Cluster structures and Hoyle-analog states in ^{11}B

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1. **Introduction**
   ^{12}C Hoyle state (2^{nd} 0^{+})
   ^{16}O structure
2. **$^{11}B=\alpha+\alpha+t$ structure**
   ($^{13}C=3\alpha+n$)
3. **Summary**

Introduction

- **Cluster picture as well as mean-field picture is important viewpoint to understand structure of light nuclei.**
- **Structure of light nuclei**

  Cluster states + Shell-model-like states

  Microscopic cluster models, AMD,....

  ^{8}Be=2α, ^{12}C=3α , ^{16}O=^{12}C+α, 4α, …

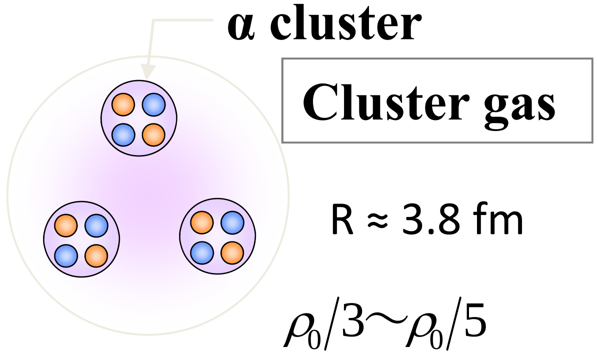
- **α-condensate states in 4n nuclei:**

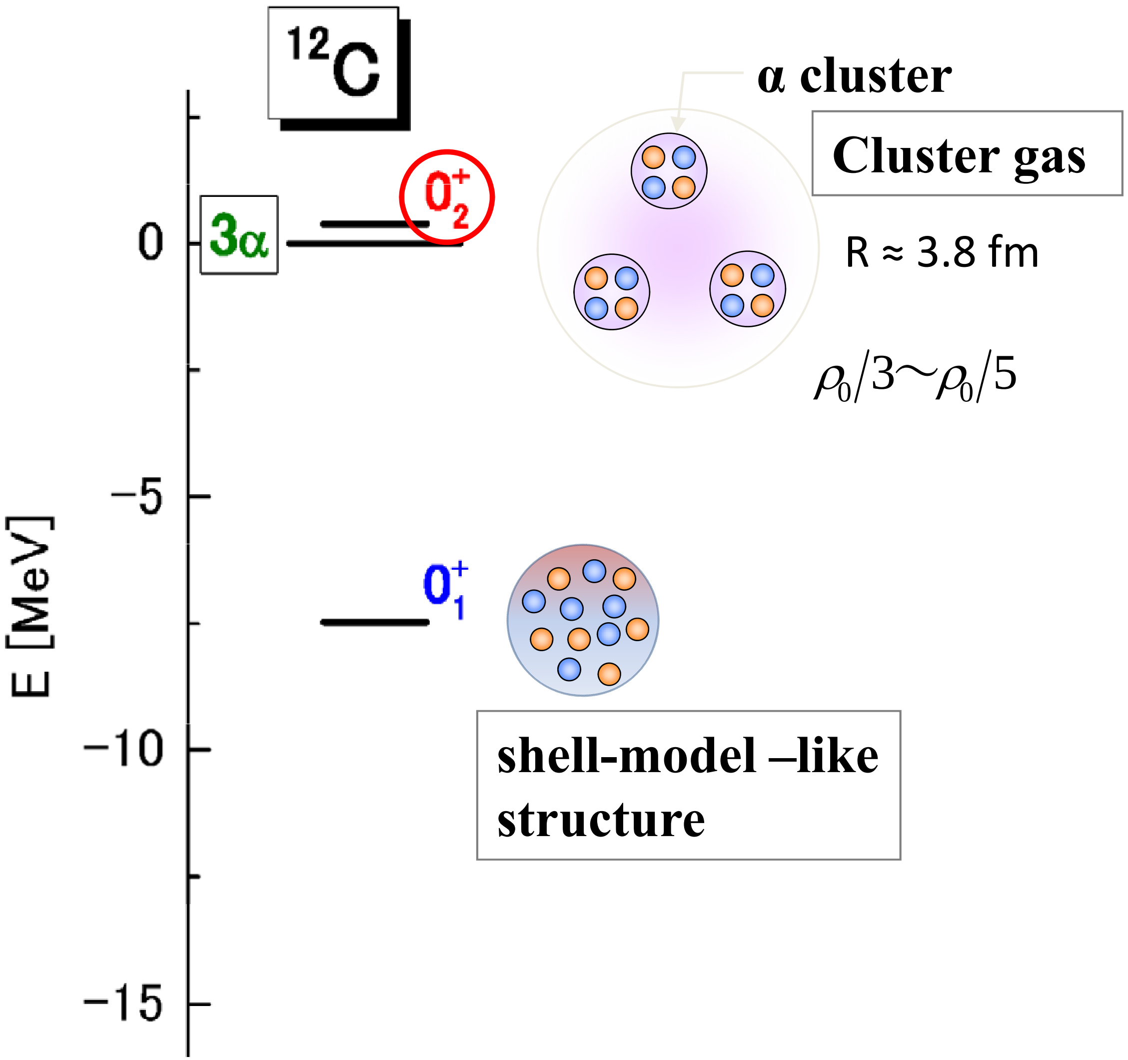
  α-gas-like state described by a product state of α-particles, all with their c.o.m. in (0S) orbit.

  Typical states:

  ^{12}C: Hoyle state (2nd 0^{+})

  Tohsaki, Horiuchi, Schuck, Roepke, Phy. Rev. Lett.87 (2001)
  Funaki et al., PRC (2003)
  Yamada et al., EPJA (2005), Matsumura et al.,NPA(2004)

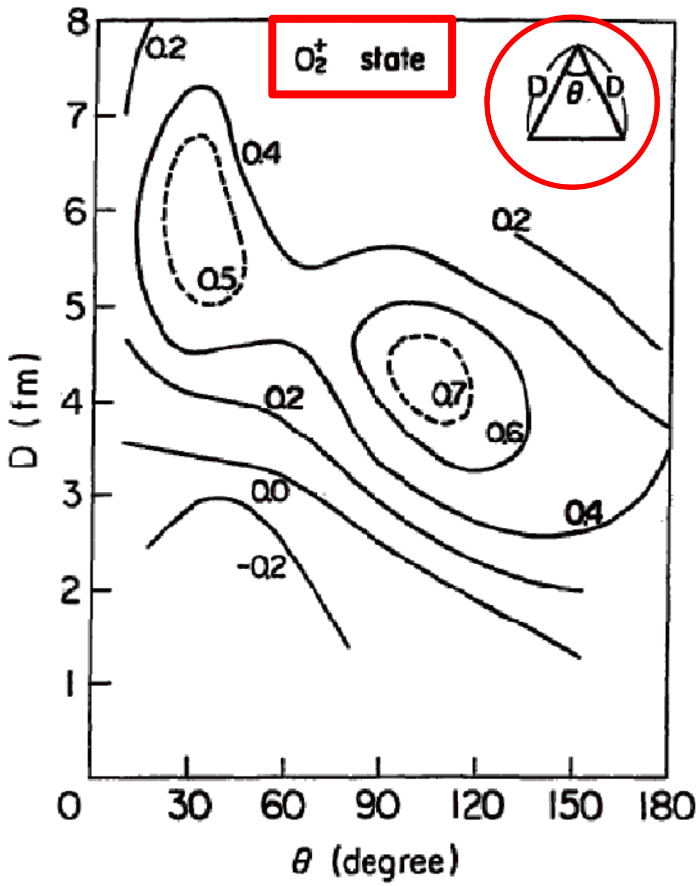




12C
α cluster
Cluster gas
3α
0_2^+
0
R ≈ 3.8 fm
$\rho_0/3 \sim \rho_0/5$
−5
0_1^+
E [MeV]
−10
shell-model –like structure
−15

Overlap 3α GCM

$$\langle \text{Brink wf} \,|\, \text{exact } 0_2^+ \text{ state} \rangle$$



Uegaki et al, PTP57(1977)

α-gas-like nature of Hoyle state

Kamimura et al., (1977): 3α RGM

Uegaki et al, PTP57(1977): 3α GCM

The 0^+_2 state has a distinct clustering and has **no definite spatial configuration**.

Chernykh, Feldmeir et al., PRL98(2007)

UCOM + FMD, 3α RGM

About **55 components** of the Brink-type wave functions are needed to reproduce the full RGM solution for the Hoyle state.

Tohsaki,Horiuchi,Schuck,Roepke, PRL87(2001)

THSR wave function: α-condensate-type cluster wf

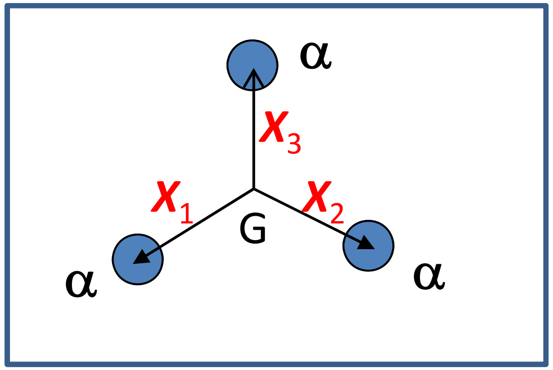
1 base THSR :

$$\left| \langle \Phi_{3\alpha}^{THSR} \,|\, \text{exact } 0_2^+ \text{ state } (3\alpha\text{RGM}) \rangle \right|^2 \approx 99\%$$

Funaki et al., PRC67, (2003)

THSR wave function

Tohsaki, Horiuchi, Schuck, Roepke, PRL (2001)

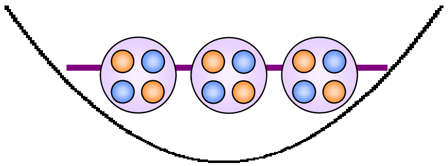


$(0S)^3_\alpha$

$$\Phi_{3\alpha}(B) = \mathcal{A}\left\{\prod_{i=1}^{3}\left[\exp\left(-\frac{2}{B^2}\vec{X}_i^{\,2}\right)\phi(\alpha_i)\right]\right\}$$

B: parameter

Condensed into the lowest orbit



$(0S)^3_\alpha$

$B \to \infty \qquad \Phi_{3\alpha}(B) \to \prod_{i=1}^{3}\left[\exp\left(-\frac{2}{B^2}\vec{X}_i^{\,2}\right)\phi(\alpha_i)\right]$

$B \to b \qquad \Phi_{3\alpha}(B) \to (0s)^4(0p)^8$

b: nucleon size parameter

Funaki et al., PRC (2003)

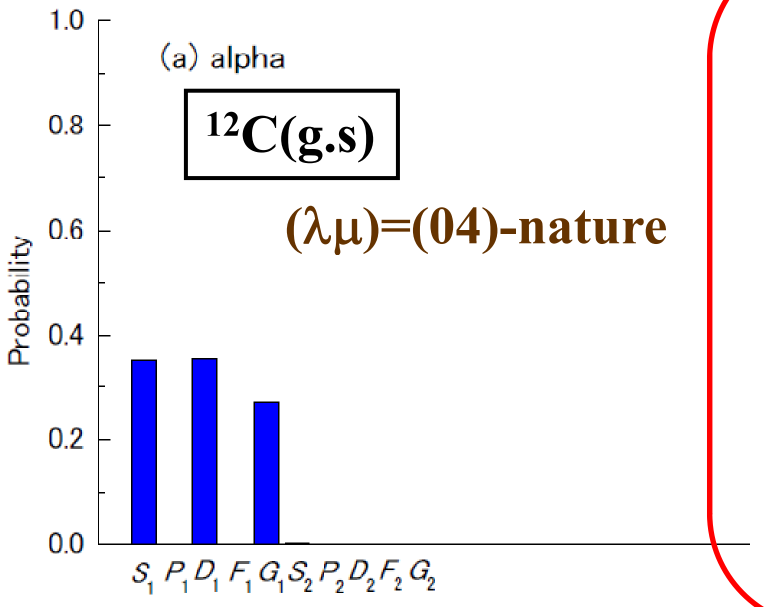
$$\left|\left\langle \Phi_{3\alpha}^{\mathrm{THSR}}(B)\middle|\,\textbf{exact } \mathbf{0}_2^+ \textbf{ state } (3\alpha\mathrm{RGM})\right\rangle\right|^2 \approx 0.9$$

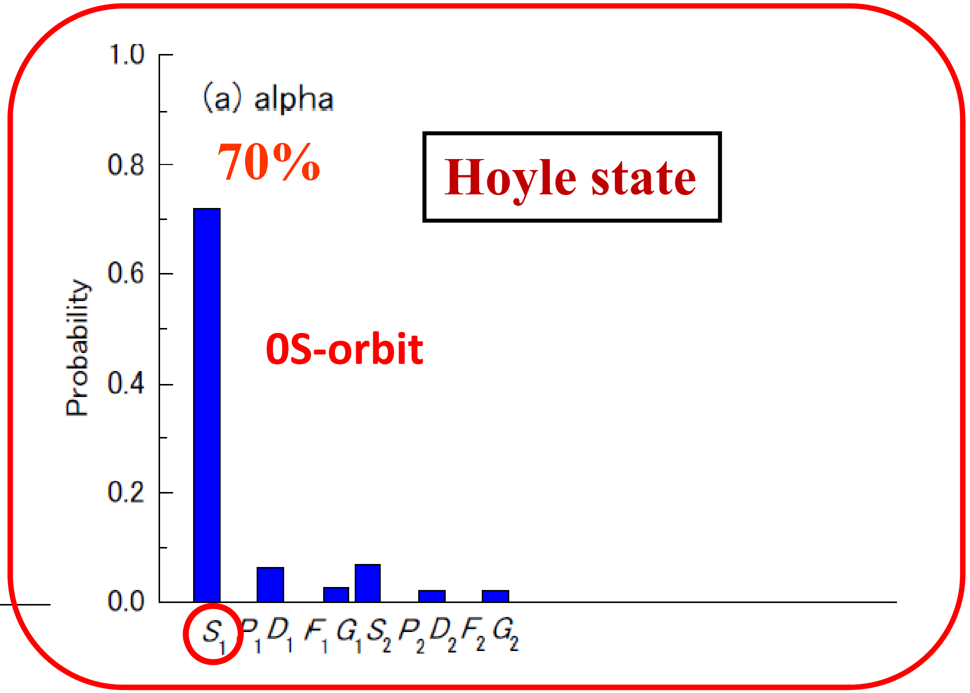
3α RGM: Kamimura & Fukushima (1978)

Deformation (B_x,B_y,B_z) ➔ 0.999

R **≈ 3.8 fm: alpha-gas-like structure**

Occupation probabilities of α-orbits in ^{12}C





SU(3) nature: confirmed by no-core shell model , Dytrych et al., PRL98 (2007)

Yamada & Schuck EPJA26 (2005) with 3α OCM wf
Matsumura & Suzuki, NPA739 (2004)
Funaki et al., PRC (2010) with 3α THSR wf

Single cluster density matrix: $\rho(\mathbf{r},\mathbf{r}')$

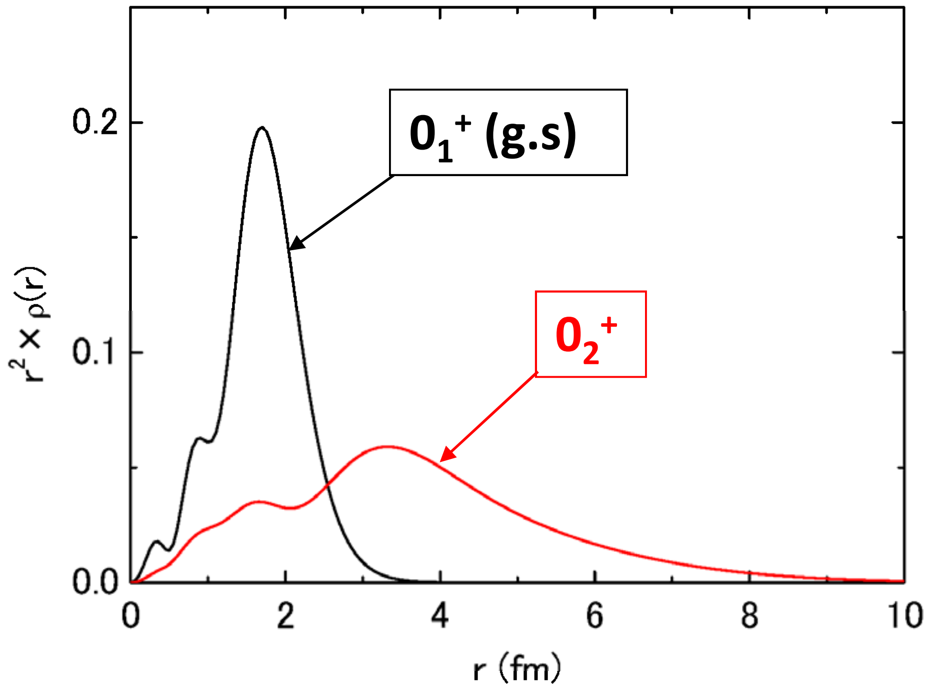
$$\int d\mathbf{r}'\rho_\alpha(\mathbf{r},\mathbf{r}')\varphi_\alpha(\mathbf{r}') = \lambda_\alpha\varphi_\alpha(\mathbf{r}),$$

$\varphi(\mathbf{r})$: **single-cluster orbital w.f.**

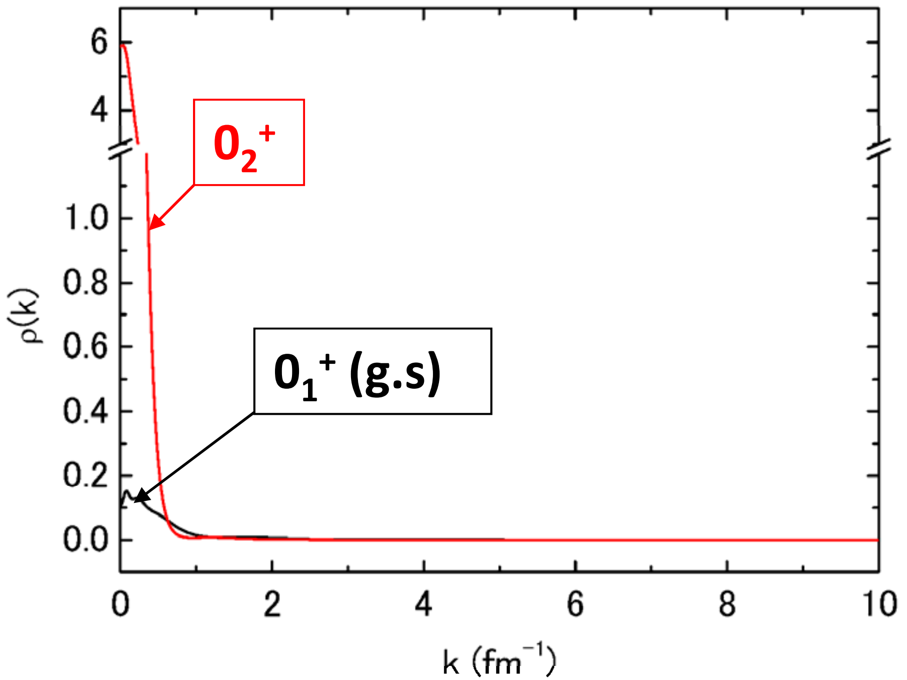
λ : **occupation probability**

α-density distribution and α-momentum distribution in ^{12}C

Density × r^2



Momentum distribution

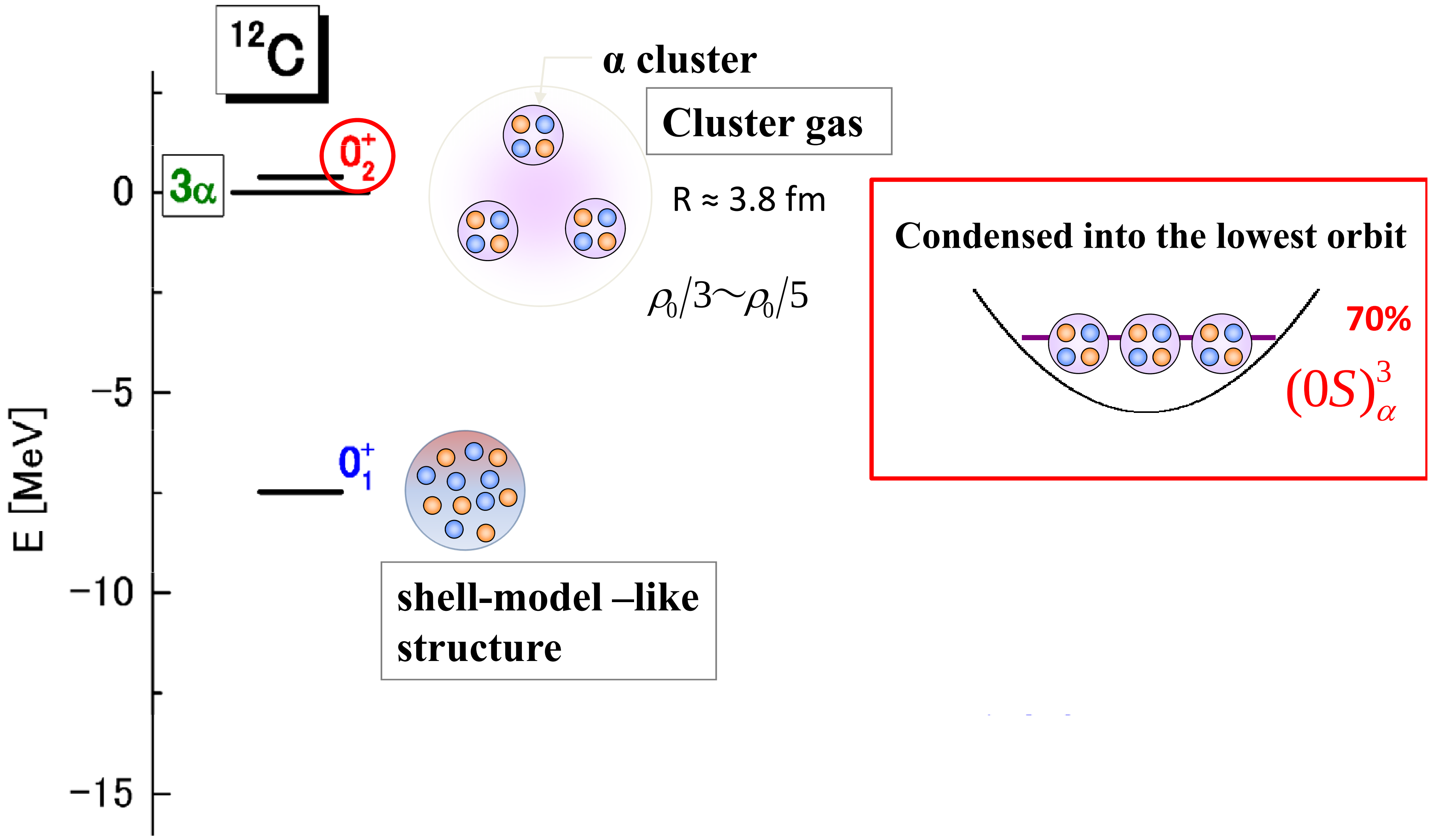


Compact (0_1^+) vs. Dilute (0_2^+)

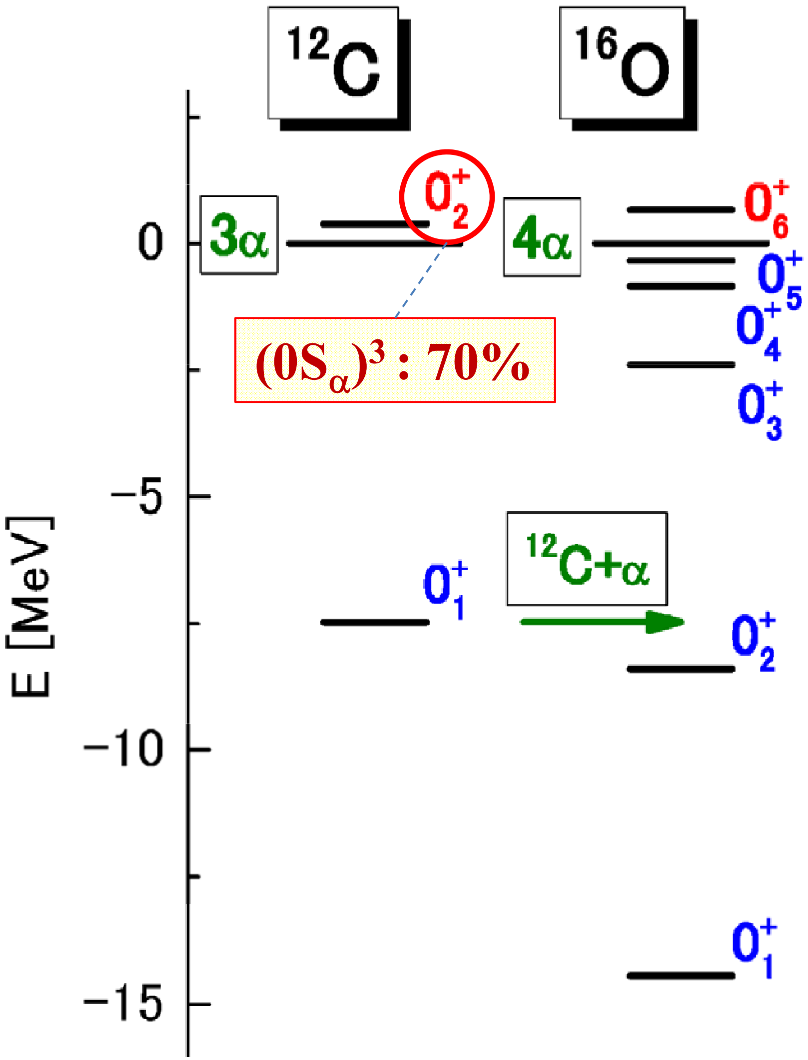
0_2^+ state: δ function-like

Similar to dilute atomic cond.

Yamada & Schuck EPJA26 (2005) with 3α OCM wf



12C
α cluster
Cluster gas
R ≈ 3.8 fm
$\rho_0/3 \sim \rho_0/5$
3α
0_2^+
0_1^+
0
−5
−10
−15
E [MeV]
shell-model –like structure
Condensed into the lowest orbit
70%
$(0S)_\alpha^3$



^{12}C
^{16}O
3α
4α
0_2^+
0_6^+
0_5^+
0_4^+
0_3^+
$(0S_\alpha)^3$: 70%
0_1^+
$^{12}C+\alpha$
0_2^+
0_1^+
E [MeV]
0
−5
−10
−15

Cluster-model analyses of ^{16}O

- $\alpha + ^{12}C$ OCM

  Y. Suzuki, PTP55 (1976), 1751

- $\alpha + ^{12}C$ GCM

  M. Libert-Heinemann, D. Bay et al., NPA339 (1980)

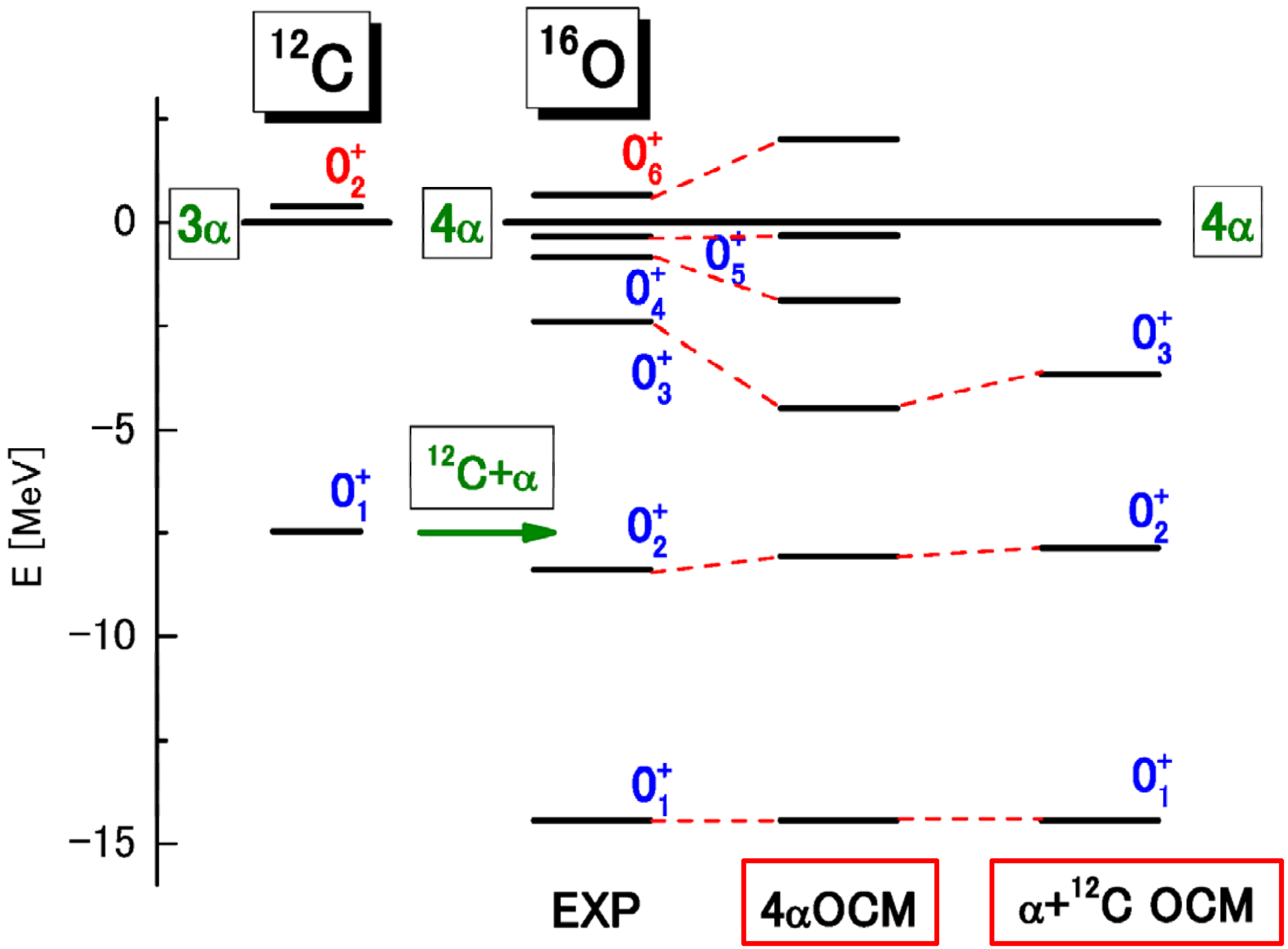
- 4α THSR wf Not include $\alpha + ^{12}C$ configuration.

  Tohsaki, Horiuchi, Schuck, Roepke, PRL87 (2001)
  Funaki, Yamada et al., PRC82(2010)

- 4α OCM 4α-gas, $\alpha + ^{12}C$, shell-model-like configurations

  Funaki, Yamada et al., PRL101 (2008)

  Reproduction of lowest six 0+ states up to 4α threshold (15MeV)



Funaki et al.,
PRL101 (2008)

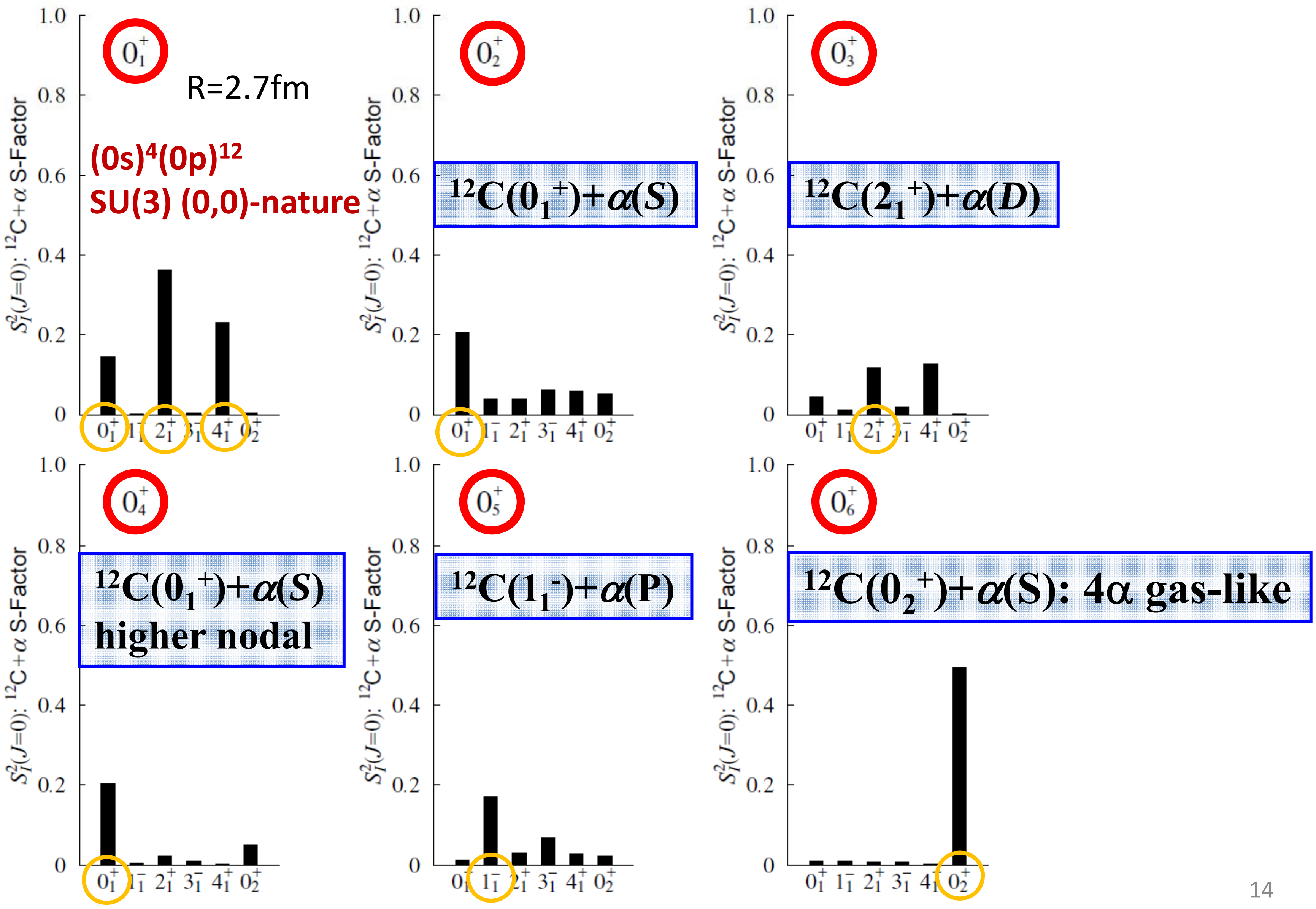
Suzuki,
PTP55 (1976)

		Experimental data				4α OCM		
	E_x [MeV]	R [fm]	M(E0) [fm²]	Γ [MeV]		R [fm]	M(E0) [fm²]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

over 15%
of total EWSR

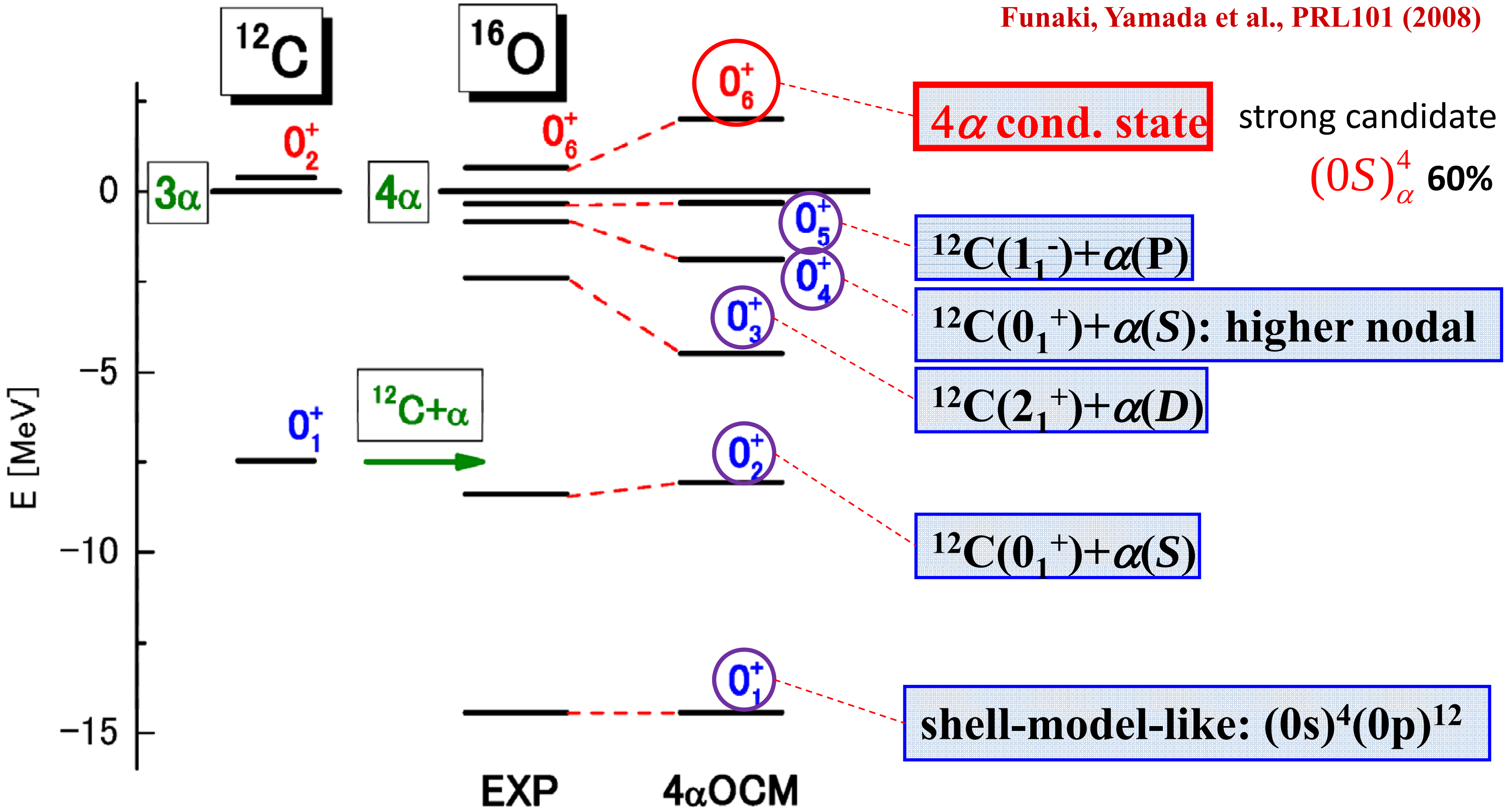
20%
of total EWSR

Components of α+^{12}C(L^{π}) channel in 0^+ states of ^{16}O



4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Hoyle-analog states in A $\neq$ 4n nuclei (^{11}B and ^{13}C)

Search for Hoyle-analogue states in A≠4n nuclei

Definition of Hoyle-analogue state:

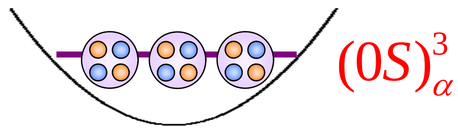
A cluster-gas-like state described mainly by a product state of clusters, all with their c.o.m. in respective 0S orbits

- **A=4n nuclei**

  ^{12}C=3α : $(0S_\alpha)^3$ 70%

  ^{16}O=4α : $(0S_\alpha)^4$ 60%

Condensed into lowest orbit



- **A≠4n nuclei, for example,**

  ^{11}B=2α+t : $(0S_\alpha)^2(0S_t)$

  ^{13}C=3α+n : $(0S_\alpha)^3(0S_n)$

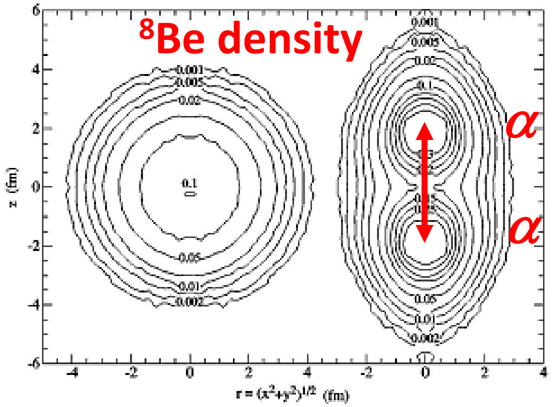
exit or not?

Purposes of present study

- **Cluster structures in A≠4N nuclei: ^{11}B (^{13}C)**
- **Hoyle-analogue states? Conditions of appearance?**

  $^{11}B = 2\alpha + t \; : \; (0S_{\alpha})^2 (0S_t)$

  $^{13}C = 3\alpha + n \; : \; (0S_{\alpha})^3 (0S_n)$



R.B. Wiringa et al., PRC(2000)

- **Which states of ^{11}B (^{13}C) correspond to the Hoyle-analogue?**
- **What happens in ^{11}B (^{13}C) when an extra triton (neutron) is added into ^{8}Be=2α (Hoyle=3α) state?**

α-*t* potentials: parity-dependent

odd waves : attractive enough to make resonances/bound states

even waves: weakly attractive

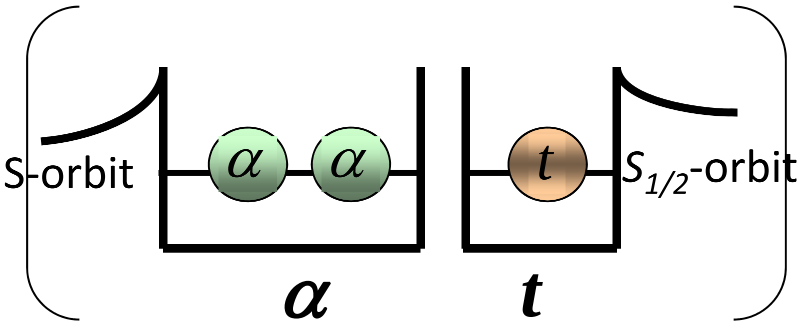
^{11}B

	triton		2α part in ^{11}B	$J^{\pi}(^{11}\mathrm{B})$
^{8}Be(2α) +	*p*-orbit	→	reduced:	1/2-,3/2$^-$
	s-orbit		not so changed:	$1/2^+$

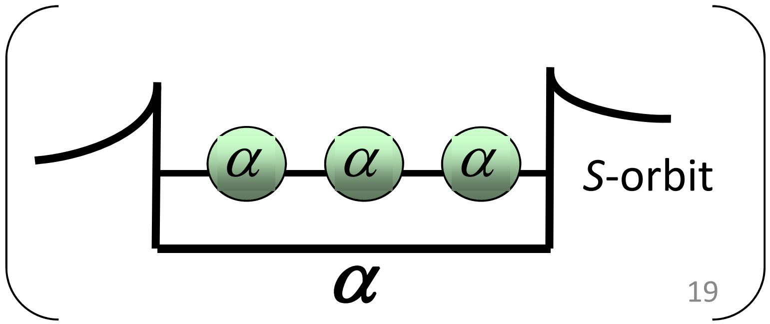
Thus, one can expect cluster-gas-like states in 1/2+.

^{11}B $1/2^+$: Hoyle-analogue

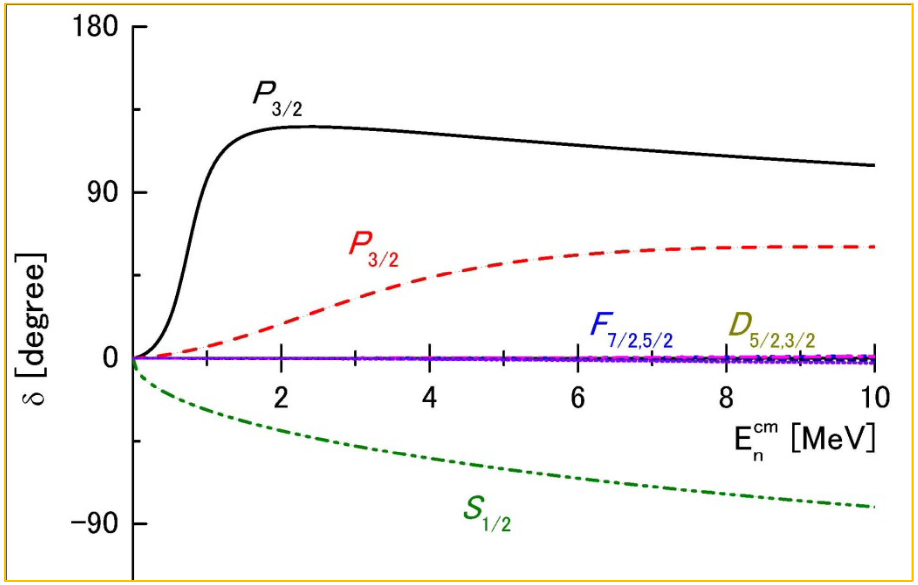
exists or not ?



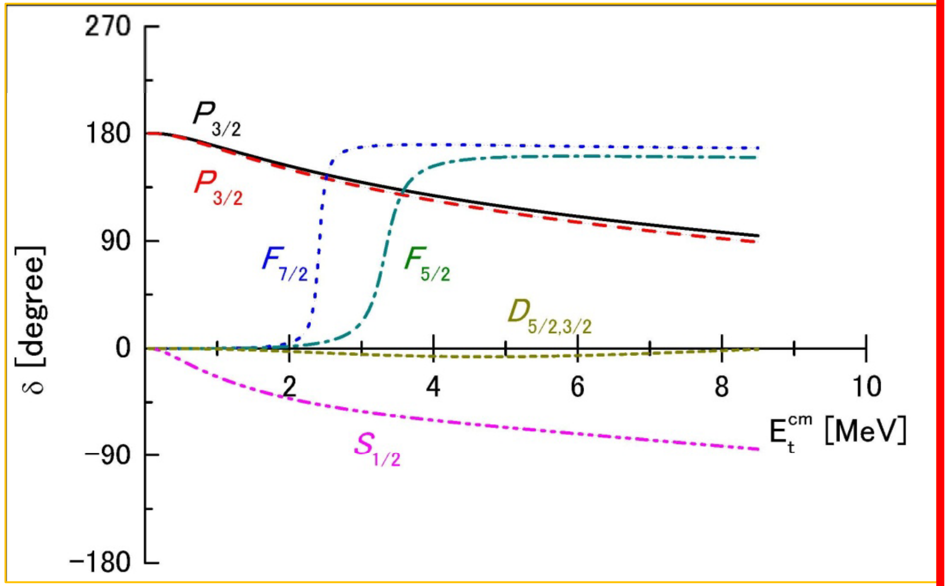
^{12}C 2nd 0^+ (Hoyle)



α-n phase shifts



α-triton phase shifts



α-n potential

(1) *P*-wave: attractive

(2) *S*-wave: weakly attractive

α-triton potential

(1) *P*-, *F*-waves: attractive

(2) *S*-, *D*-waves: weakly attractive

Parity-dependent potentials

Formulation of α-condensation
---THSR wf and OCM approach ---

THSR and OCM:
based on Resonating Group Method (RGM)

(1) THSR: fully microscopic approach

(2) OCM : semi-microscopic approach
an approximation of RGM

Present study: 2α+t OCM-GEM

OCM (Orthogonality condition model) Saito, PTP40 (1968)

with GEM (Gaussian expansion method) Kamimura et al.

huge model space:

(a) shell-model-like states (ground states etc.),
(b) ^{7}Li+α , ^{8}Be+t cluster states,
(c) 2α+t gas states

OCM (orthogonality condition model)

- An approximation of RGM (resonating group method)
- Relative motions among c.o.m. of clusters are exactly solved under an orthogonality condition arising from Pauli-Blocking effects

For example of $n\alpha$ system, S. Sato, Prog. Thor. Phys. 40 (1968)

Fermion w.f.: $\Phi^{(F)} = \mathcal{A}\left\{\prod_{i=1}^{N}\phi_{\alpha_i}^{\text{int}}\chi^{rel}\right\}$, $\quad (H-E)\Phi^{(F)}=0$, $\quad \left\langle \Phi^{(F)} \middle| \Phi^{(F)} \right\rangle = 1$,

RGM eq.: $(\mathcal{H}-E\mathcal{N})\chi^{rel}=0$, $\quad \left\langle \chi^{rel} \middle| \mathcal{N} \middle| \chi^{rel} \right\rangle = 1$,

α-cluster w.f.: $\Phi^{(B)}=\sqrt{\mathcal{N}}\chi^{rel}$, $\quad \left(\frac{1}{\sqrt{\mathcal{N}}}\mathcal{H}\frac{1}{\sqrt{\mathcal{N}}}-E\right)\Phi^{(B)}=0$, $\quad \left\langle \Phi^{(B)} \middle| \Phi^{(B)} \right\rangle = 1$,

Approximation:

$$\frac{1}{\sqrt{\mathcal{N}}}\mathcal{H}\frac{1}{\sqrt{\mathcal{N}}} \Rightarrow T+\sum_{i<j}V_{2\alpha}^{eff}(i,j)+\sum_{i<j<k}V_{3\alpha}^{eff}(i,j,k)=T+V^{eff}$$

OCM equation:

$$(T+V^{eff}-E)\Phi^{(B)}=0 \quad \text{with} \quad \left\langle u_F \middle| \Phi^{(B)} \right\rangle = 0, \quad \left\langle \Phi^{(B)} \middle| \Phi^{(B)} \right\rangle = 1$$

Orthogonality condition

u_F : **Pauli forbidden states,** $\quad \mathcal{N}u_F=0 \ \text{ or } \ \mathcal{A}\left\{\prod_{i=1}^{N}\phi_{\alpha_i}^{\text{int}}u_F\right\}=0$

$\Phi^{(B)}$: **Symmetrized w.f. with relative (Jacobi) coordinates**

Easy to formulate 2α+t OCM and 3α+n OCM based on GEM

$3/2^{-}, 1/2^{+}$ states in ^{11}B

Results of OCM calculations

Yamada and Funaki, PRC82 (2010)

Structure study of ^{11}B

- α+α+t OCM with H.O. basis

  Nishioka et al., PTP62 (1979)

- AMD calculation

  Enyo-Kanada, PRC75 (2007)

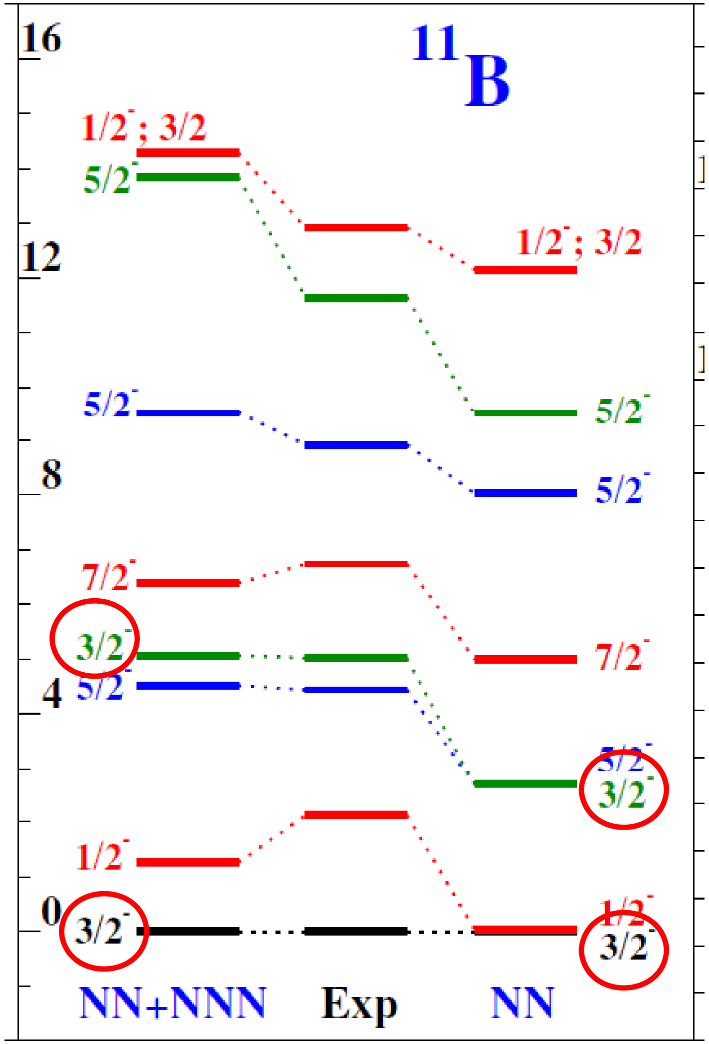
- No-core shell model

  Navratil et al., JPG36 (2009)

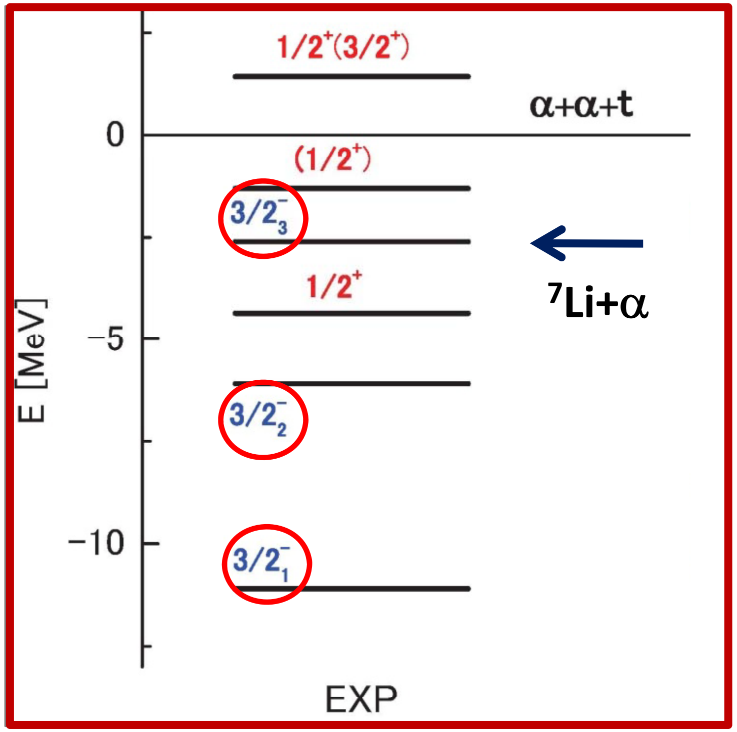
Not well understood for even-odd states of ^{11}B

No-core shell model

Navratil et al., JPG36(2009)



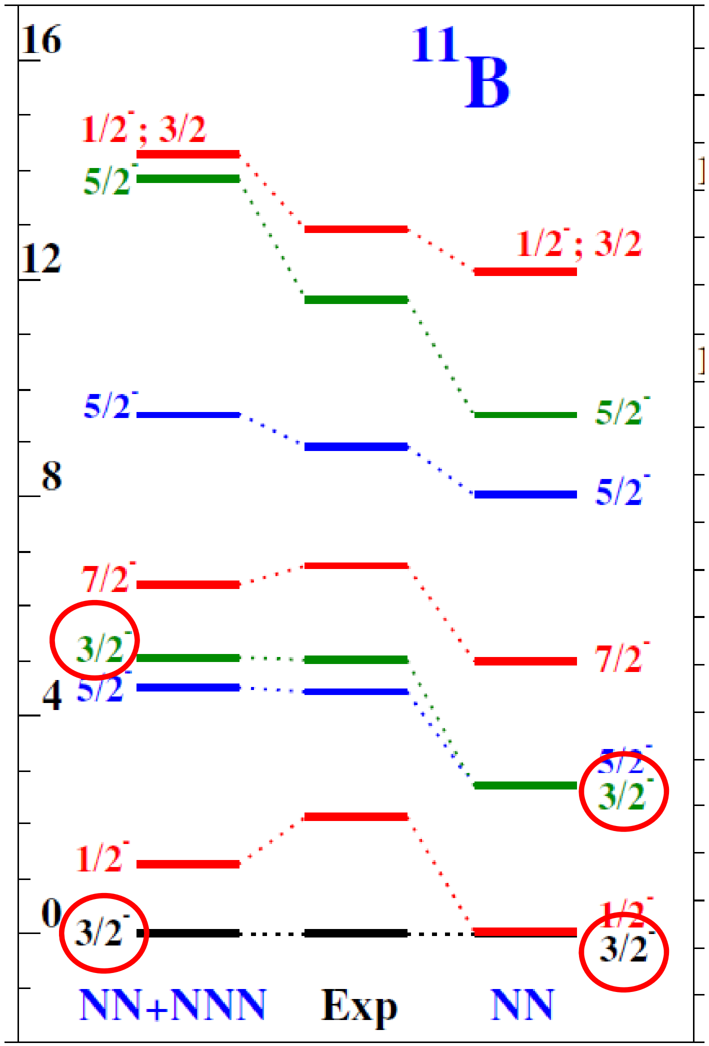
Experimentally, three 3/2- states



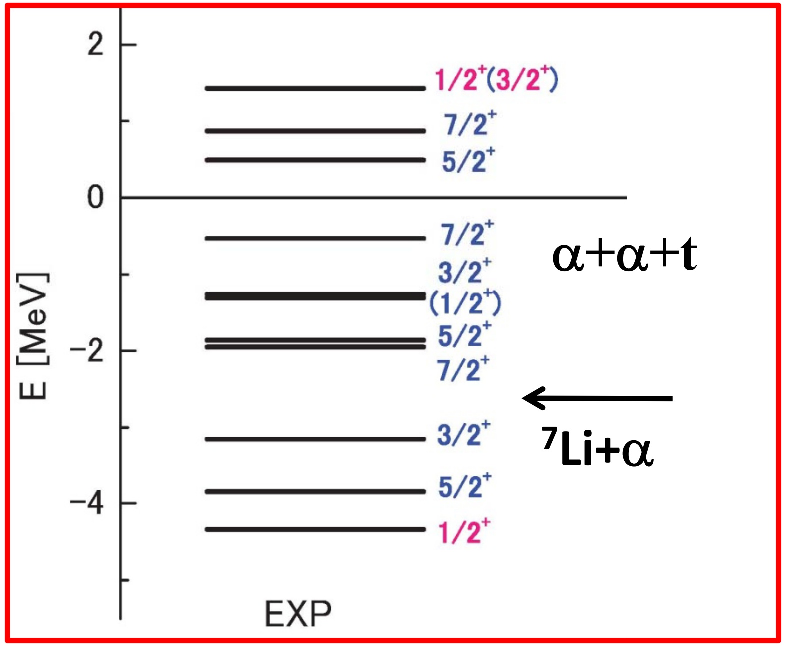
^{11}B energy levels (T=1/2)

No-core shell model

Navratil et al., JPG36(2009)

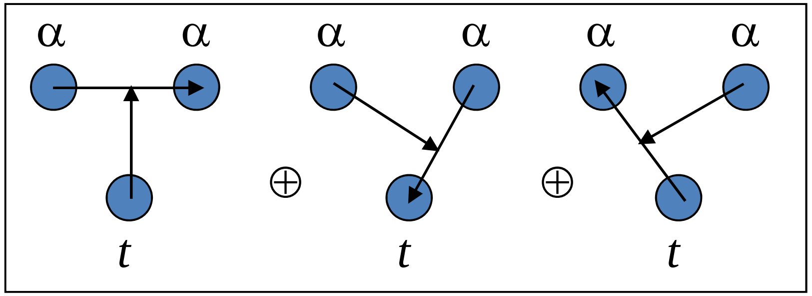


Not studied well for even-parity states



^{11}B energy levels
(T=1/2)

Total wave function: $2\alpha+t$ OCM



$$\Phi_L(^{11}\mathrm{B}) = \sum_{c,\nu,\mu}\left[A_c(\nu,\mu)\Phi_c^{(12,3)}(\nu,\mu)\right] + \sum_{c,\nu,\mu}\left[B_c(\nu,\mu)\Phi_c^{(23,1)+(31,2)}(\nu,\mu)\right],$$

$$\Phi_c^{(23,1)+(31,2)}(\nu,\mu) = \Phi_c^{(23,1)}(\nu,\mu) + \Phi_c^{(31,2)}(\nu,\mu),$$

OCM+GEM (Gaussian expansion method)

$$\Phi_c^{(ij,k)}(\nu,\mu) = \left[\varphi_\ell(\mathbf{r}_{ij},\nu)\varphi_\lambda(\mathbf{r}_k,\mu)\right]_L, \quad \varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r}),$$

Hamiltonian

$$H = T + V_{2\alpha}(r_{12}) + V_{2\alpha}^{Coul}(r_{12}) + \sum_{(ij)=(23),(31)}\left[V_{\alpha+t}^{c}(r_{ij}) + V_{\alpha+t}^{LS}(r_{ij})\mathbf{l}_{ij}\cdot\mathbf{s} + V_{\alpha+t}^{Coul}(r_{ij})\right] + V_{2\alpha t} + V_{Pauli},$$

$$V_{\alpha+x}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right), \qquad V_{\alpha+x}^{Coul}(r) = \frac{2xe^2}{r}\,\mathrm{erf}\,(ar)$$

$\alpha+\alpha$, α+t phase shifts

^{7}Li:3/2$^-$, 1/2$^-$, 7/2$^-$, 1/2$^-$

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda\left[\sum_{2n+\ell<4}\left|u_{n\ell}(r_{12})\right\rangle\left\langle u_{n\ell}(r_{12})\right| + \sum_{2n+\ell<3}\sum_{ij=(23),(31)}\left|u_{n\ell}(r_{ij})\right\rangle\left\langle u_{n\ell}(r_{ij})\right|\right],$$

: Pauli blocking operator

$$V_{2\alpha t} = \sum_{L^\pi,Q(=7,8)} \eta \left|SU3(\lambda\mu):L^\pi Q\right\rangle\left\langle SU3(\lambda\mu):L^\pi Q\right|$$

: effective 2α+t force

Equation of motion

$$\delta\left[\left\langle\Phi\left|E-H\right|\Phi\right\rangle\right] = 0$$

Single-cluster motions in ^{11}B(α+α+t)

Single-cluster density matrix:

alpha: $\rho_\alpha(\mathbf{r},\mathbf{r}') = \langle \Phi_J(^{11}\mathrm{B}) | \; \frac{1}{2}\sum_{n=1}^{2} | \delta(\mathbf{r}_n^{(G)} - \mathbf{r}') \rangle \langle \delta(\mathbf{r}_n^{(G)} - \mathbf{r}) | \; | \Phi_J(^{11}\mathrm{B}) \rangle,$

triton: $\rho_t(\mathbf{r},\mathbf{r}') = \langle \Phi_J(^{11}\mathrm{B}) | \; | \delta(\mathbf{r}_3^{(G)} - \mathbf{r}') \rangle \langle \delta(\mathbf{r}_3^{(G)} - \mathbf{r}) | \; | \Phi_J(^{11}\mathrm{B}) \rangle,$

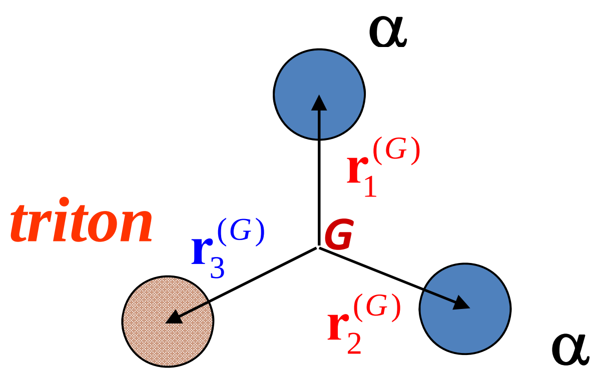
$$\int d\mathbf{r}' \rho_\alpha(\mathbf{r},\mathbf{r}') \varphi_\alpha(\mathbf{r}') = \lambda_\alpha \varphi_\alpha(\mathbf{r}), \qquad \sum \lambda_\alpha = 1$$

$$\int d\mathbf{r}' \rho_t(\mathbf{r},\mathbf{r}') \varphi_t(\mathbf{r}') = \lambda_t \varphi_t(\mathbf{r}), \qquad \sum \lambda_t = 1$$

$\varphi(\mathbf{r})$: **single-cluster orbital w.f.**

λ : **occupation probability**

$\Phi_J(^{11}\mathrm{B})$: **α+α+t OCM w.f.**



^{11}B=α+α+t

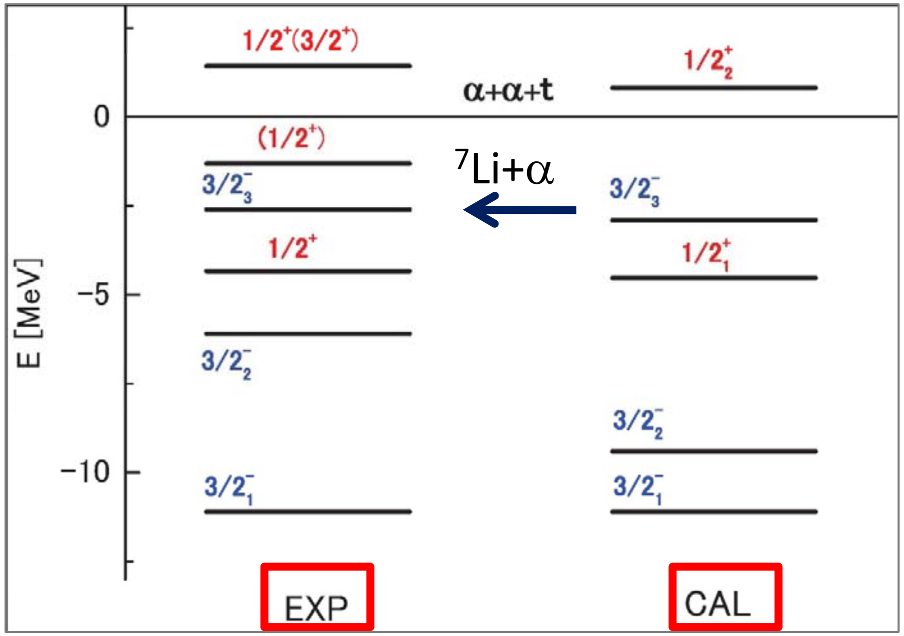
Suzuki et al., PRC65 (2002); Matsumura et al., NPA739 (2004)

Yamada et al., EPJA 26 (2005); Funaki, Yamada et al., PRL101 (2008)

Yamada et al., PRA (2008), PRC(2009)

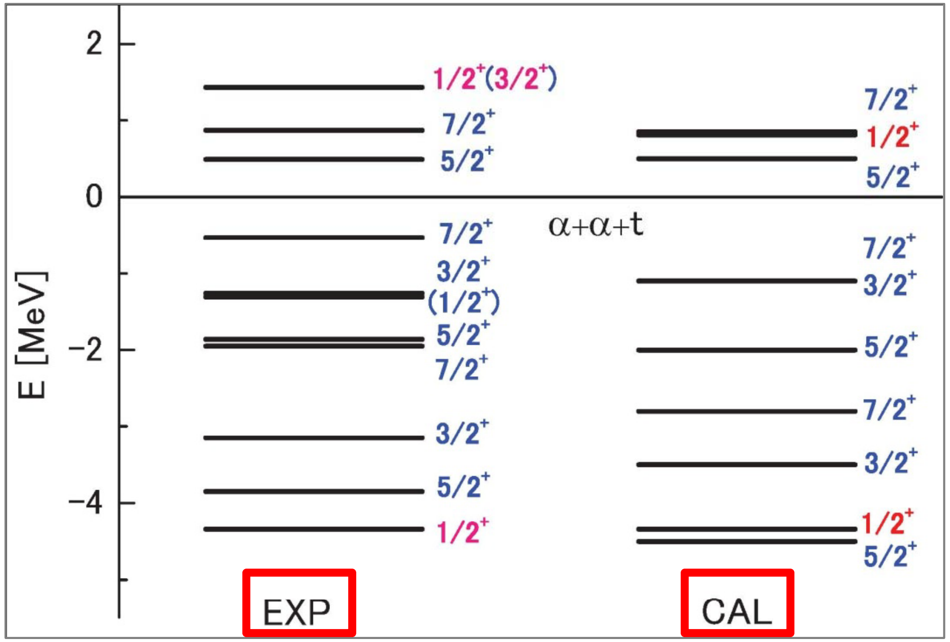
Energy levels of ^{11}B

3/2- and 1/2+ states



$\alpha + \alpha +$ t OCM

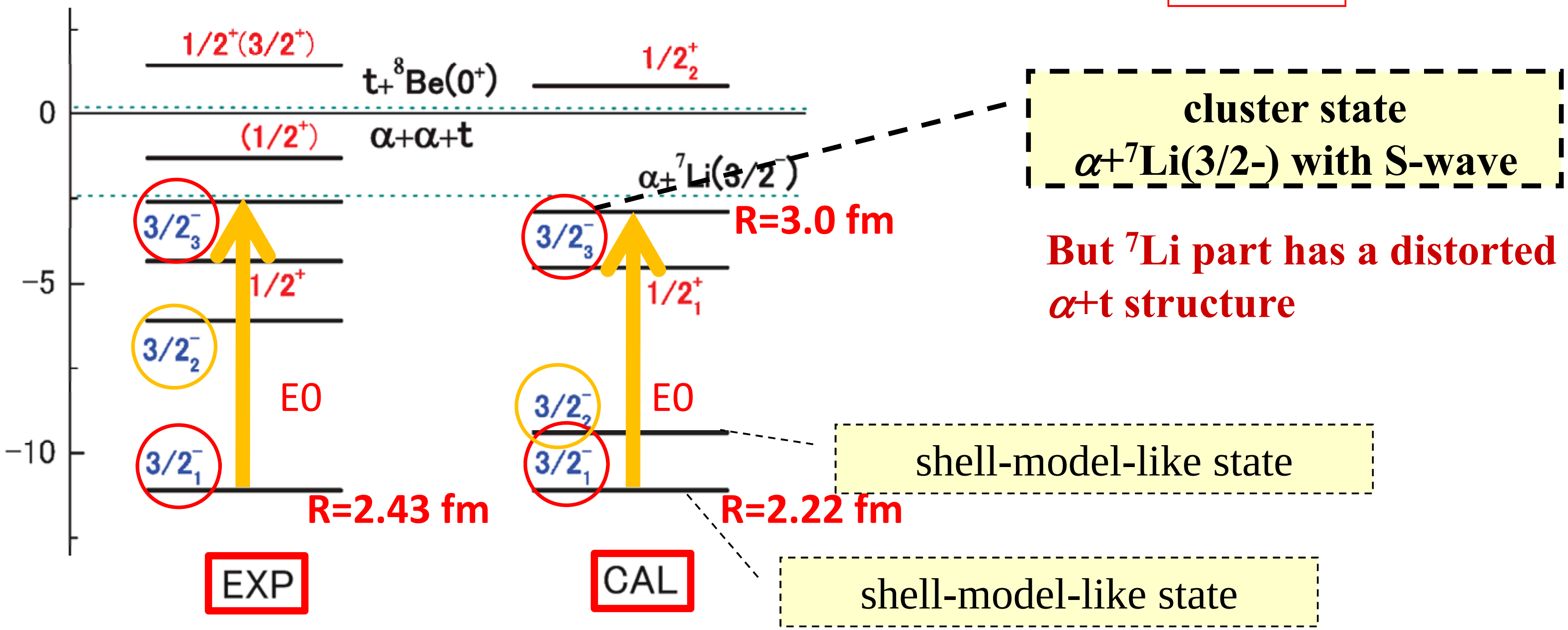
Even-parity states



$\alpha + \alpha +$ t OCM

$^{11}B = \alpha + \alpha + t$ OCM

$3/2^-$



B(E0,IS)=96±16 fm^4 B(E0,IS)=92 fm^4

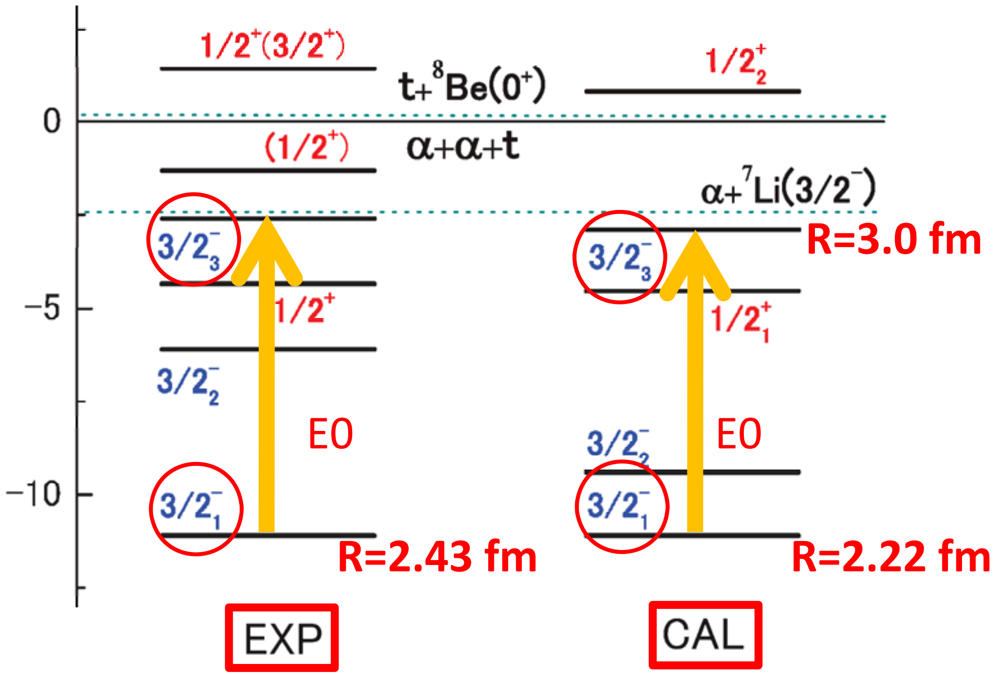
T. Kawabata et al., PRC70 (2004)

vs. Hoyle state:

B(E0,IS)=120±9 fm^4

$^{11}B = \alpha + \alpha + t$ OCM

$3/2^-$

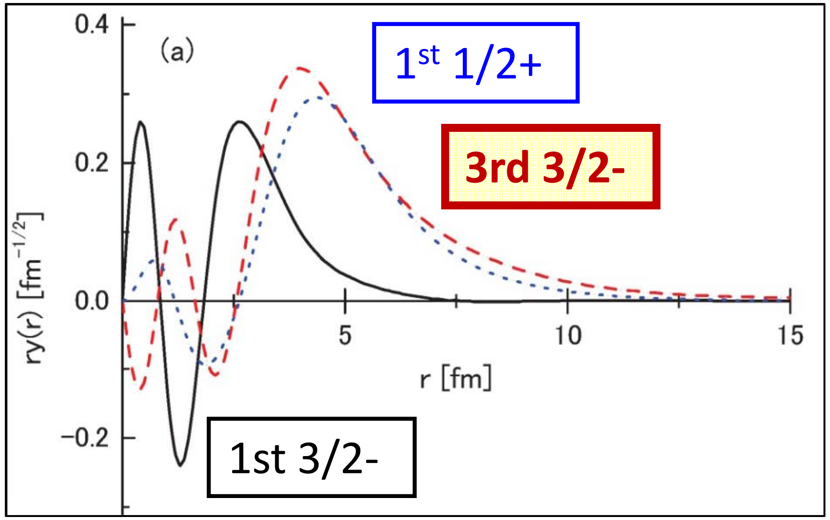


B(E0,IS)=96±16 fm^4 B(E0,IS)=92 fm^4

T. Kawabata et al., PRC70 (2004)

vs. Hoyle state:
B(E0,IS)=120±9 fm^4

Overlap amplitude with α+ ^{7}Li(g.s) channel



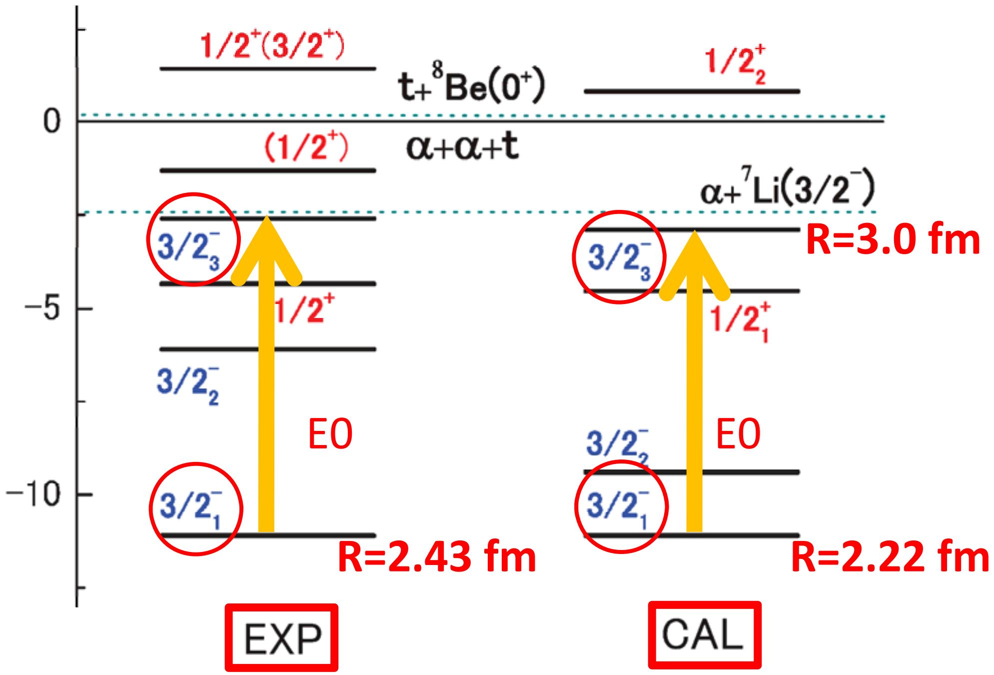
main configuration

$3/2^-_3$: α+^{7}Li(3/2-) with S-wave

But ^{7}Li part has a distorted α+t structure

Yamada & Funaki, PRC82(2010)

$^{11}B = \alpha + \alpha + t$ OCM



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vs. Hoyle state:
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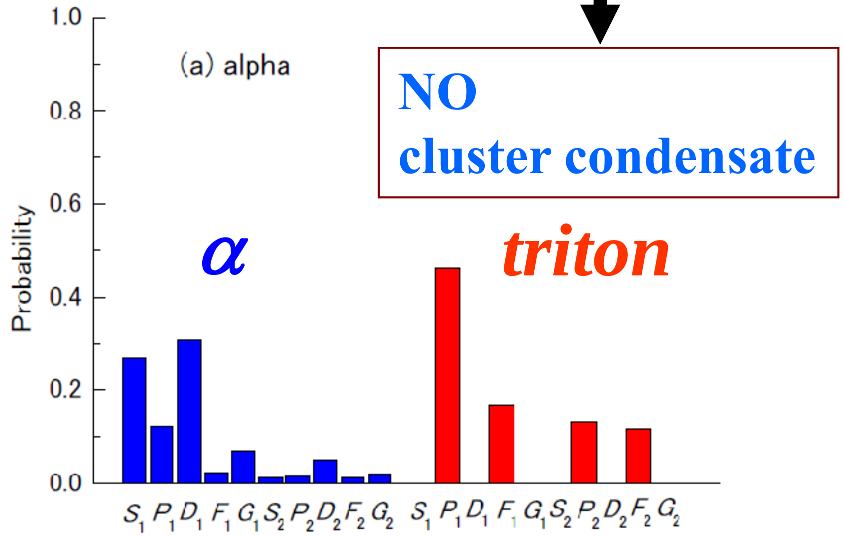
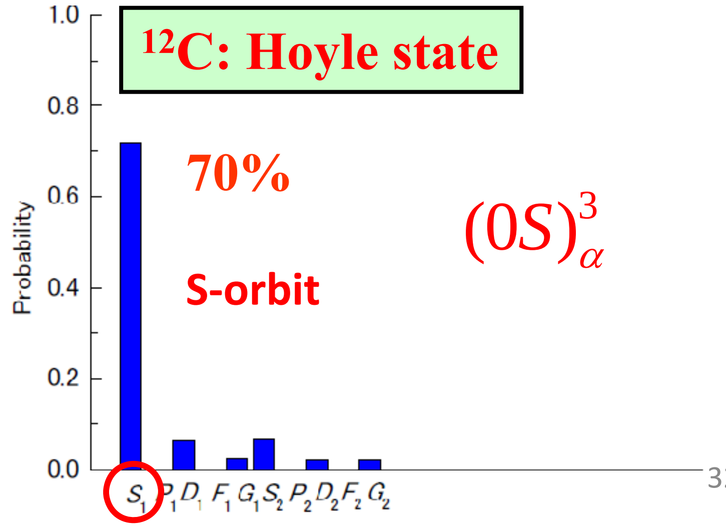
Occupation probabilities

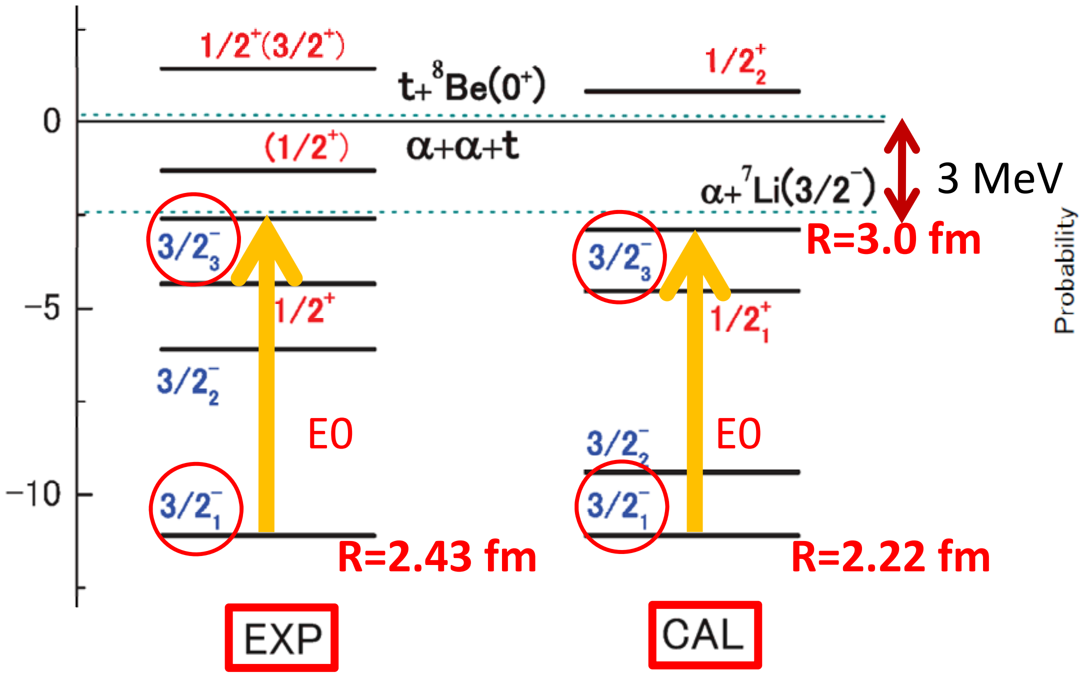
^{11}B: 3rd 3/2$^-$

No concentration on a single orbit !





$^{11}B = \alpha + \alpha + t$ OCM



B(E0,IS)=96±16 fm^4 B(E0,IS)=92 fm^4

T. Kawabata et al., PRC70 (2004)

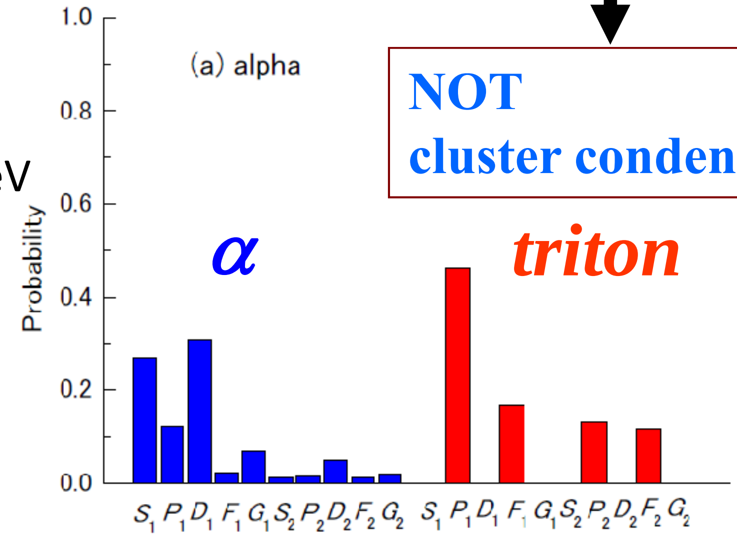
vs. Hoyle state:
B(E0,IS)=120±9 fm^4

Occupation probabilities

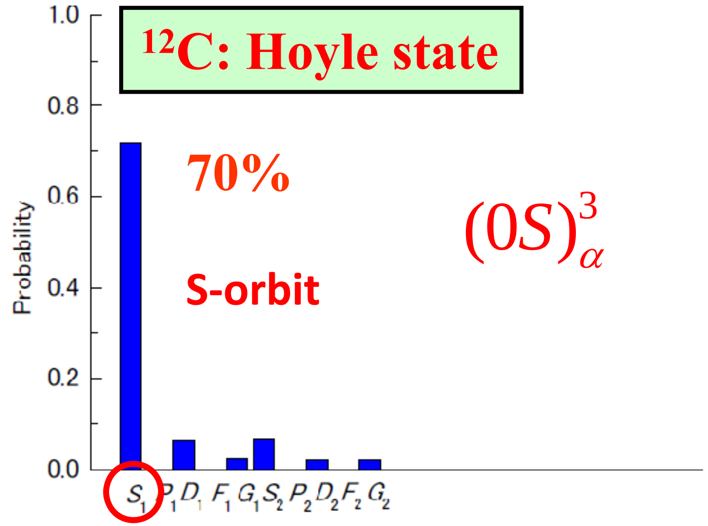
^{11}B: 3rd 3/2$^-$

No concentration on a single orbit !

NOT cluster condensate



^{12}C: Hoyle state

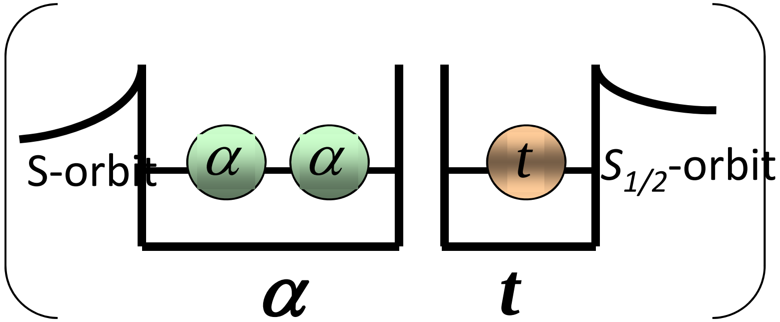


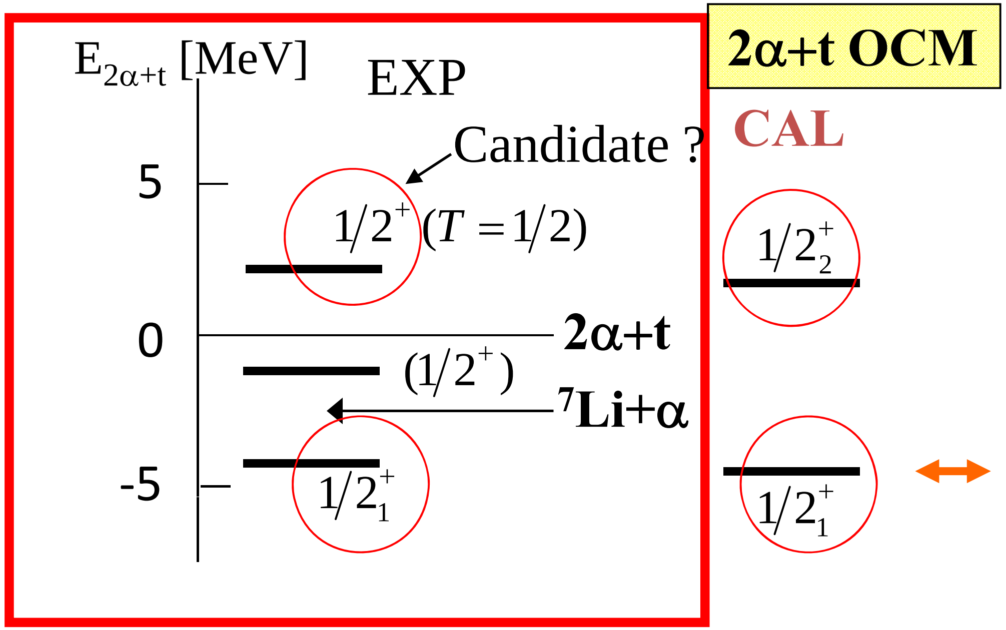
$1/2^{+}$ states in ^{11}B

Structure of $1/2^+$ states

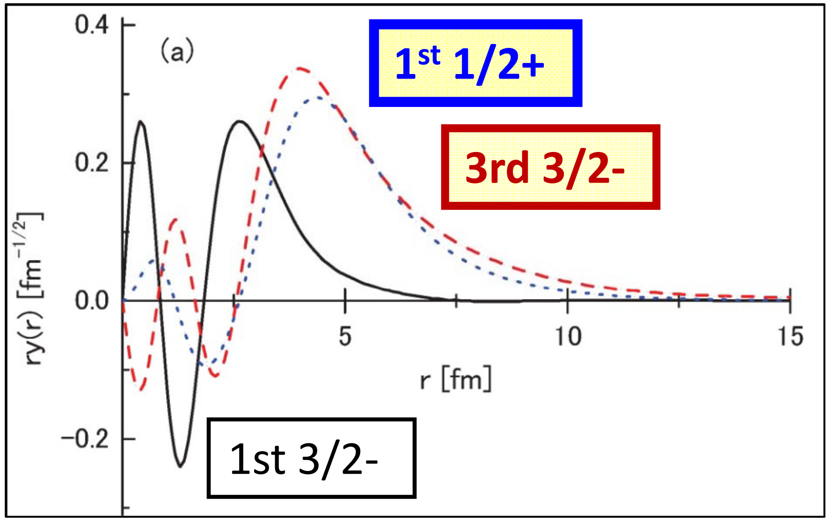
^{11}B $1/2^+$: Hoyle-analogue

exists or not ?





Overlap amplitudes with α+ ^{7}Li(g.s) channel



R_{rms} = 3.14[fm]

vs. 3.00 fm for 3rd 3/2-

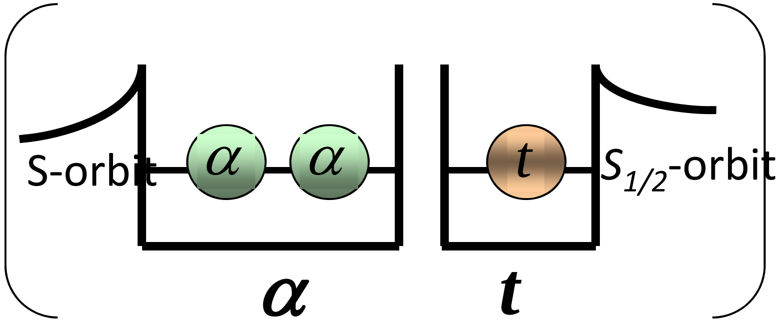
^{7}Li($3/2^-$) + α : P-wave

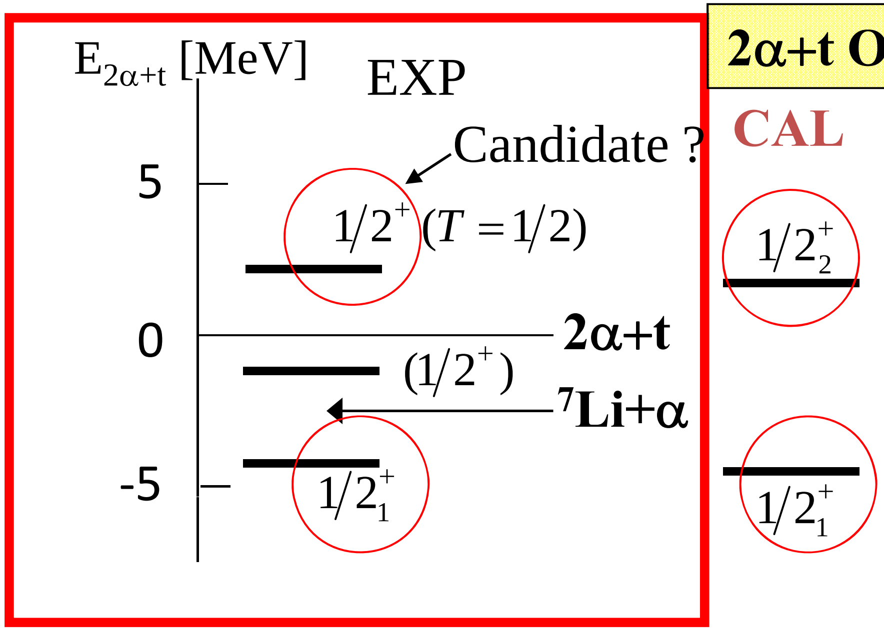
Parity doublet partner of 3rd 3/2-

Structure of 1/2$^+$ states

^{11}B 1/2$^+$: Hoyle-analogue

exists or not ?





2α+t OCM

1/2$^+$: E_x=12.6 MeV (E_{2a+t}=1.5 MeV)

^{7}Li(^{9}Be,α^{7}Li)^{5}He ***T=1/2***

Soic et al. NPA742 2004

^{7}Li(^{7}Li,^{11}B*)t ***T=1/2***

^{11}B* → α+^{7}Li Curis et al. PRC72 2006

^{10}Be(p,γ), ^{9}Be(^{3}He,p) ***T=3/2***

Goosman et al. PRC1 1970

Zwiegliski NPA389 1982

⇔Ajenberg-Selove

↔ **R_{rms} = 5.98[fm] : dilute !**

↔ **R_{rms} = 3.14[fm]**

vs. 3.00 fm for 3rd 3/2-

^{7}Li(3/2$^-$) + α : P-wave

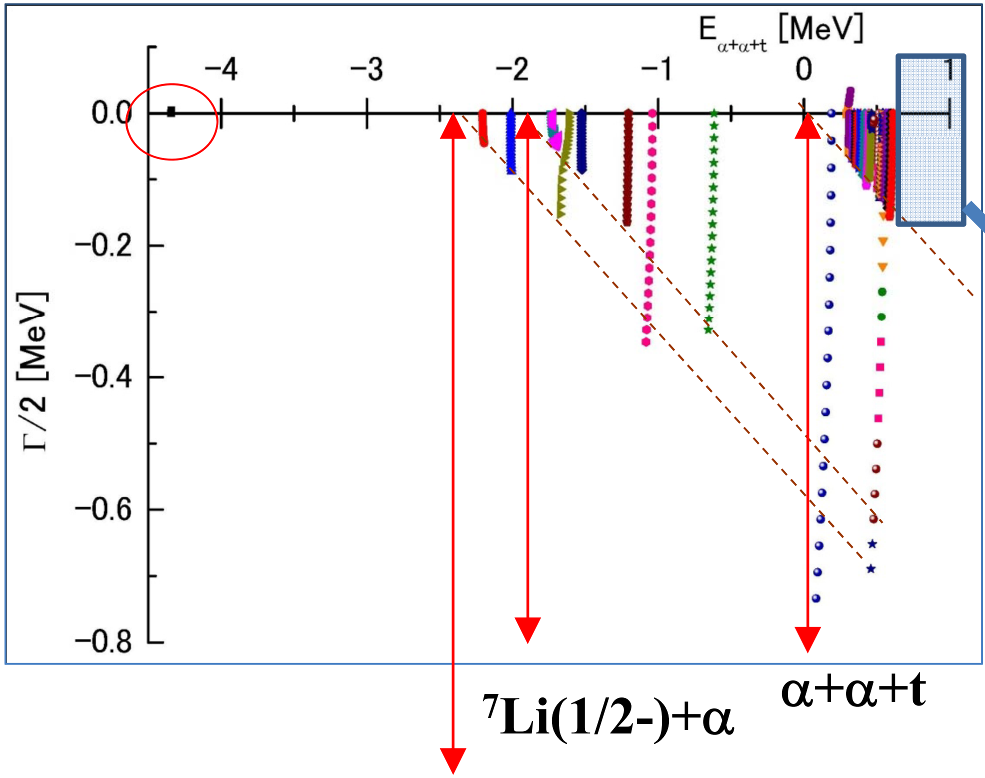
Parity doublet partner of 3rd 3/2-

Complex-scaling method for 1/2^{+} with α+α+t OCM

Bound state

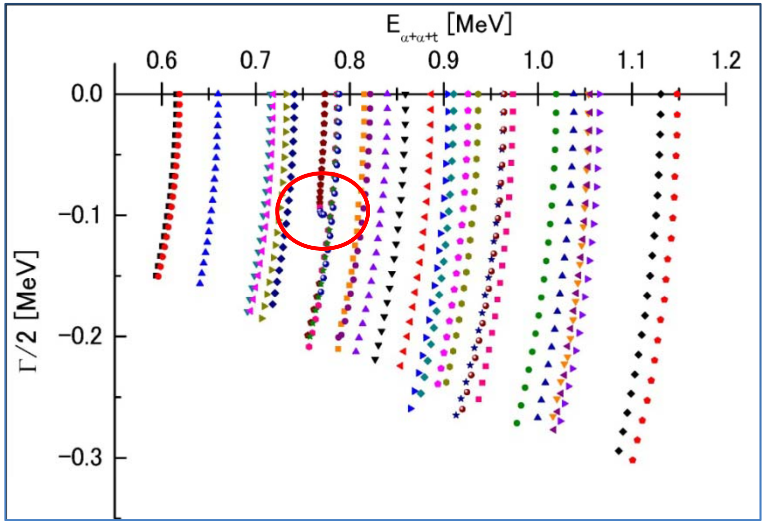
1st 1/2+

2nd 1/2+ exists as a resonant state.



Resonant state

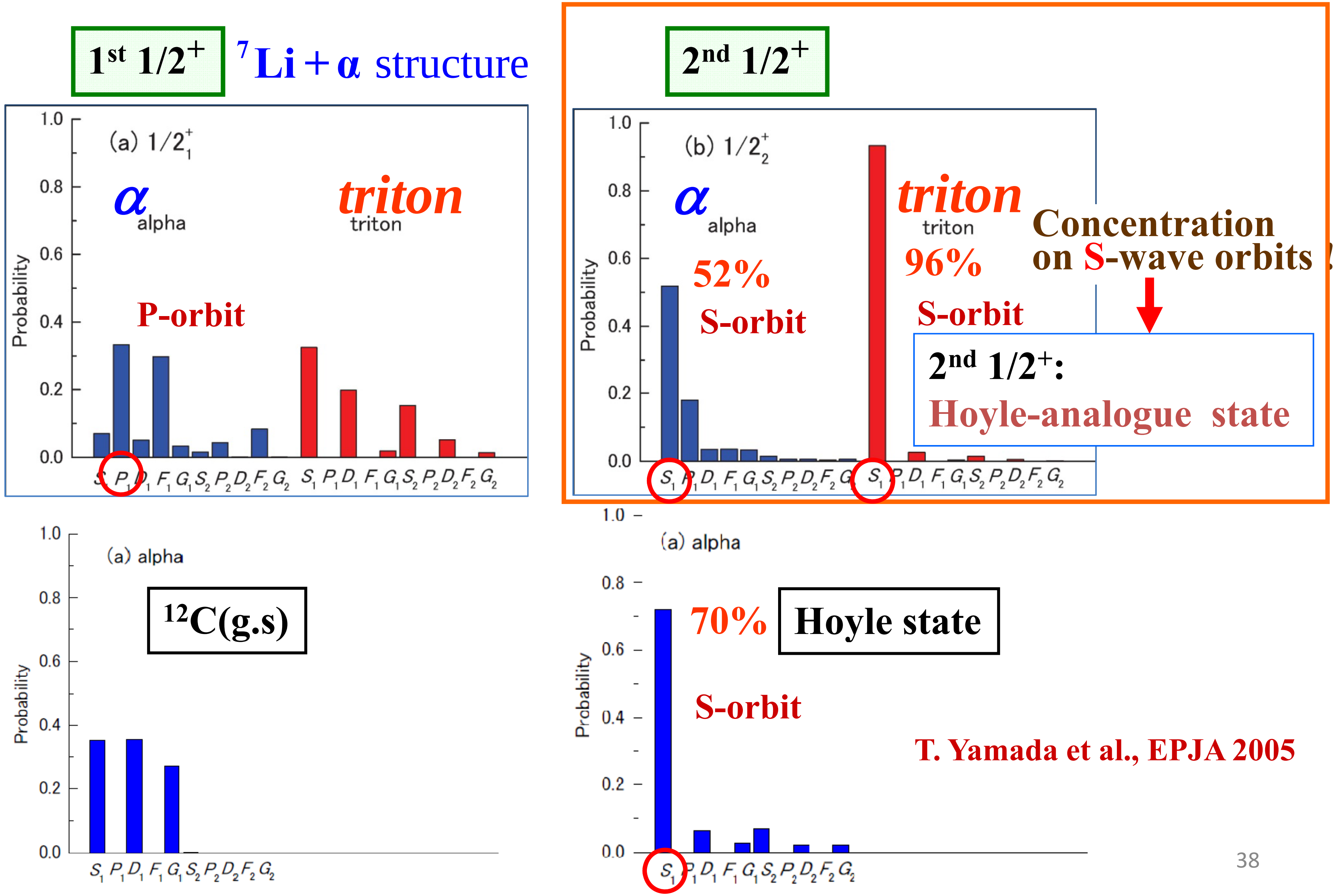
2nd 1/2+



E_x(cal)= 11.9 MeV, Γ=190 keV

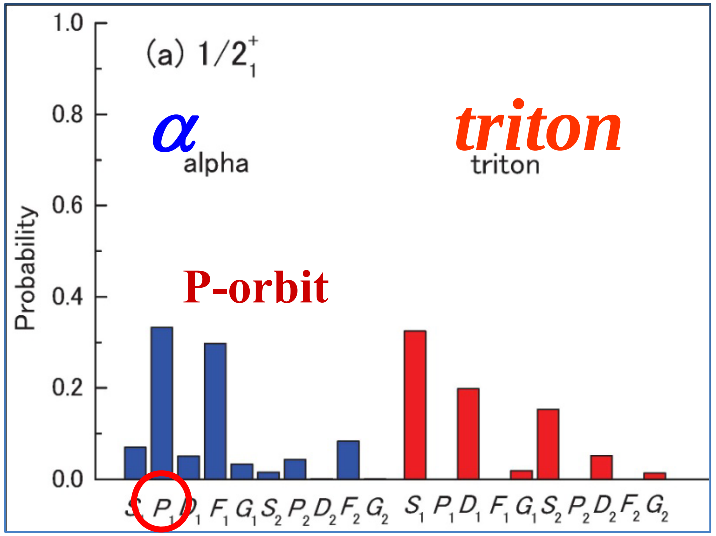
E_x(exp)=12.56 MeV, Γ=210±20 keV

Occupation probabilities of cluster orbits

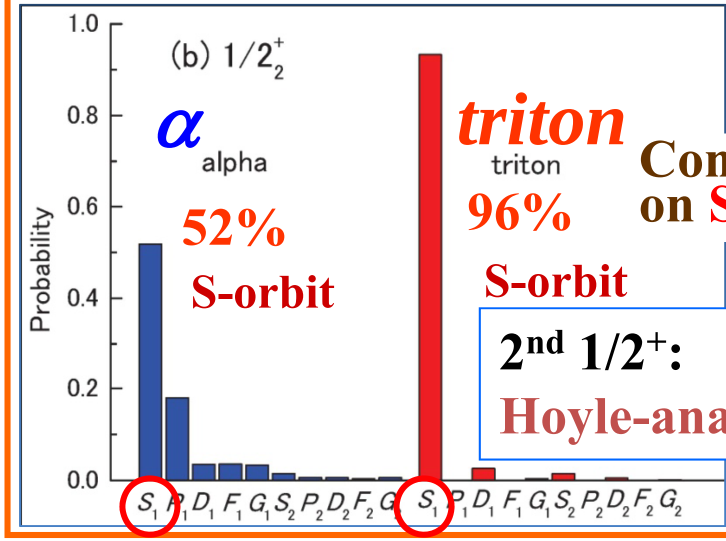


Occupation probabilities of cluster orbits

1st 1/2$^+$ ^{7}Li + α structure

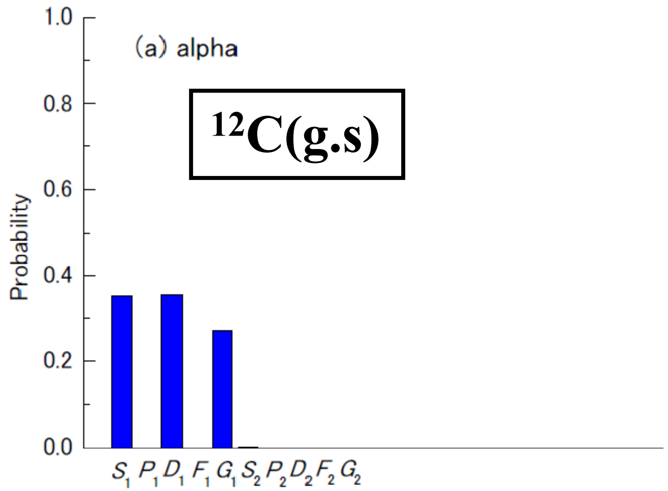


2nd 1/2$^+$



Concentration on S-wave orbits !

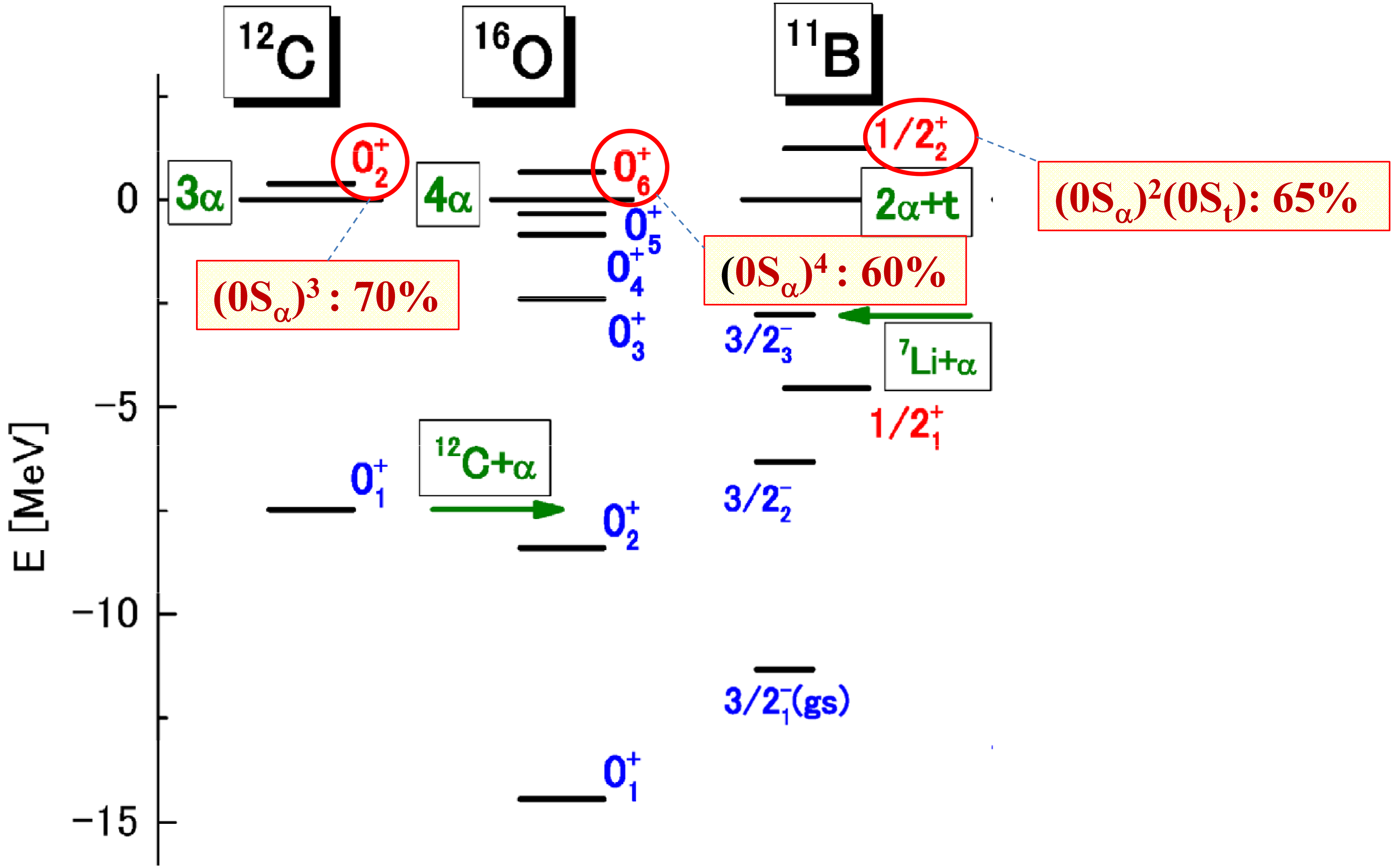
2nd 1/2$^+$:
Hoyle-analogue state



2nd 1/2+: 65% $(0S_\alpha)^2(0S_t)$

65% = (2*52%+96%)/3

Hoyle-analogue states



Summary

- Structure study of ^{11}B with α+α+t OCM using GEM
- 1st and 2nd 3/2- states : shell-model-like compact structure
- 3rd 3/2- state: M(E0) ≈ M(E0) of Hoyle-state

  Cluster structure: α+^{7}Li(g.s) with S-wave,

  but distorted α+t structure in the ^{7}Li part

  No similarity to cluster-condensate nature

  ⇔ bound by 3 MeV from α+α+t threshold
- 1st 1/2+ : bound by 3MeV from α+α+t threshold

  α+^{7}Li(g.s) with P-wave: parity-partner of 3rd 3/2-
- 2nd 1/2+ (E_x=12.6 MeV; 1.5 MeV above 2α+t threshold)

  Strong candidate of Hoyle-analogue: $(S_\alpha)^2(S_t)$

  Complex-scaling method

E_x(cal)= 11.9 MeV, Γ=190 keV

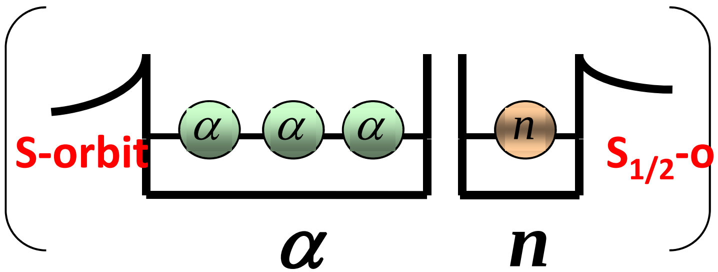
E_x(exp)=12.56 MeV, Γ=210±20 keV

Collaborators:

^{12}C, ^{16}O : Funaki, Horiuchi, Roepke, Schuck, Tohsaki
^{11}B : Funaki

$1/2^-, 1/2^+$ states in ^{13}C

- What happens in ^{12}C when an extra neutron (n) is added into the Hoyle state (3α) ?
- Hoyle-analogue states?
- 3α+n OCM with GEM



$(0S_\alpha)^3(S_n)$

$J^\pi = 1/2^+$

α-n and α-t potentials: parity-dependent

odd waves : attractive enough to make resonances/bound states

even waves: weakly attractive

^{13}C

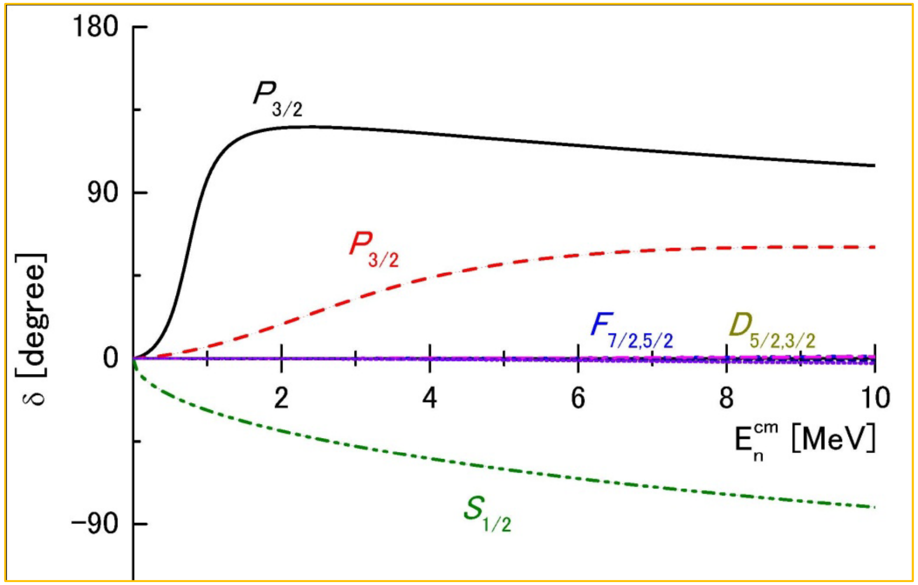
	neutron		3α part in ^{13}C	$J^\pi(^{13}C)$
^{12}C(Hoyle) +	p-orbit	→	reduced:	$1/2^-, 3/2^-$
	s-orbit		not so changed:	$1/2^+$

^{11}B

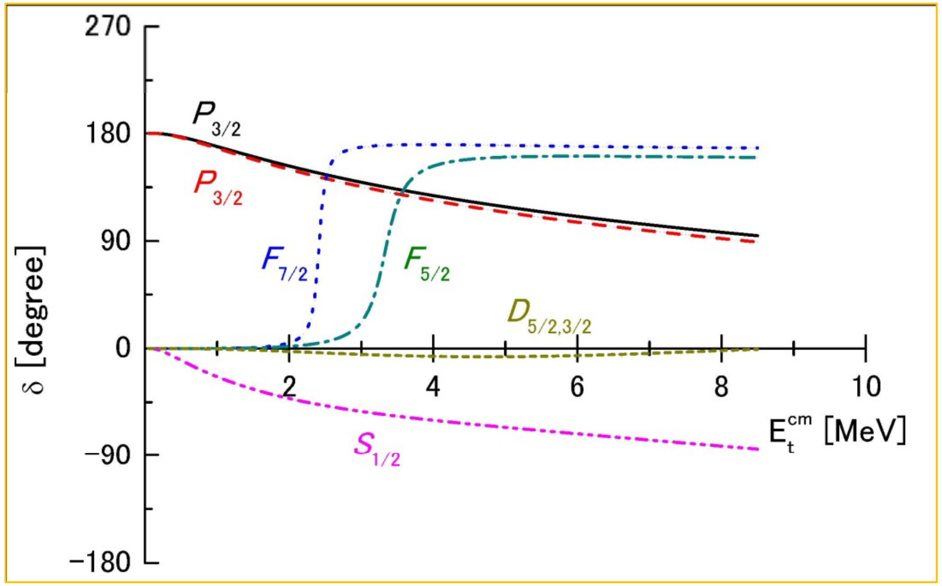
	triton		2α part in ^{11}B	$J^\pi(^{11}B)$
$^8Be(2\alpha)$ +	p-orbit	→	reduced:	$1/2^-, 3/2^-$
	s-orbit		not so changed:	$1/2^+$

Thus, one can expect cluster-gas-like states in 1/2+.

α-n phase shifts



α-triton phase shifts



α-n potential

(1) ***P*-wave: attractive**

(2) ***S*-wave: weakly attractive**

α-triton potential

(1) ***P*-, *F*-waves: attractive**

(2) ***S*-, *D*-waves: weakly attractive**

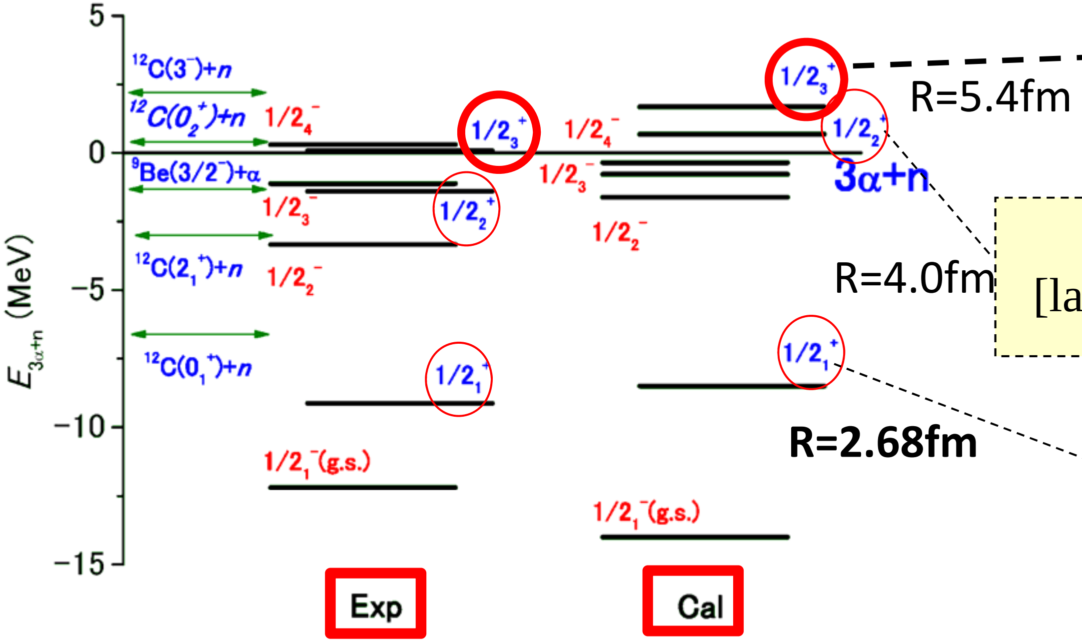
Parity-dependent potentials

3α + n OCM for ^{13}C

***J*=1/2-, 1/2+ :**

reproduction of M(E0), Hoyle-analogue state

energy spectra of ^{13}C



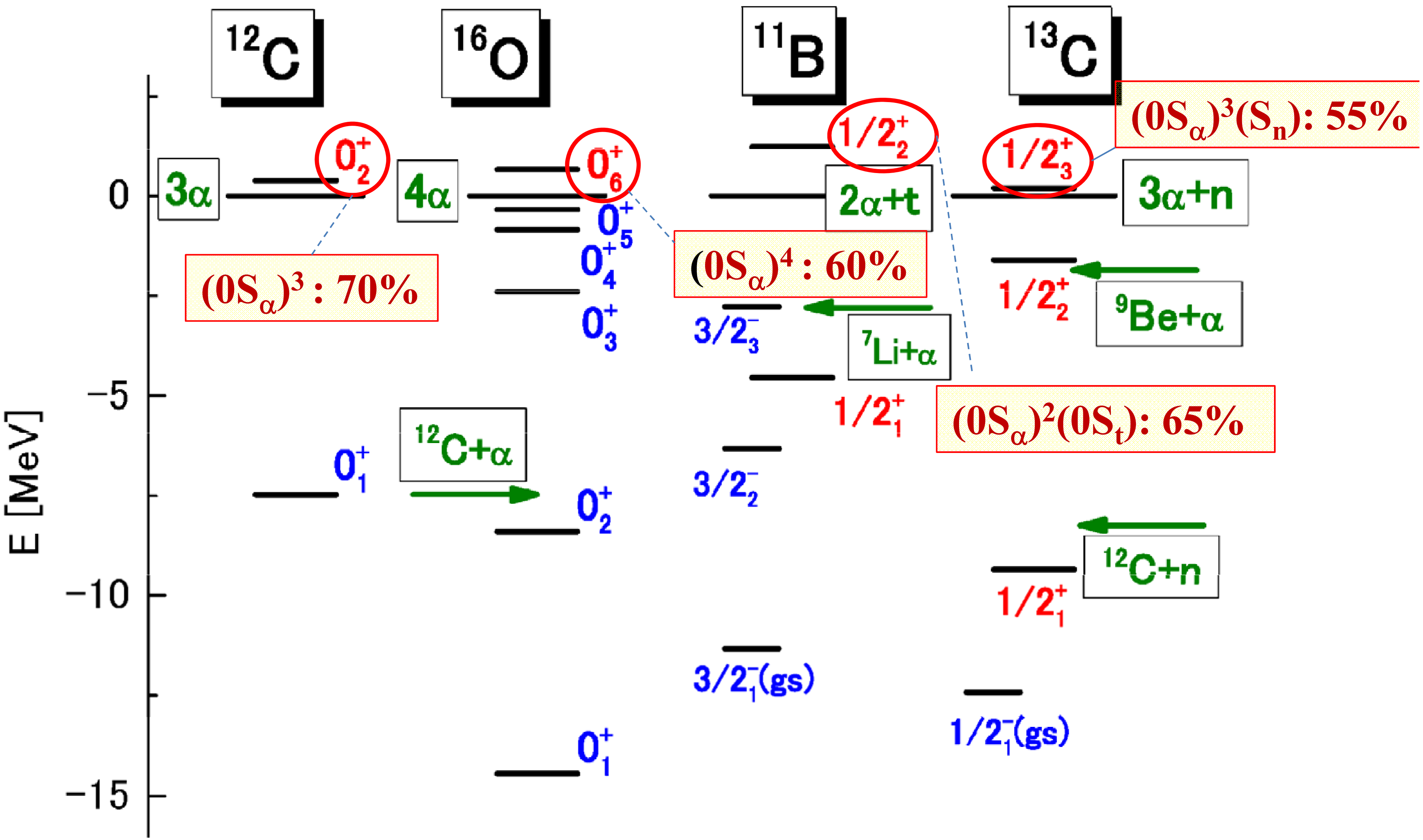
1/2+ states

Probability

S_α-orbit

^{12}C (Hoyle) + n ^{9}Be(1/2+) + α	**55%**
^{9}Be(3/2-,1/2-) + α [larger than ^{12}C (Hoyle) + n]	28%
^{12}C (g.s) + n neutron-halo-like	20%

Hoyle-analogue states



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  Complex-scaling method

  E_x(cal)= 11.9 MeV, Γ=190 keV

  E_x(exp)=12.56 MeV, Γ=210±20 keV

- **$^{13}C = 3\alpha + n$ OCM**

^{12}C(Hoyle) + *p*-wave neutron: ^{13}C(1/2-)

Size of 3α part is shrunk, due to attractive *p*-wave α-n interaction

^{12}C(Hoyle) + *s*-wave neutron: ^{13}C(1/2+)

3rd 1/2+ is the candidate of Hoyle-analog state in the present study

Reflecting weakly attractive α-n interaction,
3α+n gas-like state appears in 1/2+ above 3α + n threshold

We predict cluster-gas-like states exit in A$\neq$ 4n nuclei as well as A=4n nuclei around their cluster disintegrated threshold.

Need experiments.

Gross-Pitaevskii-equation approach

- **Total wave function:** $\Phi(N\alpha)=\prod_{i=1}^{N}\varphi(\mathbf{r}_i)$ **Symmetrized, $(0s)^N$**

- **Gross-Pitaevskii equation**

$$-\frac{\hbar^2}{2m}\left(1-\frac{1}{N}\right)\nabla^2\varphi(\mathbf{r})+U(\mathbf{r})\varphi(\mathbf{r})=\varepsilon\varphi(\mathbf{r}),$$

Single-α particle energy

↓

$$\varepsilon=\langle t\rangle+(N-1)\langle\upsilon_2\rangle+\frac{(N-1)(N-2)}{2}\langle\upsilon_3\rangle$$

$$U(\mathbf{r})=(N-1)\int d\mathbf{r}'|\varphi(\mathbf{r}')|^2\upsilon_2(\mathbf{r},\mathbf{r}')+\frac{(N-1)(N-2)}{2}\int d\mathbf{r}''d\mathbf{r}'|\varphi(\mathbf{r}'')|^2|\varphi(\mathbf{r}')|^2\upsilon_3(\mathbf{r}'',\mathbf{r}',\mathbf{r})$$

$$\langle t\rangle=\left(1-\frac{1}{N}\right)\langle\varphi|-\frac{\hbar^2}{2m}\nabla^2|\varphi\rangle$$

$$\langle\upsilon_2\rangle=\langle\varphi\varphi|\upsilon_2(\mathbf{r},\mathbf{r}')|\varphi\varphi\rangle$$

$$\langle\upsilon_3\rangle=\langle\varphi\varphi\varphi|\upsilon_3(\mathbf{r},\mathbf{r}',\mathbf{r}'')|\varphi\varphi\varphi\rangle$$

- **Total energy of Nα**

$$E(N\alpha)=N\left[\langle t\rangle+\frac{1}{2}(N-1)\langle\upsilon_2\rangle+\frac{1}{6}(N-1)(N-2)\langle\upsilon_3\rangle\right]$$

- **Rms radius for nucleon**

$$\sqrt{\langle r_N^2\rangle}=\sqrt{\langle r_\alpha^2\rangle_{GP}+1.71^2},\qquad \langle r_\alpha^2\rangle_{GP}=(1-\frac{1}{N})\langle\varphi|r^2|\varphi\rangle$$

Effective α-α potential

- **Density-dependent potential (Gogny-type)**

$$\upsilon_2(\mathbf{r},\mathbf{r}') = \upsilon_0 e^{-0.7^2(r-r')^2} - 130 e^{-0.475^2(r-r')^2}$$
$$+ (4\pi)^2 g\delta(\mathbf{r}-\mathbf{r}')\rho\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right) + \frac{4e^2}{|\mathbf{r}-\mathbf{r}'|}\mathrm{erf}\,(a|\mathbf{r}-\mathbf{r}'|)$$

υ_0=271 MeV, g = 1650 MeV · fm^3 (cf: Ali-Bodmer, 500 MeV)

^{12}C(0_2^+), E^{exp}=0.38 MeV, R_{rms}=4.29 fm (Tohsaki et al, PRL 87, 192501,('01))

- **Phenomenological 2α and 3α potential**

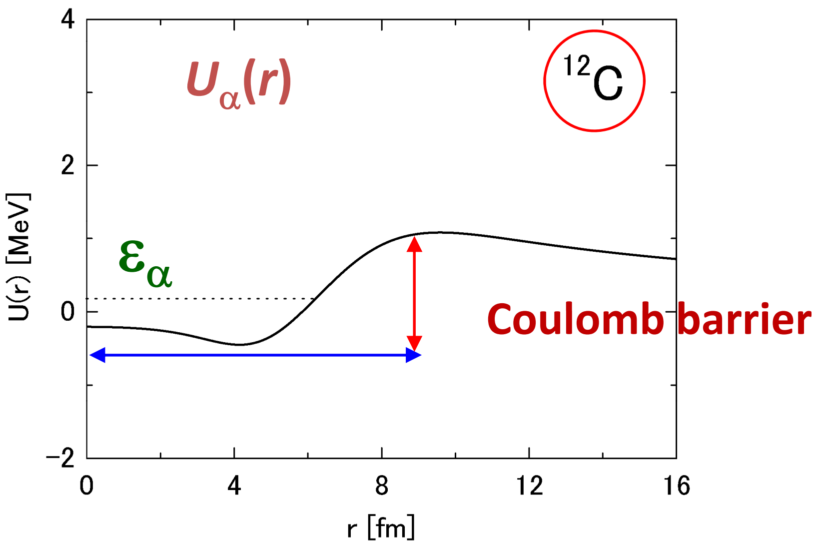
$$\upsilon_2(\mathbf{r},\mathbf{r}') = 50 e^{-0.4^2(\mathbf{r}-\mathbf{r}')^2} - 34.101 e^{-0.3^2(\mathbf{r}-\mathbf{r}')^2} + \frac{4e^2}{|\mathbf{r}-\mathbf{r}'|}\mathrm{erf}\,(a|\mathbf{r}-\mathbf{r}'|)$$

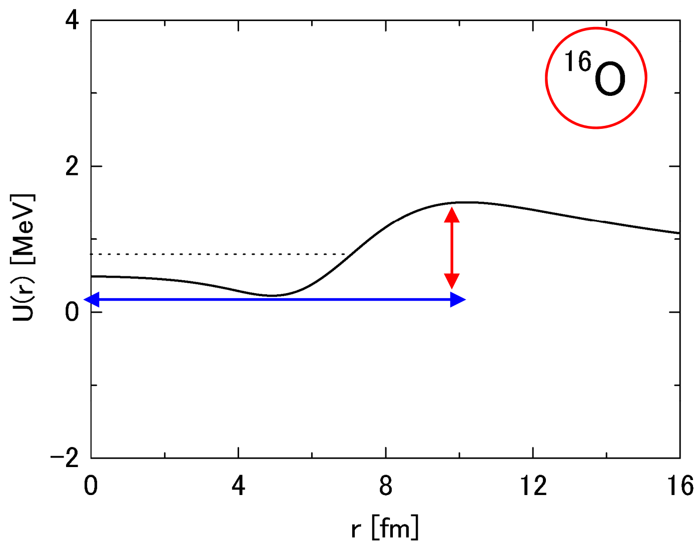
Resonant energy of ^{8}Be(0^+), E=0.092 MeV
Maximum value at r∼4 fm for α-α relative wf.
Low energy α-α scattering phase shift

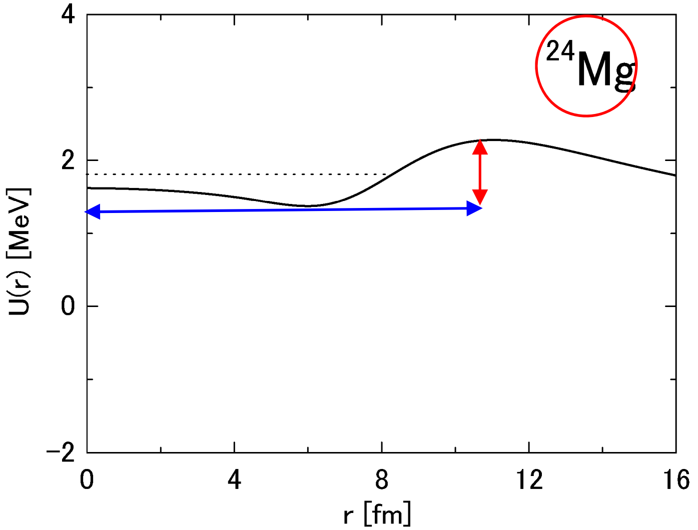
$$\upsilon_3(\mathbf{r},\mathbf{r}',\mathbf{r}'') = 151.5 e^{-0.15[(\mathbf{r}-\mathbf{r}')^2+(\mathbf{r}'-\mathbf{r}'')^2+(\mathbf{r}''-\mathbf{r})^2]}$$

Needed in 3α and 4α OCM calculations by Fukatsu & Kato
(similar role to DD term)

Radial behavior of $U_\alpha(r)$







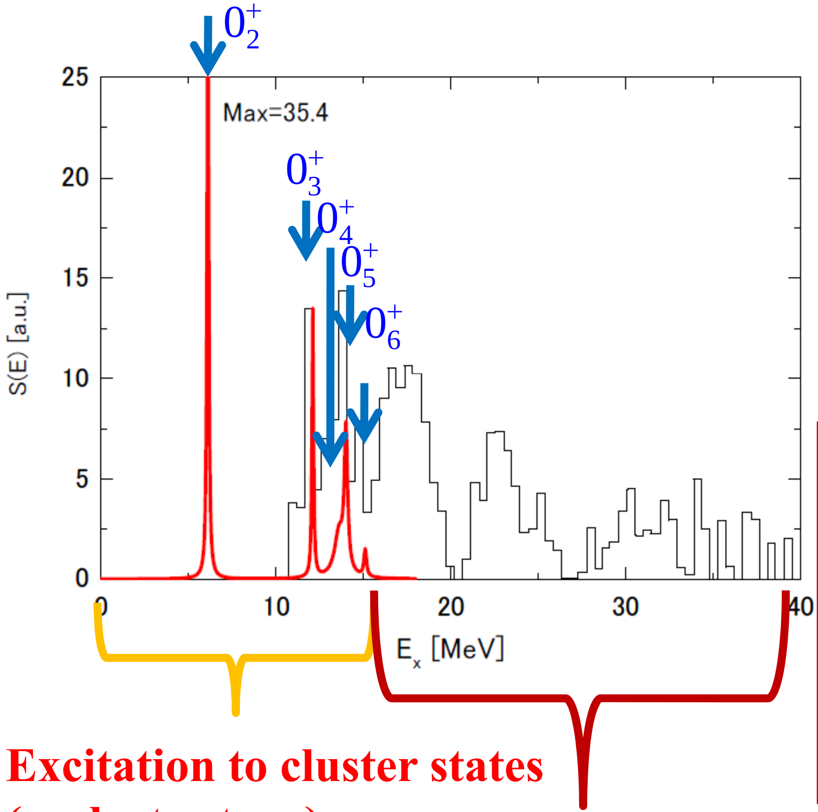
Estimated for Maxinum number

$$N^{\alpha}_{\mathrm{Limit}} \sim 10 \quad \Rightarrow \quad {}^{40}\mathrm{Ca}^{**}$$

Yamada & Schuck, PRC69, (2003)

Two features of IS monopole excitations in ^{16}O

IS monopole S(E): 4α OCM



Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

Two features in IS monpole exci.

	Fine structure $E_x < 16$ MeV	3-bump structure $16 < E_x < 40$ MeV
4α OCM	OK	difficult
RPA	difficult	OK

Origin: dual nature of G.S. of ^{16}O

(1) α-clustering degree of freedom

(2) mean-field-type one

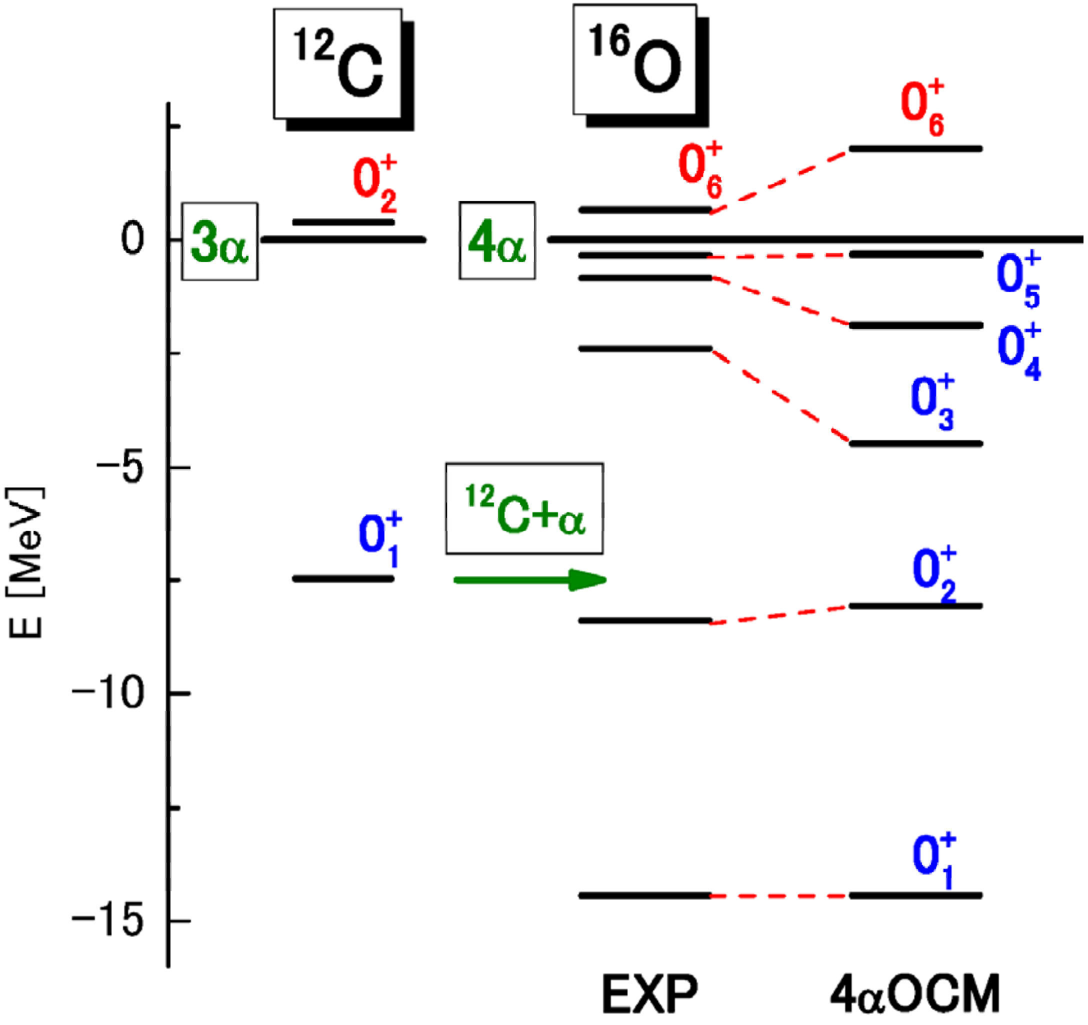
$(0s)^4(0p)^{12}$: SU(3)(00)= $^{12}C+\alpha$:

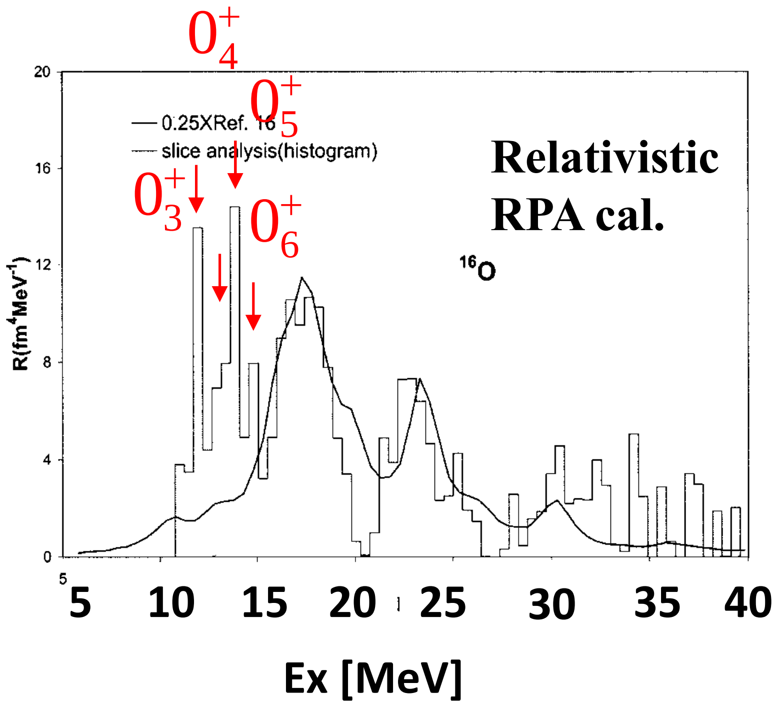
Bayman-Bohr theorem

Huge model space is needed to reproduce S(E) in the whole energy region.

cluster basis + collective basis

IS-monopole strength function of $^{16}O(\alpha,\alpha')$

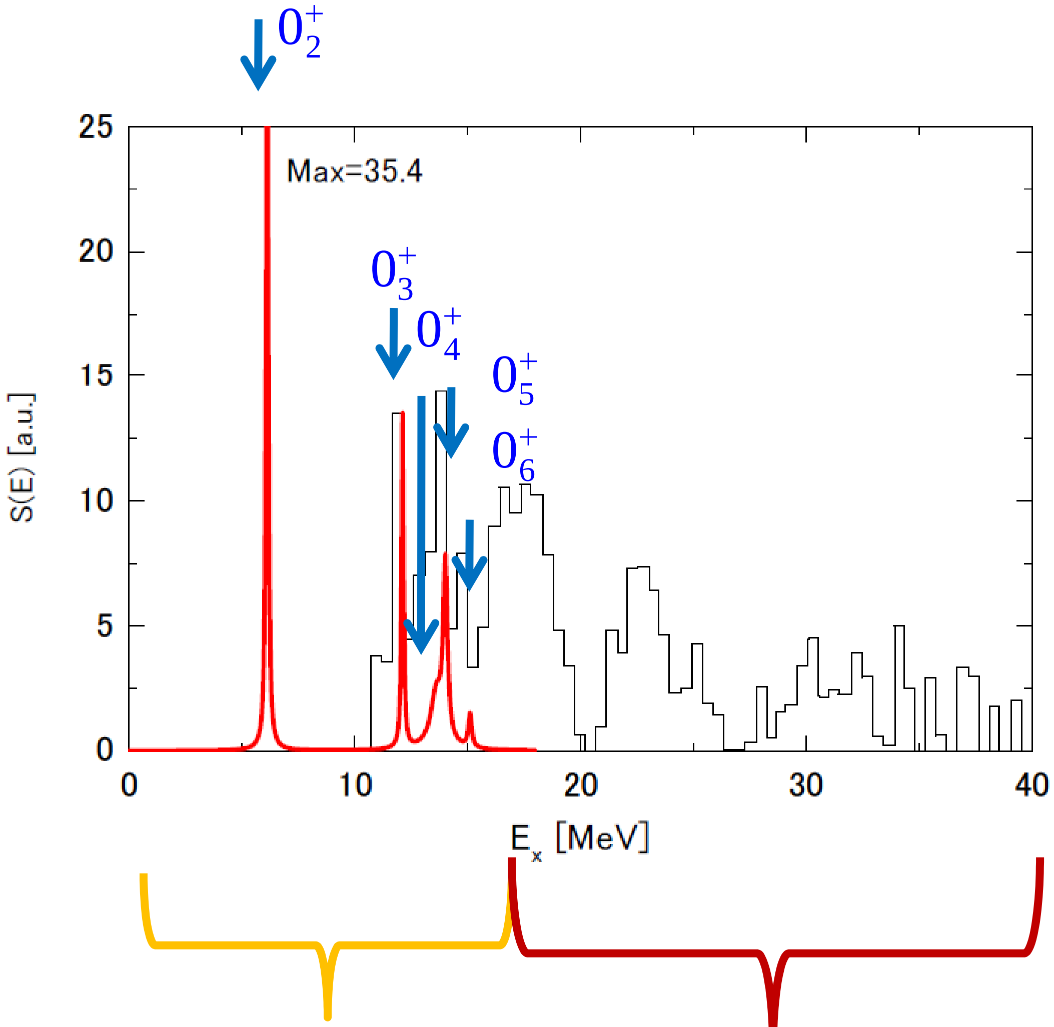




Exp: histogram
Lui et al., PRC 64 (2001)
Cal: real line
Ma et al., PRC 55 (1997)

Exp. condition: E_x > 10 MeV

IS monopole S(E) with 4α OCM



^{16}O(α,α')

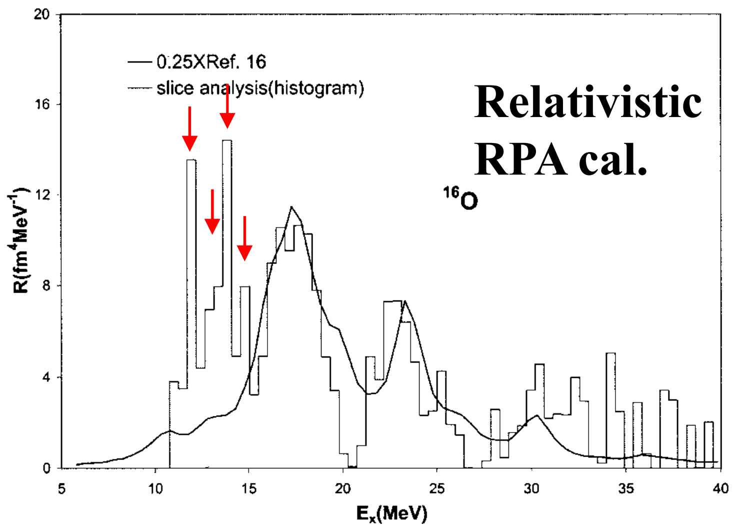


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the monopole response function from Ref. [16] multiplied by 0.25 and shifted by 4.2 MeV.

Exp. condition: E_x > 10 MeV

Excitation to cluster states (α-cluster type)

Monopole excitation of mean-field type (RPA)

4α OCM and EWSR of IS-monopole transitions

Ratio of OCM-EWSR to total EWSR in IS monopole transitions:

T. Yamada et al.

$$\frac{\text{OCM-EWSR}}{\text{total EWSR}} = 1 - \left(\frac{1.71}{R}\right)^2 = 0.60$$

4α OCM shares about 60% of the total EWSR value.

This is one of the important reasons that the 4α OCM works rather well in reproducing the IS monopole transitions in low-energy region of ^{16}O.

OCM-EWSR: $$\sum_n (E_n^{\text{OCM}} - E_1^{\text{OCM}}) \left| M^{\text{OCM}}(0_n^+ - 0_1^+) \right|^2 = \frac{2\hbar^2}{m} \times 16 \times (R^2 - 1.71^2)$$

Total EWSR: $$\sum_n (E_n - E_1) \left| M(0_n^+ - 0_1^+) \right|^2 = \frac{2\hbar^2}{m} \times 16 \times R^2$$

$$R^2 = \frac{1}{16} \left\langle 0_1^+ \left| \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2 \right| 0_1^+ \right\rangle$$

Bayman-Bohr theorem

$$\frac{1}{\sqrt{16!}}\det\left|(0s)^4(0p)^{12}\right|\times\left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1}$$
$$=N_0\sqrt{\frac{12!4!}{16!}}A\left\{\left[u_{40}(\xi_3,3\nu)\phi_{L=0}({}^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}$$
$$=N_2\sqrt{\frac{12!4!}{16!}}A\left\{\left[u_{42}(\xi_3,3\nu)\phi_{L=2}({}^{12}\mathrm{C})\right]_{J=0}\phi(\alpha)\right\}$$

$$\phi_{\rm cm}(\mathbf{R}_{\rm cm})=\left(\frac{32\nu}{\pi}\right)^{3/4}\exp(-16\nu\mathbf{R}_{\rm cm}^2)$$

c.o.m. w.f. of ^{16}O

➔ **G.S. has α+^{12}C(0^+,2^+) cluster degrees of freedom.**

$$\frac{1}{\sqrt{16!}}\det\left|(0s)^4(0p)^{12}\right|\times\left[\phi_{\rm cm}(\mathbf{R}_{\rm cm})\right]^{-1}$$
$$=\widehat{N}_0\sqrt{\frac{4!4!4!}{16!}}A\left\{\left[u_{40}(\xi_3,3\nu)\left[u_{40}(\xi_2,\frac{8}{3}\nu)u_{40}(\xi_1,2\nu)\right]_{L=0}\right]_{J=0}\phi(\alpha)\phi(\alpha)\phi(\alpha)\phi(\alpha)\right\}$$

➔ **G.S. has a 4α-cluster degree of freedom.**

Interesting characters of IS monopole operator

$$\hat{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\rm cm})^2$$

$$= \sum_{k=1}^{4}\sum_{i=1}^{4} (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 \quad + \quad \sum_{k=1}^{4} 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\rm cm})^2$$

internal part of each α-cluster

relative parts acting on relative motions of 4α's with respect to c.o.m. of ^{16}O

$$= \sum_{i=1}^{4} (\mathbf{r}_i - \mathbf{R}_{\alpha})^2 \quad + \quad \sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{12\rm C})^2 \quad + \quad 3(\mathbf{R}_{\alpha} - \mathbf{R}_{12\rm C})^2$$

internal part of α

internal part of ^{12}C

relative part acting on relative motion of α and ^{12}C

Decomposition into Internal part and relative part plays an important in monopole excitations of ^{16}O.