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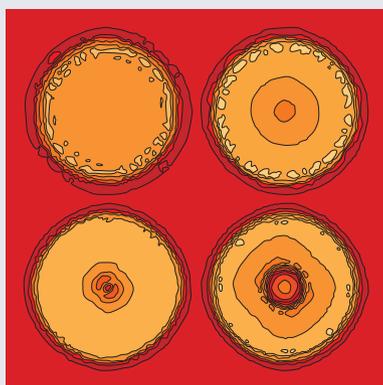
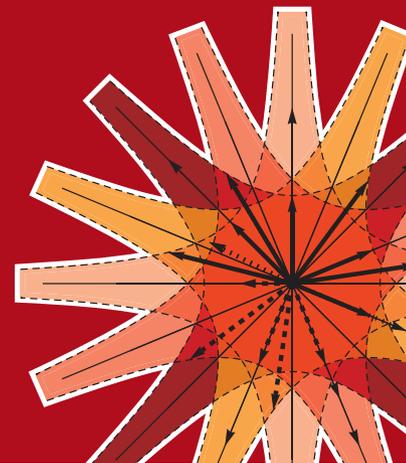
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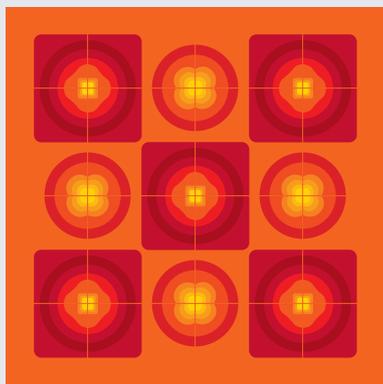
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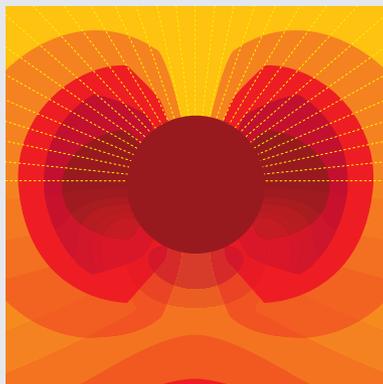
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Systemic risk in dynamical networks with stochastic failure criterion

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Abstract – Complex non-linear interactions between banks and assets we model by two time-dependent Erdős-Renyi network models where each node, representing a bank, can invest either to a single asset (model **I**) or multiple assets (model **II**). We use a dynamical network approach to evaluate the collective financial failure —systemic risk— quantified by the fraction of active nodes. The systemic risk can be calculated over any future time period, divided into sub-periods, where within each sub-period banks may contiguously fail due to links to either i) assets or ii) other banks, controlled by two parameters, probability of internal failure p and threshold T_h (“solvency” parameter). The systemic risk decreases with the average network degree faster when all assets are equally distributed across banks than if assets are randomly distributed. The more inactive banks each bank can sustain (smaller T_h), the smaller the systemic risk —for some T_h values in **I** we report a discontinuity in systemic risk. When contiguous spreading becomes stochastic ii) controlled by probability p_2 —a condition for the bank to be solvent (active) is stochastic— the systemic risk decreases with decreasing p_2 . We analyse the asset allocation for the U.S. banks.

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Phase transitions, critical points, hysteresis and regime shifts are basic blocks describing the phase flipping of a complex dynamical system between two or more phases [1]. One of the systems having these properties is the financial system that can be considered as a system flipping over time between the mainly stable phase, representing good years, and the mainly unstable phase, representing bad years. In the financial system, the transition from the mainly stable to the mainly unstable phase can be triggered either by an outside sudden event such as a war or by a bankruptcy of a huge bank when the financial contagion can spread due to interlinks between financial units. The nature of this contagion spreading

implies that a network approach can be the best suitable framework to describe not only financial contagion but also financial crises [1–17]. In networks, just like in financial systems, the existing nodes can be rewired, and new links and nodes can be added and removed as time elapses and node’s properties can change over time [18–24].

In a seminal work on the network approach in finance, Allen and Gale [3] argued that a more inter-related network may help that the losses of a distressed bank are shared among more creditors reducing the impact of negative shocks to each individual bank. Using a network model of epidemics, ref. [9] showed that a large rare shock may have different consequences depending on where it hits in the network and what is the average degree of the network. In contrast, beyond a certain point, such

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interconnections may serve as a mechanism for propagation of large shocks leading to a more fragile financial system. Reference [10] investigated the relationship between the risk taken by individual banks and the systemic risk associated with multiple bank failures. Reference [11] reported how imposing tougher capital requirements on larger banks than smaller ones can increase the resilience of the financial system. Reference [15] reported how integration (each organization becoming more dependent on its counterparts) and diversifications (each organization interacting with a larger number of counterparts) have different effects on cascading failure. Reference [13] reported that financial contagion exhibits a phase transition as interbank connectivity increases. If shocks are smaller than some threshold, more linked networks enhance the stability of the system. For shocks larger than the threshold, more linked networks facilitate the financial contagion.

The global financial crisis has urged the need for analysing the systemic risk representing the collective financial failure [1,4,7–12,15]. The majority of the literature on the systemic risk focuses on how the financial system responds to the failure of a single bank. However, in real finance many banks can fail inherently either at equal or at different times. Here, in order to estimate the collective financial failure when multiple initial failures are possible, occurring presumably at different times, we model a complex non-linear interaction between banks and firms by two variants of the dynamical Erdős-Renyi network proposed in ref. [25], where nodes (banks) contiguously fail due to links to both i) assets and ii) other banks, and iii) possibly recover. In ref. [25], the collective phenomena reported in a financial system—phase transitions, critical points, hysteresis and phase flipping—have been described by the dynamical network approach. It has been explained how the network, due to ii) stochastic contiguous spread among the nodes, may lead to the spontaneous emergence of macroscopic phase-flipping phenomena. In our dynamical network approach, bank i can internally fail at any time t_i , and once it fails, the contagion spreads at t_{i+1} on i 's nearest neighbours, at t_{i+2} on i 's second neighbours, and so on. This approach allows us to calculate the systemic risk at different future time horizons. The probability parameter, controlling the macroscopic phase-flipping phenomena [25], determines also that the condition for a bank to be active is not deterministic [9], but stochastic.

Qualitatively it is known that more links between banks may reduce the risk of contagion [3]. Recently a model of contagion in financial networks has been proposed [9] where each bank i has interbank assets A_i^B , interbank liabilities L_i^B , deposits D_i and illiquid assets A_i^M , and the condition for the bank to be solvent (active) is $(1 - \phi)A_i^B + A_i^M - L_i^B - D_i > 0$ —a bank's assets must exceed its liabilities—where ϕ is the fraction of inactive neighbouring banks. To account for the possibility that many banks can fail at any moment—not only at the initial moment—and to account for the possibility to estimate the bank risk

for different future time horizons, highly volatile financial networks in this work we model by a variant of the dynamical Erdős-Renyi network [25]. We define the world model **I** in the following: i) nodes represent banks, and bank i at time t fails if $(1 - \phi_{t,i})A_i^B + A_{t,i}^M - L_i^B - D_{t,i} \leq 0$. The previous time-dependent condition can be accomplished if i) bank i internally fails randomly and independently of other nodes with probability of failure p (see “Methods” section)—for each bank i , with probability p , $A_{t,i}^M - D_{t,i}$ becomes negative and i fails, regardless of interbank assets. ii) Besides internally, bank i can also fail with probability p_2 if it has less than or equal to $100T_h\%$ active neighbours (where i sets its interbank assets) [25,26]. If not stated differently, here we assume that $p_2 = 1$, implying that if bank i has $< 100T_h\%$ active banks, it deterministically fails. The parameter T_h measures the robustness of the bank network—the smaller the parameter T_h , the more robust the bank network. Note that T_h can be related with the criterion for bank failures of ref. [9]. To this end, let us assume that for each bank there is a linear dependence between asset A_i^B and network degree k —*e.g.* $A_i^B = k_i$. Since the number of incoming and outgoing links is equal, it holds $A_i^B = L_i^B$. Expressing A_i^M and D_i at $t = 0$ as proportional to $k_i(A_i^B)$ say $A_{t=0,i}^M = 0.6k_i$ and $D_{t=0,i} = 0.3k_i$, then bank i is inactive if $\phi_{t,i} \geq 0.3 \equiv \phi_h$ (if at least 30% neighbouring banks are inactive, or alternatively, if less than 70% neighbouring banks are active). Thus,

$$1 - T_h = \phi_h. \quad (1)$$

iii) After a time period τ , the nodes recover from internal failure (see “Methods” section). In finance, this τ is comparable with an average time a firm spends in financial distress that is approximately two years for U.S. firms [27].

To calculate how the systemic risk—among many different definitions [14], here defined as the fraction of failed banks—depends on the model parameters, first we present an analytical result that holds for the mean-field approximation, which is generally valid for a network with a large number of nodes and degrees. If internal (X) and external (Y) failures are independent, ref. [25] calculated the probability, $a = a(p, p_2, T_h) \equiv P(X \cup Y)$, that a randomly chosen bank node i is inactive $a = p + p_2(1 - p)\Sigma_k P(k)E(k, m, a)$. This probability is equal to the fraction of inactive bank nodes, $a = 1 - \langle f_n \rangle$. Here $P(k)$ is the degree distribution of the interbank links, and parameters p , T_h , and p_2 are explained in **I**. $E(k, m, a) \equiv \Sigma_{j=0}^m a^{k-j}(1-a)^j \binom{k}{k-j}$ is the probability that node i 's neighbourhood is critically damaged [25], where k is the number of links of node i , and $m \equiv T_h k$. For $m = 1$ we provide an analytical form of $a = 1 - \langle f_n \rangle$ when $P(k)$ is the Poisson distribution $P(k) = (\langle k \rangle)^k e^{-\langle k \rangle} / (k!)$ —we obtain $a = p + p_2(1 - p)(1 + \langle k \rangle - a\langle k \rangle)e^{\langle k \rangle(a-1)}$.

The application of analytical results based on the mean-field approximation in finance—where generally there are either a small or a moderately large number of banks—is highly limited since for these cases the mean field holds

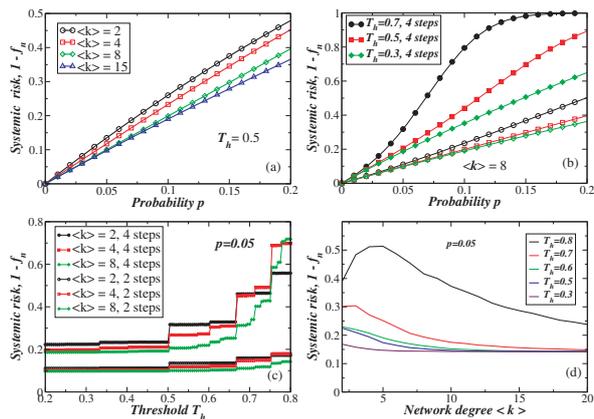


Fig. 1: (Colour on-line) For model **II** we show the dependence of the network systemic risk of banks, $1 - f_n$ (fraction of failed banks), on network parameters, individual probability of bank failure p , threshold T_h , and network degree, $\langle k \rangle$. (a), (b): $1 - f_n$ increases with p for varying T_h and $\langle k \rangle$. (c) $1 - f_n$ non-continuously increases with T_h . (d) $1 - f_n$ decreases with $\langle k \rangle$ for large values of $\langle k \rangle$.

only approximately [25]. For these cases, in practice, the numerical approach helps us estimate how the systemic risk quantitatively depends on each model parameter and finally it enables us to estimate, using regression, what the systemic risk is for a given set of empirically estimated parameters.

For the world model **I** with 1000 banks, in numerical simulations each of 10000 runs is used to estimate, *e.g.*, the systemic risk a year ahead expressed as the fraction of failed banks, $1 - f_n$. Each run we accomplish in two time steps and each bank can internally fail in both time steps with no recovery ($\tau > 2$). By fixing parameters $\langle k \rangle$ and T_h , in fig. 1(a), (b) we find that the bank systemic risk, $1 - f_n$, increases non-linearly with individual bank failure p . In fig. 1(a) the systemic risk decreases with the average degree $\langle k \rangle$. Thus, the larger the number of links between banks, the smaller the systemic risk. This result is in agreement with ref. [2] where it was shown that networks with more links are less vulnerable than networks with a few links, since the fraction of the losses in one bank is transferred to more banks through interbank links. Note that our choice $\langle k \rangle = 15$ is due to ref. [5] reporting that the average bank in the U.S. is linked to 15 others; however most banks have only few connections while a small number of huge banks have thousands.

In fig. 1(b) we report that the more failed neighbouring banks any bank can sustain (the smaller T_h), the smaller the systemic risk. In economics, the existence of a threshold assumes that when an organization's value (say asset minus debt) hits a failure threshold, the organization discontinuously can lose part of its value [15]. Due to interdependences among nodes, individual failures may trigger collective cascade of failures. To this end, with increasing T_h , the systemic risk in fig. 1(c) exhibits a non-linear discontinuity where $1 - f_n$ suddenly jumps at some critical

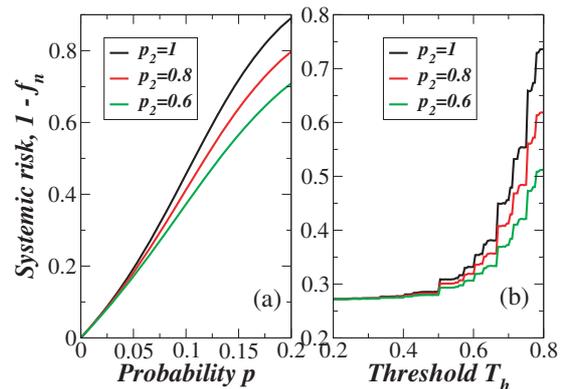


Fig. 2: (Colour on-line) For fixed T_h in (a) and p in (b) with decreasing parameter p_2 , the systemic risk decreases. We set $\langle k \rangle = 8$ and $T_h = 0.5$ in (a).

points in T_h such as $1/2, 2/3, \dots$, since link values are integer numbers. Note that ref. [25] reported a discontinuity in the fraction of active nodes, f_n , when increasing (decreasing) p and p_2 , together with the hysteresis property that is the characteristic feature of a first-order phase transition [25,26]. As stated previously, recently ref. [13] reported a phase transition as interbank links increases. Next, fig. 1(d) confirms again that the systemic risk in the dynamical network approach decreases with $\langle k \rangle$.

For many parameter sets ($\langle k \rangle, T_h, p, 1 - f_n$) obtained from two-period runs (where $\tau > 2$, thus no recovery), we perform regression analysis $1 - f_n = \alpha + \alpha_p p + \alpha_T T_h + \alpha_k k$, and obtain $\alpha = -0.016 \pm 0.001$, $\alpha_p = 2.20 \pm 0.03$, $\alpha_T = 0.067 \pm 0.005$, and $\alpha_k = -0.0017 \pm 0.0002$. The systemic risk significantly increases with p and T_h , while it decreases with degree $\langle k \rangle$. Note that these values we obtained using $p \in (0, 0.1)$, $T_h \in (0.2, 0.8)$, and $\langle k \rangle \in (2, 20)$.

The dynamical network approach reveals one more forecasting benefit. It provides us with forecasting power for generally any future time horizon. For the world model **I** in fig. 1(b) we show the expected systemic risk where each of 10000 runs is composed of four steps. If each time step represents, say a semi-year period, then a two-step systemic risk represents our estimation for the systemic risk a year ahead, while a four-step systemic risk represents our estimation for the systemic risk two years ahead. As expected for the case with no recovery, the four-step systemic risk is larger than the two-step systemic risk. A similar result we obtain in fig. 1(c) where for $\langle k \rangle = 8$ we show that the four-step systemic risk is substantially larger than the two-step systemic risk. Note that these results we obtain under the assumption that banks and assets, once failed, do not recover. Clearly, in a more reliable dynamical network approach one should also define how assets and banks recover over time after, for example, the government intervenes in the market.

Figure 2 shows that for fixed p and T_h , the systemic risk decreases with decreasing parameter p_2 (shown from 1 to 0.6), that is a reasonable result since $p_2 = 1$ implies that a bank deterministically fails when a criterion for

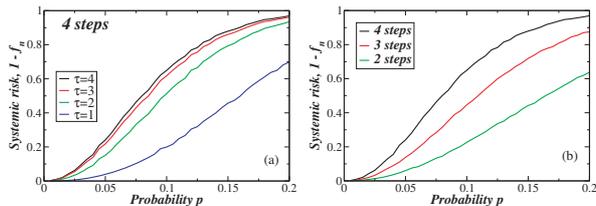


Fig. 3: (Colour on-line) (a) The systemic risk for bank network increases with the time spent in failure τ . (b) Systemic risk for different time scales. With increasing time horizon, the systemic risk also increases.

insolvency $(1 - \phi_{t,i})A_i^B + A_{t,i}^M - L_i^B - D_{t,i} \leq 0$ is fulfilled, while when $p_2 \neq 1$, there is some chance that the bank will not fail. This unpredictability is something that really occurs in the real market, since bankruptcy is not a deterministic event. However, stochasticity is not important only at microscopic (microeconomic, bank) level. Reference [25] reported that stochasticity leads to the emergence of macroscopic phase-flipping between “active” and “inactive” macroscopic phases that are demonstrated in ref. [26] for “expansion” and “recession” phases in macroeconomics.

Next we propose another world model **II** (see “Methods” section), where each bank can put its money not only in other banks, but also in different illiquid asset classes $\mathcal{A}_{t,i}^M$. Since different banks can invest their money in equal assets $\mathcal{A}_{t,i}^M$, banks’ failures are now not independent. In contrast to banks’ failures, $\mathcal{A}_{t,i}^M$ ’s failures are assumed to be independent of each other. For simplicity, we define that firms can affect banks, but not vice versa as in [9,11,28].

In the following simulations we include a recovery process. In the world model **II** there are four time steps in our analysis, but this time we change the time period needed for recovery, τ . For i)–iii) ER dynamical networks with a finite number of banks, $N_b = 1000$, and assets, $N_f = 10$, our numerical simulations in fig. 3(a) confirm that the systemic risk calculated for bank network —defined by the fraction of banks in failure— increases with the individual probability of asset failure p , chosen to be equal for each asset. We assume as in ref. [11] that both large and small banks hold the same number of asset classes, 10. We perform 10^5 simulations in order to estimate the expected systemic risk where each simulation itself we perform in four steps with τ meaning that once a bank or an asset is failed, it stays failed for the entire period τ . Figure 3 reveals that the world model **II** exhibits highly non-linear properties. By first fixing parameter T_h , in fig. 3(a) we find that the systemic risk for bank networks increases with τ . The longer time a bank stays in failure, the larger the systemic risk. In fig. 3(b) we show how a dynamical approach enables us to estimate the systemic risk for different time horizons. As expected, with increasing time horizon, the systemic risk also increases.

In order to analyse how banks and assets interact with each other over time, next in the world model **II** we

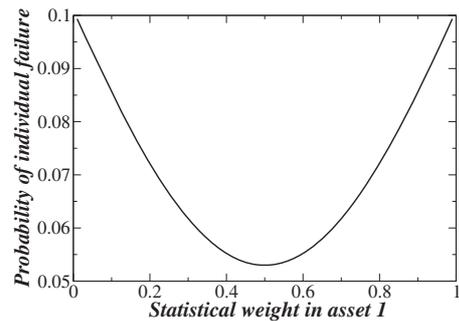


Fig. 4: Smile-form in the probability of individual bank failure with two assets with allocations W_1 and W_2 . The smallest risk occurs as expected for $W_1 = W_2 = 0.5$. We use $\tau = 2.5$.

evolve simultaneously both banks ($N_b = 1000$) and assets ($N_f = 10$). The world model **II** is partially motivated by Beale *et al.* [10]. As known in finance, each bank can reduce its probability of failure by diversifying its risk [29]. However, when many banks diversify their risks in a similar way, the probability of multiple failure increases [10]. For the case with N banks and M assets, ref. [10] defined the total loss incurred by bank i after one period $Y_i = \sum W_{ij}V_j$, where the failure occurs if $Y_i > \gamma_i$. Here, W_{ij} denotes bank i ’s allocation in asset j , V_j is the loss in asset j ’s value taken from a student t distribution, and γ_i is a threshold.

In the world model **II** we use not the t distribution [10], but a simpler Laplace distribution and derive a probability of bank failure with two assets with allocations W_1 and W_2 :

$$P = \frac{\exp(-\frac{\gamma}{W_2\tau})}{4(1 + \frac{W_1}{W_2})} + \frac{\exp(-\frac{\gamma}{W_2\tau})}{4(1 - \frac{W_1}{W_2})} \left(1 - \exp\left(-\frac{(1 - \frac{W_1}{W_2})\gamma}{W_1\tau}\right) \right) + \frac{\exp(-\frac{\gamma}{W_1\tau})}{2} + \frac{\exp(\frac{\gamma}{W_2\tau})}{4(1 + \frac{W_1}{W_2})} \exp\left(-\frac{(1 + \frac{W_1}{W_2})\gamma}{W_1\tau}\right).$$

This expression, numerically tested in fig. 4, gives the same smile-form for the probability of bank failure as numerically found in ref. [10]. Next we analyse how diversification of asset allocations affects systemic risk within the dynamical network approach. To this end, ref. [10] proposed a measure for the level of asset allocation,

$$\Delta = \frac{1}{2N(N-1)} \sum_{i=1}^N \sum_{j=1}^N \sum_{l=1}^M |W_{i,l} - W_{j,l}|, \quad (2)$$

in order to quantify the average of the distances between each pair of banks’ asset allocations, where $\Delta = 0$ if each bank invests equally in each asset, thus if $W_{i,j} = 1/M$ for each i and j . Thus, when many banks decide to invest with similar portfolios, they may increase the chance to fail simultaneously. In fig. 5(a) within the dynamical network approach, we obtain that the lowest systemic risk occurs when all assets are equally distributed across banks, which is in agreement with the result obtained in ref. [10] where banks do not affect each other, and

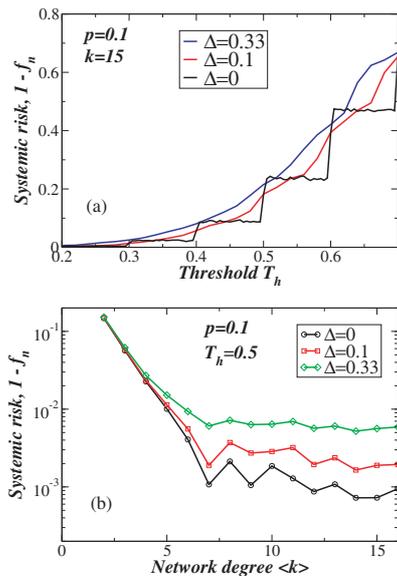


Fig. 5: (Colour on-line) Dependence of the diversification of assets on the systemic risk within the dynamical network approach. (a) The systemic risk increases with T_h for varying Δ value. Δ quantifies the level of asset allocation. With increasing randomness in asset allocation, where Δ increases from zero to $1/3$, the systemic risk increases. We use $p_2 = 1$ and $T_h = T'_h$ (see “Methods” section). (b) The systemic risk decreases with the average network degree $\langle k \rangle$.

the systemic risk is defined as the expected number of failures. With increasing randomness in asset allocation, where Δ increases from zero to $1/3$, the systemic risk also increases. As a new result arising from the network approach, in fig. 5(b) we obtain that the systemic risk decreases with the average degree $\langle k \rangle$ faster when the asset allocation is more homogeneous.

To see how well the real market can withstand the systemic risk, we examine the level of diversification of the asset allocation for the U.S. commercial banks. Here, based on 64289 commercial banks’ balance sheet, we analyze their loan allocations in 10 different sectors [30]. The data are collected from 1/1/1976 to 12/31/2008 and a considerable fraction of the banks have disappeared during the period. We use eq. (2) to calculate the average diversification of the commercial banks, and find $\Delta = 0.51$, a larger value than $\Delta = 0.33$, obtained for random diversification.

Generally, the systemic risk may arise either because banks diversify their risks in firms in similar ways [10] or because banks are linked within a bank network and thus failures can spread contiguously [2–4]. For the dynamical (time-dependent) network [25] where nodes i) inherently fail, ii) contiguously fail, and iii) recover, when applied to finance, the stochastic contiguous spreading controlled by parameter p_2 corresponds to a stochastic condition for the bank to be solvent (active in network terms). We demonstrated how the systemic risk in the dynamical (time-dependent) network depends on dynamical network parameters: the probability of individual asset failure p ,

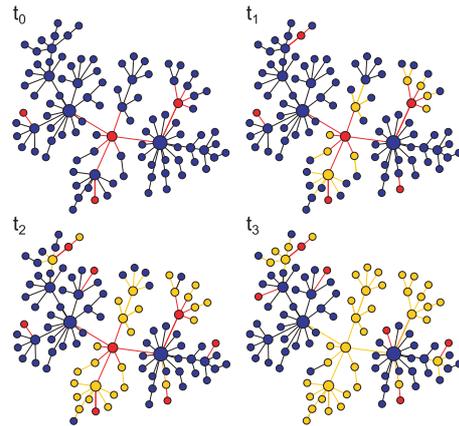


Fig. 6: (Colour on-line) Contiguous spread in model **I** with the stochastic solvency criterion, $p_2 \neq 1$. Active nodes are blue, internally inactive nodes are red and externally inactive nodes are denoted by orange. Nodes can become internally inactive at any moment where contagion is spread each step towards the nearest neighbours. For steps $t = 0, 1, 2$, and 3 shown is a network for $\tau = 3$ and $p_2 = 0.8$.

bank vulnerability parameter T_h , and average network degree $\langle k \rangle$. We found that the more inactive banks each bank can sustain (T_h closer to zero), the smaller the systemic risk. For fixed p and T_h , the systemic risk decreases with decreasing parameter p_2 (increasing stochasticity). This result is reasonable because when a criterion for a bank’s insolvency is met and $p_2 \neq 1$, there is some chance that the bank will not fail. In practice, the parameter T_h can be controlled by the government or banks. Our analysis showed that having $T_h < 0.4$ enables the system to function with low risk for a large set of parameters p and $\langle k \rangle$. We demonstrated how the dynamical network approach enables us to estimate the systemic risk for different future time scales. We also showed how the systemic risk depends on the level of asset allocation. The dynamical network approach enables to estimate the systemic risk in a wide range of financial systems from pension, mutual, and hedge funds to the banking system.

Methods. – We previously defined the world model **I** and here we explain the model graphically. In fig. 6 we choose $\tau = 3$ and $p_2 = 0.8$ where the last choice for p_2 implies that the solvency criterion is stochastic. Due to the dynamical time-dependent network approach, we see how different nodes become internally failed at different times. We choose $T_h = 0.8$, meaning that each bank is highly dependent on failures of its neighbours. Note that, due to stochasticity in the solvency criterion, some nodes, which would be externally inactive due to the deterministic criterion ($p_2 = 1$), are externally active.

We define the world model **II** in the following: i) At each time t , each of the N_f assets $\mathcal{A}_{t,i}^M$ can independently fail where the probability p of failure for each $\mathcal{A}_{t,i}^M$ is equal and is taken from a Laplace (double) exponential distribution, $\mathcal{L}(x)$. We define that asset $\mathcal{A}_{t,i}^M$ fails if the

Laplace variable x is smaller than some threshold, P_h —thus, $p = \int_{-\infty}^{P_h} dx \mathcal{L}(x)$. For simplicity, assets do not influence each other. Once the asset $\mathcal{A}_{t,i}^M$ fails, it stays in failure with no possibility of recovery. ii) Each of the N_b nodes representing banks, for simplicity reasons, has the same asset and liability values, as for example, in **I**—so, bank j at time $t = 0$ has $A_j^B = k_j$, $A_{t=0,j}^M = 0.6k_j$ and $D_j = 0.3k_j$ (here D_j does not change in time), but the results are robust to different allocations. However, this time the total illiquid asset $A_{t=0,j}^M$ is allocated across N_f assets. Precisely, at initial time $t = 0$ every bank j determines how much money $W_{j,i}$ to invest in asset \mathcal{A}_i^M , where $W_{j,i}$ is taken randomly from the homogeneous distribution after proper normalization, where clearly for each j , $\sum_{i=1}^{N_f} W_{j,i} = A_{t=0,j}^M$, where the sum runs over all assets. Since at each moment assets can be either active or inactive (failed), at each t , bank j has the total illiquid asset equal to $A_{t,j}^M = \sum_{i=1}^{N_f} W_{j,i} \delta_i(t) \leq A_{0,j}^M$, where $\delta_i(t)$ at time t can be either one or zero depending whether asset \mathcal{A}_i^M is active or inactive. The more robust the bank, the larger the fraction of failed assets the bank can sustain without getting failed. Here we define that bank j 's node becomes internally contiguously failed due to links to assets if $A_{t,j}^M \leq T'_h A_{t=0,j}^M$, where T'_h is the given threshold. Bank j internally fails if $A_{t,j}^M - D_j < 0$ and from our choice for $A_{t=0,j}^M = 0.6k_j$ and $D_j = 0.3k_j$ the failure occurs when $T'_h = 0.5$. We define bank node j 's internal failure state by spin $|s_j\rangle$. iii) In our financial network, a bank can fail either due to illiquid assets failures or due to banks' failures. Here the concept of a network is used to model interbank lending and to study the phenomena of financial stability and contagion [2–4]. At the initial time, banks create links among themselves through exchanging deposits to insure themselves against contagion [3]. To this end, the external failure state of bank node j denoted by spin $|S_j\rangle$ is $|0\rangle$ (during a time $\tau' = 1$) with probability p_2 if less than $100 \cdot T_h\%$ of j 's neighbouring links are active [25]. Bank node j —described by the two-spin state $|s_i, S_i\rangle$ —is active only if both spins are 1, *i.e.*, $|s_i, S_i\rangle = |1, 1\rangle$. Links between banks are bidirectional. Having two spins, representing the financial health of a bank, assumes that a bank fails either if it made bad investments in firms or in other banks—alternatively if it is surrounded by bad neighbours. We assume $A_{0,j}^M = 0.6k_j$, $D_j = 0.3k_j$ (the same choices as in **I**), and $A_j^B = k_j$. For this choice of allocations, if we assume that all illiquid assets where j invested are active, we obtain $T_h = 0.7$ ($\phi_h = 0.3$) (see eq. (1)). Suppose that at time t some illiquid assets (20%) are inactive and $A_{t,j}^M = \sum_{i=1}^{N_f} W_{j,i} \delta_i(t) = 0.8A_{0,j}^M = 0.48k_j$. From the insolvency criterion $(1 - \phi_{t,i})A_i^B + A_{t,i}^M - L_i^B - D_{t,i} \leq 0$ we obtain $\phi_{t,i} \geq 0.18 \equiv \phi_h$. Hence, the larger the fraction

of inactive illiquid assets, the smaller the fraction of neighbouring banks required to cause the external failure.

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