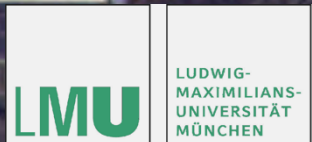


The Density Dependence of the Symmetry Energy from Heavy Ion Collisions

Hermann Wolter

Ludwig-Maximilians-Universität München (LMU)



Int. Symposium „Advances of Nuclear Many-Body Theory“ in honor of Peter Ring's 70th birthday
Primosten, Croatia, June 7-10, 2011

Peter Ring and I have been together in Munich for many years, even though at competing universities.

Many thanks for sympathy and encouragement

Some recent pictures.



Dubna 2005



Beijing 2005



Beijing 2005

Points to discuss

- I. Density (and momentum) dependence of the nuclear Symmetry Energy (SE); uncertainties, importance, astrophysics
- II. Situation from many-body theory, particularly in the RMF theory
- III. Study of Symmetry Energy in heavy ion collisions, choice of asymmetry and density regime.
theoretical tool: transport theory
- IV. Discuss specific examples:
 - < ρ_0 , isospin transport, Fermi energies
 - > ρ_0 momentum distributions („flow“), meson production
- V: What do we know?

collaborators:

Massimo Di Toro, Maria Colonna, V. Greco, Lab. Naz. del Sud, Catania

Theodoros Gaitanos, Univ. of Giessen

Vaia Prassa, Georgios Lalazissis, Aristotle Univ. Thessaloniki

Malgorzata Zielinska-Pfabe, Smith Col., USA

Equation-of-State and Symmetry Energy

BW mass formula

$$E(A, Z) / A = a_v - a_s A^{-1/3} - a_c Z(Z-1)A^{-4/3} - a_s (N-Z)^2 / (N+Z)^2 + \delta_{pair}$$

density-asymmetry dep. of nucl.matt.

$$E(\rho_B, \delta) / A = E_{nm}(\rho_B) + E_{sym}(\rho_B) \delta^2 + O(\delta^4) + \dots$$

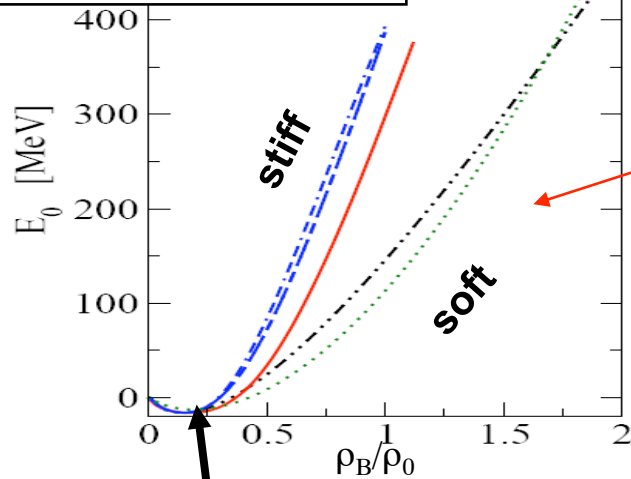
$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$a_s = E_{sym}(\rho_0)$$

rather unknown

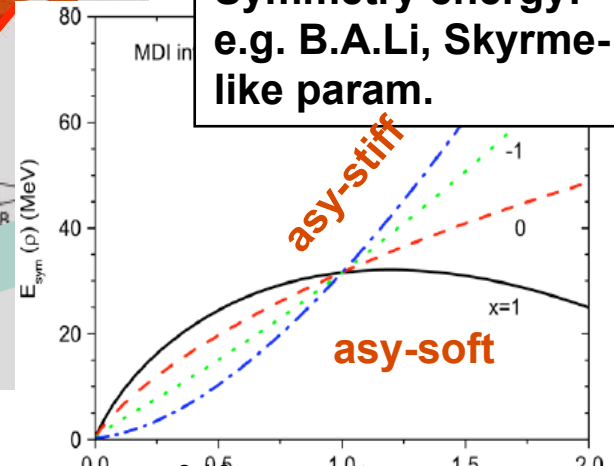
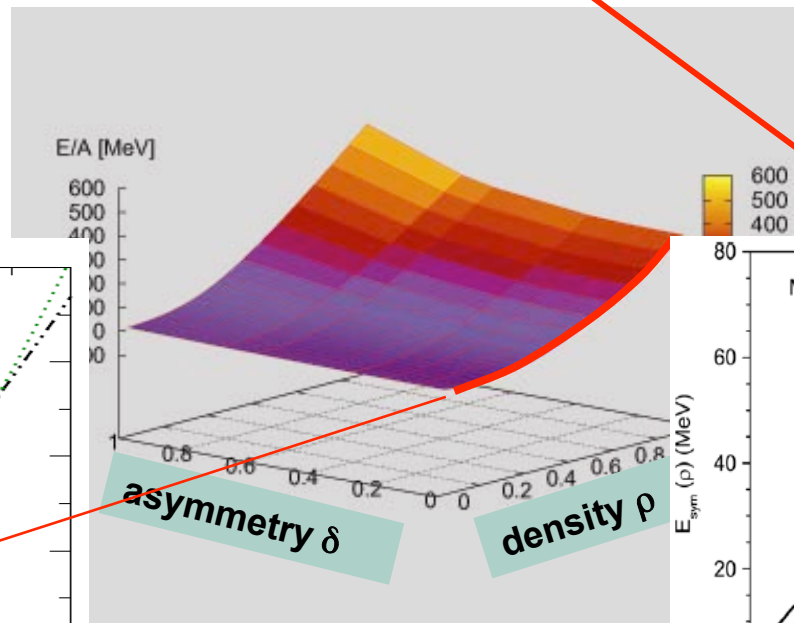
Symmetry energy: e.g. B.A.Li, Skyrme-like param.

EOS of symmetric nuclear matter



saturation point

Fairly well fixed! Soft!



$$E_{sym}(\rho) = \frac{1}{3} \varepsilon_F (\rho / \rho_0)^{2/3} + E_{sym}^{pot}(\rho)$$

\$E_{sym}^{pot}(\rho)\$ often parametrized as $C(\rho/\rho_0)^\gamma$ useful around \$\rho=\rho_0\$. Expansion around \$\rho_0\$:

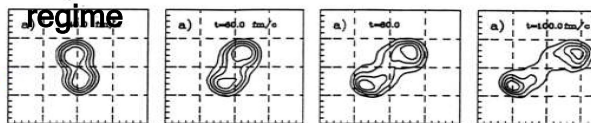
$$E_{sym}(\rho) = J + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$

Importance of Nuclear Symmetry Energy

$$E(\rho_B, I) / A = E(\rho_B) + E_{sym}(\rho_B) I^2 + O(I^4) + \dots$$

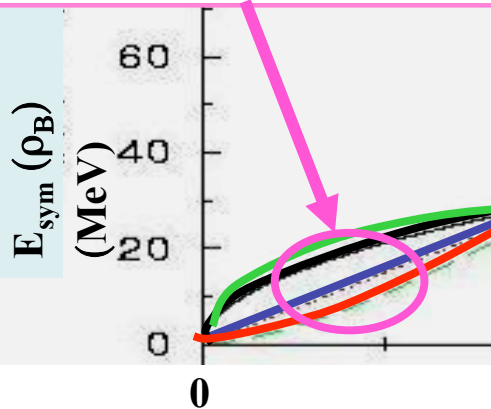
(supernovae, neutron stars)

heavy ion collisions in the Fermi energy regime

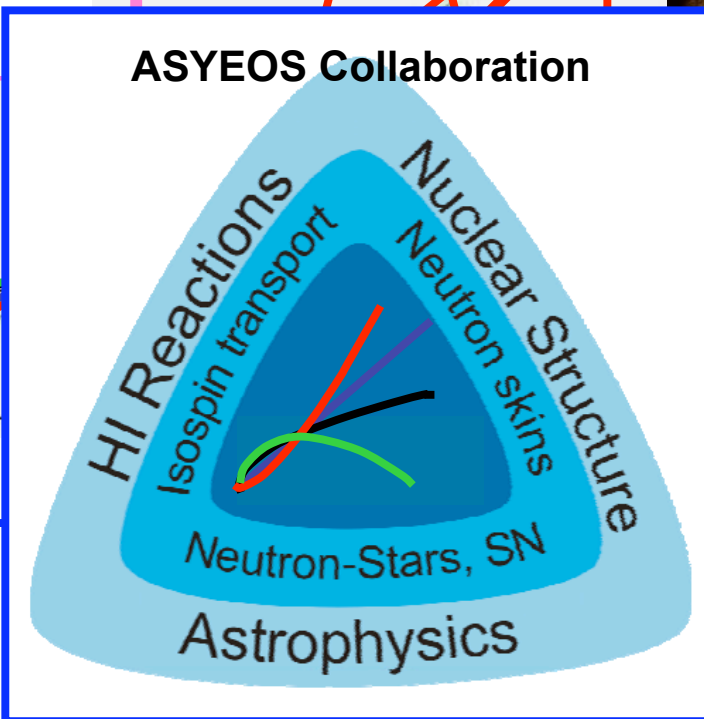


$$I = \frac{N - Z}{N + Z}$$

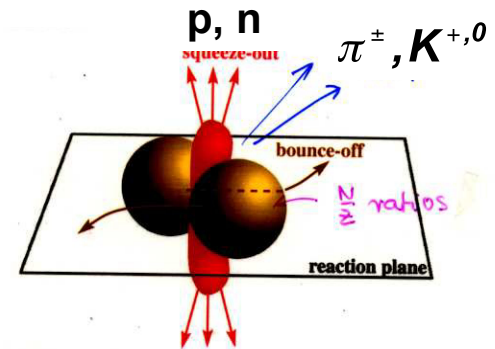
Isospin Transport properties, (Multi-)fragmentation (diffusion, fractionation, migration)



ASYEOS Collaboration

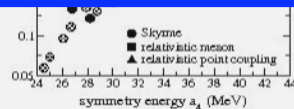


I. Heavy ion collisions



Nuclear structure (neutron skin thickness, Pygmy DR, IAS)

Slope of Symm Energy



The Nuclear Symmetry Energy in different „realistic“ models

The EOS of symmetric and pure neutron matter in different many-body approaches

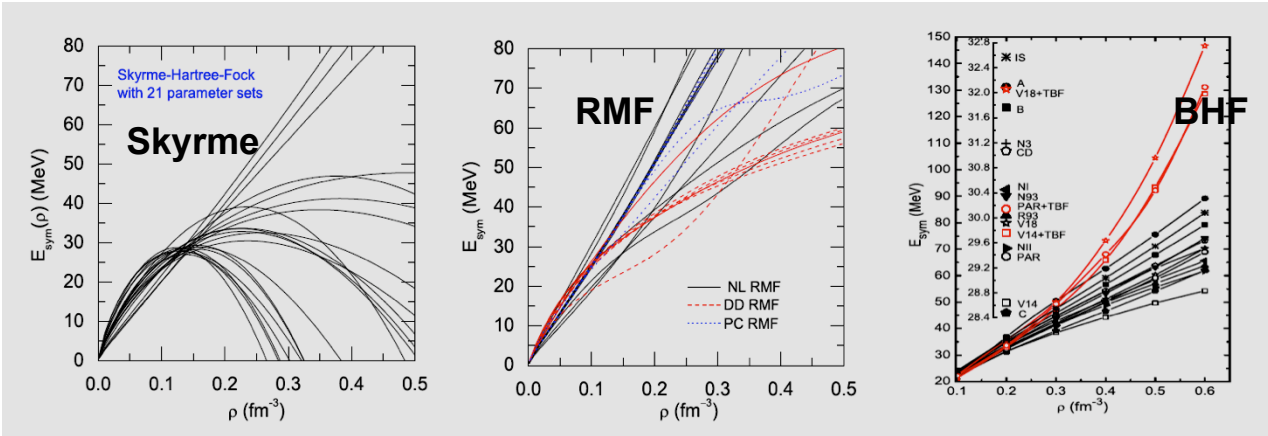
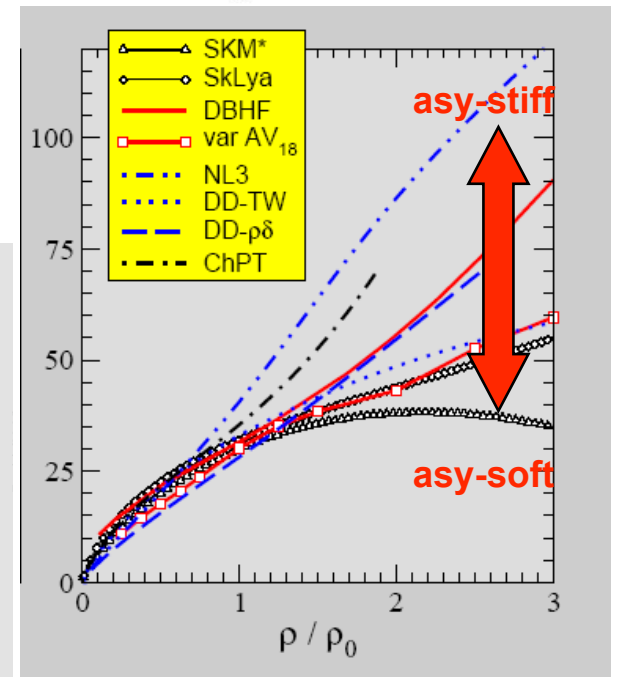
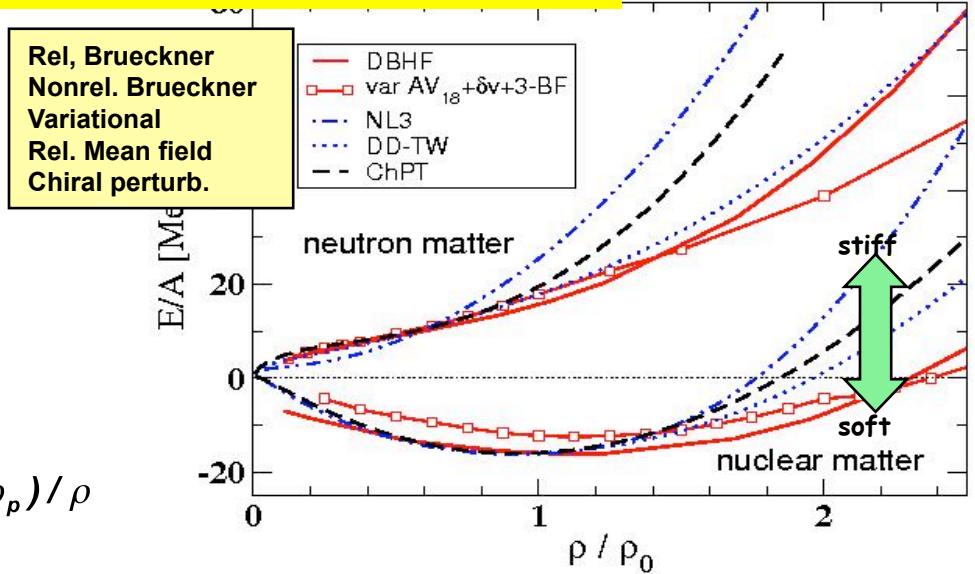
C. Fuchs, H.H. Wolter, EPJA 30(2006)5,(WCI book)

The symmetry energy as the difference between symmetric and neutron matter:

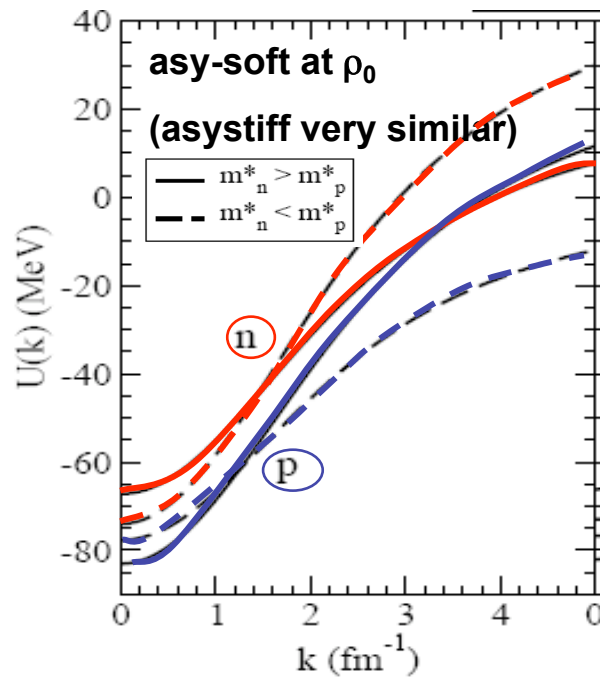
$$E(\rho, \delta) = E(\rho) + E_{sym}(\rho)\delta^2 + \dots; \quad \delta = (\rho_n - \rho_p) / \rho$$

$$E_{sym} = E_{neutr.matt} - E_{nucl.matt}$$

many more in: B.A. Li et al., Phys. Rep. 464 (2008)



Momentum Dependence of the Symmetry Energy



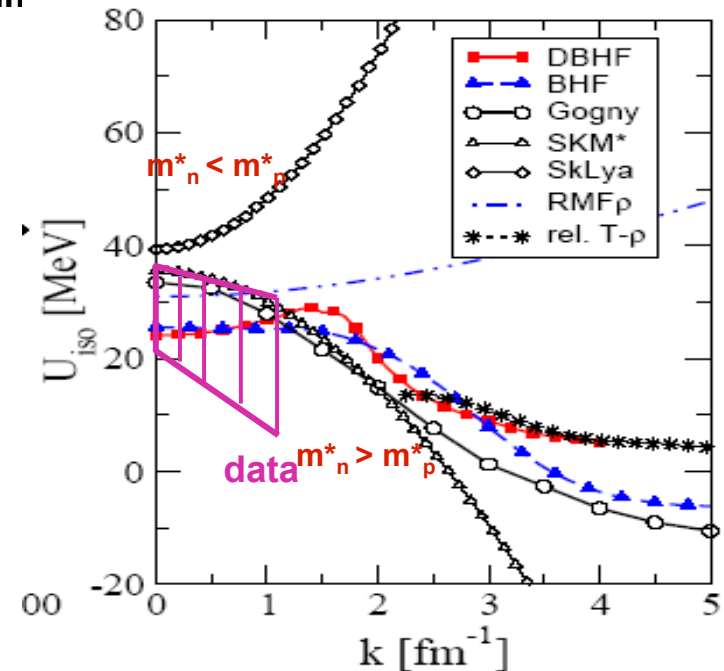
$$\frac{m_q^*}{m} = \left[1 + \frac{m}{\hbar^2 k} \frac{\partial U_q}{\partial k} \right]^{-1}$$

Different proton/
neutron effective
masses

-> crossing around
Fermi momentum

Isvector (Lane)
potential:
momentum
dependence

$$U_{\text{Lane}}(k) = \frac{1}{2I} (U_{\text{neutr}} - U_{\text{prot}})$$



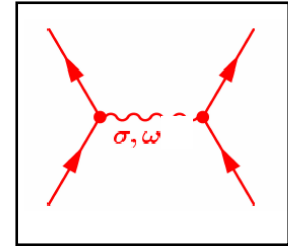
Why is symmetry energy so uncertain??

-> In-medium ρ mass, and short range tensor correlations (B.A. Li, PRC81 (2010));

-> effective mass scaling (Dong, Kuo, Machleidt, arxiv 1101.1910)

Hadronic field theory → Quantenhydrodynamics (QHD)

$\sigma\omega$ -model: however, not sufficient for nuclear matter properties:
 extensions necessary; many choices



$$L = \bar{\Psi} \left[i\gamma_\mu \left(\partial^\mu + ig_\omega \omega^\mu \right) - \left(m - g_\sigma \sigma \right) \right] \Psi + L^{mes}$$

isovector mesons: symmetry energy

V. Greco, et al., PLB562(03)215

Full Lorentz structure:

	isoscalar	isovector
scalar	σ	δ
vector	ω	ρ

non-linear meson self-interactions

Boguta, Bodmer, NPA292(77)413

$$L_\sigma^{mes} = \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) - \frac{b_3}{3} \sigma^3 - \frac{b_4}{4} \sigma^4$$

density dependent coupling vertices

Fuchs, Lenske, Wolter, PRC52(95)3043

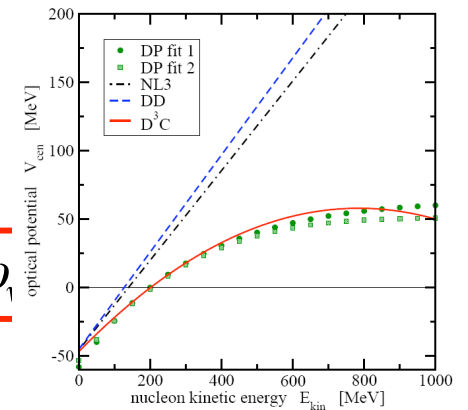
$$\Gamma_\omega(\hat{\rho})$$

$$\Gamma_\rho(\hat{\rho})$$

density dependent derivative coupling (D3C)

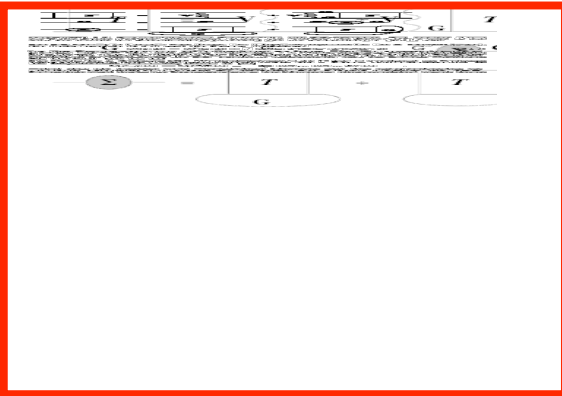
Type1, PRC71(05)064301

$$\Gamma_\mu = \gamma^\nu (g_{\nu\mu} + Y(\omega_\nu))$$



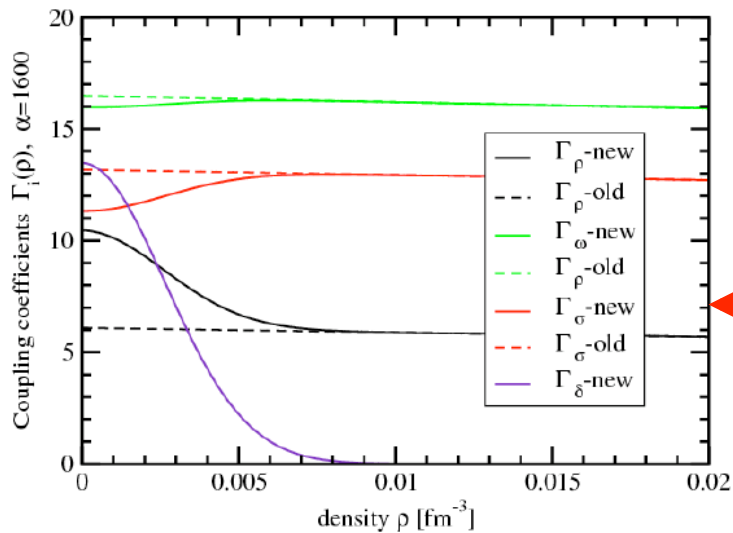
Justification for contribution of δ -meson: $a_0(980, S=1, I=1)$ very heavy!

Dirac-Brueckner (DB)

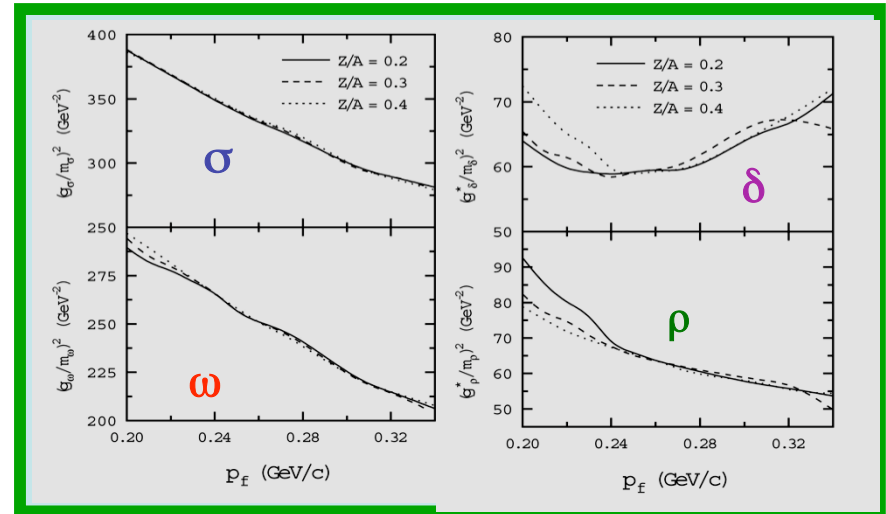


Decomposition of DB self energy

$$\Sigma(p) = \Sigma^s(p) - \gamma^0 \Sigma^0(p) + \vec{\gamma} \cdot \vec{p} \Sigma^v.$$



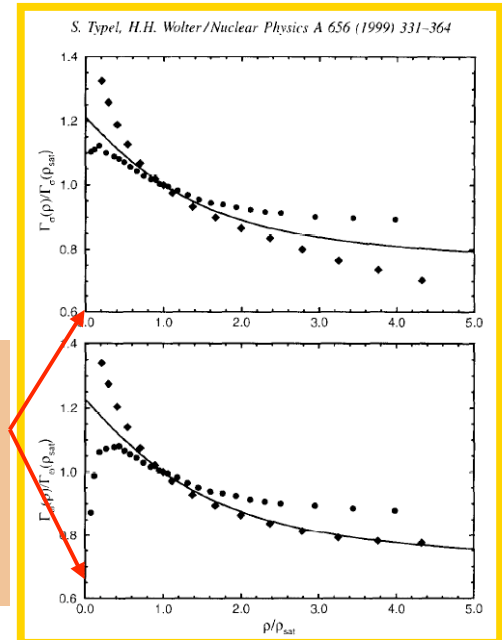
deJong, Lenske, PRC58(98)890



empirical fit of coupling coefficients (solid line)

Typel, Wolter NPA656(99)

Additional constraint at $\rho=0$ from free NN scattering (Voskresenskaya, Typel)
 δ meson necessary!



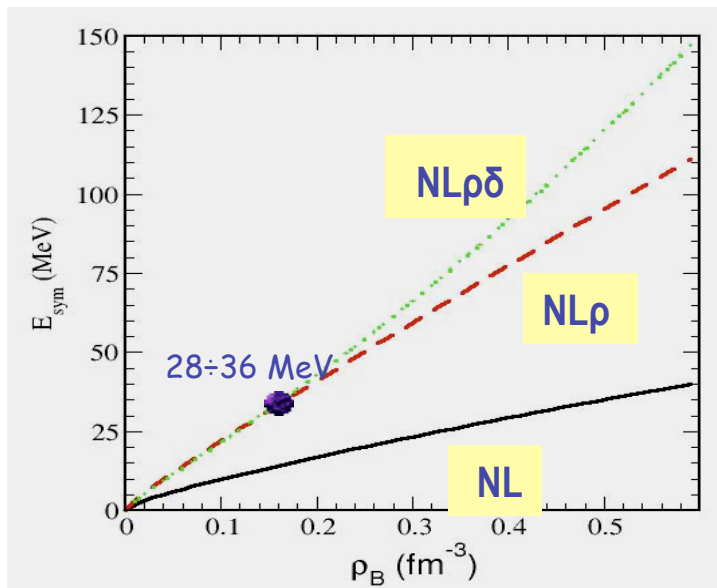
Dynamic effects of δ -meson:

	isoscalar	isovector
scalar	σ	δ
vector	ω	ρ

A cancellation between scalar and the isovector sector, similarly as for the isoscalar vector parts in potential

$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^{*2}} + \frac{1}{2} \left[f_\rho - f_\delta \left(\frac{M^*}{E^*} \right)^2 \right] \rho_B$$

\rightarrow No δ $\rightarrow f_\rho \approx 1.5 f_\rho^{FREE}$ **NL ρ**
 $\rightarrow f_\delta = 2.5 \text{ fm}^2 \rightarrow f_\rho \approx 5 f_\rho^{FREE}$ **NL $\rho\delta$**



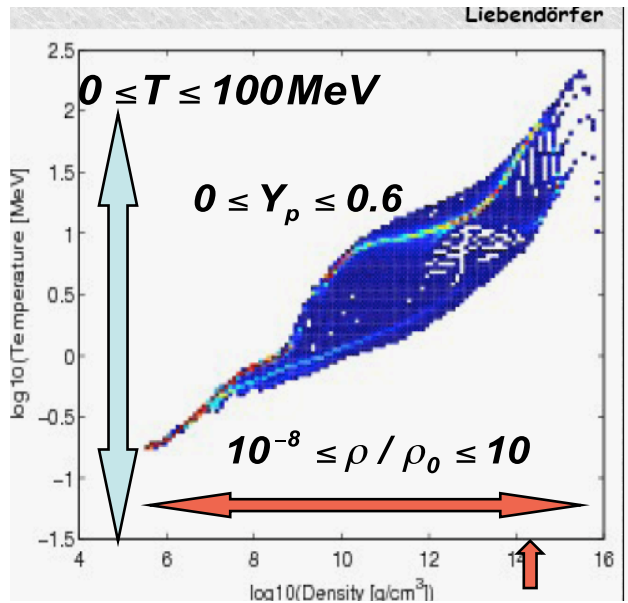
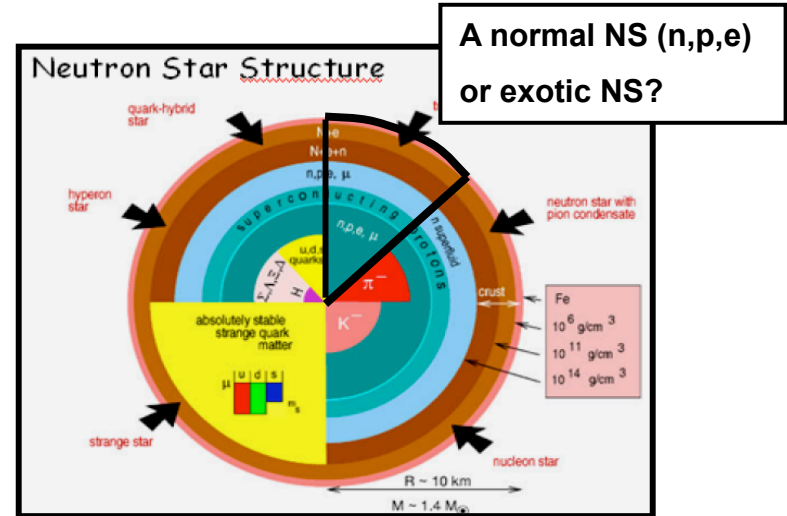
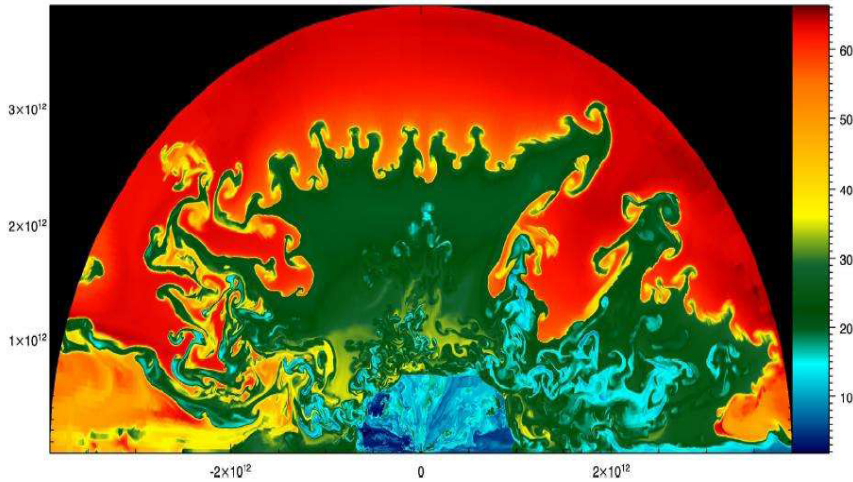
Investigation of the Symmetry Energy in Different Density Ranges

1. $\rho \ll \rho_0$: expanding fireball in Fermi-energy heavy ion collisions. cluster correlations at low density and temperature
2. $\rho < \rho_0$: Isospin transport in Fermi energy central and peripheral collisions, (multi-)fragmentation,
3. $\rho \sim \rho_0$: structure and low energy excitations of (asymmetric) nuclei: skin thickness, Pygmy resonances, IAS,
4. $\rho > \rho_0$: Relativistic heavy ion collisions: light cluster emission, flow and particle production,
5. $\rho \gg \rho_0$: Ultrarelativistic HI collisions, Dependence of mixed and deconfinement phase on asymmetry?

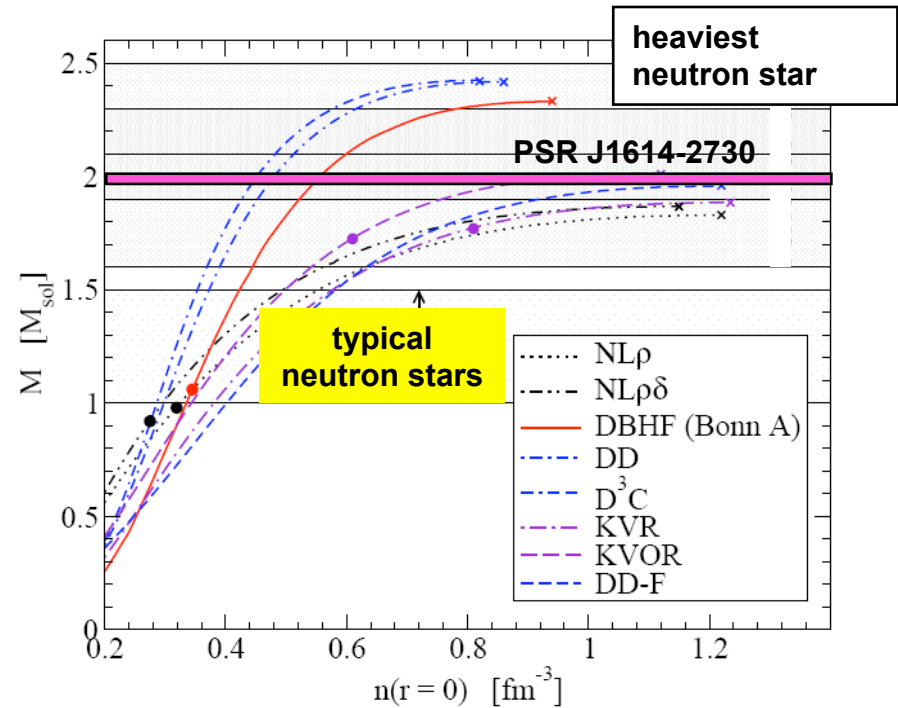
mainly
discuss these



Astrophysics: Supernovae and neutron stars



Supernova evolution: range of densities and temperatures and asymmetries:



Determination of nuclear SE in heavy ion collisions

→ Transport theory

central quantity:

1-body phase space distribution $f(r,p,t)$

Wigner transform of one-body density

→ equation of motion in semiclass.

approx: **Vlasov equation**

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}(r) f - \vec{\nabla}(r) U(r) \vec{\nabla}(p) f = 0$$

EOS

isoscalar and isovector (~10%)

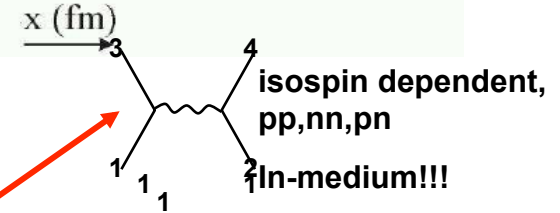
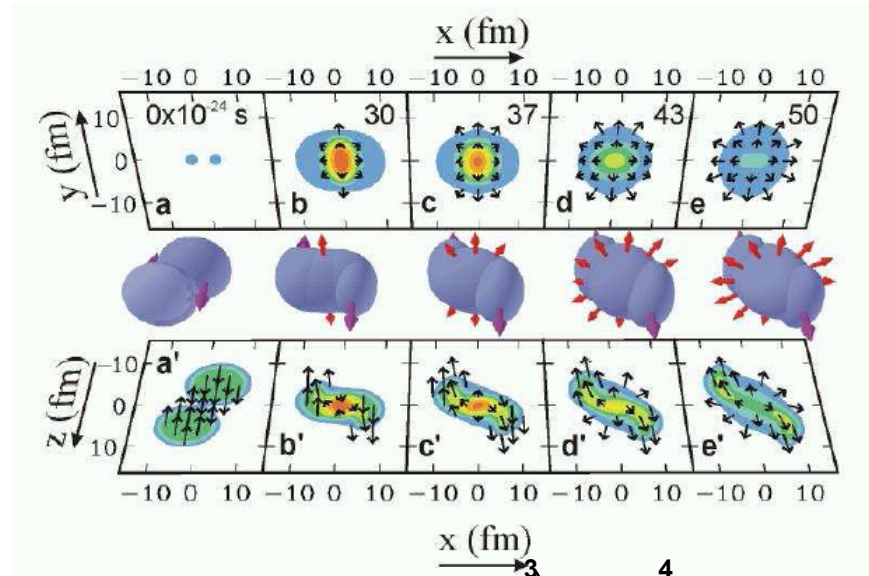
$$I_{coll} = \int d\vec{v}_2 d\vec{v}_1 d\vec{v}_2' v_{12} \sigma(\Omega) (2\pi)^3 \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2') \times [f_1' f_2' (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_1')(1 - f_2')]]$$

2-body collisions

gain term

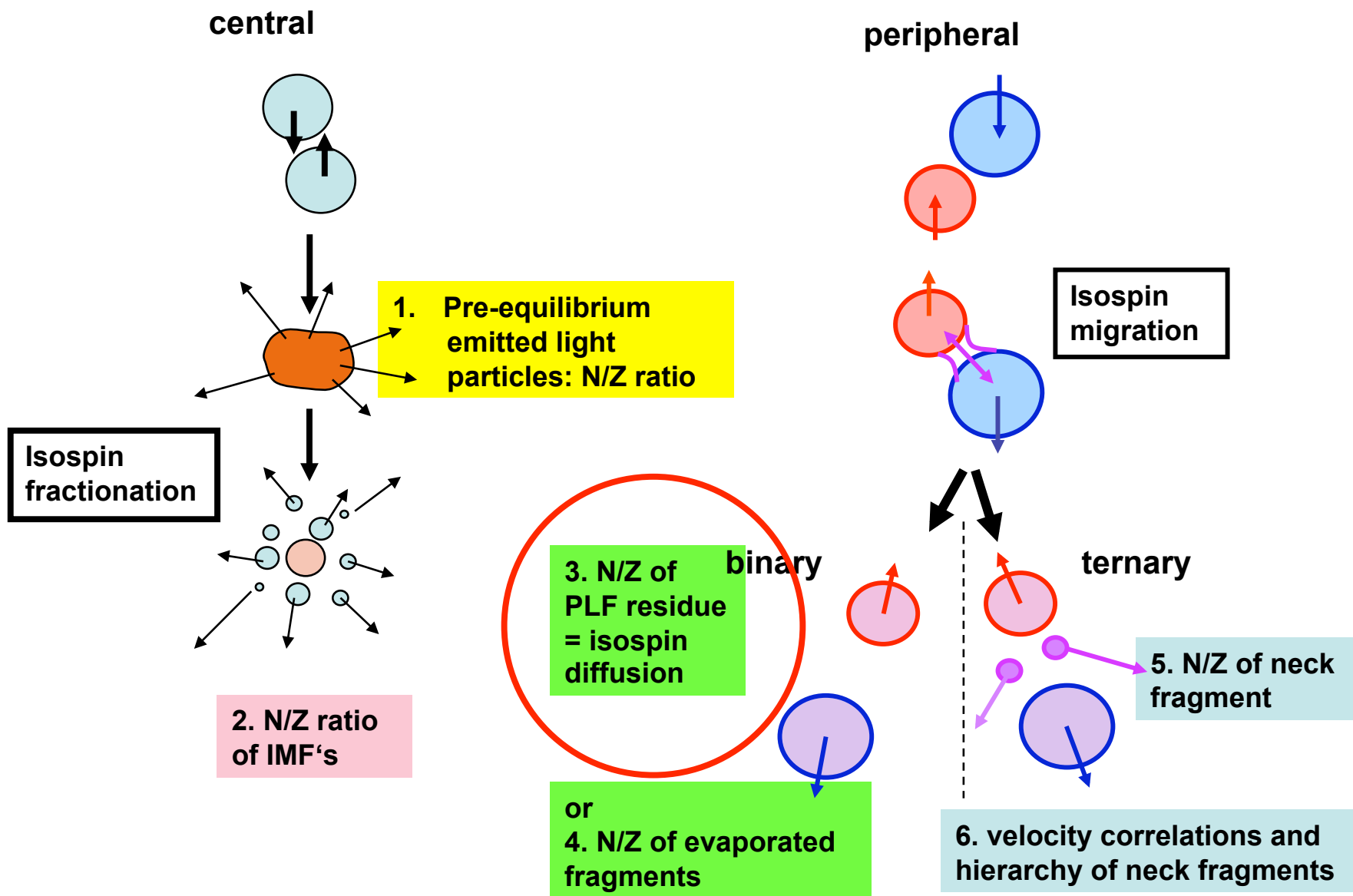
loss term

Boltzmann-Uhling-Uhlenbeck, Landau-Vlasov



- 1) Approximation to a much more complicated **non-equilibrium quantum transport equation (Kadanoff-Baym)** by neglecting finite width of particles (**quasi-particle approximation**)
- 2) Coupled transport eqs. for neutrons and protons
- 3) Isovector effects are **small relative to isoscalar quantities**
- 4) **Relativistic transport equations: RMF models**
- 5) **Inelastic collisions (particle production; mesons π, K)**

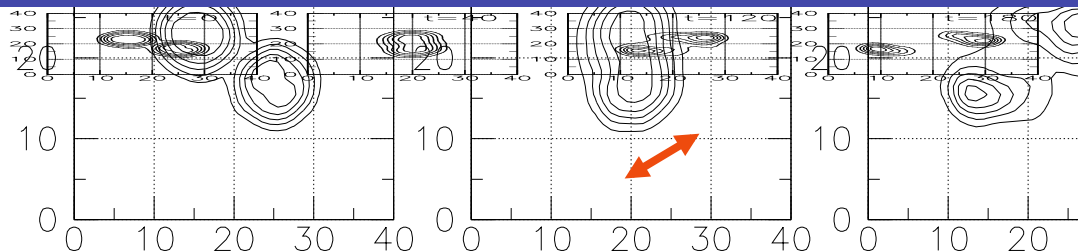
Isospin Effects in the Fermi energy domain



Isospin transport

isospin transport through „neck“ in peripheral collisions

$^{124}\text{Sn}(\text{H})+^{112}\text{Sn}(\text{L})$

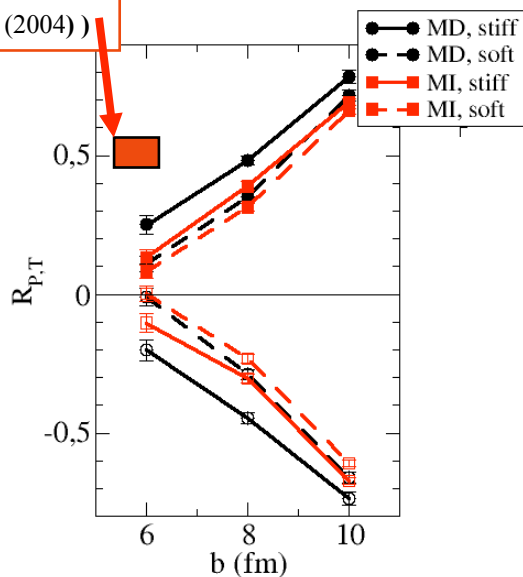


Imbalance (or Rami, transport) ratio:
 β asymmetry of residue (i=PLF,TLF)
 (also for other isospin sens.quantities)

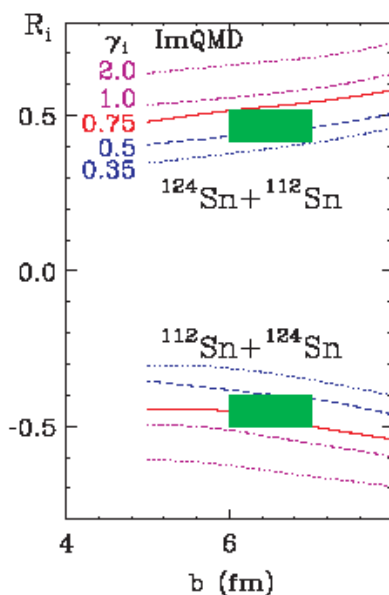
$$R_i = \frac{\beta_i^{mix} - \frac{1}{2}(\beta_i^{HH} + \beta_i^{LL})}{\frac{1}{2}(\beta_i^{HH} - \beta_i^{LL})}$$

Limiting values:
 $R=0$ complete equilibration
 $R=\pm 1$, complete transparency

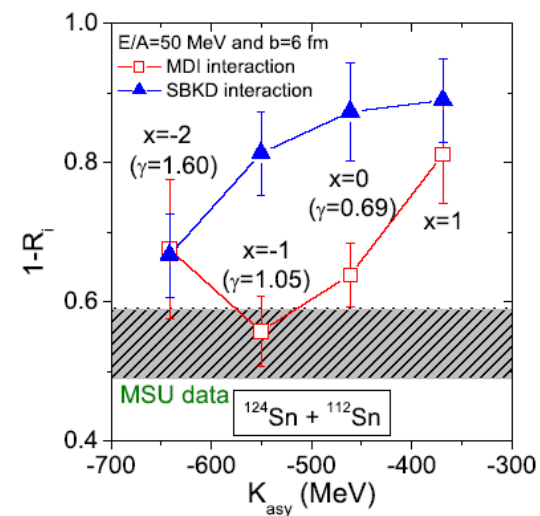
experimental data
 (B. Tsang et al.
 PRL 92 (2004))



J.Rizzo, et al., Nucl. Phys. A806 (2008) 79



M.B. Tsang, et al., PRL 102 (2008)



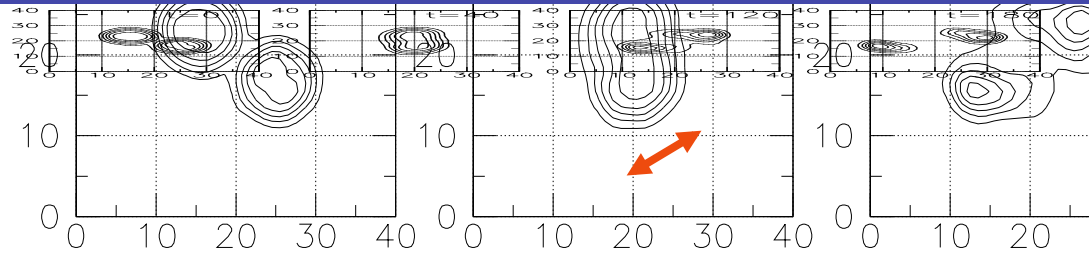
L.W.Chen, C.M.Ko, B.A.Li, PRL 94, 032701 (2005)

points toward a moderately stiff ($\gamma \sim 1$) SE, but disagreement in detail

Isopin transport

isospin transport through „neck“ in peripheral collisions

$^{124}\text{Sn}(\text{H}) + ^{112}\text{Sn}(\text{L})$



Imbalance (or Rami, transport) ratio:

β asymmetry of residue (i=PLF,TLF)
(also for other isospin sens.quantities)

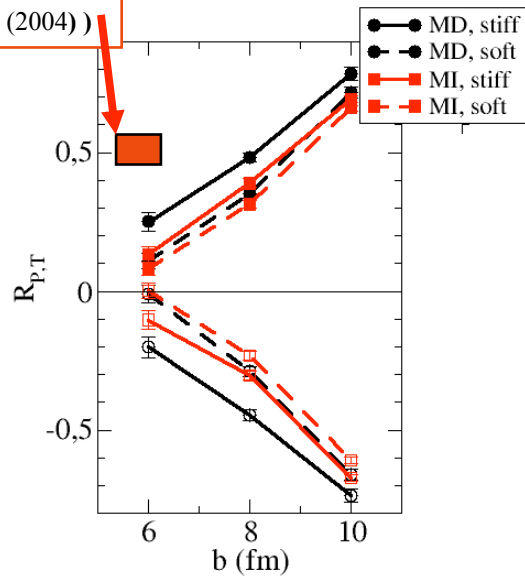
$$R_i = \frac{\beta_i^{mix} - \frac{1}{2}(\beta_i^{HH} + \beta_i^{LL})}{\frac{1}{2}(\beta_i^{HH} - \beta_i^{LL})}$$

Limiting values:

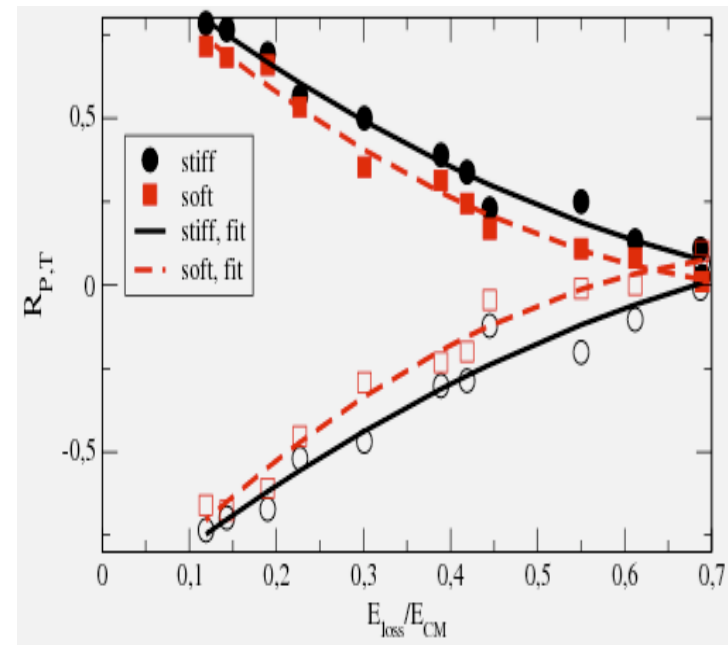
$R=0$ complete equilibration

$R=+1$, complete transparency

experimental data
(B. Tsang et al.
PRL 92 (2004))



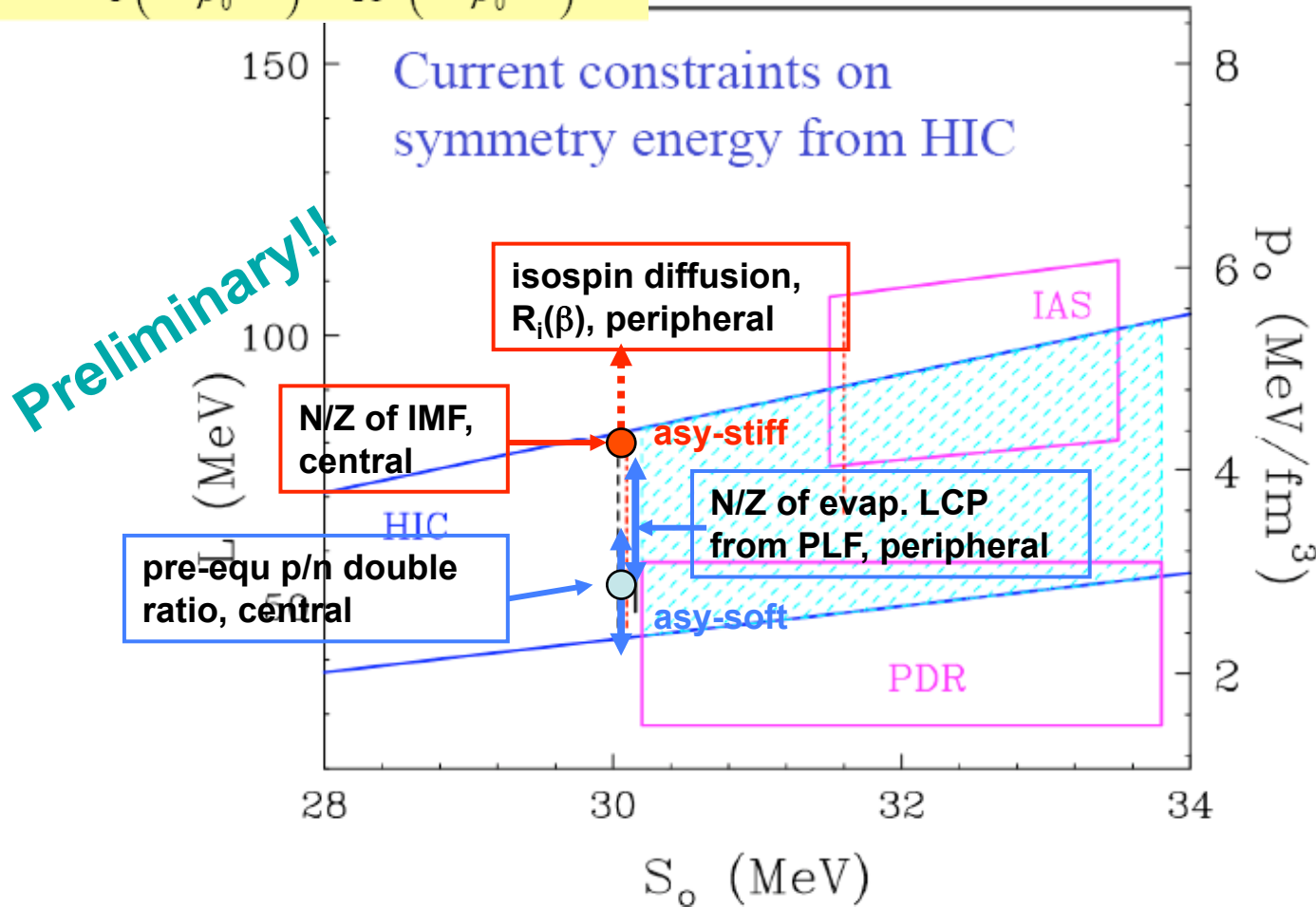
J.Rizzo, et al., Nucl. Phys. A806 (2008) 79



more equilibration (lower R) for longer interaction time ~ correlation with total energy loss

Comparison to constraints from MSU (Contribution of B. Tsang)

$$E_{sym} = S_0 + \frac{L}{3} \left(\frac{\rho_B - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left(\frac{\rho_B - \rho_0}{\rho_0} \right)^2 + \dots$$



Current state of knowledge:

Generally consistent with each other , but still rather uncertain. More work necessary, especially also on consistency of codes

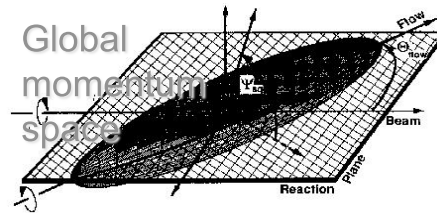
Heavy Ion Collisions at Relativistic Energies: "Flow"

Fourier analysis of momentum tensor : „flow“

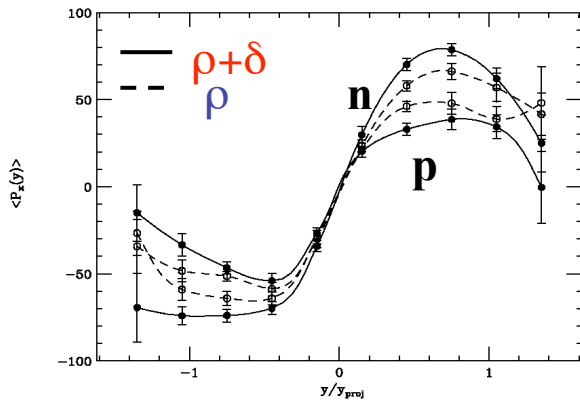
$$N(\theta, y, b) = N_0(1 + v_1(y, b) \cos \theta + v_2(y, b) \cos 2\theta + ..)$$

v_1 : sideward flow

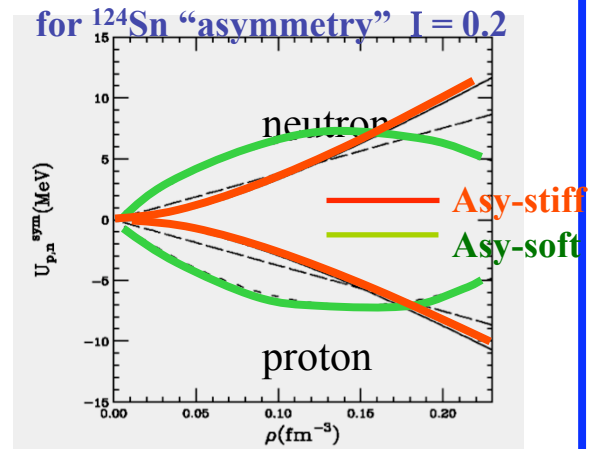
v_2 : elliptic flow



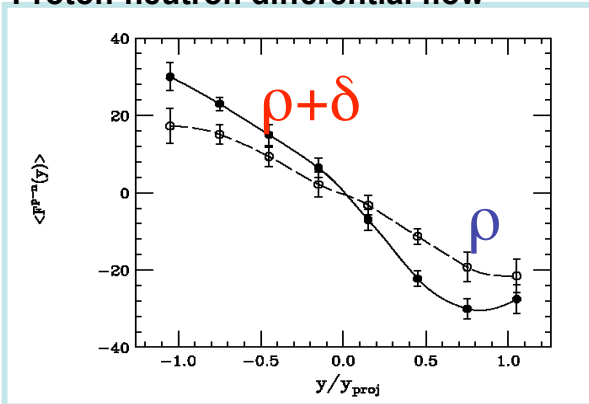
$^{132}\text{Sn} + ^{132}\text{Sn}$ @ 1.5 AGeV $b=6\text{fm}$



Proton/neutron potentials



Proton-neutron differential flow



★ dynamical boosting of vector contribution

$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \vec{\nabla} \rho_3 = \frac{4}{\rho_B} E_{sym}^* \vec{\nabla} \rho_3$$

T. Gaitanos, M. Di Toro, et al., PLB562(2003)

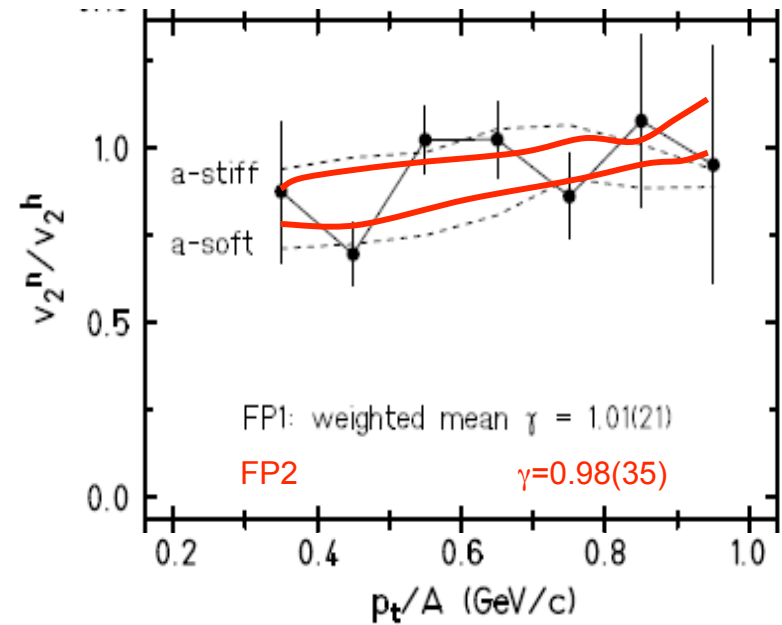
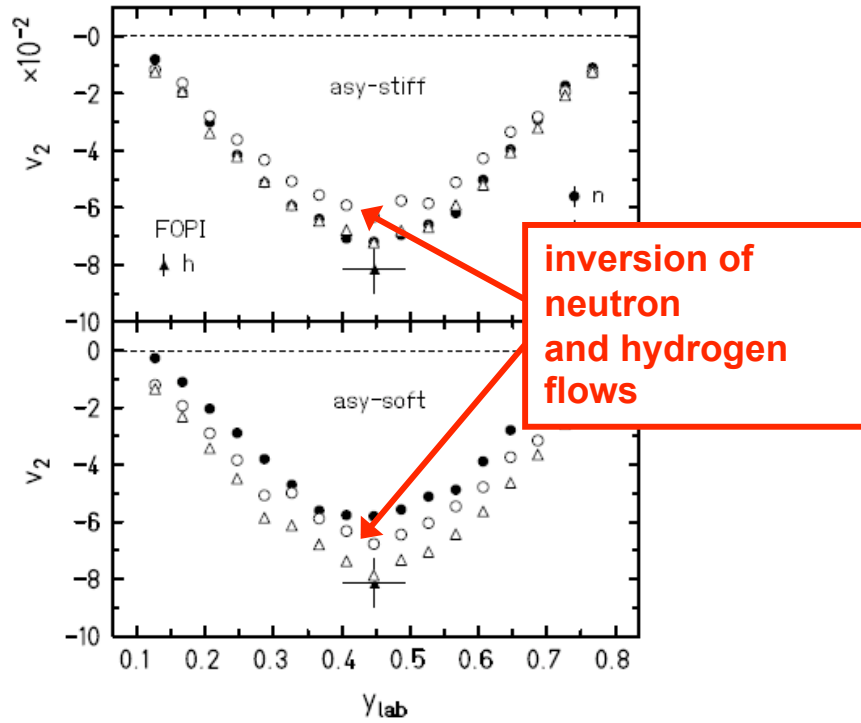
$$F_{n-p}^x(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^x w_i),$$

$$w_i = +1(-1) \text{ f omeutron (proton)}$$

and analogously for elliptic flow

ASY-EOS: Hunting the high density symmetry energy with v_2

Russotto, et al., PLB 697, 471 (2011)



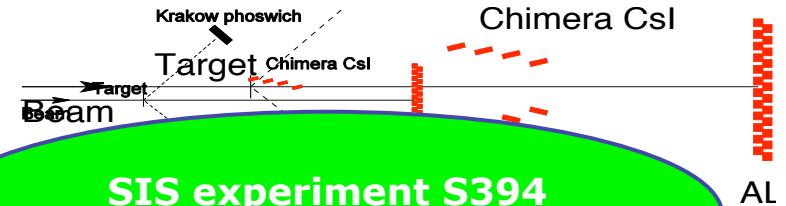
Indication of an asy-stiff symmetry energy at suprasaturation density

Constraining the Symmetry Energy at Supra-Saturation Densities
With Measurements of Neutron and Proton Elliptic Flows

Co-Spokespersons: R.C. Lemmon¹ and P. Russotto²

Collaboration

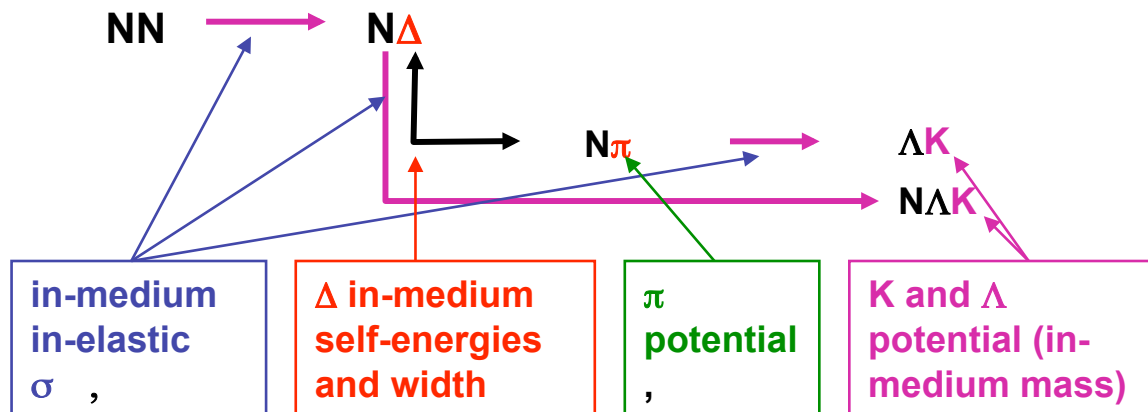
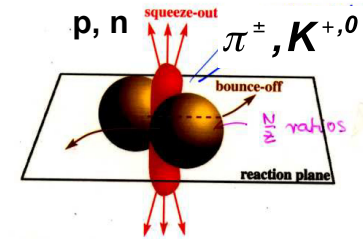
F. Amorini², A. Anzalone¹⁷, T. Aumann³, V. Avdeichikov¹², V. Baran²³, Z. Basrak⁴, J. Benlliure¹³, I. Berceanu¹¹, A. Bickley¹⁴, E. Bonnet⁶, K. Boretzky³, R. Bougault³⁰, J. Brzychczyk⁸, B. Bubak²², G. Cardella⁷, S. Cavallaro², J. Cederkall¹², M. Chartier⁵, M.B. Chatterjee¹⁰, A. Chbihi⁶, M. Colonna¹⁷, D. Cozma¹¹, B. Czech¹⁰, E. De Filippo⁷, K. Fissum¹², D. Di Julio¹², M. Di Toro², M. Famiano²⁷, J.D. Frankland⁶, E. Galichet¹⁸, I. Gasparic⁴, E. Geraci¹⁵, V. Giordano², P. Golubev¹², L. Grassi¹⁵, A. ...



High-density Symmetry Energy: Flow and particle production

Difference in neutron and proton potentials

1. „direct effects“: difference in proton and neutron (or light cluster) emission and momentum distribution
2. „secondary effects“: production of particles, isospin partners $\pi^{\pm}, K^{0,+}$



1. Mean field effect: U_{sym} more repulsive for neutrons, and more for asystiff

→ pre-equilibrium emission of neutron, reduction of asymmetry of residue

2. Threshold effect, in medium effective masses:

→ m_{N}^* , m_{Δ}^* , contribution of symmetry energy; m_{K}^* , models for K-potentials

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})} \downarrow \Rightarrow \frac{\pi^{-}}{\pi^{+}} \downarrow$$

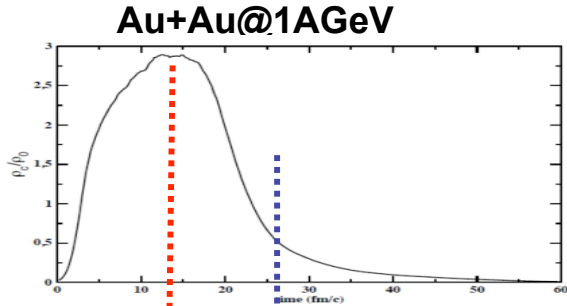
decrease : $NL \rightarrow NL\rho \rightarrow NL\rho\delta$

$$\sigma = \sigma(s_{in} - s_{th}) \frac{\pi^{-}}{\pi^{+}} \uparrow \text{increase } NL \rightarrow NL\rho\delta$$

s_{th} independent of isospin, „too simple“ model of Δ self energies

Dynamics of particle production (Δ, π, K) in heavy ion collisions

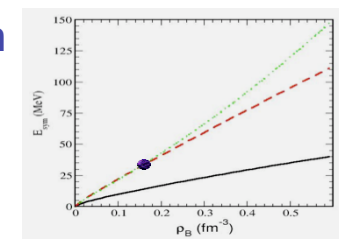
Central density



Δ and K : production in high density phase

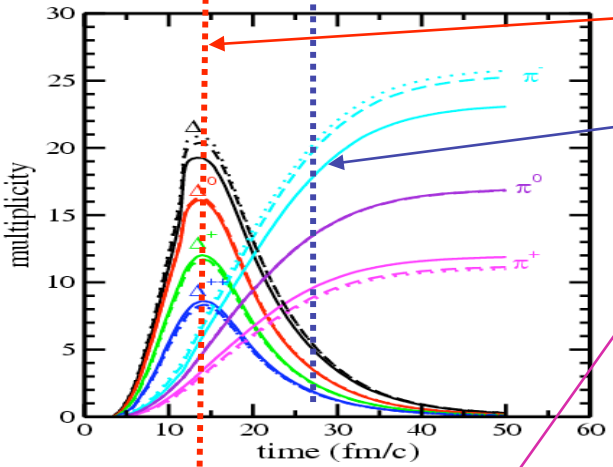
Pions: low and high density phase

Sensitivity to asy-stiffness

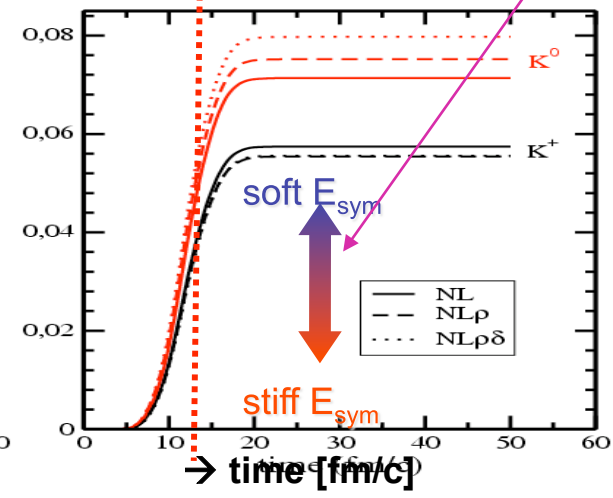


NL $\rho\delta$
NL ρ
NL

π and Δ multiplicity



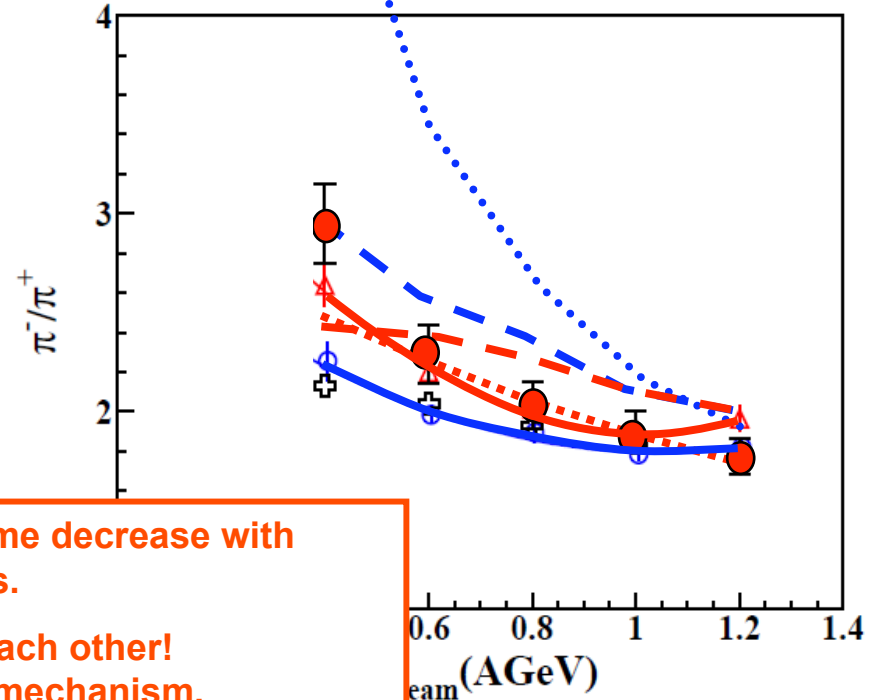
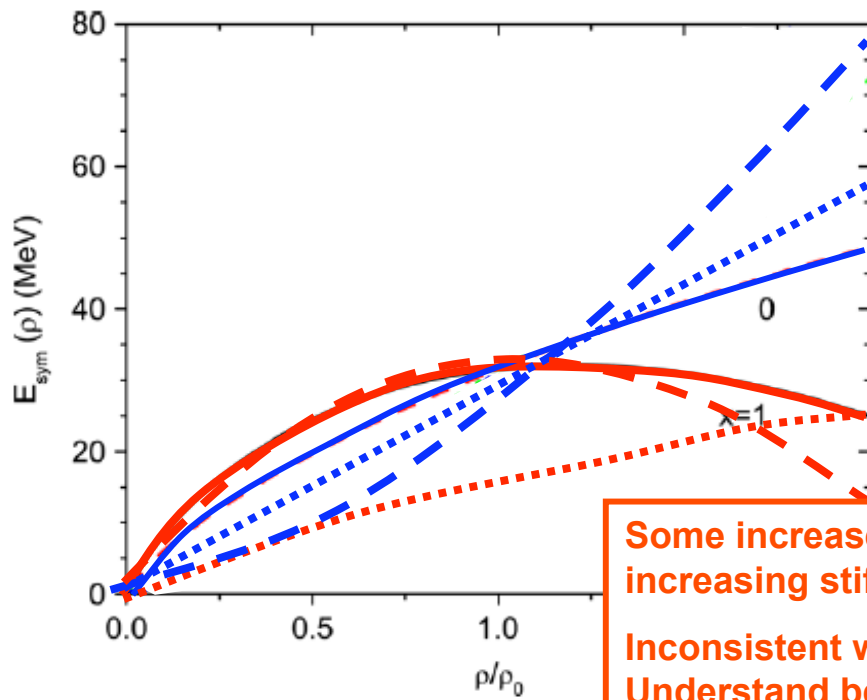
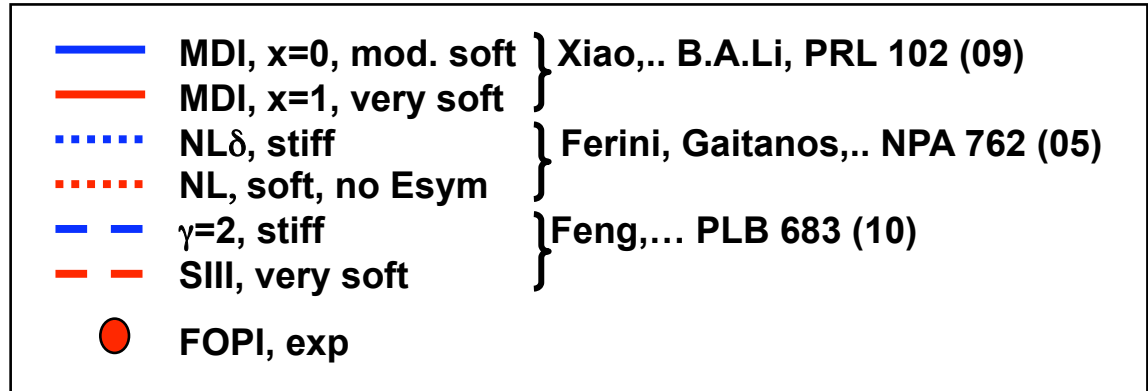
$K^{0,+}$ multiplicity



Dependence of ratios on asy-stiffness

- n/p
- $\rightarrow \Delta^{0,-}/\Delta^{+,++}$
- $\rightarrow \pi^-/\pi^+, K^0/K^+$
- \rightarrow n/p ratio governs particle ratios

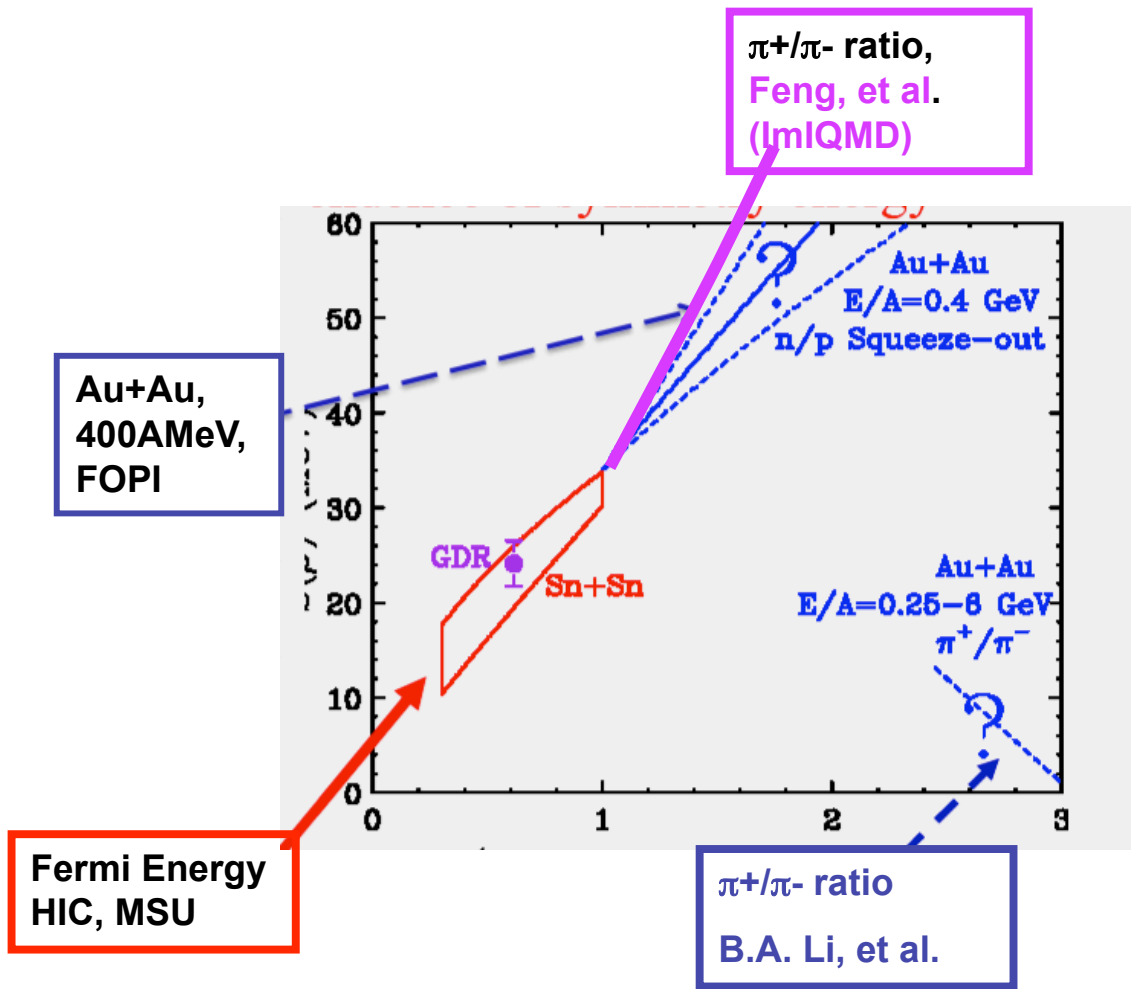
Pion Ratios in Comparison to FOPI data (W.Reisdorf et al. NPA781 (2007) 459)



Some increase, some decrease with increasing stiffness.

Inconsistent with each other!
Understand better mechanism,
esp. Δ dynamics!

Present constraints on the symmetry energy



Moving towards a better determination of the symmetry energy

Large uncertainties at higher density

Conflicting theoretical conclusions for pion observables.

Work in exp. and theory necessary!

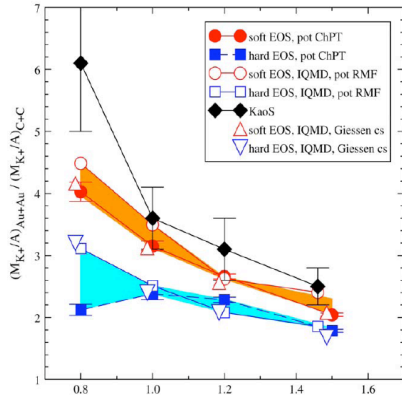
Summary and Outlook

- While the EOS of symmetric NM is now fairly well determined, the density (and momentum) dependence of the Symmetry Energy is still rather uncertain, but important for exotic nuclei, neutron stars and supernovae.
- Constraints come from neutron star observables and from HIC both at sub-saturation (Fermi energy regime) and supra-saturation densities (relativistic collisions).
- At subsaturation densities the constraints become increasingly stringent ($\gamma \sim 1$), but constraints are largely lacking at supra-saturation densities.
- Observables for the suprasaturation symmetry energy
N/Z of pre-equilibrium light clusters,
difference flows, (first hints \rightarrow ASYEOS)
part. production ratios π^-/π^+ , K^0/K^+ (FOPI,HADES)
- More work to do in exp. (more data)
and theory (consistency of transport codes, π, Δ dynamics)
- Nuclear symmetry energy: an interesting field that connects areas of nuclear structure, reactions and astrophysics

Thank you, and all the best to Peter in the future!

backup

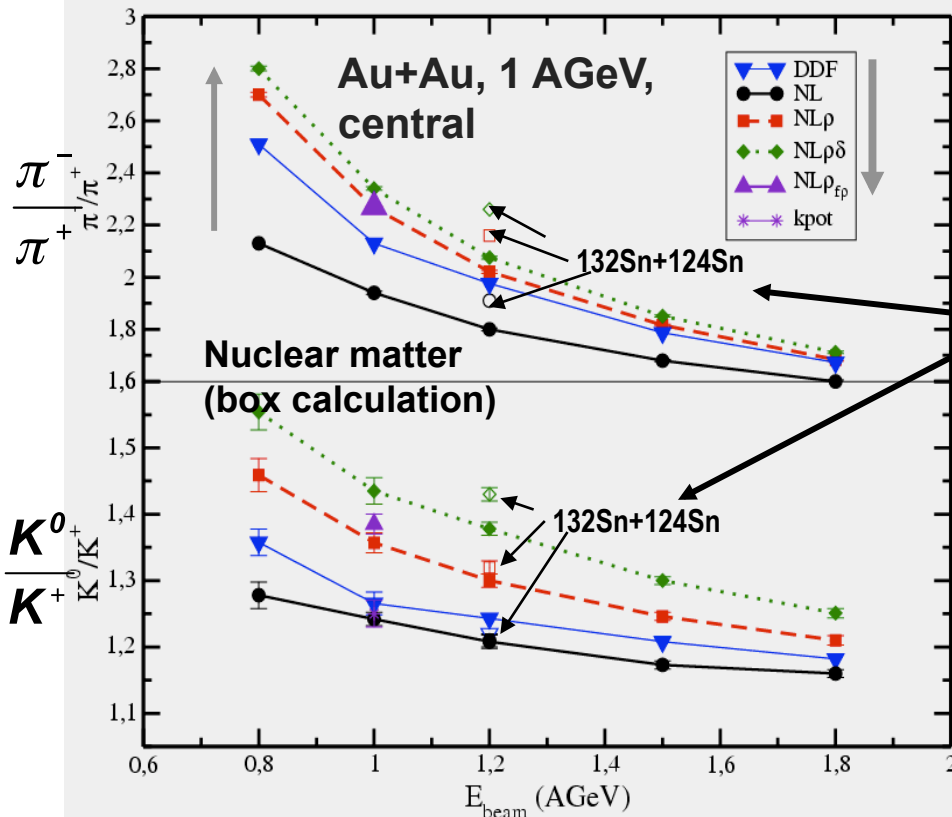
Kaons as a probe for the EOS – also for the Symmetry Energy?



Kaons are a more sensitive and clean probe of the high density EOS. Demonstrated by Fuchs, et al., PRL 86 (01), C.M. Ko & J. Aichelin, PRL55(85) for **symmetric matter in comparison with KAOS data.**

Kaons are **closer to threshold, come only **from high density**, have **large mean free path**, **small width**:**

Also a probe for the symmetry energy for different charge states?



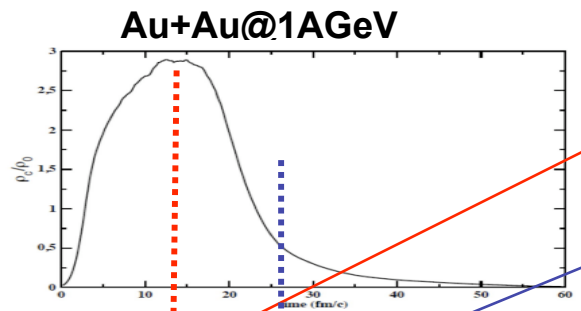
From Soft to Stiff from lower to upper curves: Stiffer iso-EOS \rightarrow larger ratio! Threshold dominates mean field effect; larger at lower energies

Effect reduced in finite nuclei (pre-equil emission reduces asymmetry)

Kaons ratio still a bit more sensitive probe: $\sim 15\%$ difference betw. very soft and stiff \rightarrow small but perhaps measurable! \rightarrow Exp.: double ratio data by FOPI (Lopez, et al., PRC75 (07))

Dynamics of particle production (Δ, π, K) in heavy ion collisions

Central density

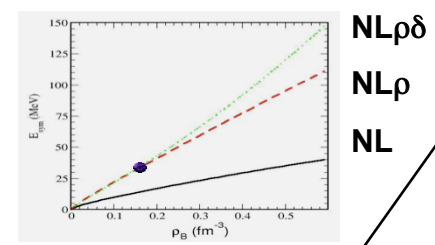
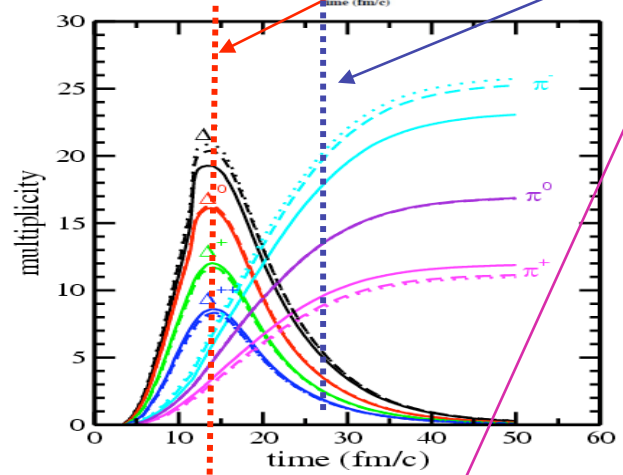


Δ and K: production in high density phase

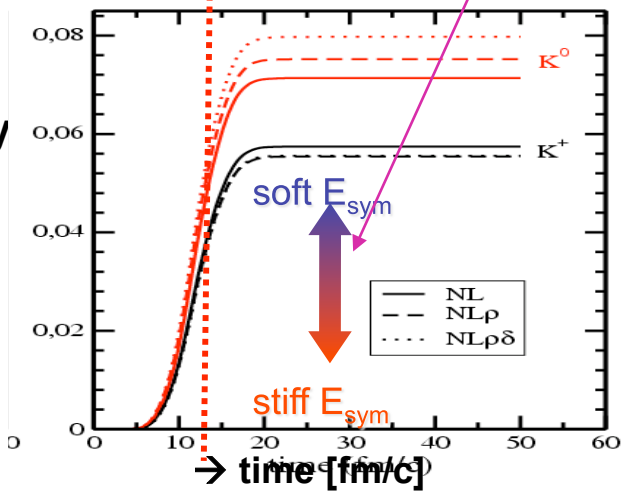
Pions: low and high density phase

Sensitivity to asy-stiffness

π and Δ multiplicity



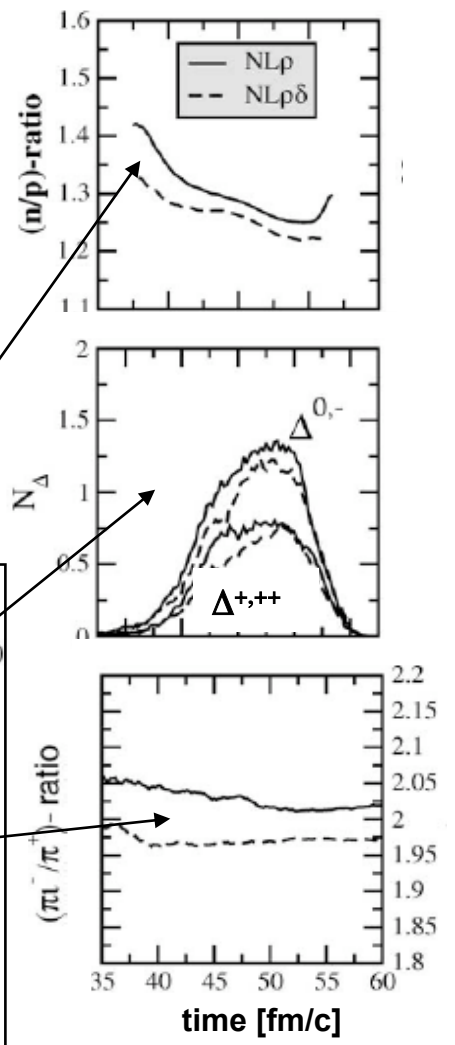
$K^{0,+}$ multiplicity



Dependence of ratios on asy-stiffness

- n/p
- $\rightarrow \Delta^{0,-} / \Delta^{+,++}$
- $\rightarrow \pi^- / \pi^+$
- $\rightarrow n/p$ ratio governs particle production effects

Au+Au, 0.6 A MeV



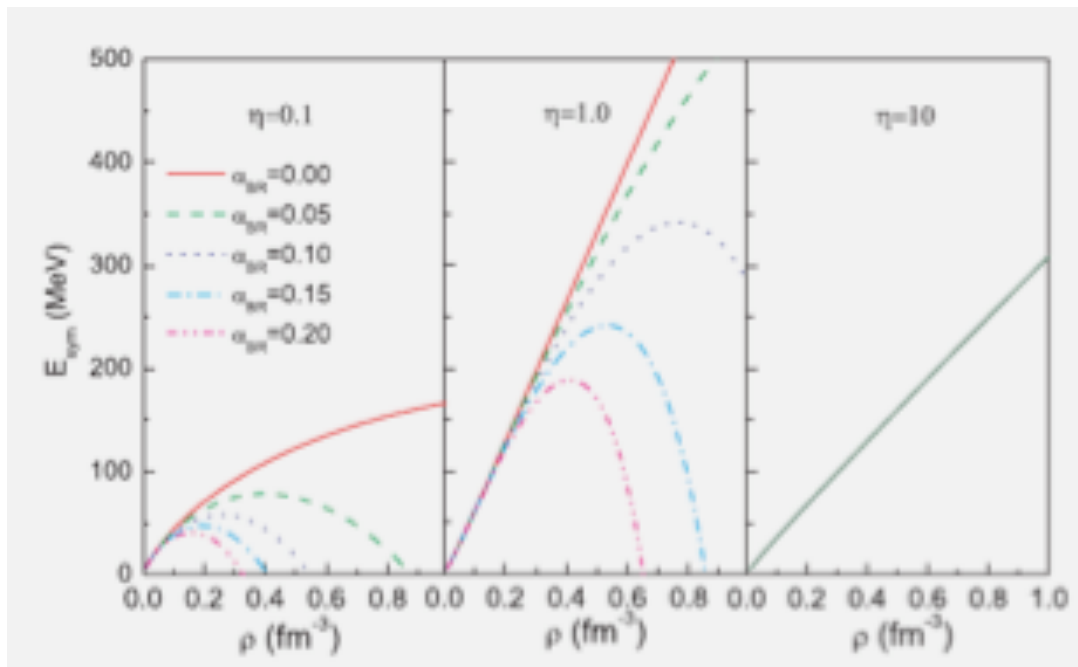
Why is the symmetry energy so uncertain at high densities?

- In-medium properties of the short-range tensor force in the n-p (T=0) channel, controlled by the in-medium rho-meson mass
- Isospin-dependence of short-range nucleon-nucleon correlations
- Effects of many-body forces

$$\bar{V}_T = \int \Theta(r_{ij} - r_c) [V_T^\pi(r_{ij}) + V_T^\rho(r_{ij})] dr_{ij}; r_c = \eta \left(\frac{3}{4\pi\rho} \right)^{1/3}$$

$$m_\rho^* / m_\rho = 1 - \alpha_{BR} \cdot \rho / \rho_0$$

Brown-Rho scaling of ρ mass



Changes of SE
without changing
the isoscalar
potential

around normal density: Structure

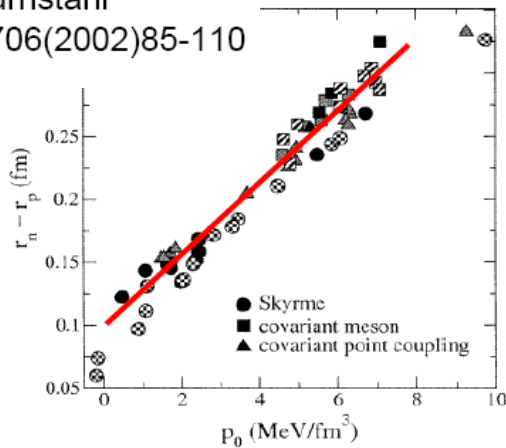
Expansion of SE around ρ_0 :

$$E_{sym} = a_4 + \frac{L}{3} \left(\frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left(\frac{\rho - \rho_0}{\rho_0} \right)^2$$

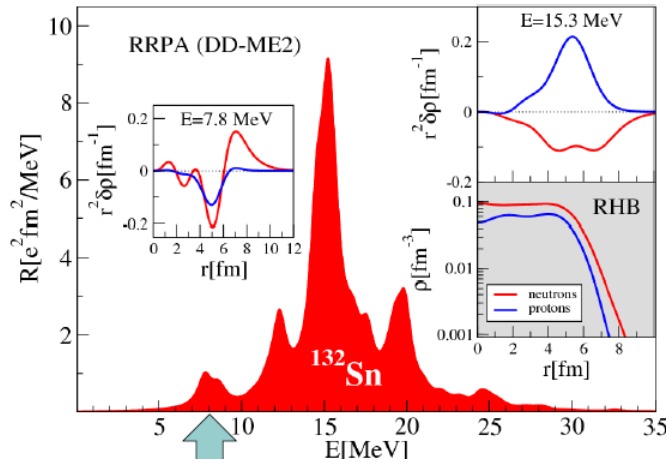
Symmetry pressure

$$P_{sym} = \frac{1}{\rho_0} \frac{dE_{sym}}{d\rho}$$

R.J.Furnstahl
NPA 706(2002)85-110

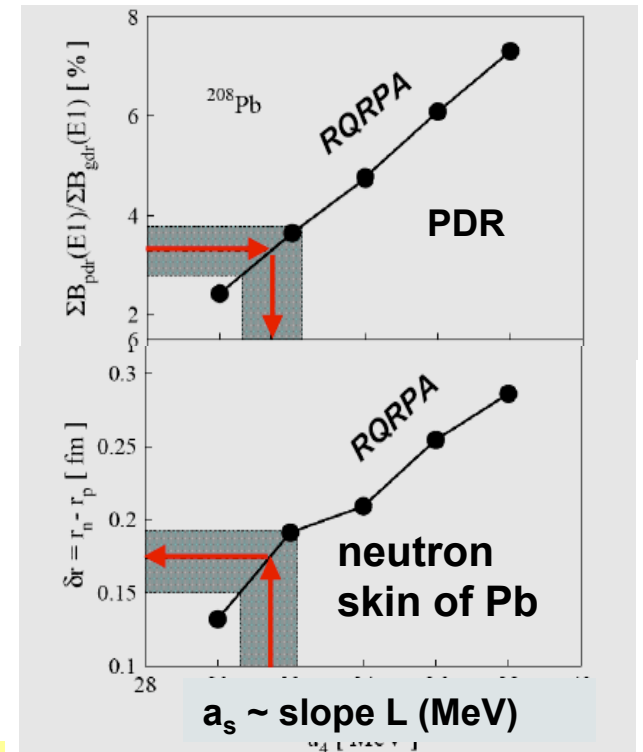


strong linear correlation between neutron skin thickness and SE slope $L \sim \rho_0$ (or also a_4)



strong correlation between Dipole strength of Pygmy resonance and SE slope L

→ possibility to determine slope of SE, but also neutron skin thickness



N. Paar, D. Vretenar, and P. Ring, Phys. Rev. Lett. 94, 182501 (2005)

Transport approaches

Transport theory describes the non-equilibrium aspects of the temporal evolution of a collision. The central quantity is the phase space density (coordinate and momentum distribution).

Demonstrate two aspects:

1. Evolution in coordinate space:

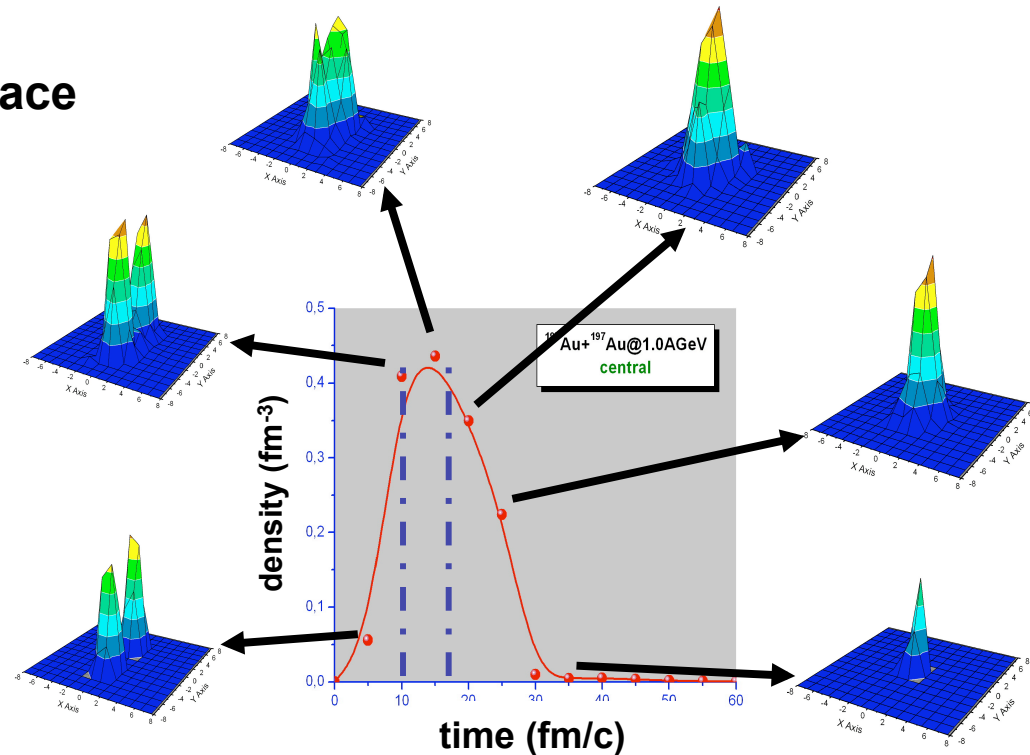
→ movies curtesy T. Gaitanos, T.Chossy



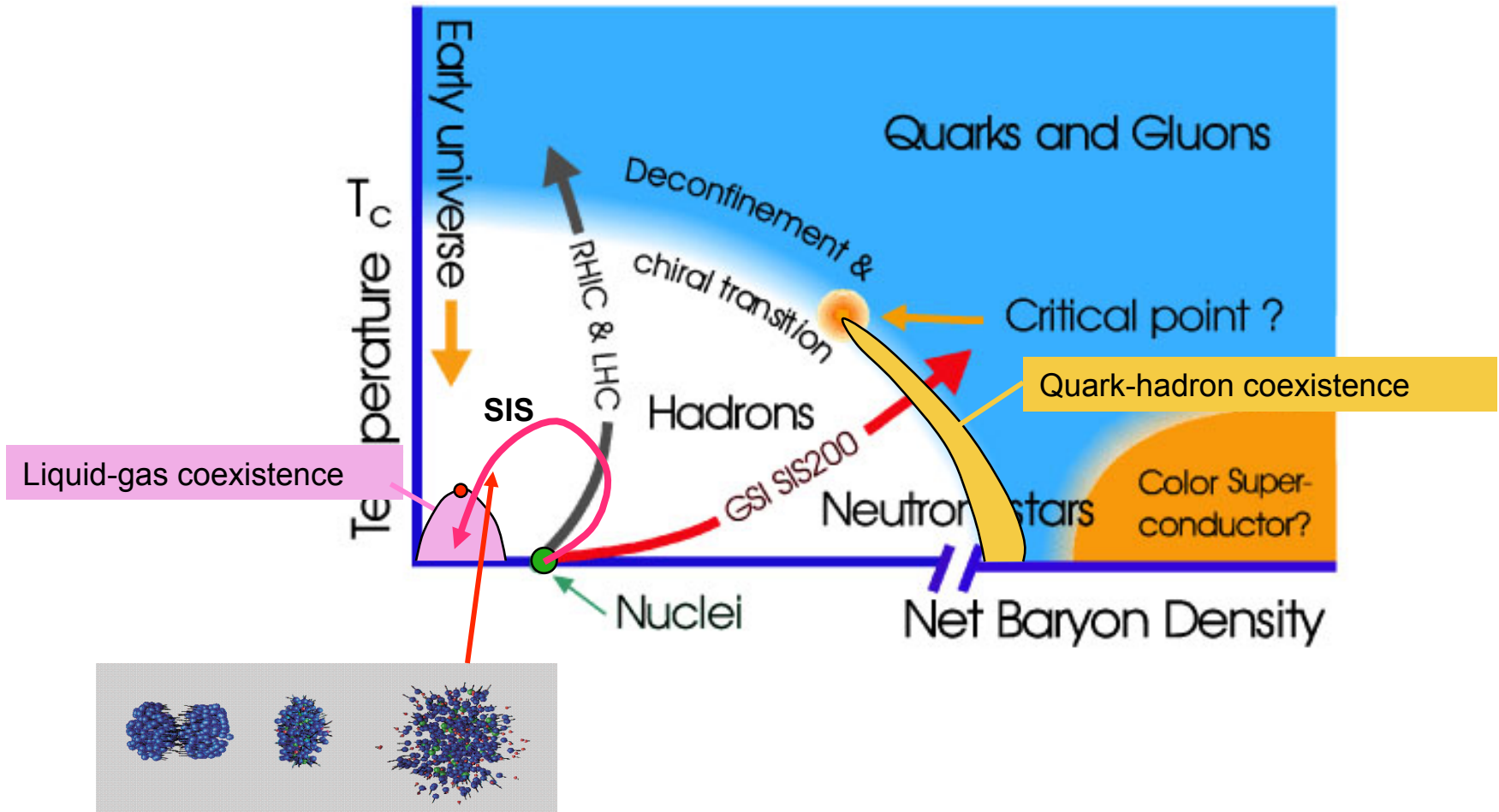
2. Evolution in momentum space

non-equilibrium,
non-sphericity of local
momentum distributions

→ Transport theory!!!

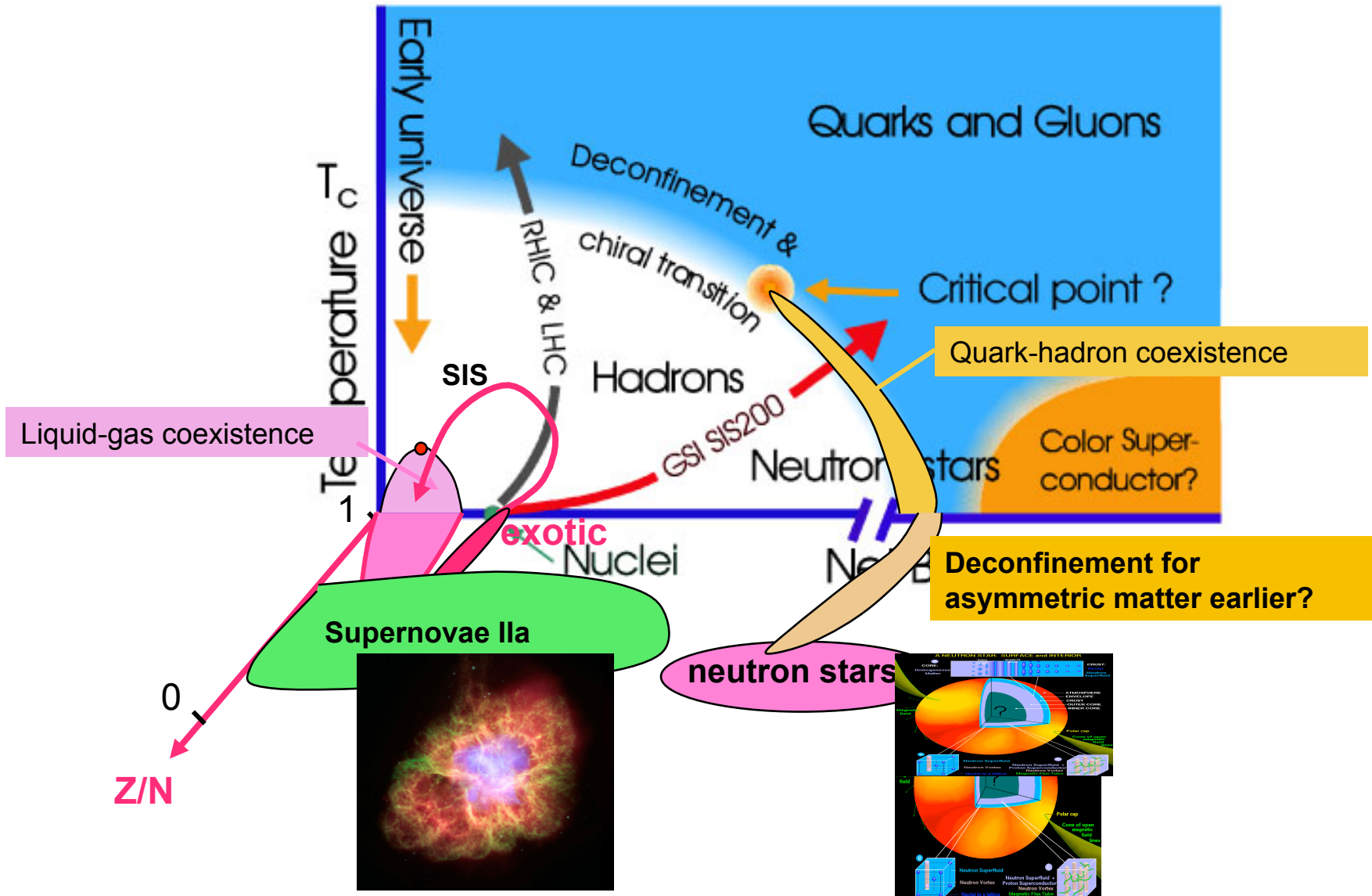


Extreme States of Strongly interacting Matter



Extreme States of Strongly interacting Matter

Asymmetry degree of freedom

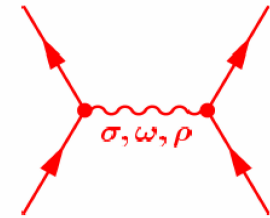


II.3

Relativistic transport theory

„Quantumhydrodynamics“ (QHD) effective field theory

$$L = \bar{\Psi} \left[\gamma_{\mu} \left(i\partial^{\mu} - g_{\omega} \omega^{\mu} \right) - \left(m - g_{\sigma} \sigma \right) \right] \Psi + L^{mes}$$



Relativistic Transport Eq.

$$\left[p^{*\mu} \partial_{\mu}^x + \left(p_{\nu}^* F^{\mu\nu} + m^* \partial_x^{\mu} m^* \right) \partial_{\mu}^{p^*} \right] f(x, p^*) = I_{coll}(f, \sigma)$$

effective mass

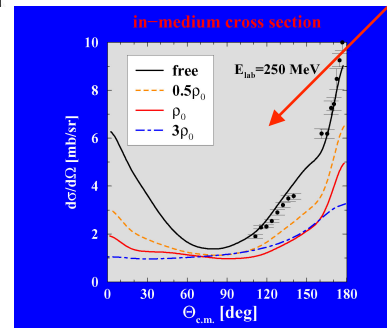
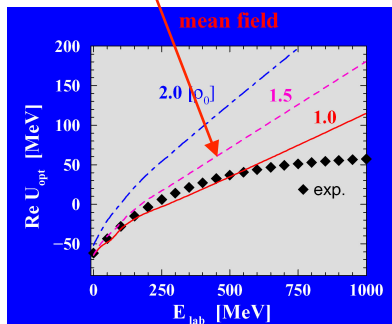
$$m^* = m$$

Kinetic momentum

$$p_{\mu}^* = p_{\mu}$$

Field tensor

$$F^{\mu\nu} = \partial^{\mu} \Sigma^{\nu} - \partial^{\nu} \Sigma^{\mu}$$



consistency between self-energies and in-medium cross sections !

e.g. in Dirac BHF $T = V + VQGGT$ Bethe – Salpeter – Eq.

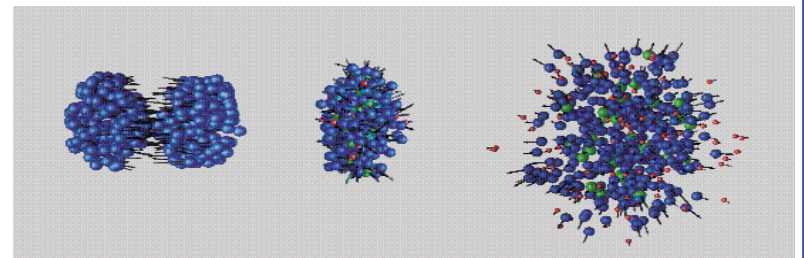
$$\Sigma \approx \text{Re Trace}(Tf); \quad \sigma \approx T^2$$

extensions: fluctuations: fluctuation-dissipation theorem

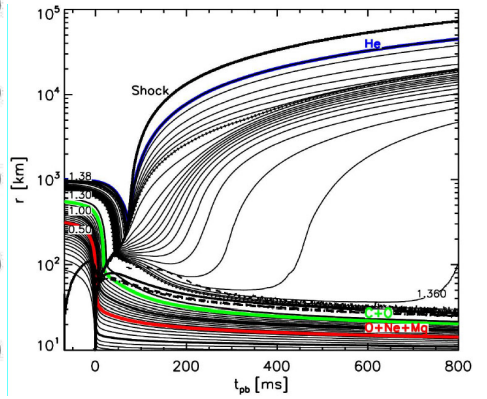
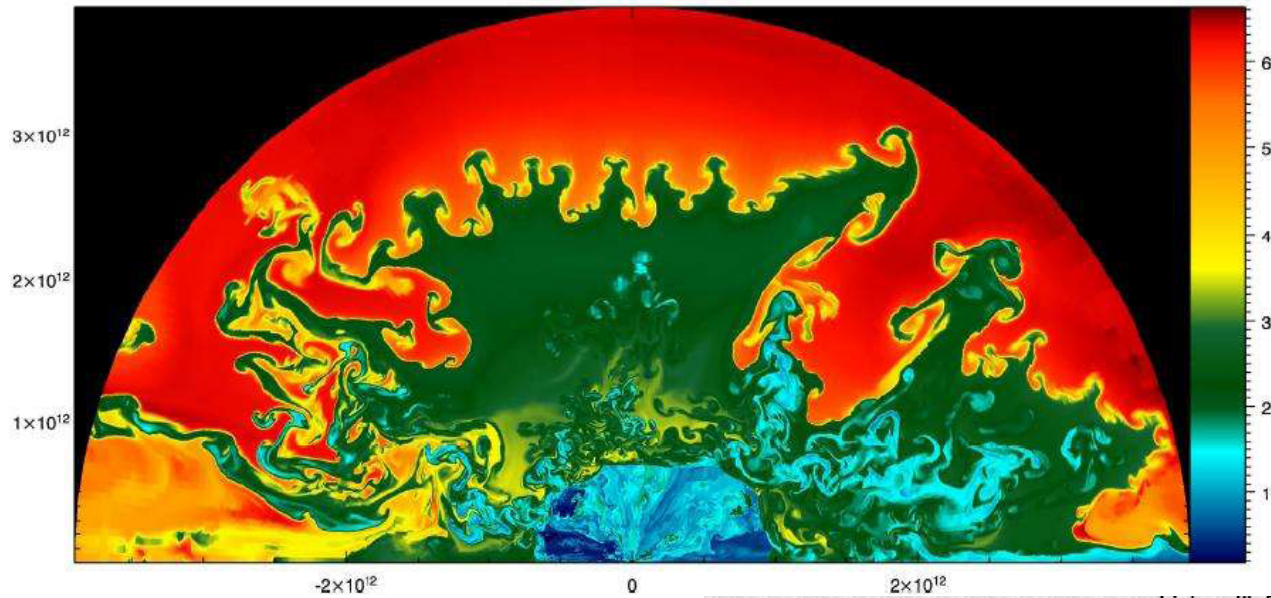
instabilities, liquid-gas phase transition

propagation of particles with finite width

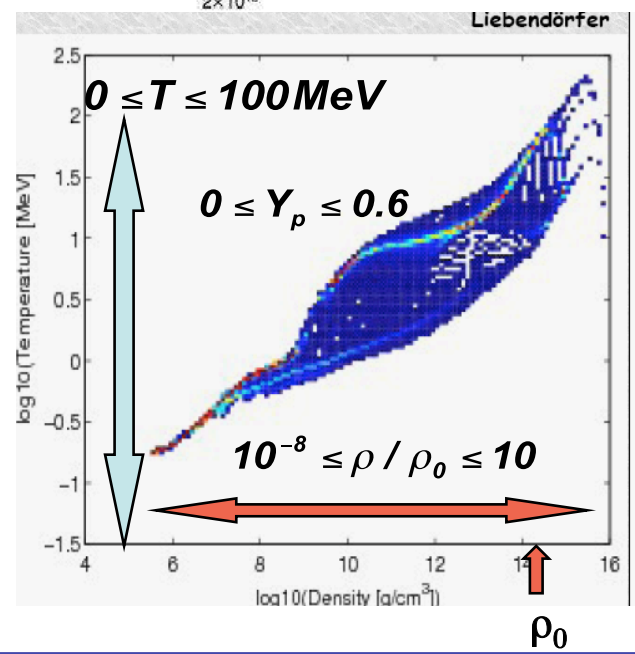
(spectral function): Kadanoff-Baym Eqs.



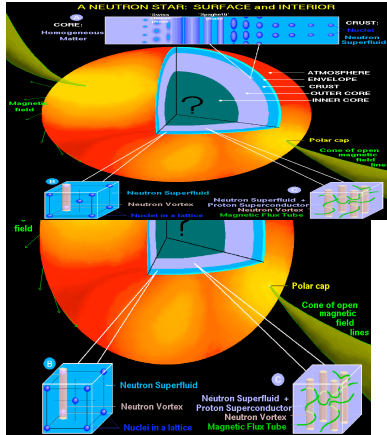
I.7 Importance of Symmetry energy: Supernovae



Supernova evolution: range of densities and temperatures and asymmetries:



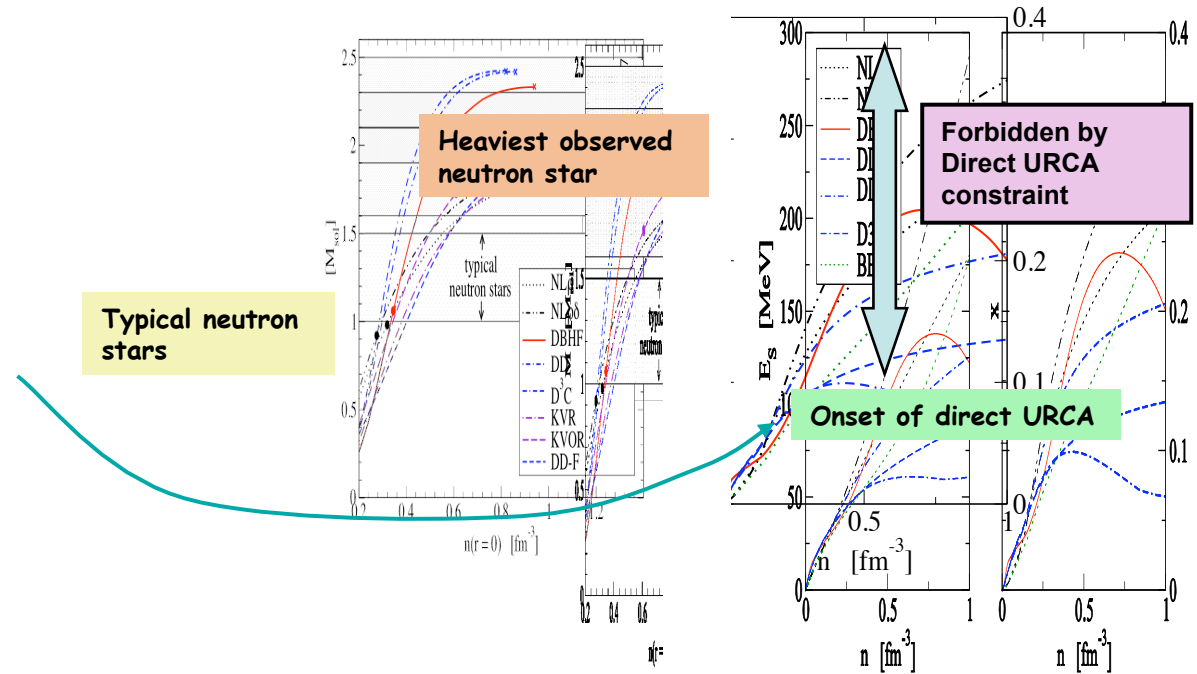
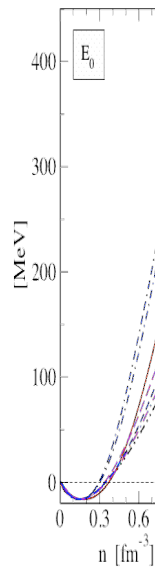
I.8 Importance of Symmetry energy: neutron stars



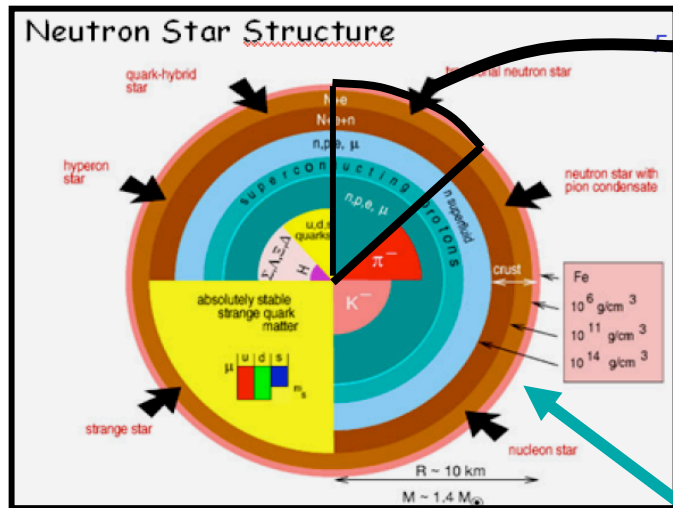
dependence of maximum neutron star mass on symmetry energy.

However, other constraints, e.g. cooling due to URCA process

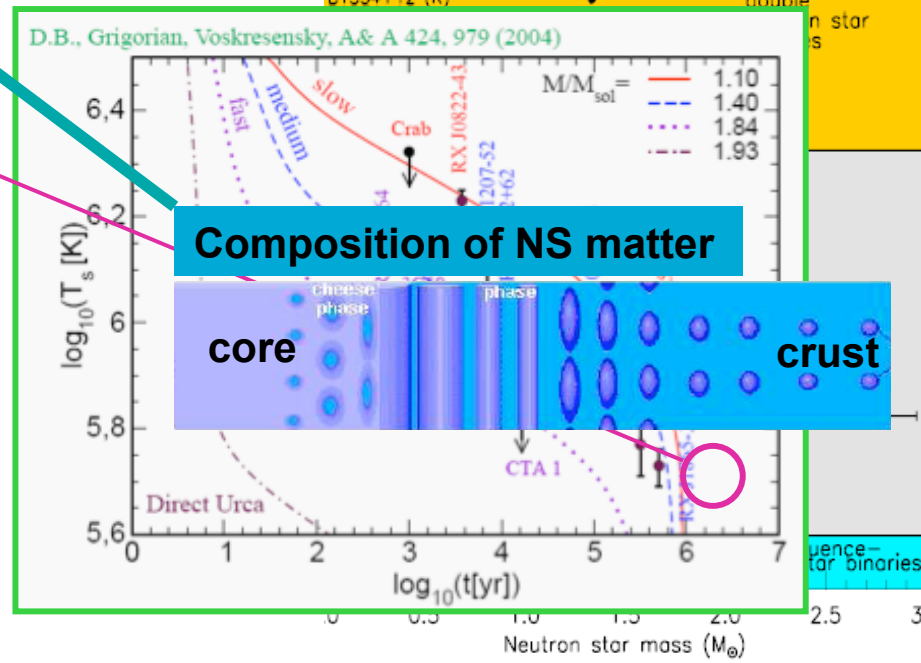
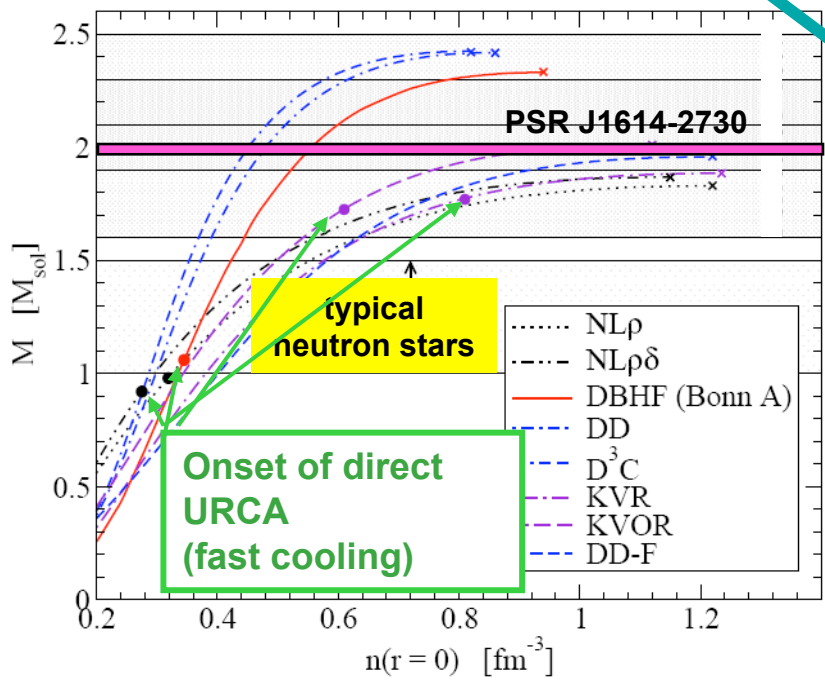
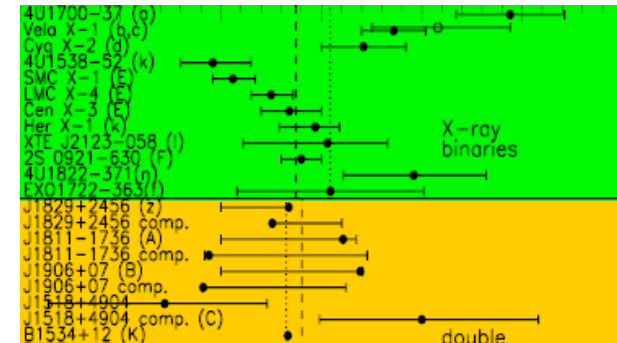
Klähn, Blaschke, Typel, Faessler, Fuchs, Gaitanos, Gregorian, Trümper, Weber, Wolter, Phys. Rev. C74 (2006) 035802



Neutron Stars: a Laboratory for the High-Density Equation-of-State

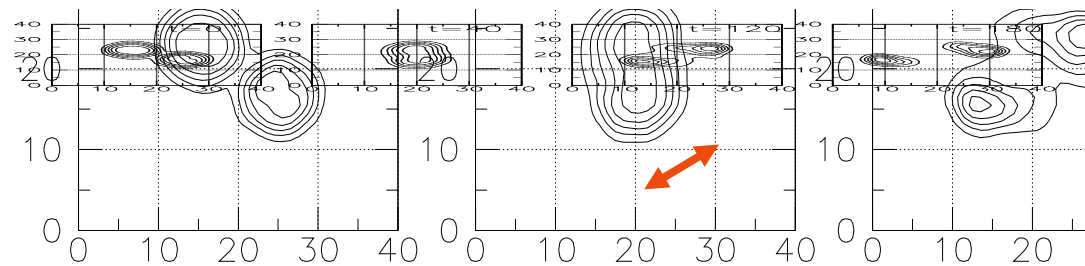


A normal NS (n,p,e) or exotic NS?



Composition of NS matter

3. Peripheral collisions: Isospin Equilibration through Neck:



Imbalance (or Rami, transport) ratio:

β asymmetry of residue ($i=PLF,TLF$)
(also for other isospin sens.quantities)

$$R_i = \frac{\beta_i^{mix} - \frac{1}{2}(\beta_i^{HH} + \beta_i^{LL})}{\frac{1}{2}(\beta_i^{HH} - \beta_i^{LL})}$$

Limiting values: $R=0$ complete equilibration

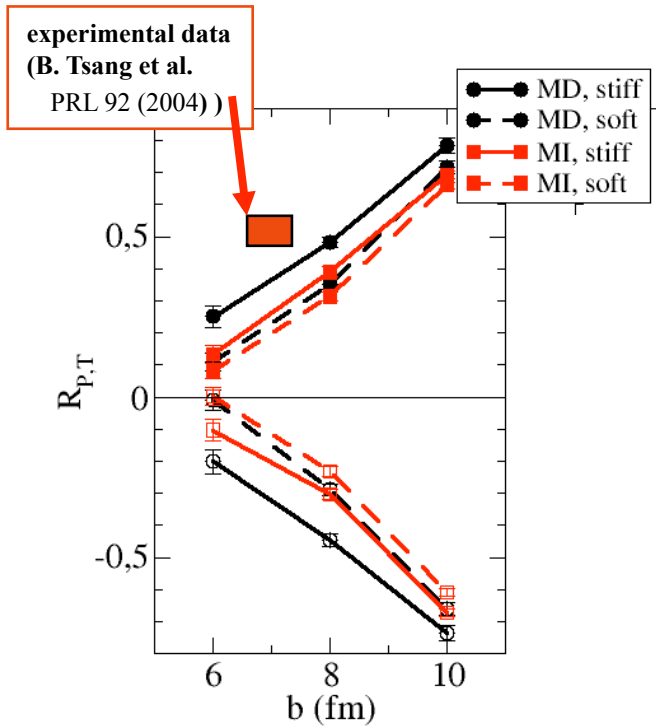
$R=+1$, complete transparency

Discussed extensively in the literature (talk by B. Tsang)

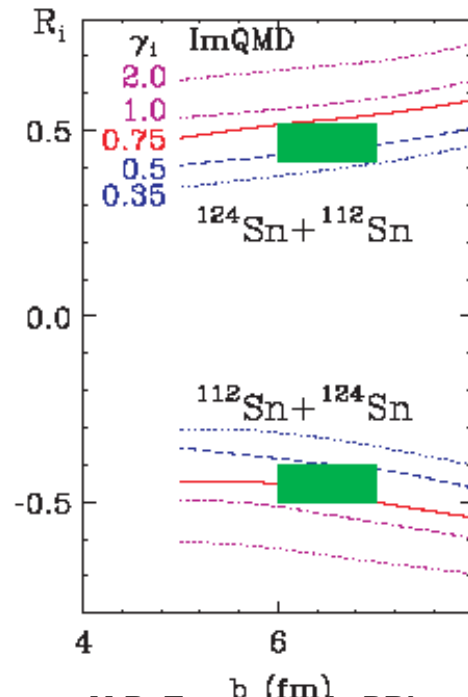
→ Momentum dependence important

3. Imbalance Ratios for Projectile/Target Residues: $^{112,124}\text{Sn} + ^{112,124}\text{Sn}$, 50 MeV

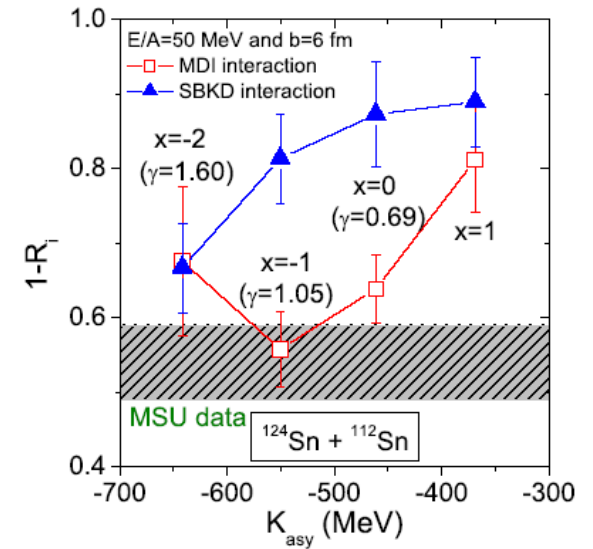
Comparison to other calculation:



J.Rizzo, et al., Nucl. Phys. A806 (2008) 79



M.B. Tsang, et al., PRL 102 (2008)



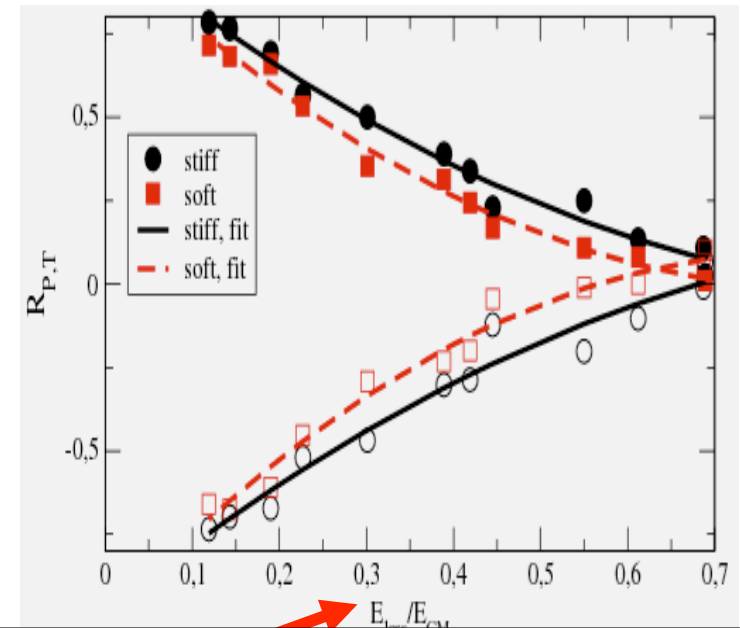
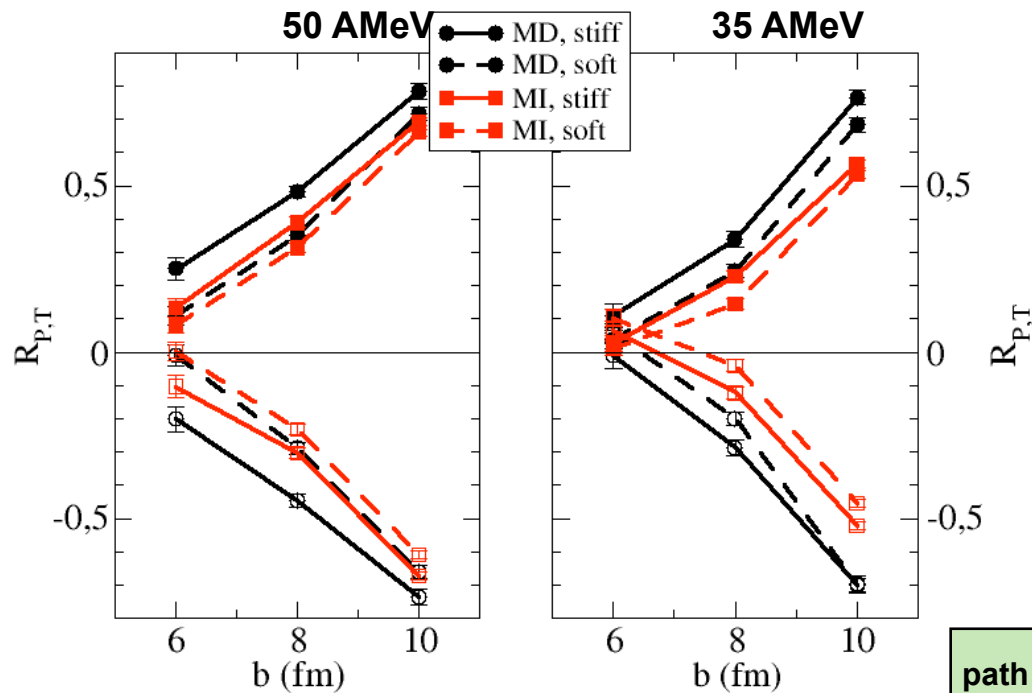
L.W.Chen, C.M.Ko, B.A.Li,
PRL 94, 032701 (2005)

1. disagreement with MSU calculation and also with B.A. Li ($\gamma \approx 1$)
3. much stronger impact parameter dependence
3. question still open

3. Peripheral collisions: Isospin Equilibration through Neck

Imbalance Ratios for Projectile/Target Residues: $^{112,124}\text{Sn} + ^{112,124}\text{Sn}$

$$R_i = \frac{\beta_i^{mix} - \frac{1}{2}(\beta_i^{HH} + \beta_i^{LL})}{\frac{1}{2}(\beta_i^{HH} - \beta_i^{LL})}$$



More equilibration (lower R) for MI interaction and lower energy, i.e. longer interaction time

Correlation with total energy loss = measure of interaction time

J.Rizzo, et al., Nucl. Phys. A806 (2008) 79

path towards equilibrium for observable x, e.g. R_i

$$\beta_{1,2}(t) - \beta_{eq} = (\beta_{1,2}(0) - \beta_{eq}) e^{-t/\tau};$$

$$\beta_{eq} = \frac{1}{2}(\beta_1 + \beta_2)$$

$$\Rightarrow R_{1,2}^\beta = \pm e^{-t/\tau}$$

$t \rightarrow$ contact time, $\tau \rightarrow$ dissipation time for obs. β

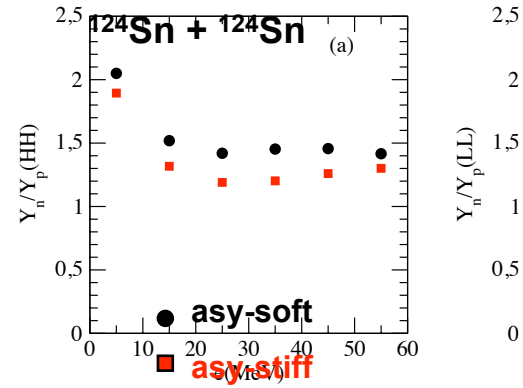
\rightarrow universal curve (for each asy_EOS)

III.7 Central Collisions: Ratios of emitted pre-equilibrium particles

Early emitted neutrons and protons reflect difference in potentials in expanded source, esp. ratio $Y(n)/Y(p)$.

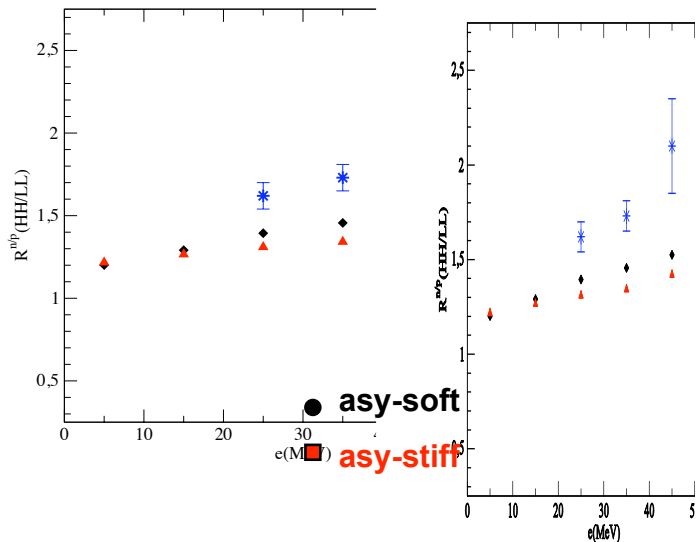
more emission for asy-soft, since symm potential higher

But magnitude of calculation too low

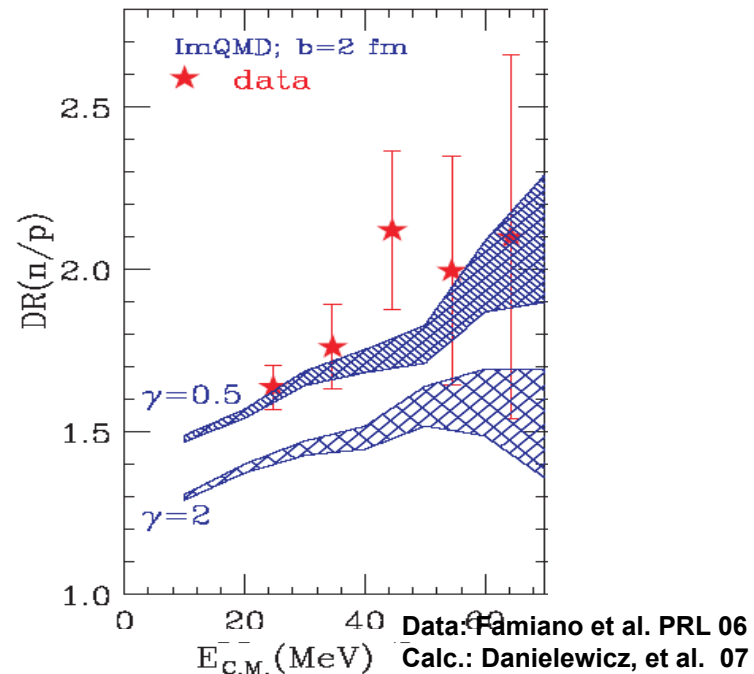


„Double Ratios“

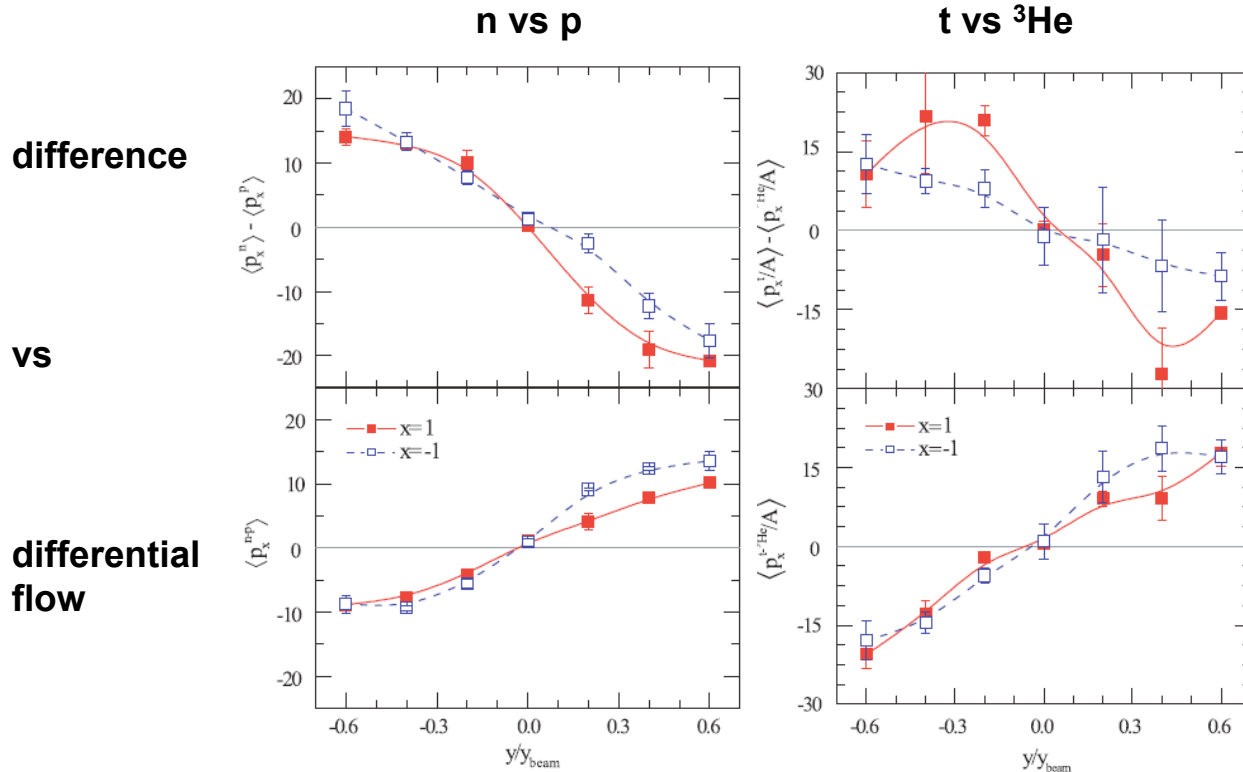
$$\frac{^{124}\text{Sn} + ^{124}\text{Sn}}{^{112}\text{Sn} + ^{112}\text{Sn}}$$



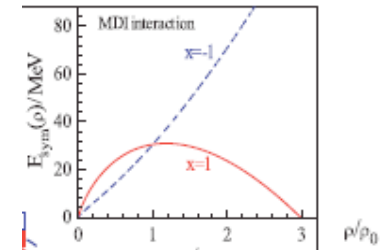
softer symmetry energy closer to data



Differential directed and transverse flow

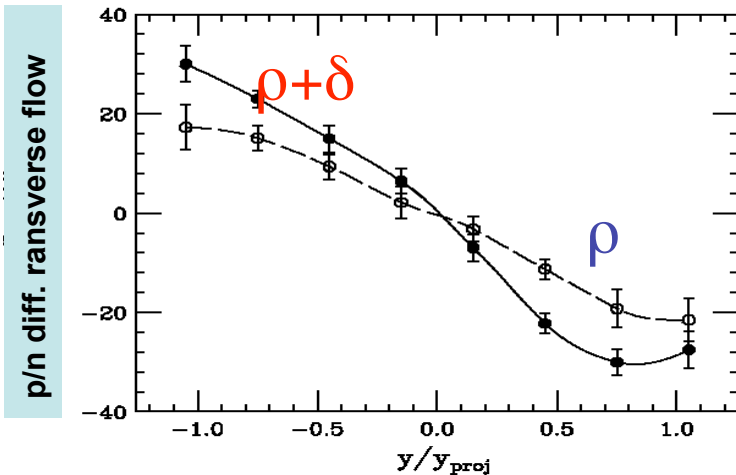


132Sn+124Sn
400 AMeV, b=5fm



Coulomb effects dominate,
→ difference flow negative,
more for soft SE

G.C.Yong, et al. PRC80 (09)



132Sn+132Sn, 1.5 AGeV, b=6fm

RMF model for SE, ρ^- and δ -meson

$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \vec{\nabla} \rho_3 = \frac{4}{\rho_B} E_{\text{sym}}^* \vec{\nabla} \rho_3$$

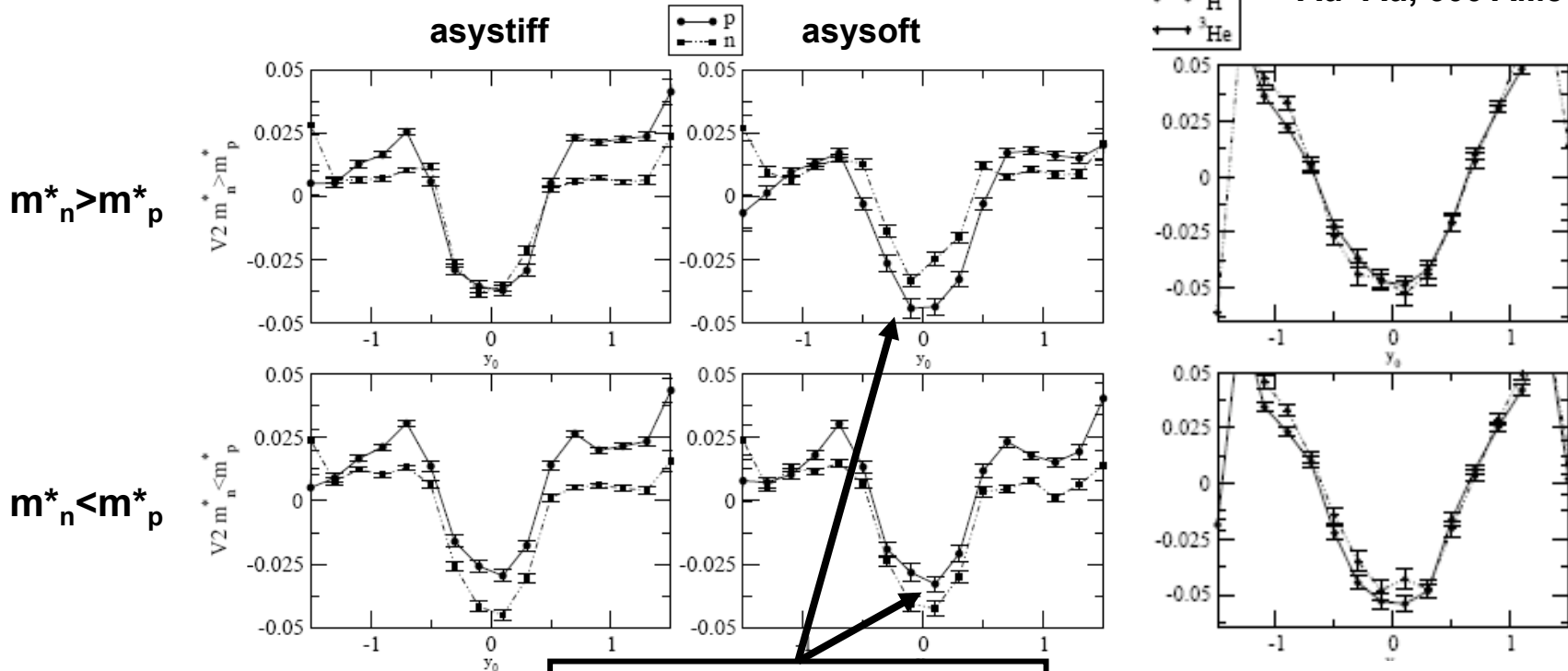
Stiffer E_{sym} \Rightarrow stiffer F_{np}

T. Gaitanos, M. Di Toro, et al., PLB562(2003)

Differential elliptic flow

Au+Au, 400 AMeV

Au+Au, 600 AMeV



$m_n^* > m_p^*$

$m_n^* < m_p^*$

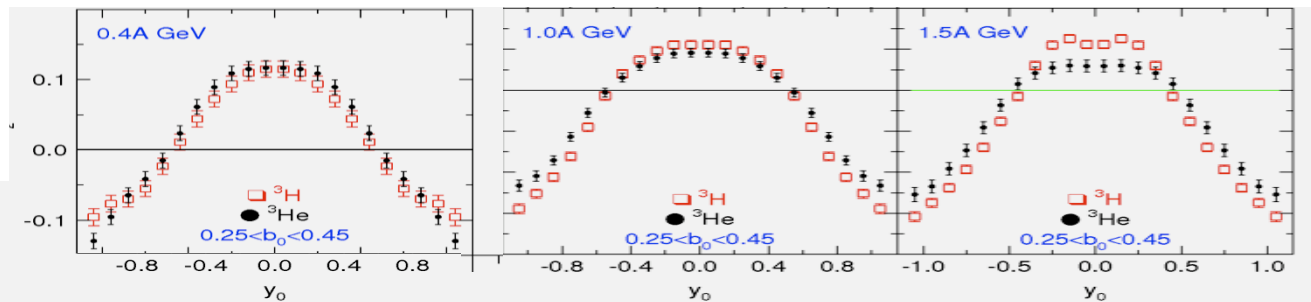
Inversion of elliptic flows because of inversion of potentials with effective mass

t-³He pair similar but weaker

V.Giordano, M.Colonna et al., arXiv 1001.4961, to appear PRC

W. Reisdorf, ECT*, May 09

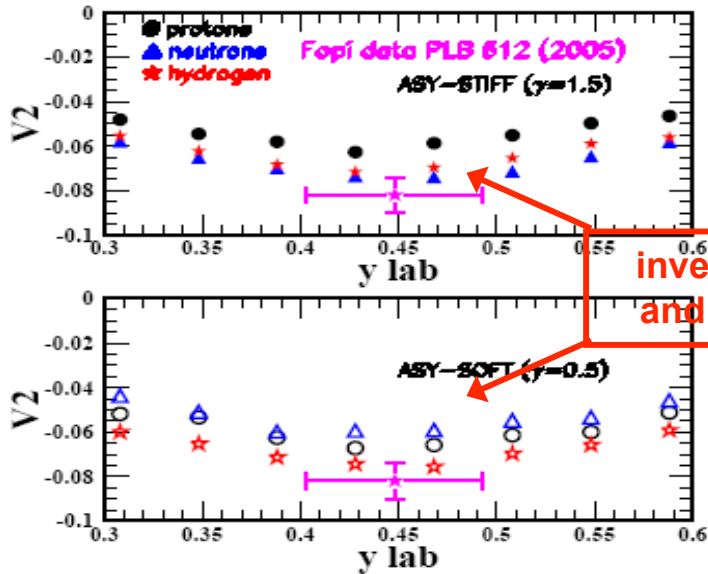
Indication of experimental effect



Elliptic flow perhaps more sensitive, since determined by particles that are emitted perp to the beam direction

ASYEOS: Hunting the high density SE with v_2

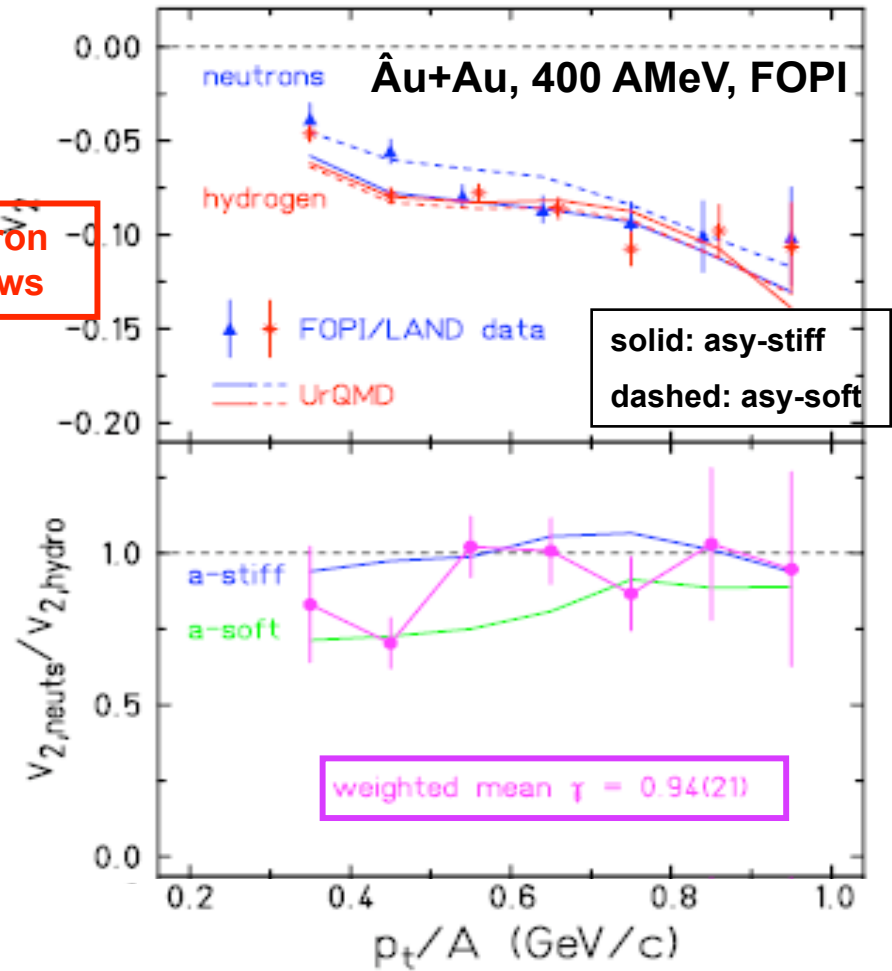
Early FOPI data (Y. Leifels)



Indication of a asy-stiff
Symmetry energy at
suprasaturation density

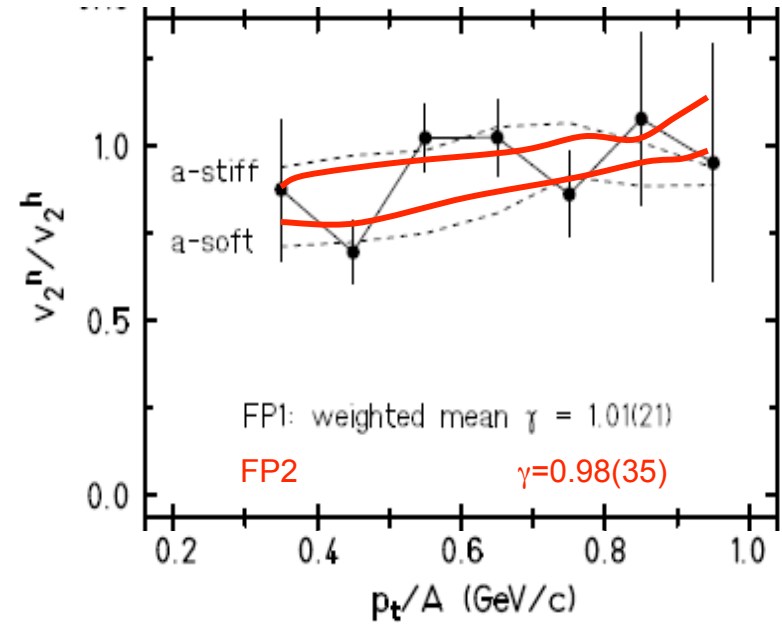
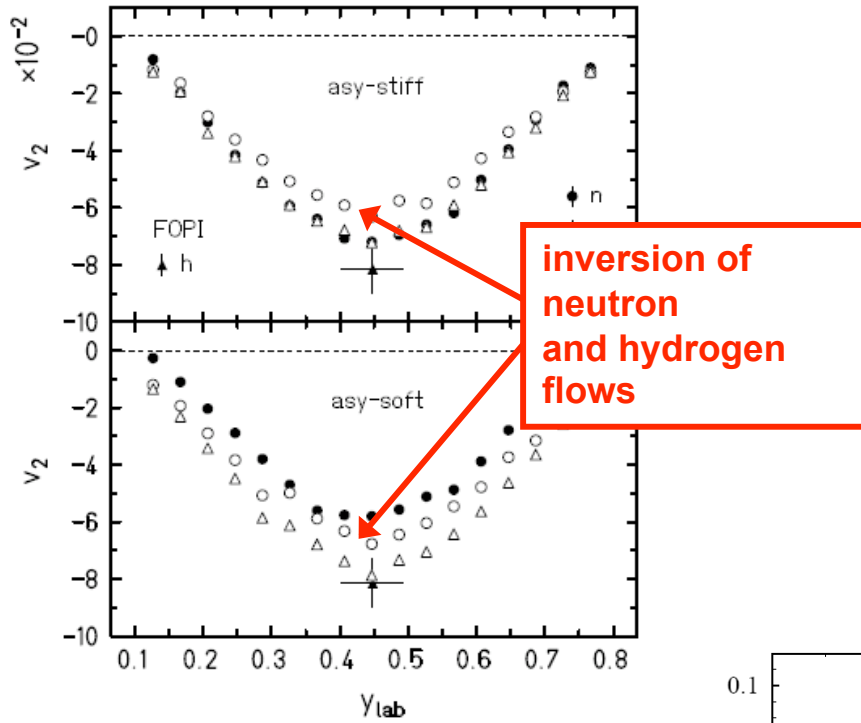
UrQMD calc.

W. Trautmann, ..., Q.F. Li, ...P. Russotto
PPNP 62 (425)



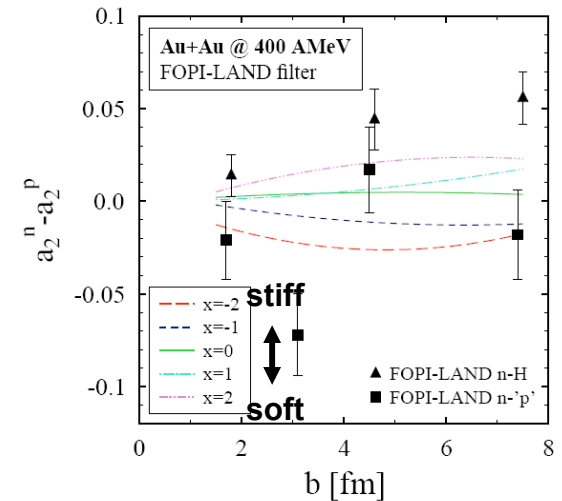
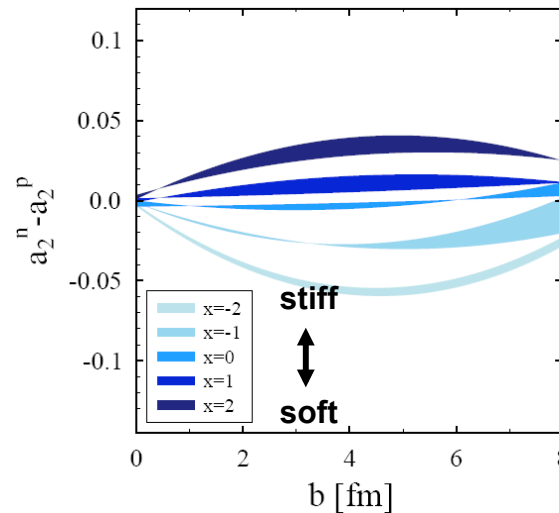
ASY-EOS: Hunting the high density symmetry energy with v_2

Russotto, et al., PLB 697, 471 (2011)



Recent study by
Cozma, arxiv
1102.2728

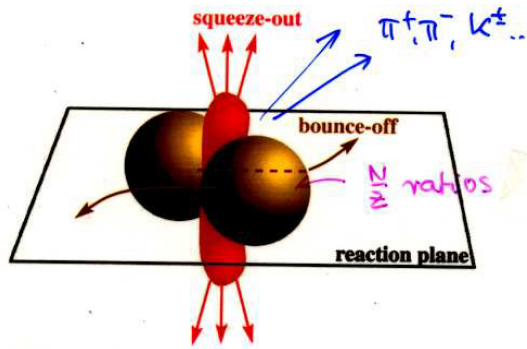
band: soft vs. stiff eos of
symmetric matter



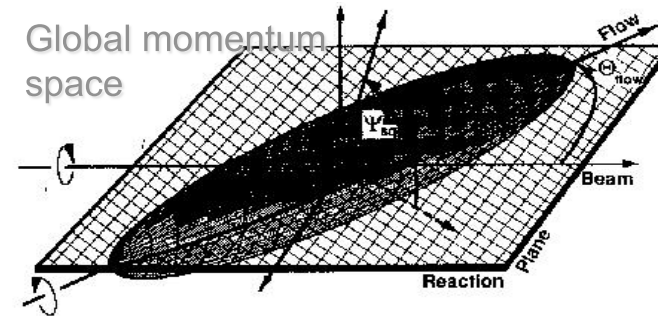
IV.1

Heavy Ion Collisions at Relativistic Energies: Observables

Observables



Analysis of momentum tensor : „flow“



$$Flow: N(\Theta; y, p_t, b) = N_0 (1 + v_1(y, p_t) \cos \Theta + v_2(y, p_t) \cos 2\Theta + \dots)$$

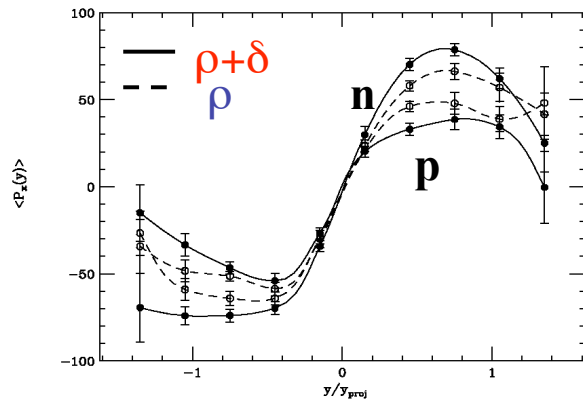
V1: Sideward flow

V2: Elliptic flow

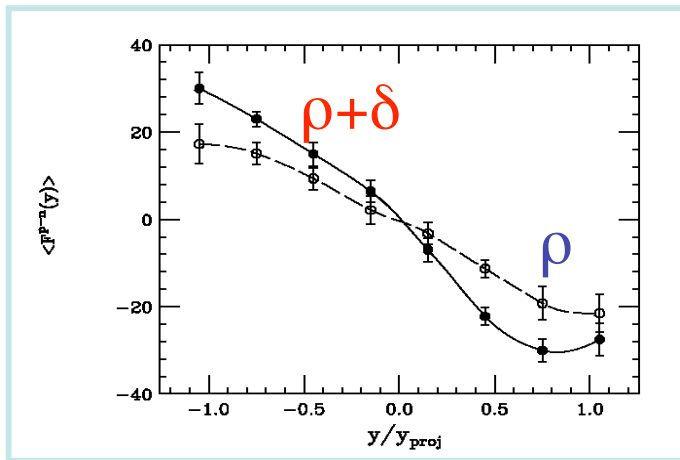
Yields of produced particles,
in particular ratios of isospin partners

Asymmetric matter: Differential directed and elliptic flow

$^{132}\text{Sn} + ^{132}\text{Sn}$ @ 1.5 AGeV $b=6\text{fm}$



differential directed flow



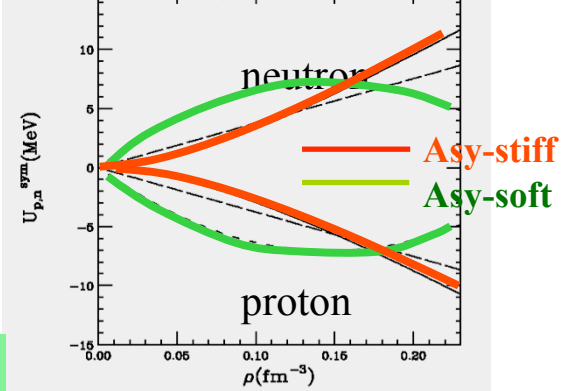
Proton-neutron differential flow

$$F_{n-p}^x(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^x w_i),$$

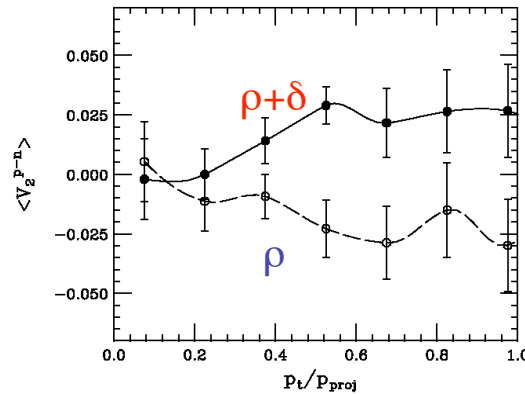
$$w_i = +1(-1) \text{ for neutron (proton)}$$

and analogously for elliptic flow

for ^{124}Sn "asymmetry" $I=0.2$



differential elliptic flow



✿ Difference at high $p_t \leftrightarrow$ first stage, dynamical boosting of vector contribution

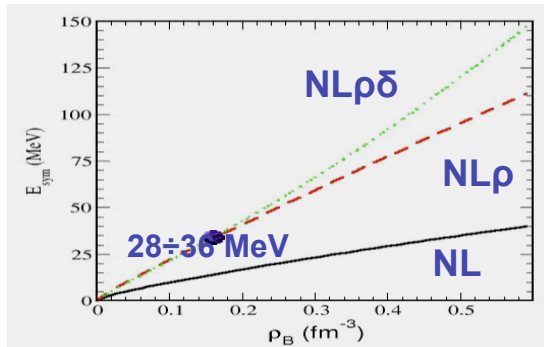
$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[\gamma f_\rho - \frac{f_\delta}{\gamma} \right] \vec{\nabla} \rho_3 = \frac{4}{\rho_B} E_{\text{sym}}^* \vec{\nabla} \rho_3$$

T. Gaitanos, M. Di Toro, et al., PLB562(2003)

IV.2 Heavy Ion Collisions at Relativistic Energies

Probing the symmetry energy with flow in HIC: Iso-flows

RMF model with ρ and σ mesons:



$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^{*2}} + \frac{1}{2} \left[f_\rho - f_\delta \left(\frac{M^*}{E^*} \right)^2 \right] \rho_B$$

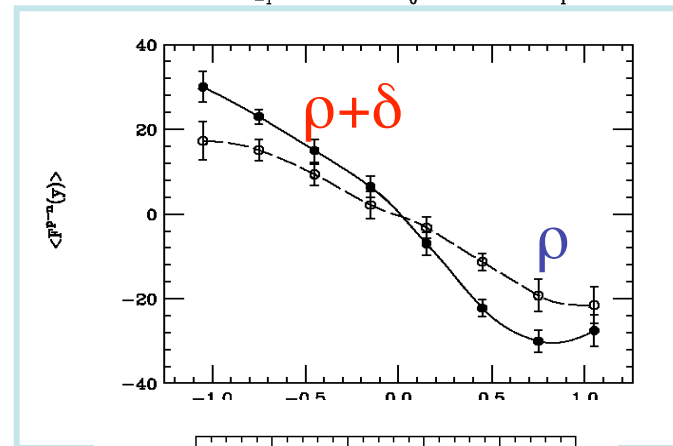
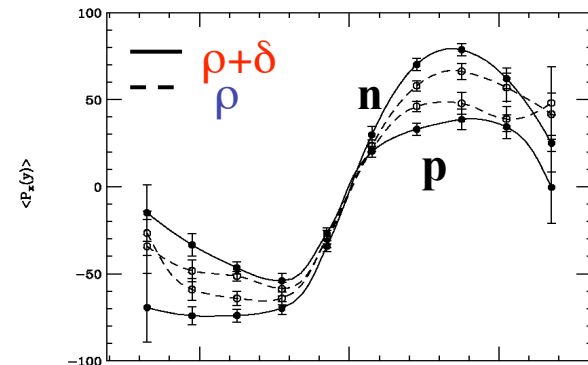
Proton-neutron differential flow

$$F_{n-p}^x(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} (p_i^x w_i),$$

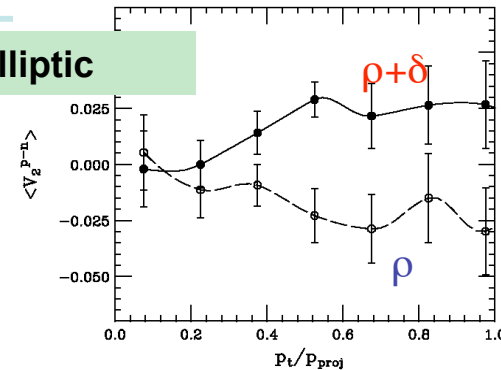
$$w_i = +1(-1) \text{ f omeutron (proton)}$$

and analogously for elliptic flow

Greater $E_{sym} \Rightarrow$ stiffer F_{np}



differential elliptic flow



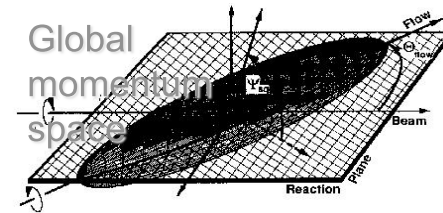
Heavy Ion Collisions at Relativistic Energies: "Flow"

Fourier analysis of momentum tensor : „flow“

$$N(\theta, y, b) = N_0(1 + v_1(y, b)\cos\theta + v_2(y, b)\cos 2\theta + ..)$$

v_1 : sideward flow

v_2 : elliptic flow



or transverse flow $\langle p_x/A \rangle(y) \equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_x^i/A(y),$

To investigate symmetry energy:

differences of flow (more sensitive for clusters):

or differential flow

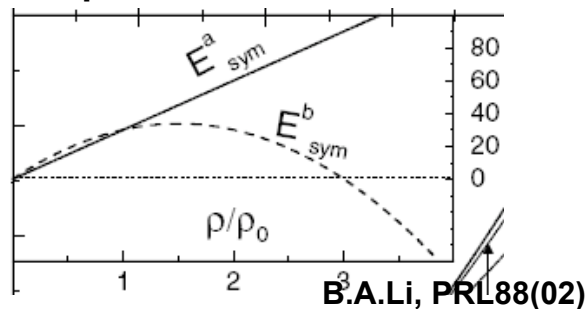
$$\langle p_x^t/A \rangle - \langle p_x^{3\text{He}}/A \rangle = \frac{1}{N_t} \sum_{i=1}^{N_t} p_x^i/A - \frac{1}{N_{3\text{He}}} \sum_{i=1}^{N_{3\text{He}}} p_x^i/A.$$

$$\langle p_x^{t-3\text{He}}/A \rangle = \frac{1}{N_t + N_{3\text{He}}} \left(\sum_{i=1}^{N_t} p_x^i/A - \sum_{i=1}^{N_{3\text{He}}} p_x^i/A \right)$$

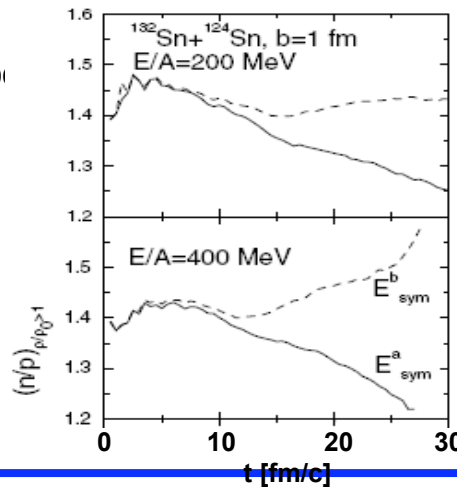
$$= \frac{N_t}{N_t + N_{3\text{He}}} \langle p_x^t/A \rangle - \frac{N_{3\text{He}}}{N_t + N_{3\text{He}}} \langle p_x^{3\text{He}}/A \rangle,$$

(analogous for $v_1, v_2, ..$)

Effects in flow complimentary with resp. pre-equilibrium emission:



B.A.Li, PRL88(02)



Stiffer SE emits more pre-eq neutrons, thus residual system is less neutron rich

→ Isospin fractionation at high energies