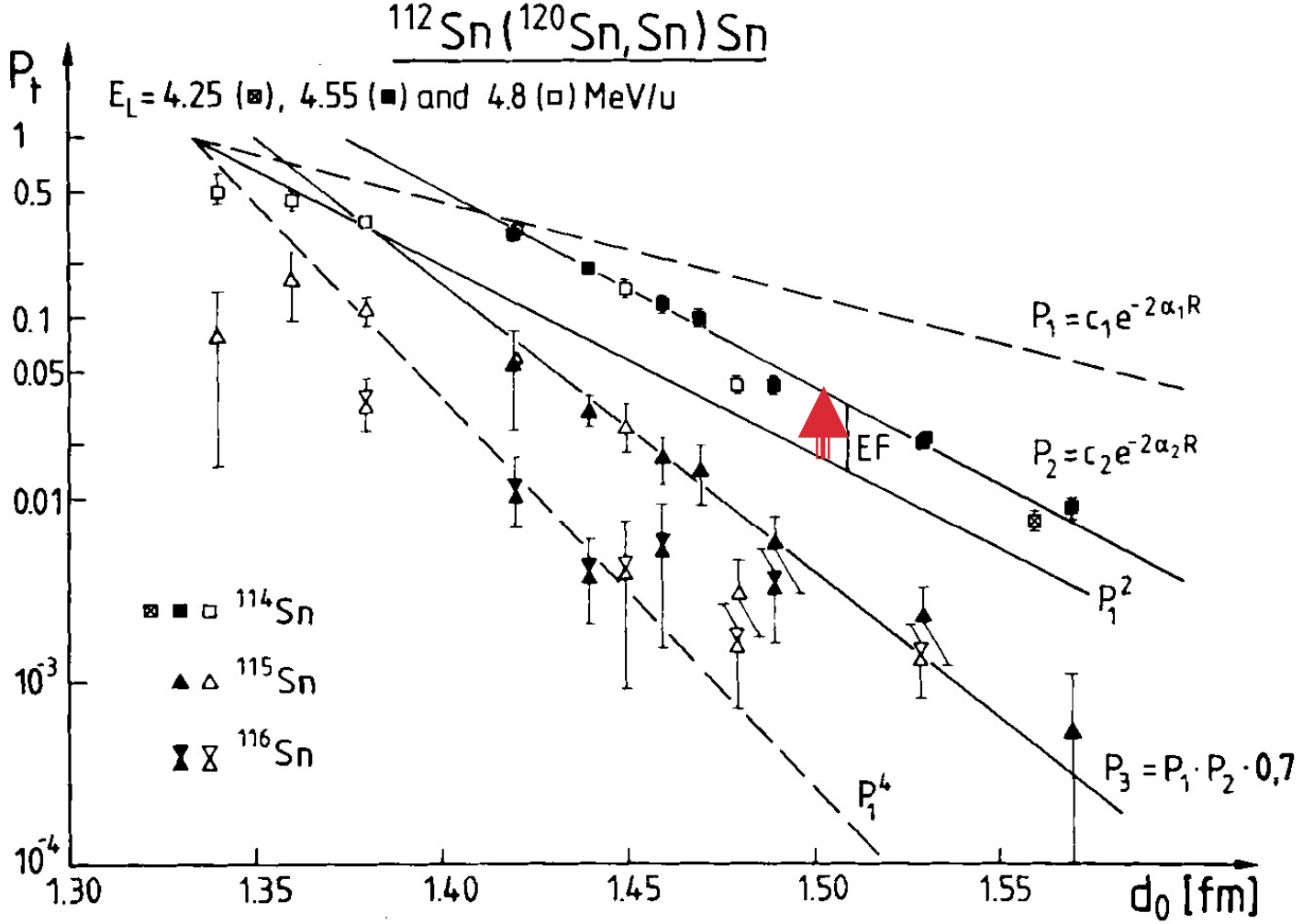
P_1

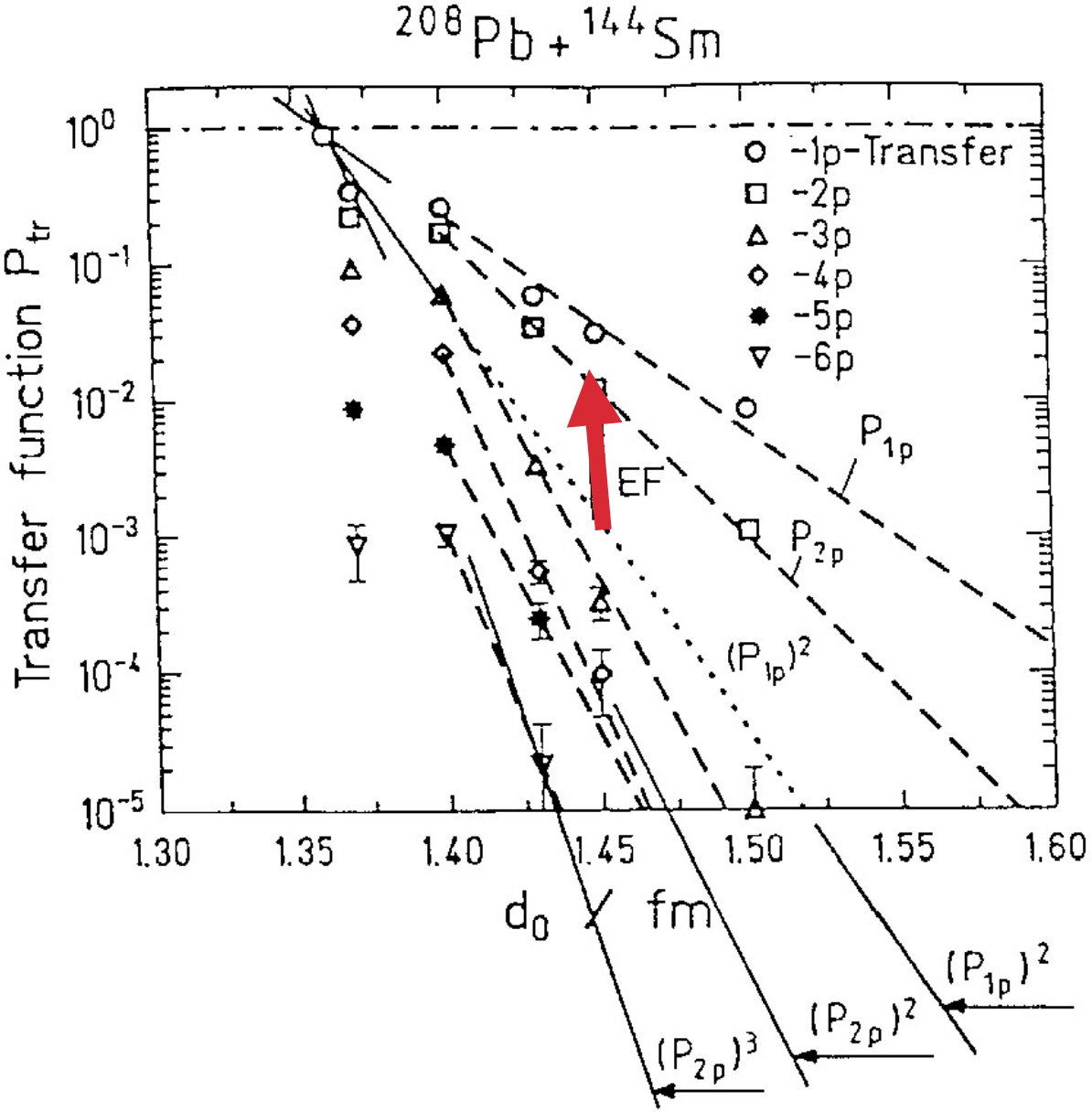
P_2

$(P_1)^2$

A way to define a pairing "enhancement" factor, by plotting transfer probabilities not as function of the scattering angle, but as function of the distance of closest approach of the corresponding classical trajectory




^{112}Sn (^{120}Sn, Sn) Sn
E_L = 4.25 (⊠), 4.55 (■) and 4.8 (□) MeV/u
P_t
1
0.5
0.1
0.05
0.01
10^{-3}
10^{-4}
$P_1 = c_1 e^{-2\alpha_1 R}$
$P_2 = c_2 e^{-2\alpha_2 R}$
EF
P_1^2
$P_3 = P_1 \cdot P_2 \cdot 0{,}7$
P_1^4
⊠ ■ □ ^{114}Sn
▲ △ ^{115}Sn
⧗ ⧖ ^{116}Sn
1.30
1.35
1.40
1.45
1.50
1.55
d_0 [fm]


$^{208}Pb + ^{144}Sm$
Transfer function P_{tr}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1.30
1.35
1.40
1.45
1.50
1.55
1.60
d_0 fm
-1p-Transfer
-2p
-3p
-4p
-5p
-6p
EF
P_{1p}
P_{2p}
$(P_{1p})^2$
$(P_{2p})^3$
$(P_{2p})^2$
$(P_{1p})^2$

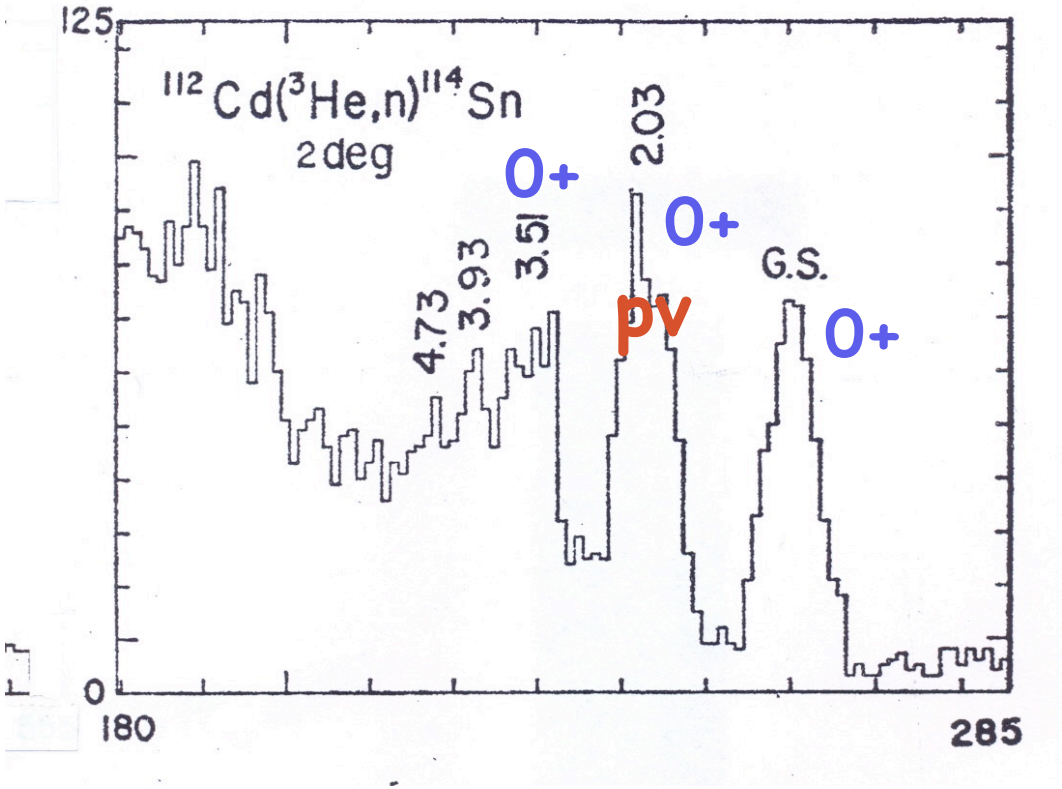
Reaction mechanism and models for two-particle transfer processes

Large number of different approaches, ranging from macroscopic to semi-microscopic and to fully microscopic. They all try to reduce the actual complexity of the problem, which is a four-body scattering (the two cores plus the two transferred particles).

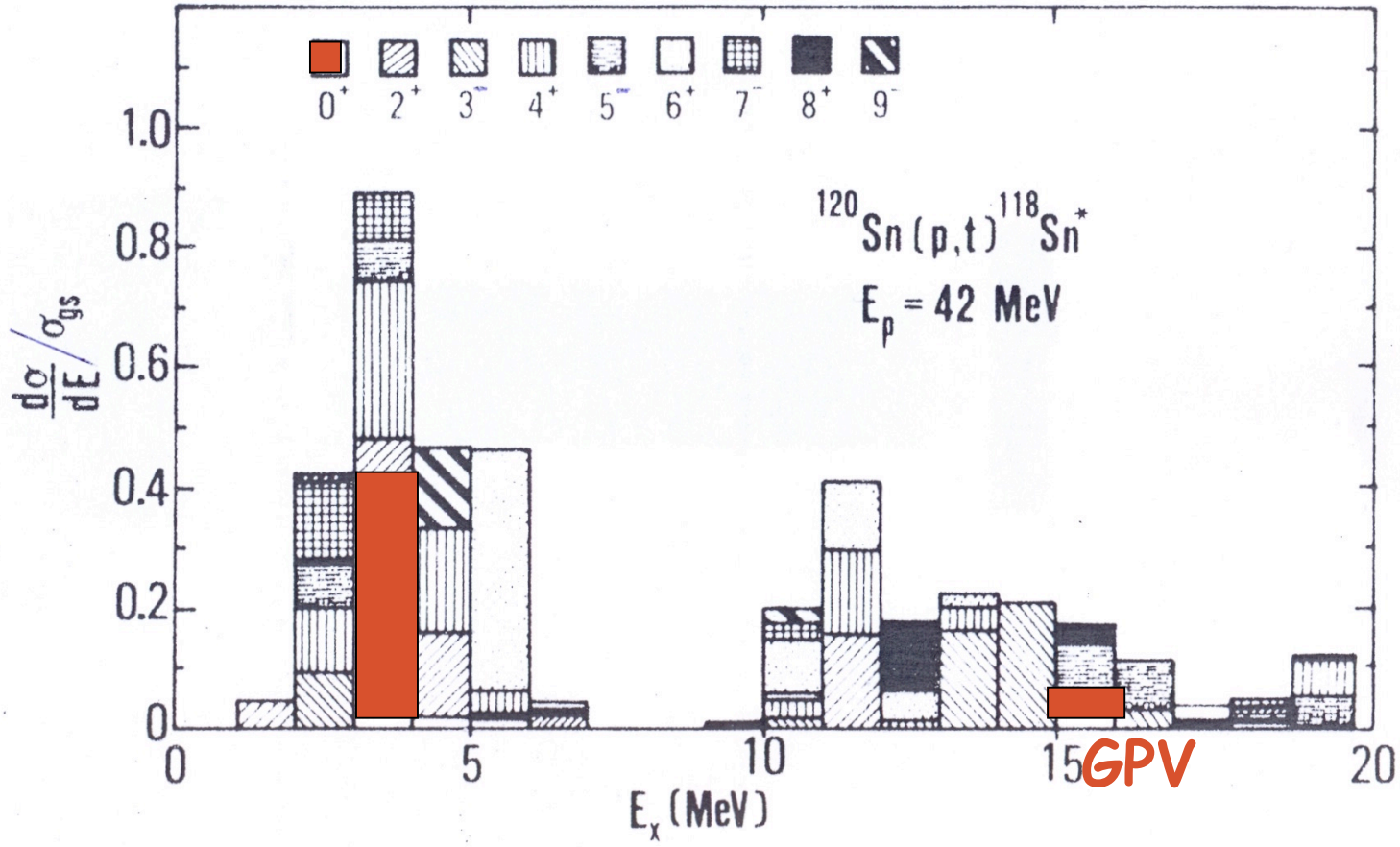
Note that pairing correlations should effect 0+ states,
but these are often overwhelmed by other multipole states
(only for light ions at forward angles one excites selectively
0+ states)

Note that pairing correlations should effect 0+ states,
but these are often overwhelmed by other multipole states
(only for light ions at forward angles one excites selectively
0+ states)

Example: The excited states in 114Sn are of proton character at Z=50 closed shell



Example:
predicted total cross sections in
$^{120}Sn(p,t)^{118}Sn^*$ reaction



Bortignon and Vitturi

Models for two-particle transfer reactions

Example

Semi-microscopic approach

Reaction mechanism: one-step di-neutron (cluster) transfer

Microscopy: Formfactor obtained by double-folding the microscopic pair densities of initial and final states with some nucleon-nucleon interaction

or

Simple folding of microscopic pair density in the target with the one-body mean field of the projectile

Macroscopic approach

Complete parallelism with inelastic excitation of collective surface modes

Reaction mechanism: one step transfer produced by a new generalized pair field

$F(r) = \beta_P \, dU/dA = \beta_P \, (R/3A) \, dU/dr$

Where the "deformation" parameter β_P is the pair-transfer matrix element and contains all the microscopy of the approach

Very simple, appropriate for situations with many other coupled open channels
Problem: recoil? Relative cross sections?

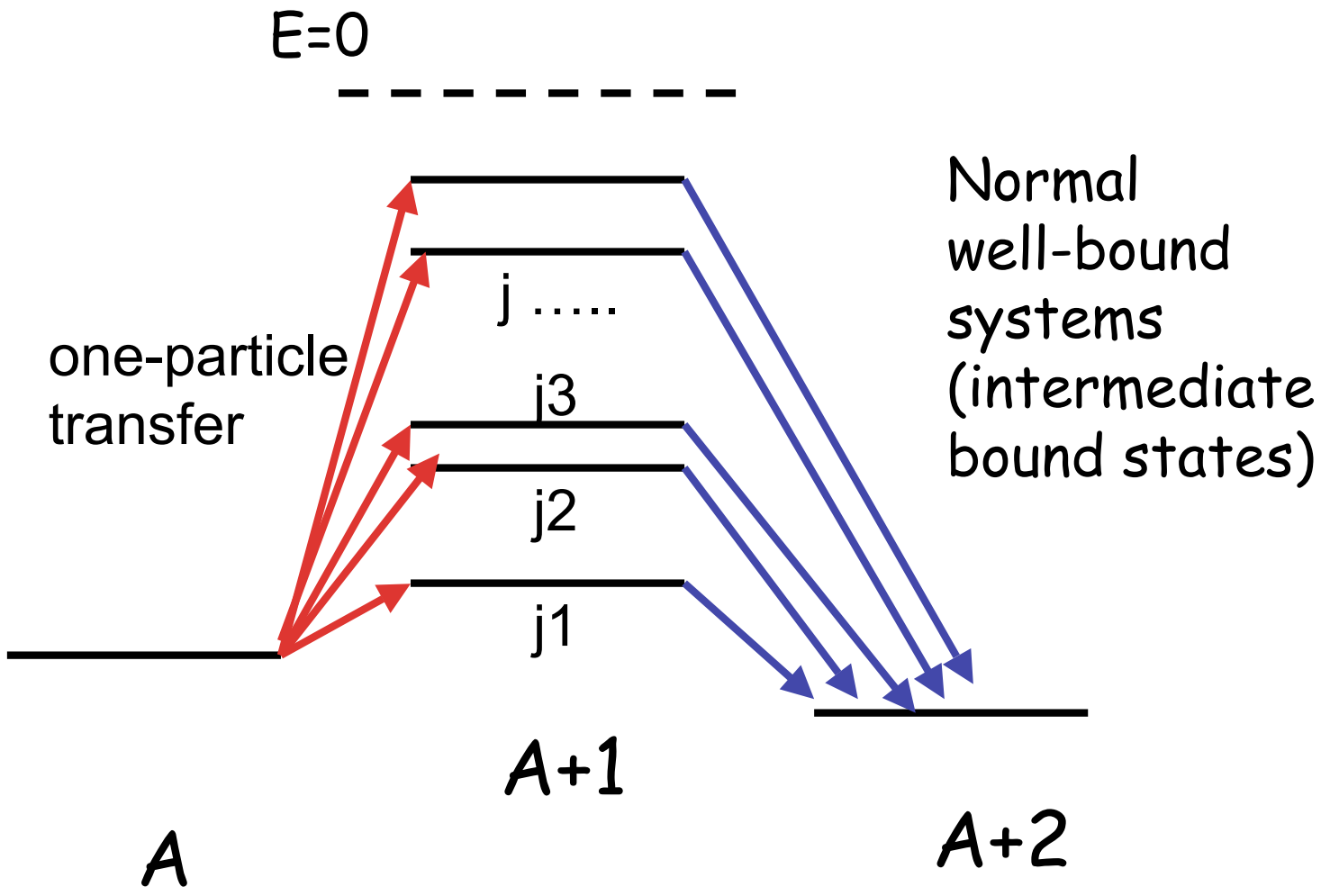
Fully microscopic approach ☺ (cf. talk by Vigezzi)

Reaction mechanism: Sequential two-step process (each step transfers one particle)

Microscopy: Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A+1) nuclei, due to the correlations in initial and final wave functions

Building blocks: single-particle formfactors and wf's

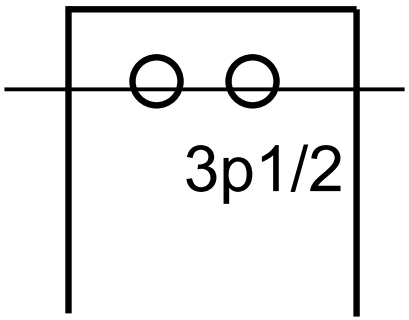
Problems: quantal calculations rather complex (taking into account full recoil), semiclassical more feasible (but approximate treatment of recoil)



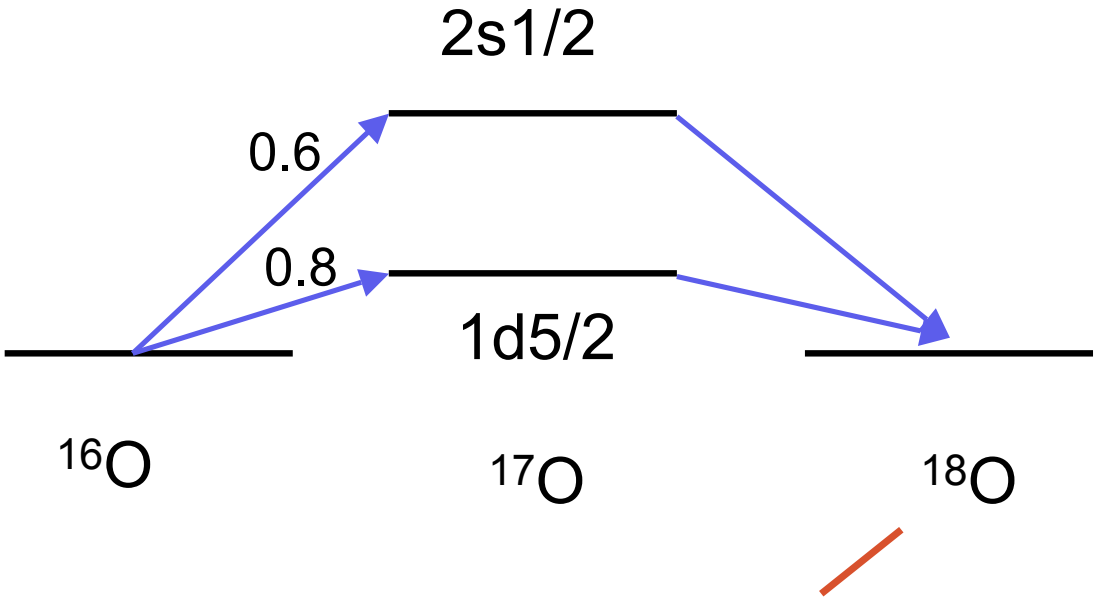
Example $|A=2\rangle = \Sigma_i X_i [a_i^+ a_i^+]_0 |A\rangle$

Example

$^{208}Pb(^{16}O,^{18}O)^{206}Pb$

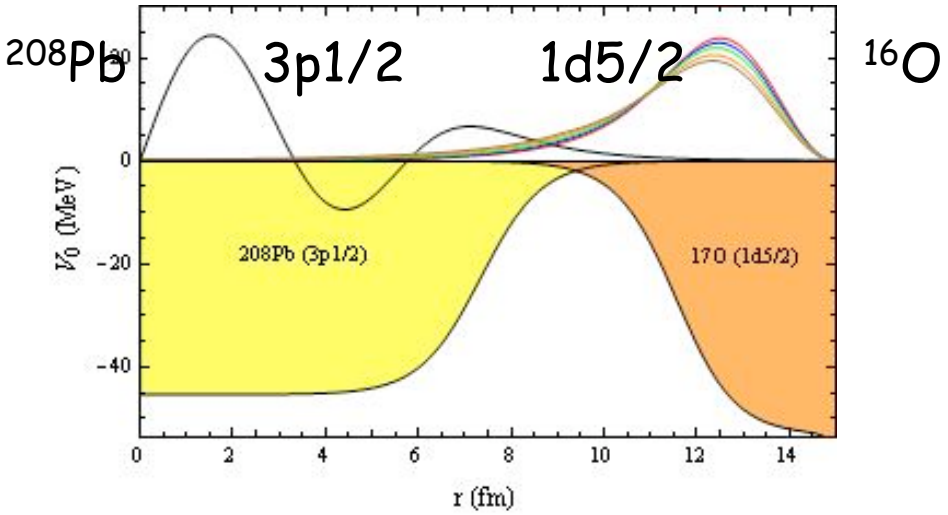


^{208}Pb



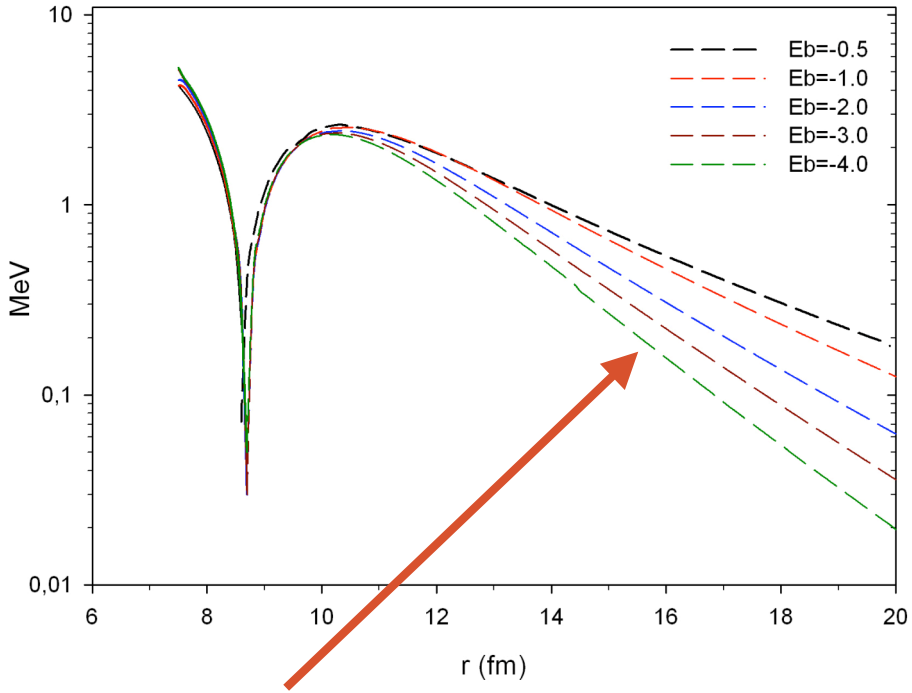
$0.8\ (1d5/2)^2 + 0.6(2s1/2)^2$

Basic blocks: single particle formfactors



3p1/2 (in Pb) ⇒ 1d5/2 (in O)

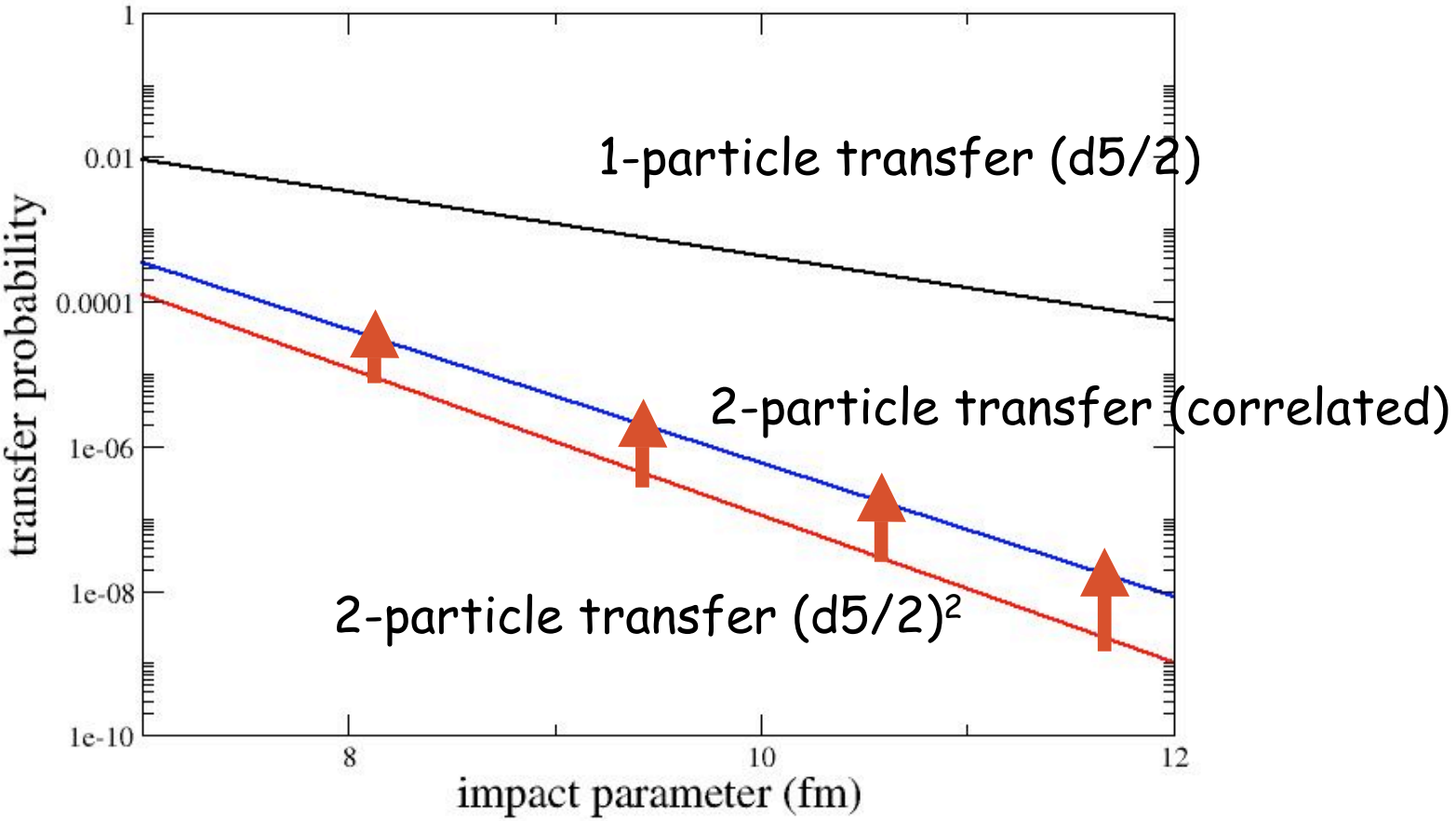
$\lambda = 3$



$$F_\lambda(r) = \int \phi_{in}\, V\, \phi_{fin}\, ds$$

dependence on the binding energy

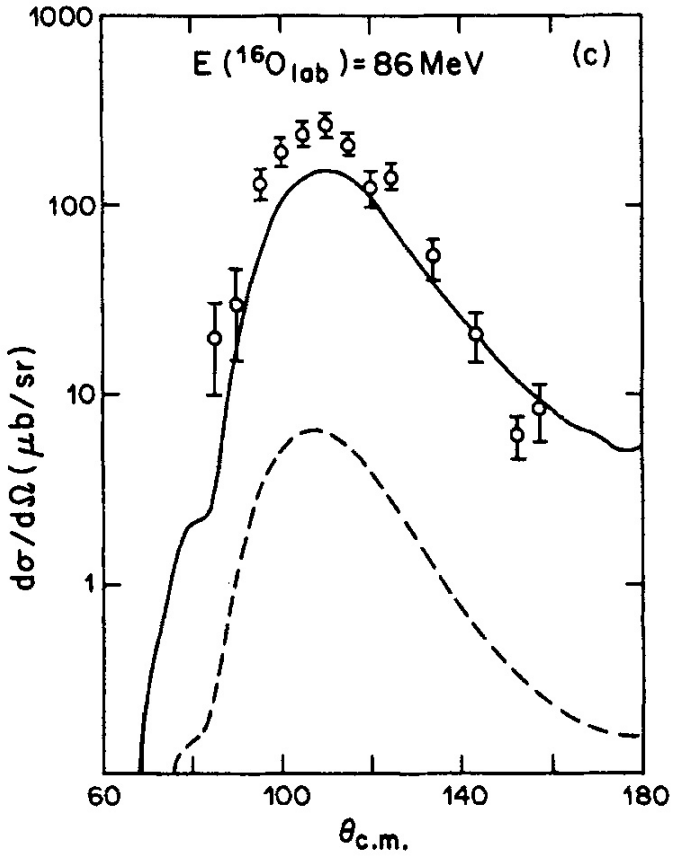
$^{208}Pb(^{16}O,^{17,18}O)^{207,206}Pb$



$^{208}Pb(^{16}O,^{18}O)^{206}Pb$ *gs*

——— two-step

- - - - - - - - - - one-step

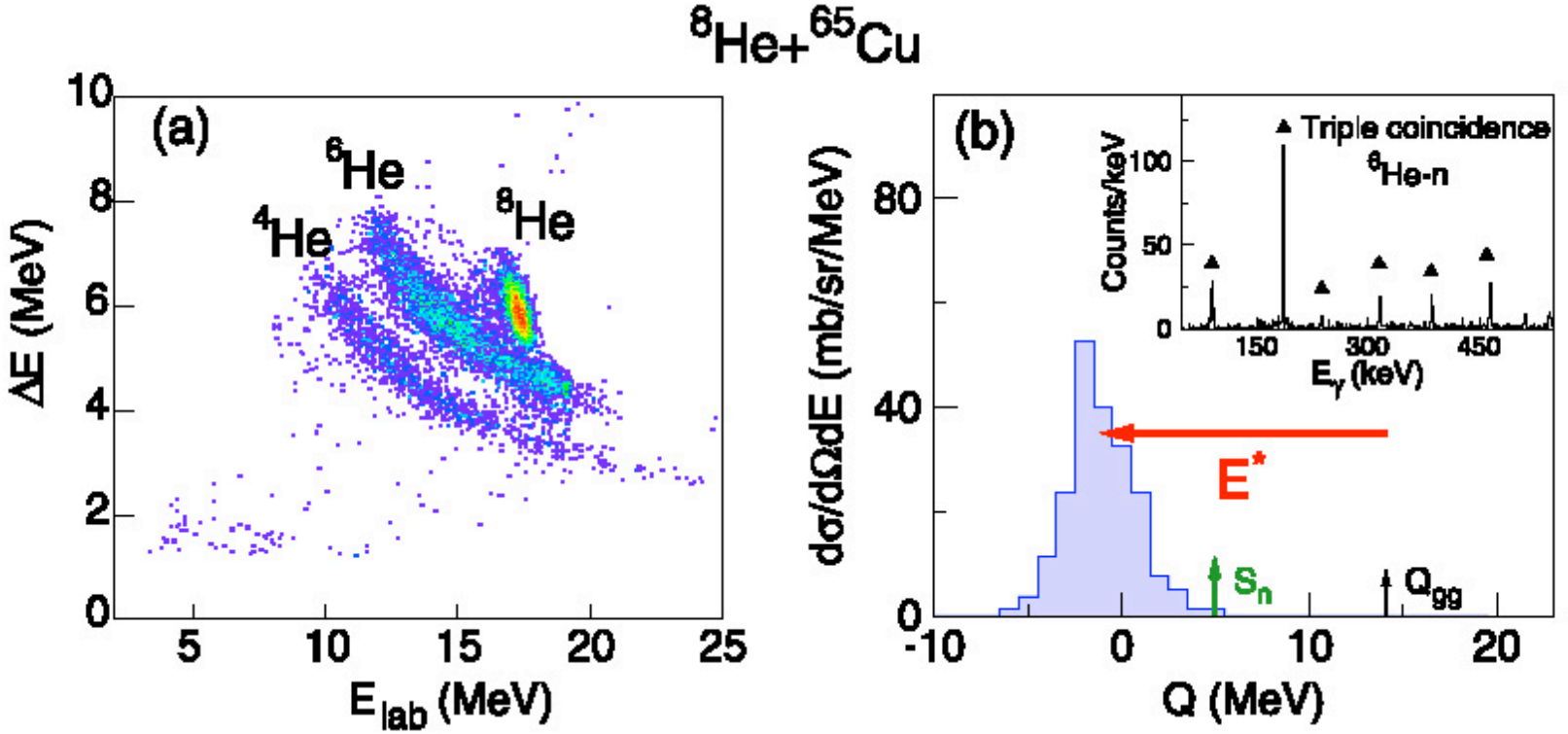


Maglione, Pollarolo, Vitturi, Broglia, Winther

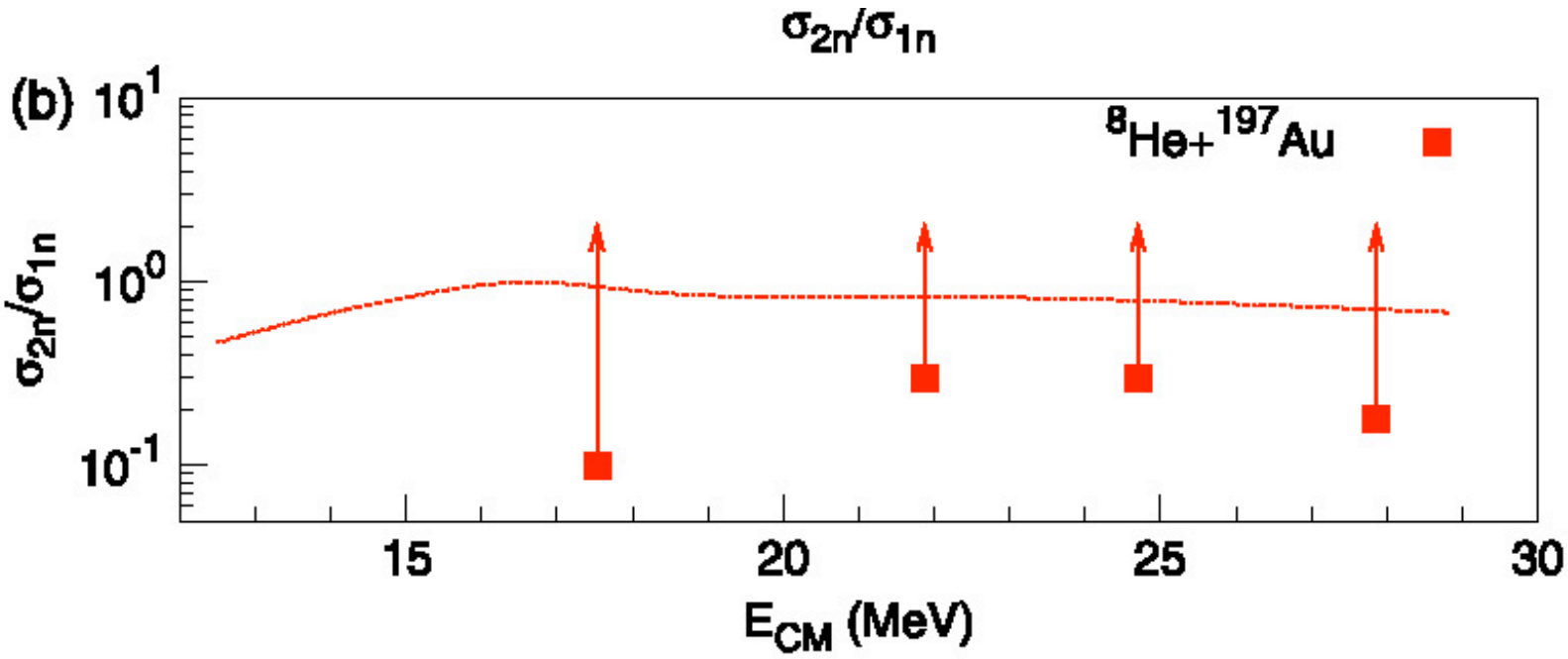
Basic problem:

how is changed the picture as we move closer or even beyond the drip lines?

Data from GANIL, Navin etal, 2011



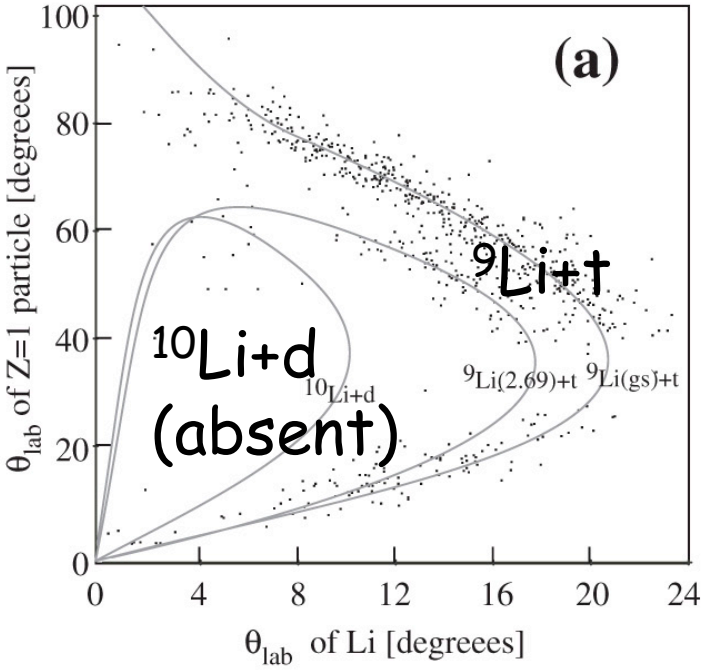
Extremely difficult to extract the fundamental $\sigma 2/\sigma 1$ ratio

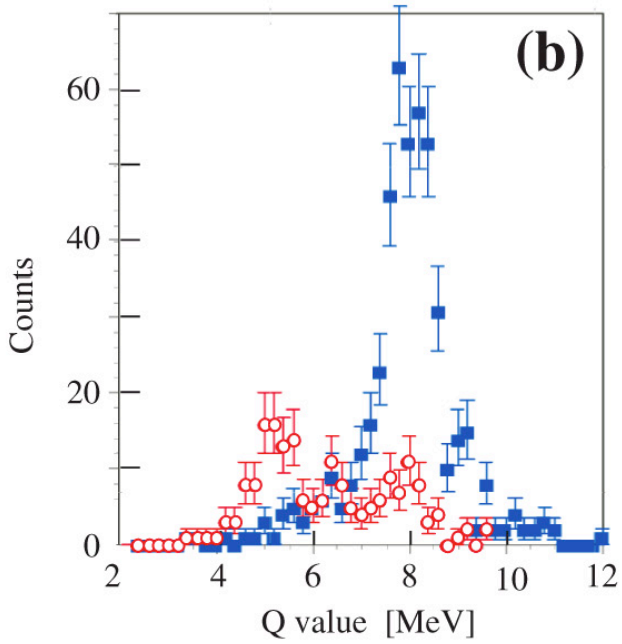


^{11}Li+p -> ^{9}Li+t

Data from ISAC-2, TRIUMF

Isao Tanihata etal



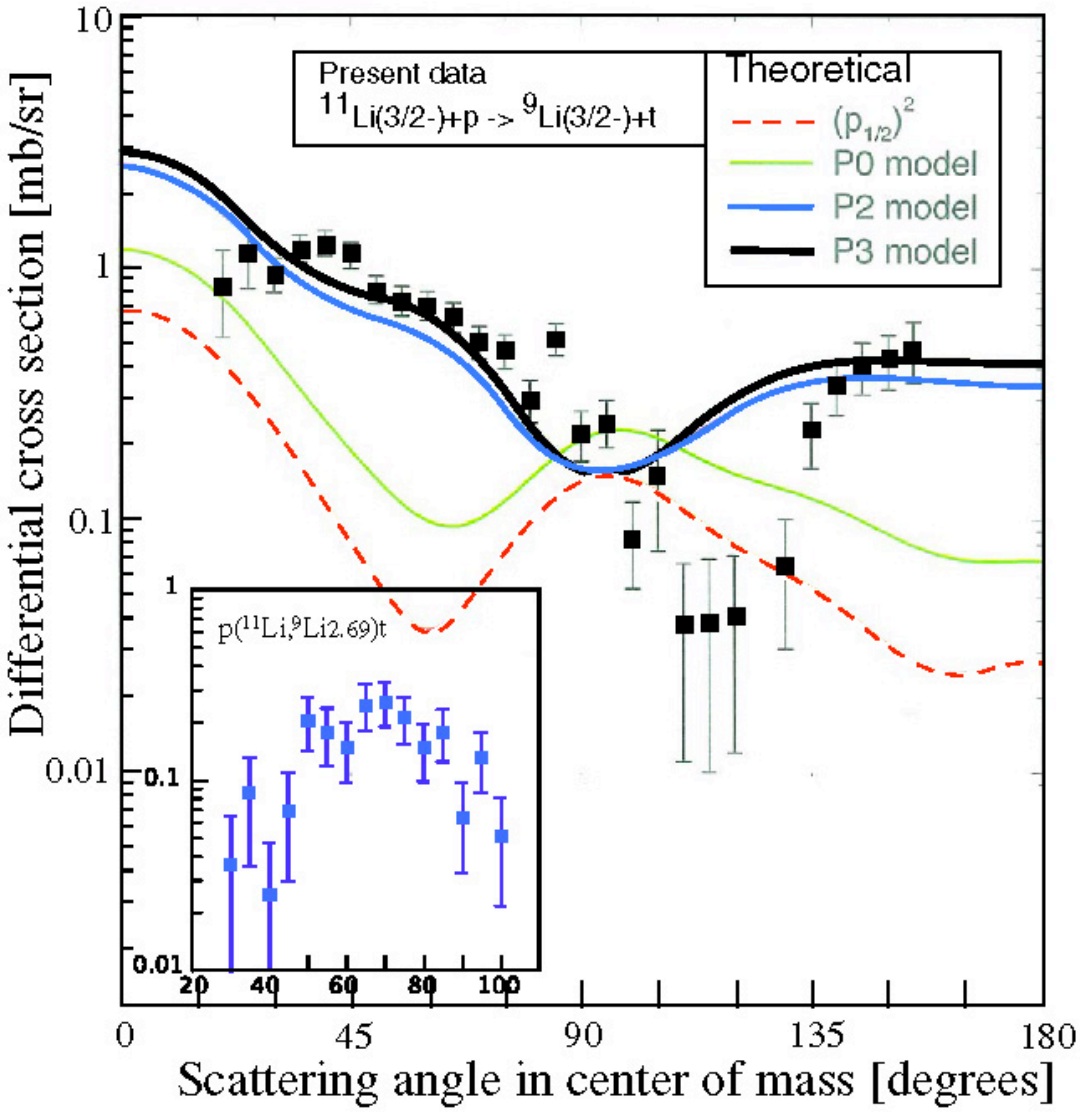


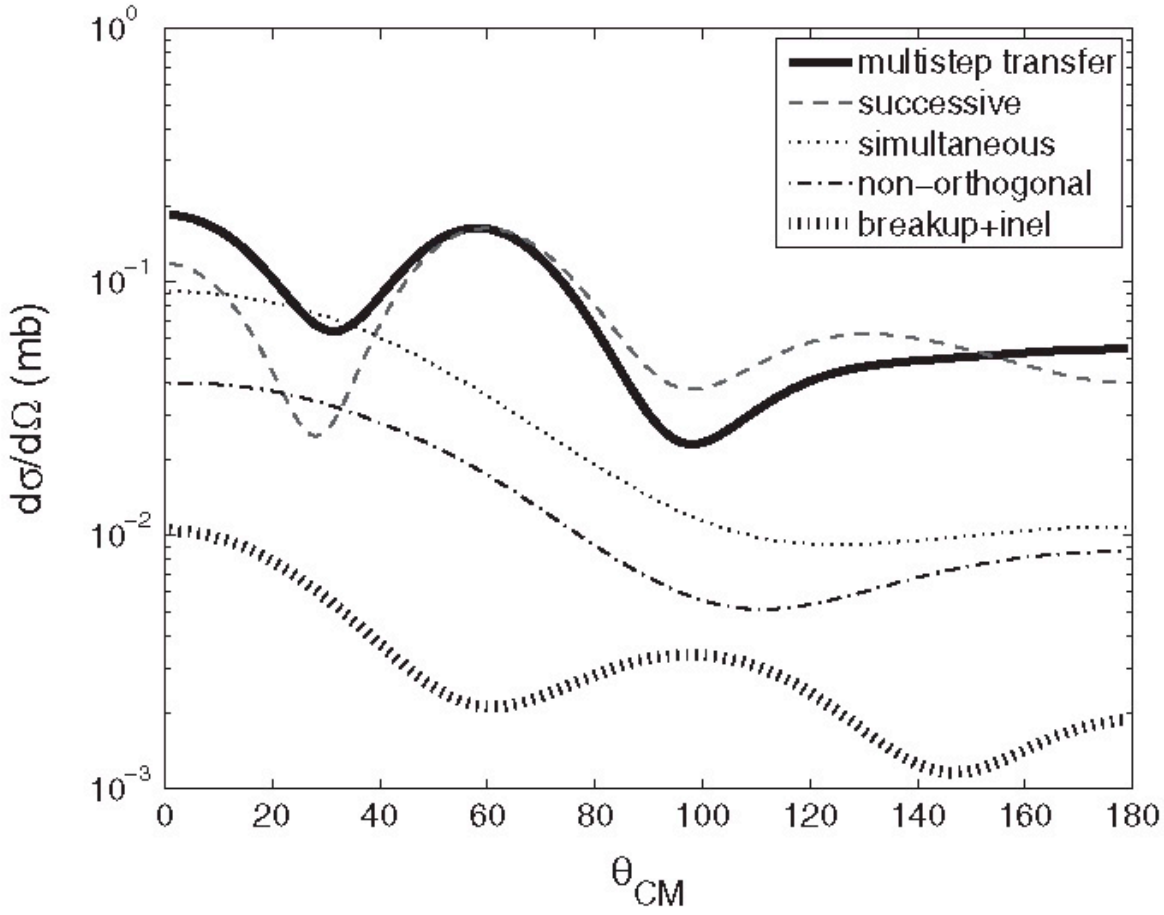
Sensitivity to the pairing function in ^{11}Li

P0: 3% of $(s_{1/2})^2$

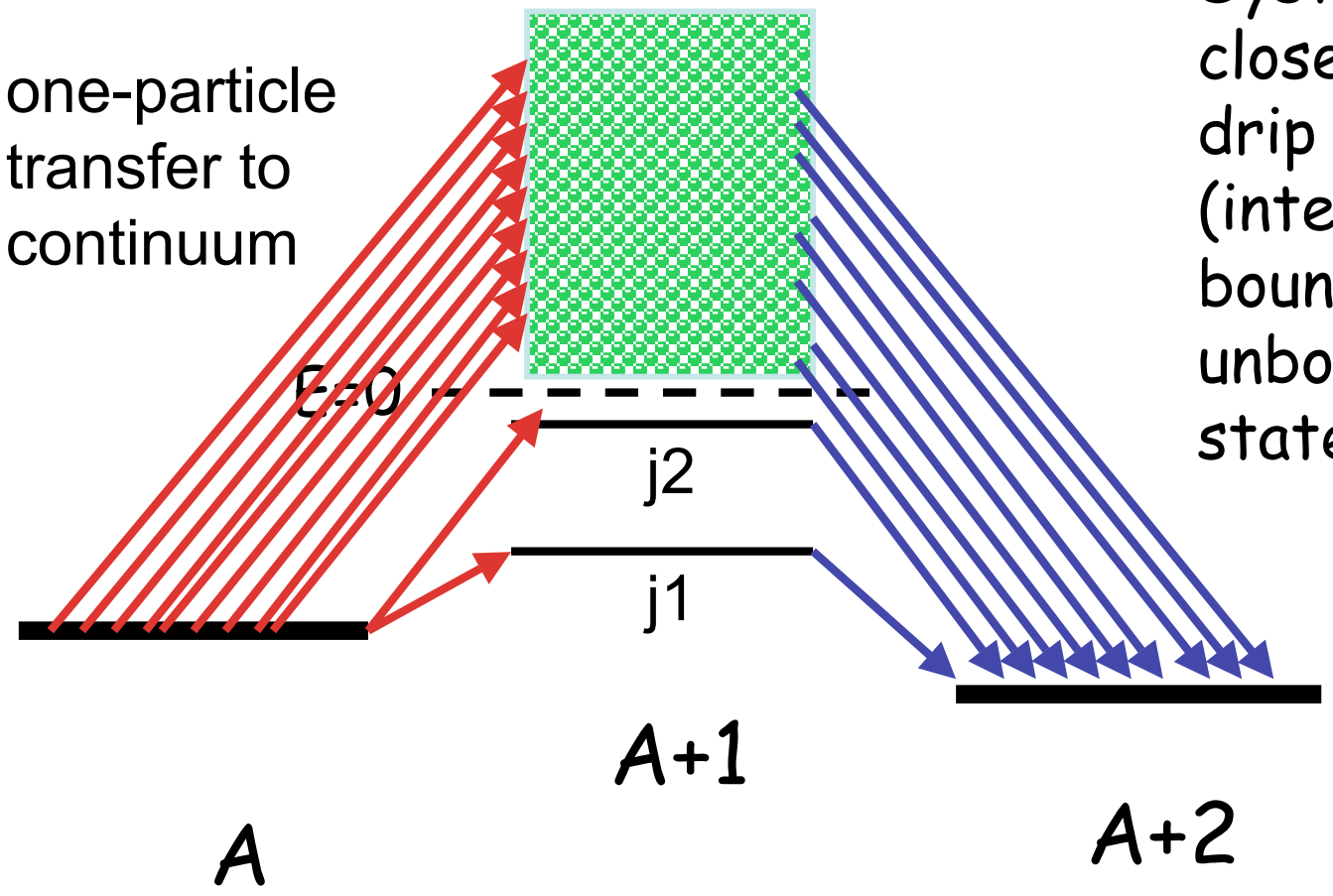
P2: 31% of $(s_{1/2})^2$

P3: 45% of $(s_{1/2})^2$





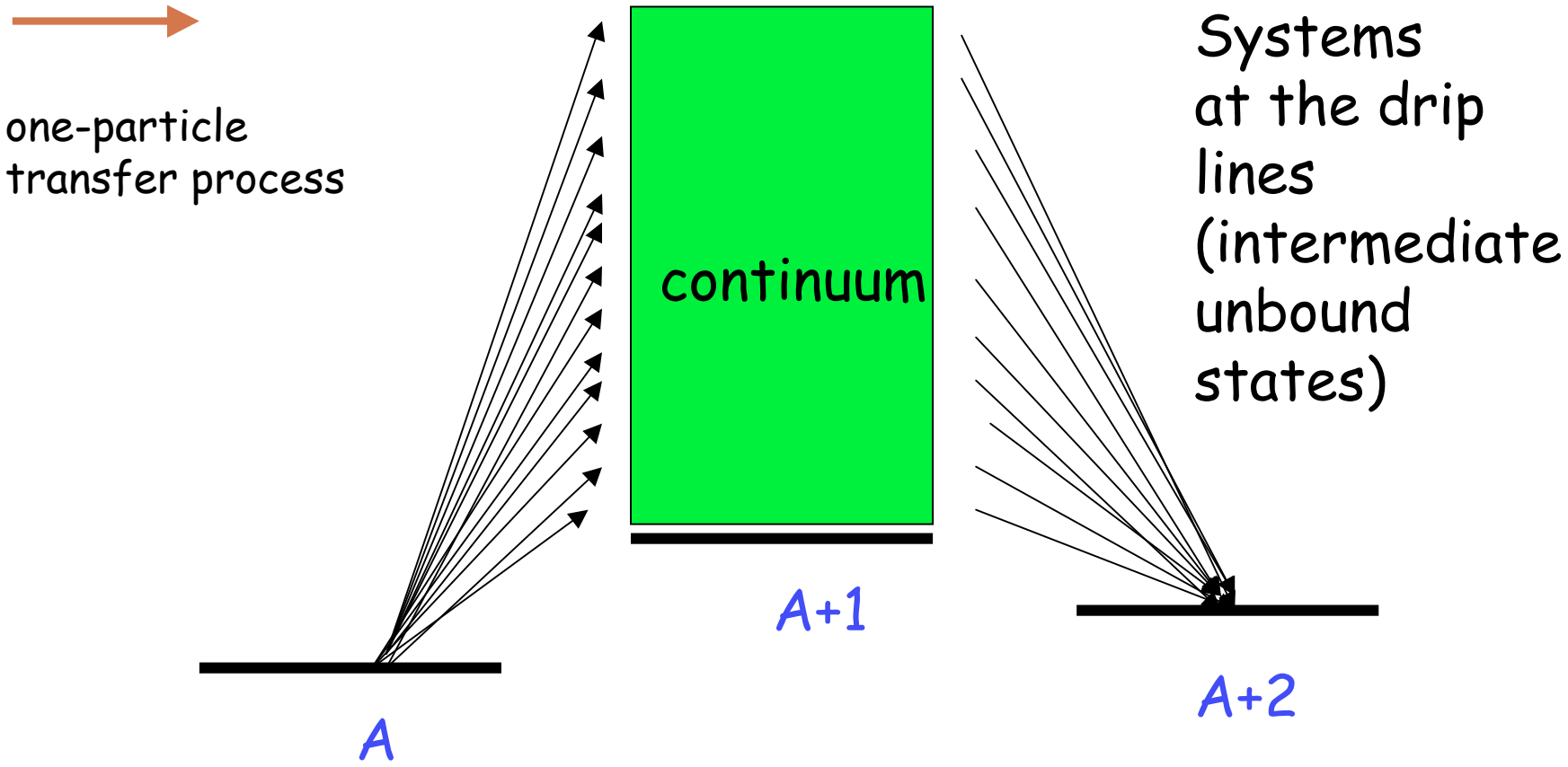
Potel etal, 2010



Systems closer to the drip lines (intermediate bound and unbound states)

Example

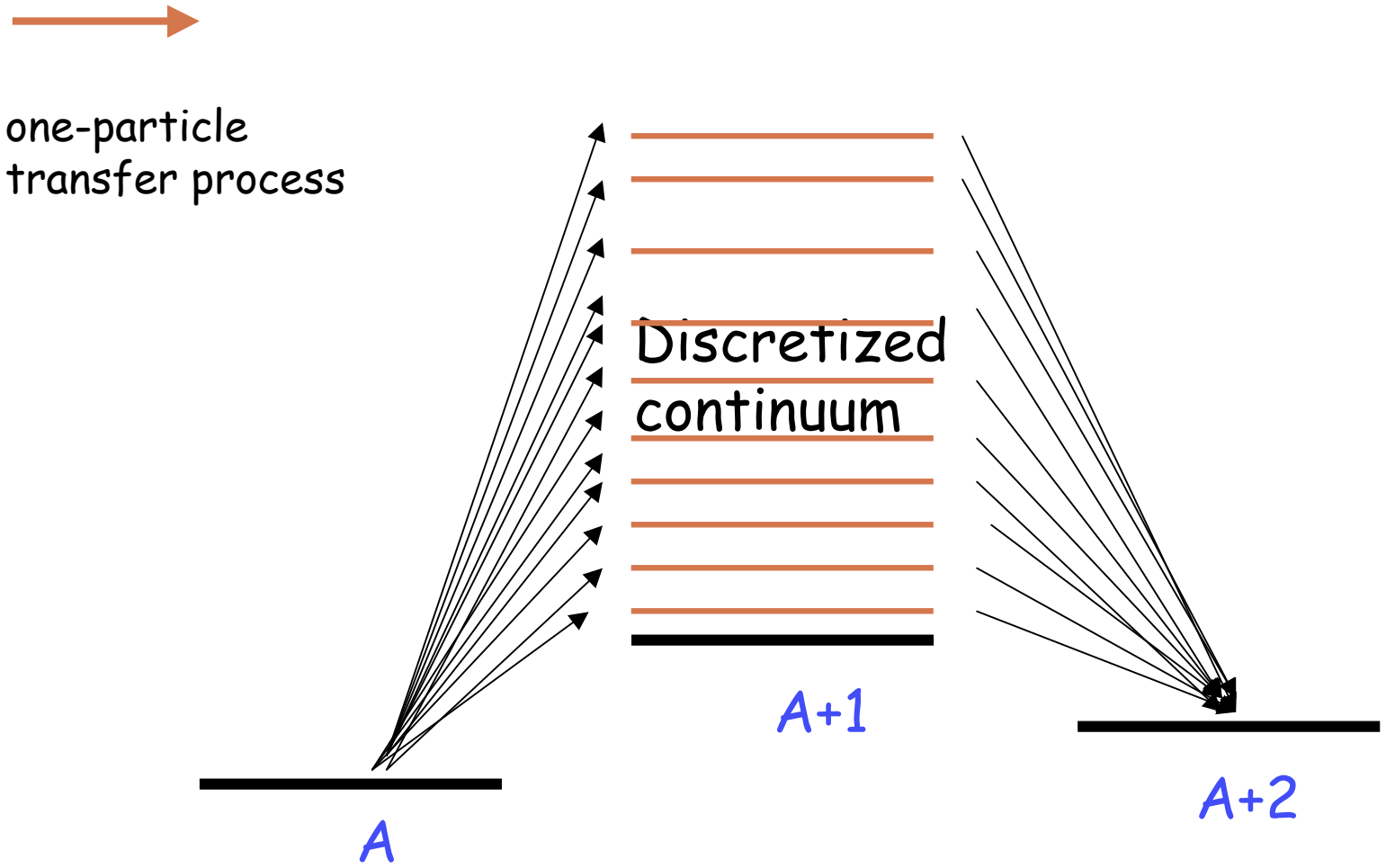
$$|A=2\rangle = \{ \Sigma_i X_i [a_i^+ a_i^+]_0 + \int dE\, X(E)\, [a^+(E) a^+(E)]_0 \} |A\rangle$$



Systems at the drip lines (intermediate unbound states)

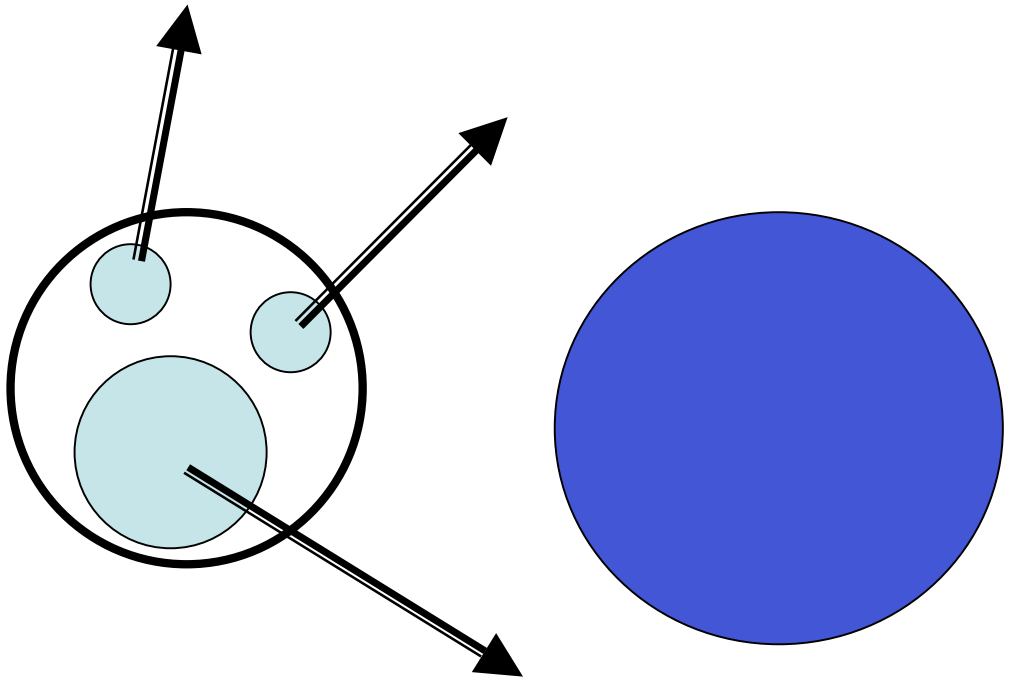
$$|A=2\rangle = \int dE\, X(E)\, [a^{+}(E)a^{+}(E)]_0\, |A\rangle$$

Two-particle trasfer will proceed mainly by constructive interference of successive transfers through the (unbound) continuum intermediate states



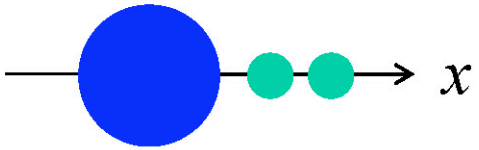
The integration over the continuum intermediate states can becomes feasible by continuum discretization:
but how many paths should we include? Thousands or few.
for example only the resonant states?

Two-particle break-up



Break-up of a two-particle halo system is a rather complex 4-body process. To make it simpler let us consider an one-dimensional case
(Hagino, Vitturi, Sagawa, Perez Bernal.
Cf also Denis Lacroix)

One-dimensional three-body model

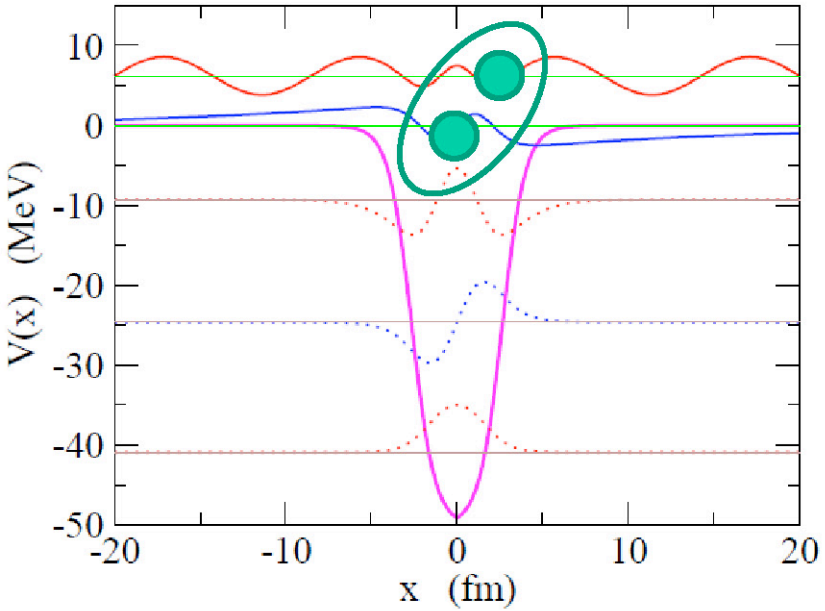


Two interacting neutrons in a one-dimensional potential well:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx_1^2} + V(x_1) - \frac{\hbar^2}{2m}\frac{d^2}{dx_2^2} + V(x_2) + v_{nn}(x_1, x_2)$$

density-dependent contact interaction:

$$v_{nn}(x, x') = -g\left(1 - \frac{1}{1 + e^{(|x|-R)/a}}\right)\delta(x - x')$$



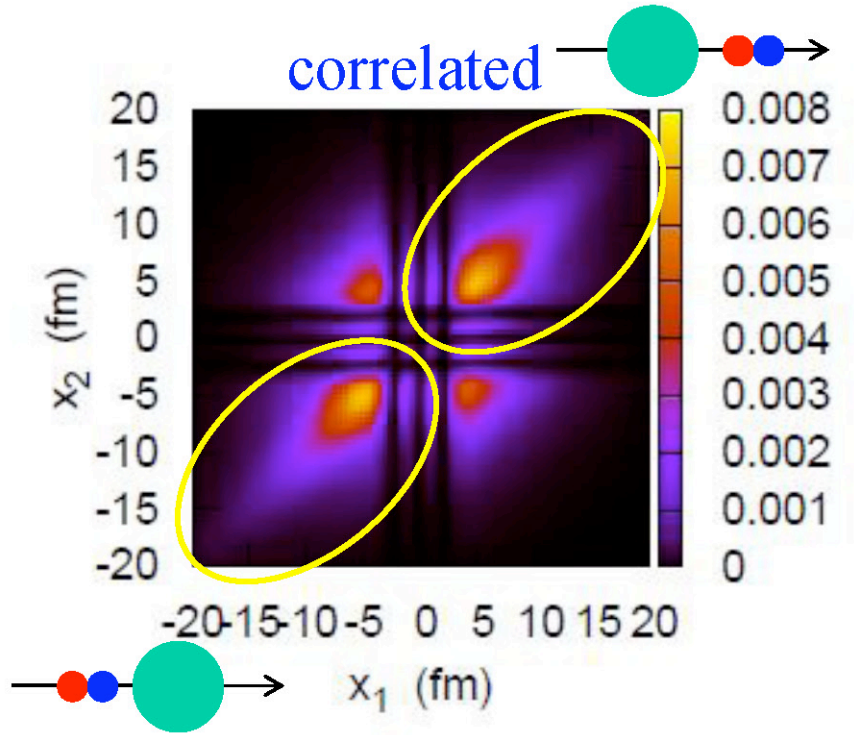
$$\Psi_{\rm gs}(x_1, x_2) = \sum_{n \le n'} \alpha_{nn'} \Psi_{nn'}(x_1, x_2)$$

$$\Psi_{nn'}(x_1, x_2) \propto \mathcal{S}[\phi_n(x_1)\phi_{n'}(x_2)] \times |S = 0\rangle$$

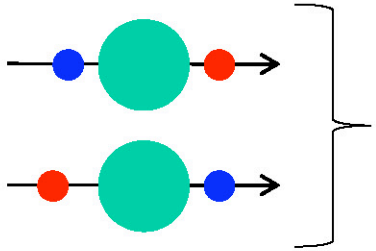
- $S = 0$ state: symmetric for the spatial part of wf
- n, n': the same parity

Ground state properties

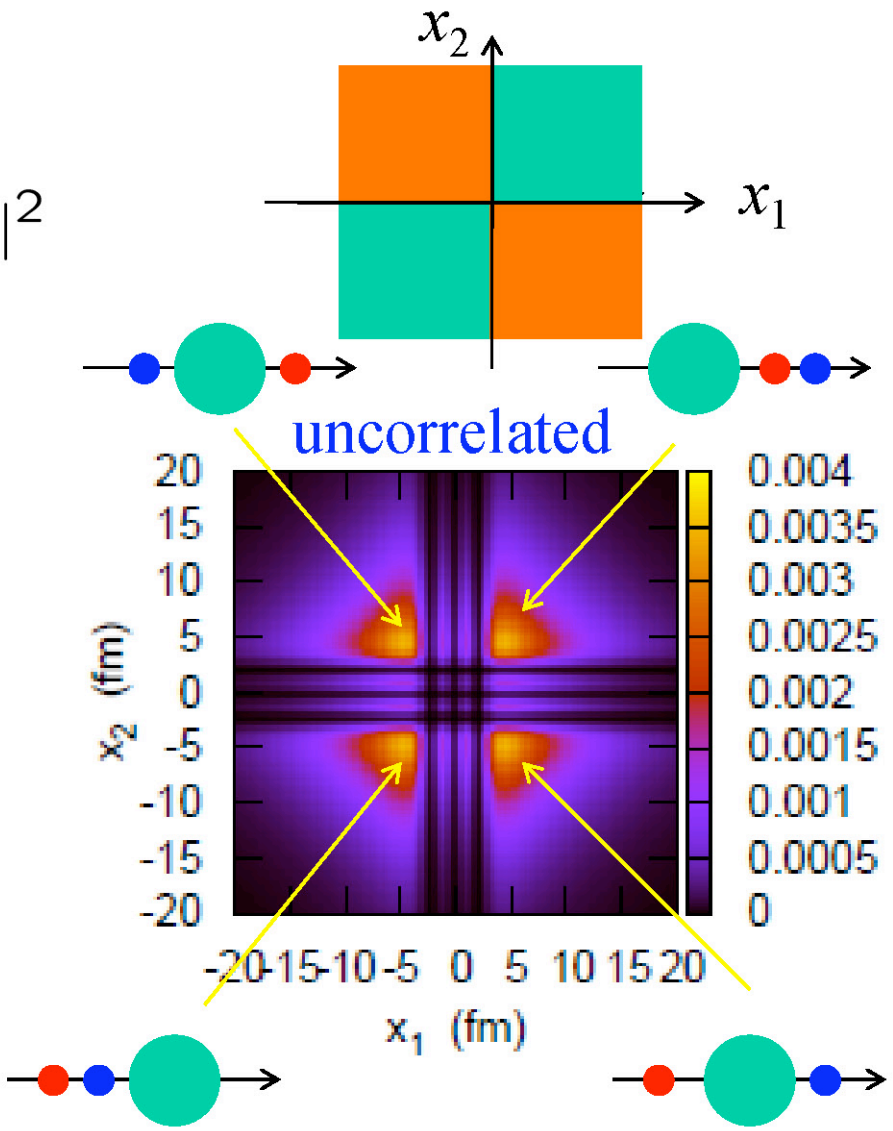
two-particle density: $|\Psi_{gs}(x_1, x_2)|^2$



dineutron correlation



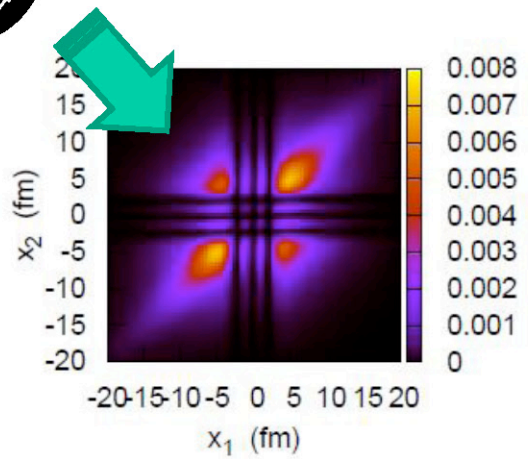
largely suppressed



four symmetric peaks

Nuclear Breakup Process

(one-body) external field



$\Psi_{gs}(x_1, x_2)$

Time-dependent two-particle Schroedinger equation:

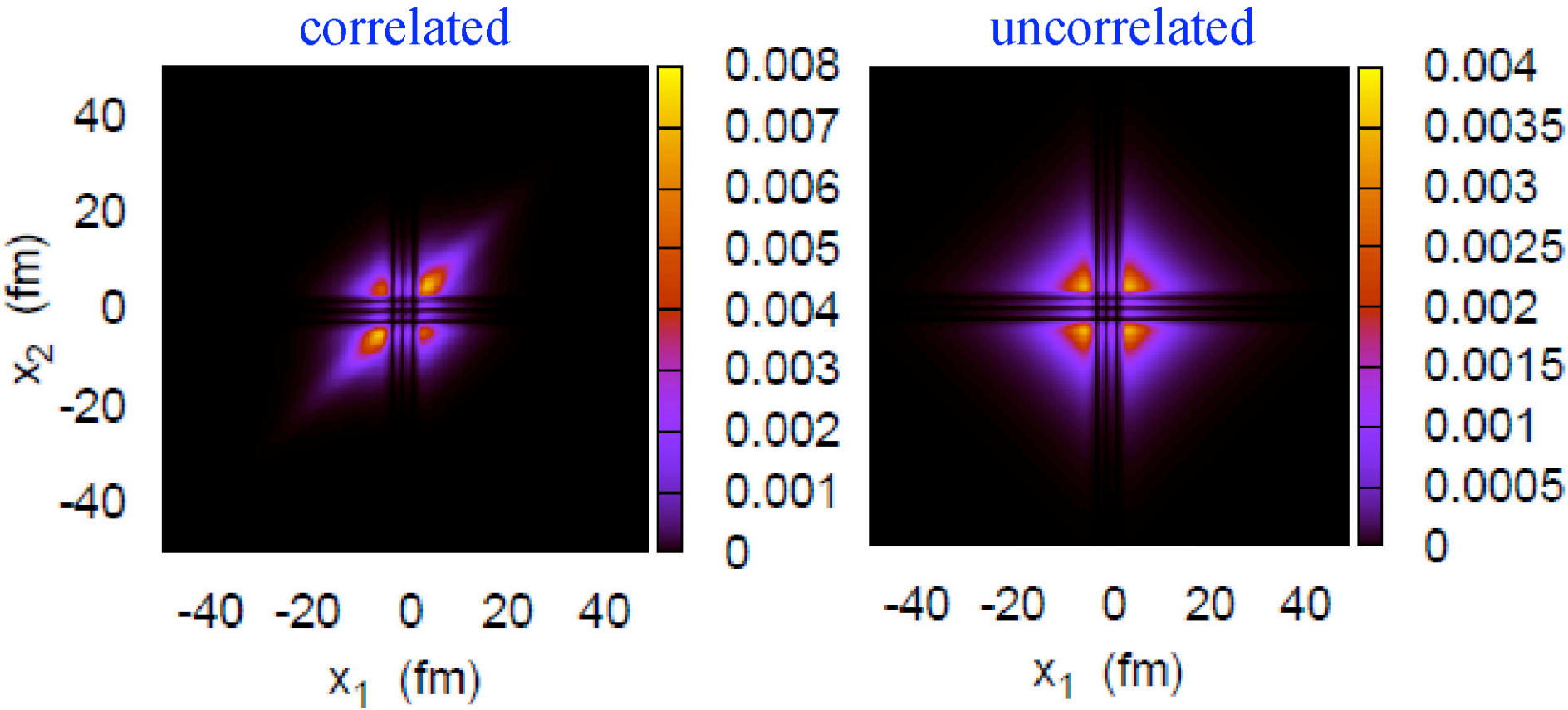
$$i\hbar\frac{\partial}{\partial t}\Psi(x_1, x_2, t) = [H + V_{ext}(x_1, x_2, t)]\Psi(x_1, x_2, t)$$

$$V_{ext}(x_1, x_2, t) = \sum_{i=1,2} V_c\, e^{-t^2/2\sigma_t^2} e^{-(x_i - x_0)^2/2\sigma_x^2}$$

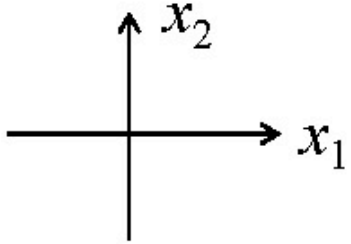
V_c = 3 MeV, σ_t = 2.1 hbar/MeV, x_0 = 0

The perturbing interaction (that produces the break-up) is a one-body field (i.e. acting individually on each of the two particles). The enhanced two-particle break-up originates from the correlation in the two-particle wave function, and not from the reaction mechanism

two-particle density at $t = t_{ini}$

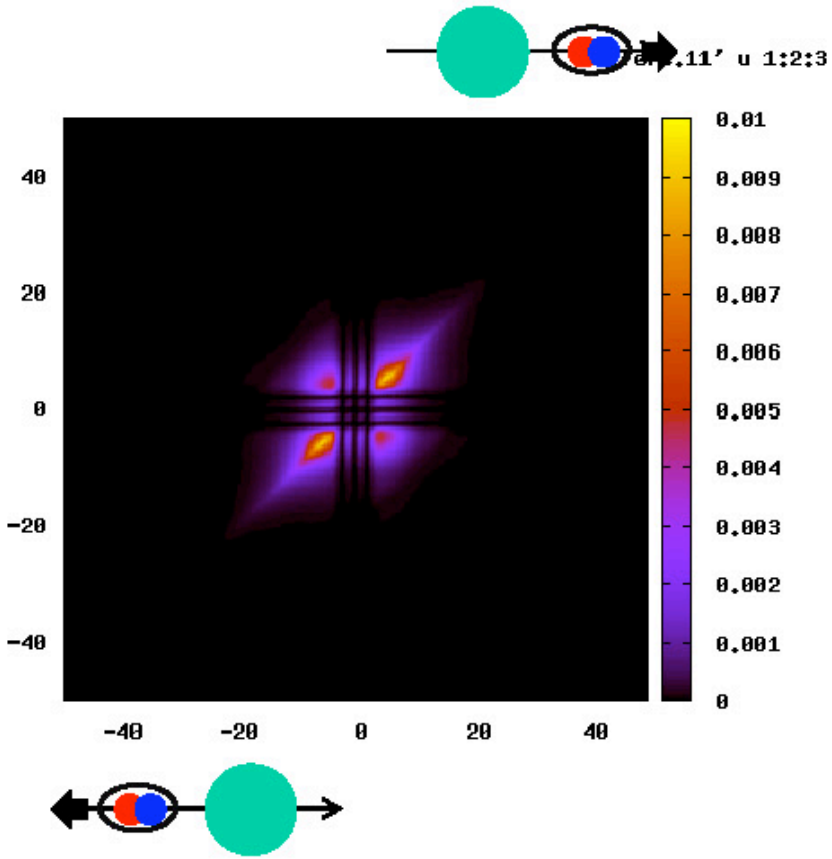


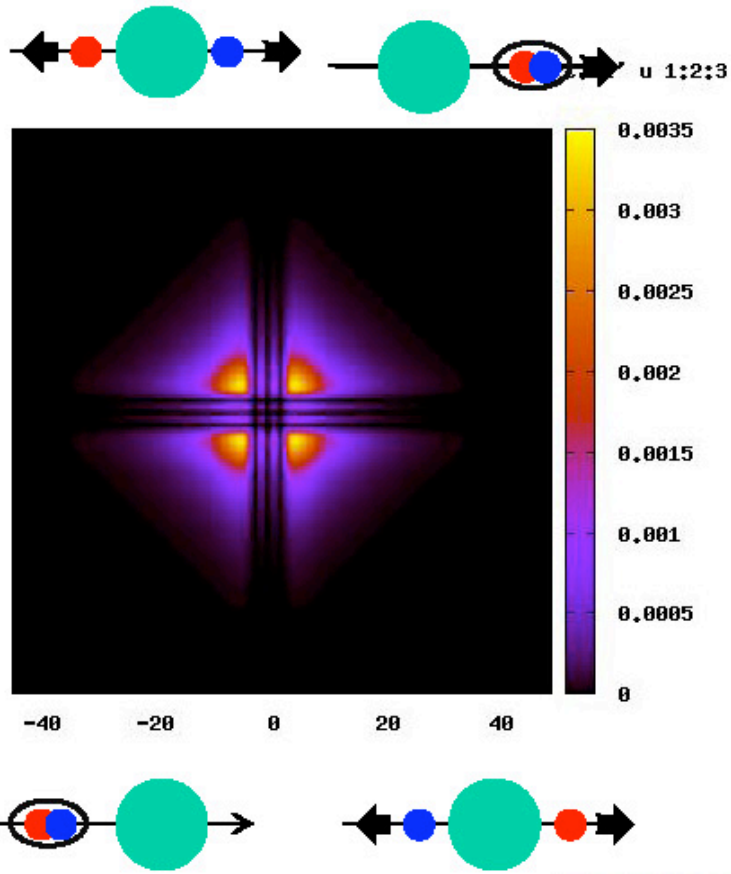
time evolution



correlated

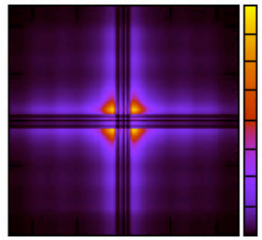
uncorrelated

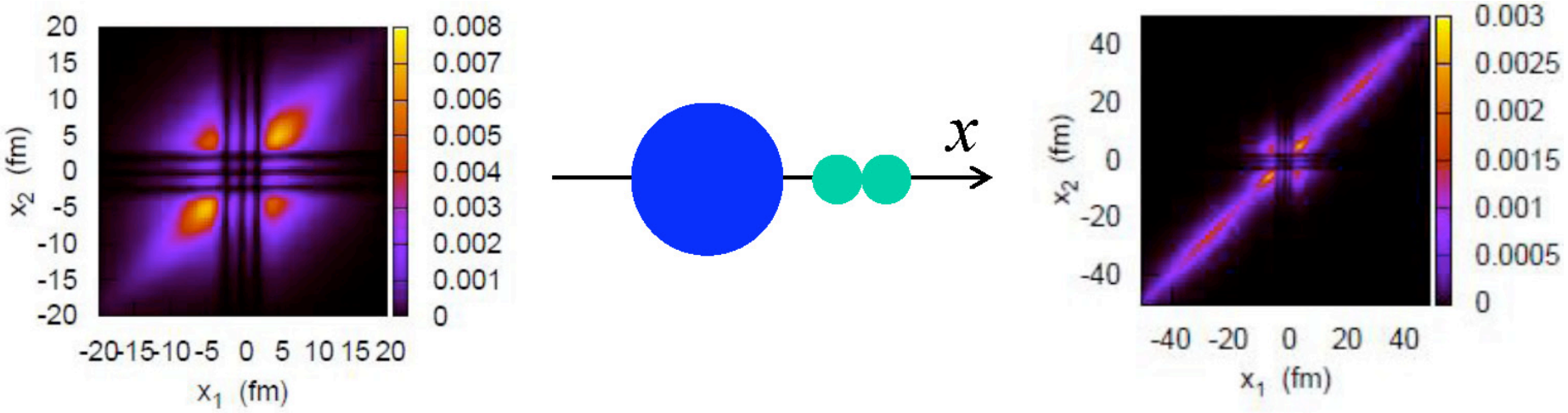




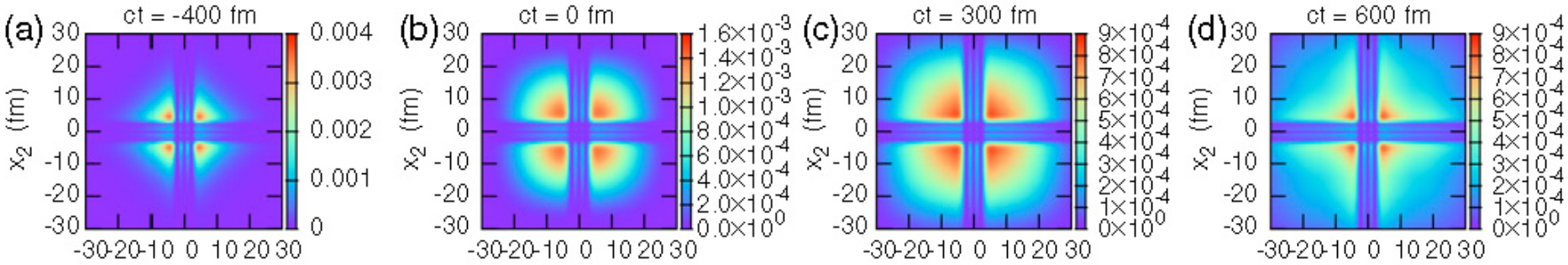
"dineutron emission"

large (bc) component

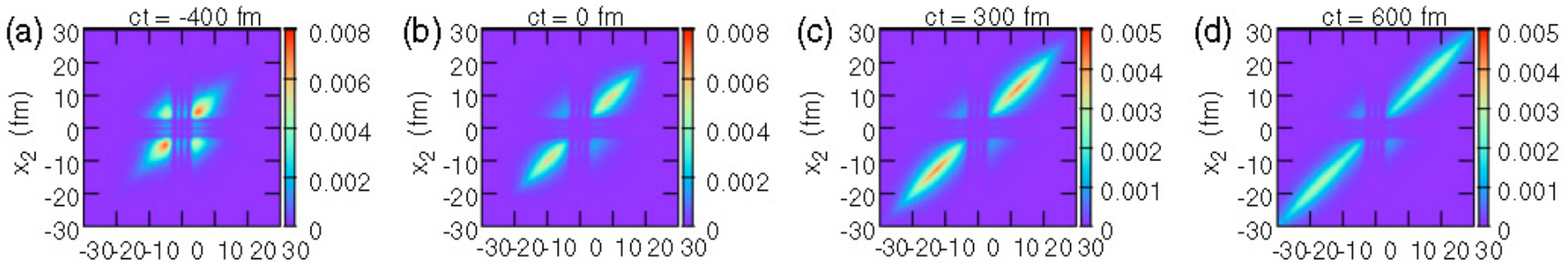


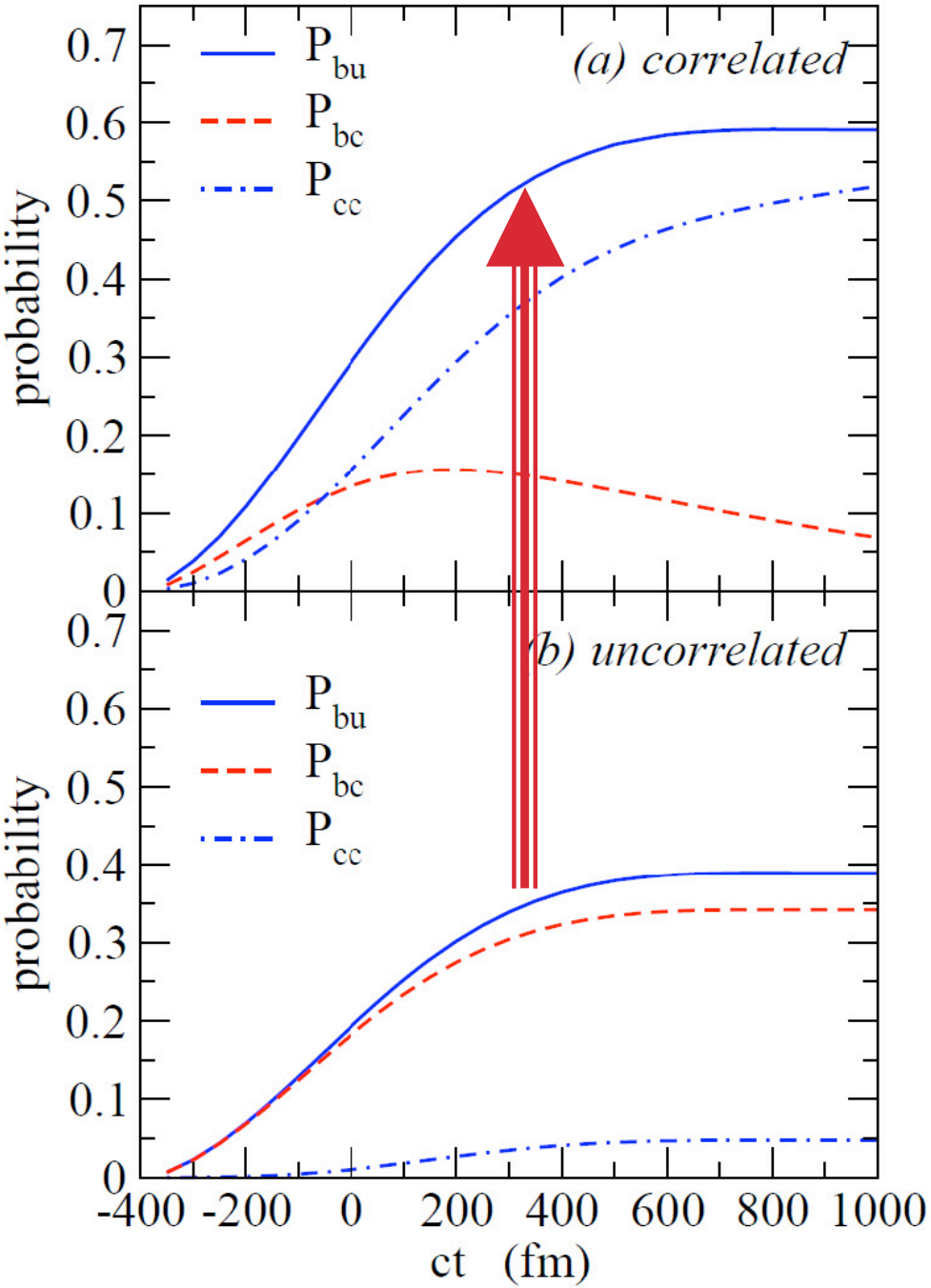

20
15
10
5
0
-5
-10
-15
-20
x_2 (fm)
-20 -15 -10 -5 0 5 10 15 20
x_1 (fm)
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
x
40
20
0
-20
-40
x_2 (fm)
-40 -20 0 20 40
x_1 (fm)
0.003
0.0025
0.002
0.0015
0.001
0.0005
0

Time evolution (uncorrelated case)



Time evolution (correlated case)





- Pairing: enhances the breakup
- Correlated: (cc) process
- Uncorrelated: (bc) process

P_{cc}: 2 neutron breakup
P_{bc}: 1 neutron breakup