Confinement and Landau gauge QCD Green Functions

Gluon positivity violation,

dynamical scalar quark confinement,

and the nucleons' quark core

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Rab, August 31, 2008

Outline

Some Selected Approaches to Confinement:

see *e.g.* R.A. and J. Greensite, *Quark Confinement: The Hard Problem of Hadron Physics* , J. Phys. **G34** (Special focus issue on Hadron Physics) (2007) S3.

- chromomagnetic **monopoles**
  't Hooft, diGiacomo, . . .
- center **vortices**
  Greensite, Olejnik, . . .
- **AdS$_5$** / QCD correspondence
  Maldacena, Brodsky, . . .
- **Coulomb** confinement
  Gribov, Zwanziger, . . .
- Landau gauge **Green Functions**
  Smekal, Fischer, . . .

Theories of Confinement

Properties to be explained:

- String formation
  - Casimir scaling at intermediate distances
  - N-ality at large distances
- Positivity violation
  - BRST quartet mechanism
  - Oehme–Zimmermann superconvergence relations
- Rôle of Gribov copies
- Conformal IR-YM sector
- DχSB
- $U_A(1)$ anomaly

Note: Not yet understood relations between different approaches,
definitely not mutually exclusive.

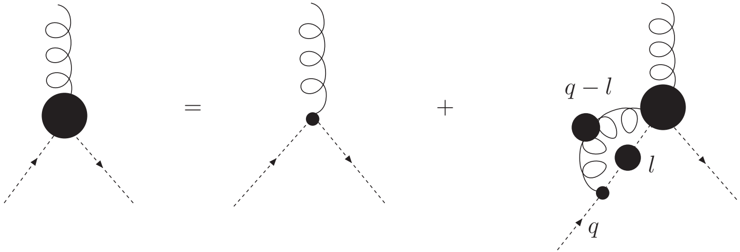
Functional Approaches to Confinement

!!! Infrared behaviour of Green functions;
e.g. in linear covariant gauges:
7 primitively divergent Green functions in QCD,
5 primitively divergent Green functions in Yang-Mills theory.

- gluon and ghost [and quark] propagators as well as
- 3-gluon, 4-gluon and gluon-ghost [and quark-gluon] vertices

Infrared Structure of Landau gauge YM theory

- Starting point in gauges with transverse gluon propagator: Ghost-Gluon-Vertex fulfills Dyson-Schwinger eq.

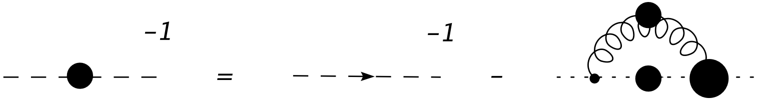


- $l_\mu D_{\mu\nu}(l-q) = q_\mu D_{\mu\nu}(l-q) \Rightarrow$ Bare Vertex for $q_\mu \to 0$
- No anomalous dimensions in the IR

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.
A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).
W. Schleifenbaum, A. Maas, J. Wambach and R. A., Phys.Rev.D72 (2005) 014017.

Infrared Exponents for Gluons and Ghosts

- Dyson-Schwinger eq. for the ghost-propagator:



Ansatz for Gluon, $Z(p^2) \sim (p^2)^\alpha$,
and Ghost Ren. Fct., $G(p^2) \sim (p^2)^\beta$.
L. v. Smekal, A. Hauck, R. A., Phys. Rev. Lett. **79** (1997) 3591

- Selfconsistency $\Rightarrow \quad -\beta = \alpha + \beta =: \kappa$ i.e.

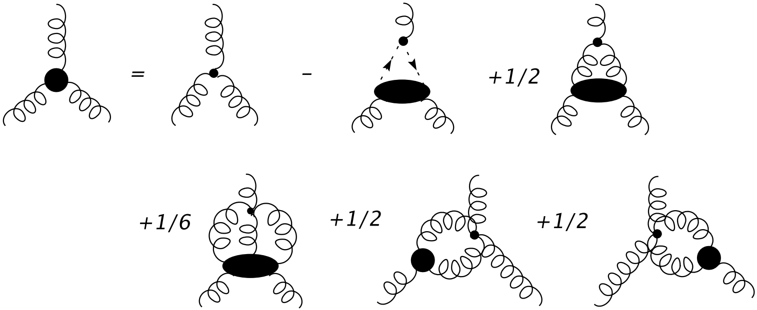
$$Z(p^2) \sim (p^2)^{2\kappa}\,, \quad G(p^2) \sim (p^2)^{-\kappa}$$

- IR enhanced ghost propagator: $0.5 \leq \kappa < 1$
Kugo–Ojima confinement criterion
and Gribov–Zwanziger horizon condition fulfilled!
P. Watson and R. A., Phys. Rev. Lett. **86** (2001) 5239

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Example: DSE for 3-gluon-vertex



Use DSEs and ERGEs:

→ Two different towers of equations for Green functions

E.g. ghost propagator

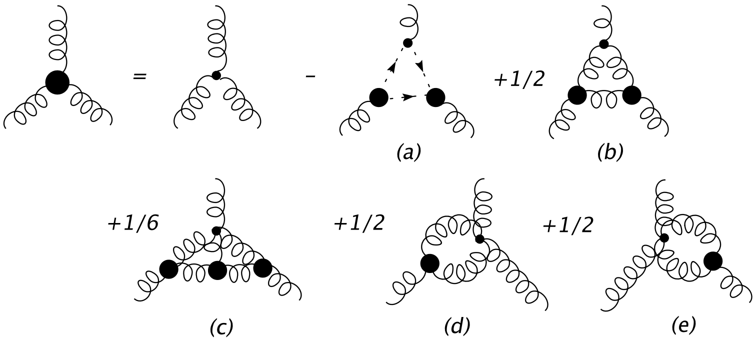
Infrared Exponents for Gluons and Ghosts:

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Skeleton expansion &

generalized formulas (neg. dim.) for Feynman integrals:

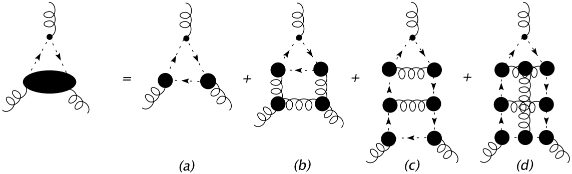


Use DSEs and ERGEs:

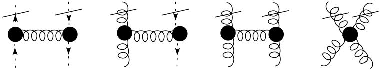
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Three-gluon vertex: **higher order** in skeleton expansion



built from insertions



insertions have zero IR anomalous dimensions $\Rightarrow$

IR-analysis valid to all orders in skeleton expansion

Use DSEs and ERGEs:

Infrared Exponents for Gluons and Ghosts:

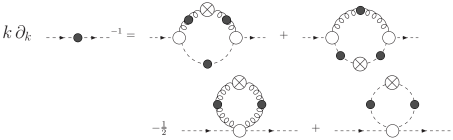
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Use DSEs and ERGEs:

$\longrightarrow$ Two different towers of equations for Green functions

E.g. ghost propagator



IR-Analysis of whole tower of equations $\Rightarrow$

Solution unique [C.S. Fischer and J.M. Pawlowski, PRD **75** (2007) 025012]

except a solution with IR trivial Green functions.

General Infrared Exponents for Gluons and Ghosts

n external ghost & antighost legs and m external gluon legs (one external scale p^2; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite if all external momenta vanish
- 3- & 4- Gluon vertex IR divergent if external momenta vanish
- IR fixed point for the coupling from each vertex
- Conformal nature of Infrared Yang-Mills theory!
- Ghost sector of YM-theory dominates IR!

  D. Zwanziger, Phys. Rev. D **69** (2004) 016002

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General Infrared Exponents for Gluons and Ghosts

n external ghost pair legs and m external gluon legs in $d \leq 4$ dim.:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa+(1-n)(d/2-2)}$$

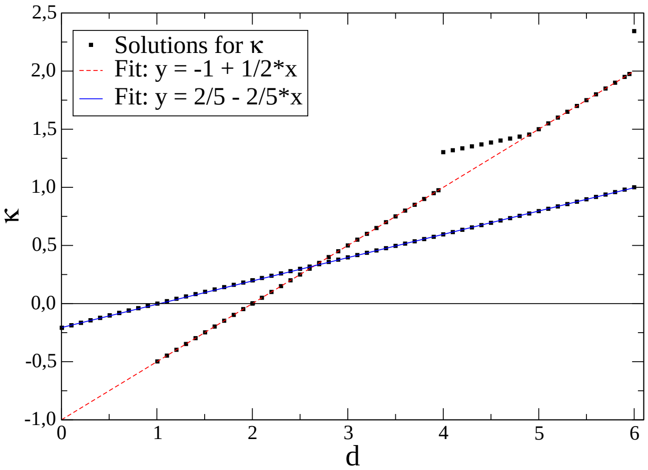
M. Huber, R.A., C.S. Fischer und K. Schwenzer, Phys.Lett. **B659** (2008) 434.

verified in 2 and 3 dimensions in MC calculations on large lattices

A. Maas, to be published.

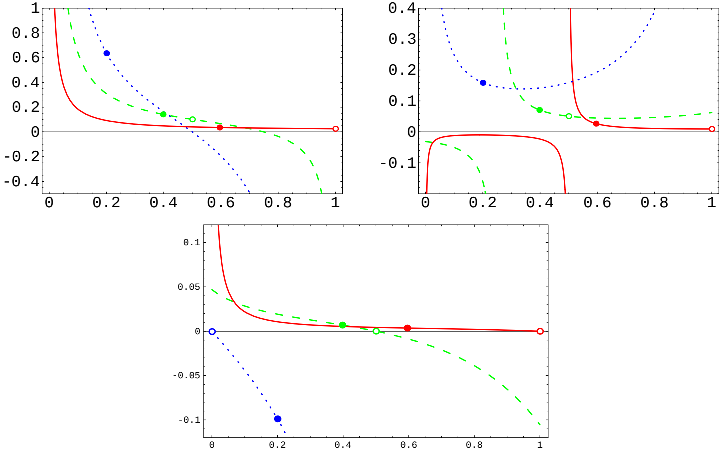
General Infrared Exponents for Gluons and Ghosts

Numerical values for the ghost exponent:



General Infrared Exponents for Gluons and Ghosts

Coefficients of gluon & ghost prop., 3-gluon vertex (symm.):



YM Running Coupling: IR fixed point

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}\,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$

$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2) \sim \frac{const_{gh-gl}}{N_c}$$

$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2) \sim \frac{const_{3g}}{N_c}$$

$$\alpha^{4g}(p^2) = \alpha_\mu \, [\Gamma^{4g}(p^2)]^2 \, Z^4(p^2) \sim \frac{const_{4g}}{N_c}$$

Running Coupling

Ghost-Gluon-Vertex UV finite:

$$\alpha_S(\mu^2) = \frac{g^2(\mu^2)}{4\pi} = \frac{1}{4\pi\beta_0} g_0^2 Z(\mu^2) G^2(\mu^2)$$

With known IR behavior of gluon (Z) and ghost (G) renormalization function:

IR fix point

$$\alpha_c = \alpha_S(k^2 \to 0) \simeq 2.972^*$$

$^*\alpha_S(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)}$

Infrared fix point also in Coulomb gauge!

[C.S. Fischer and D. Zwanziger, PR **D72** (2005) 054005]

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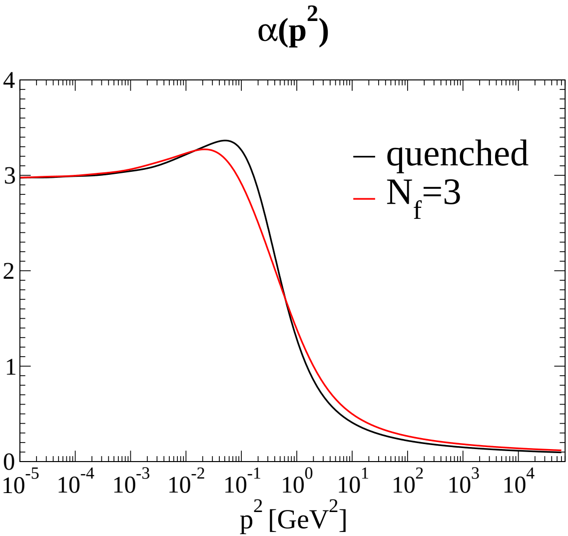


$\alpha_S(M_Z^2) \stackrel{!}{=} 0.118$

Positivity violation for the gluon propagator

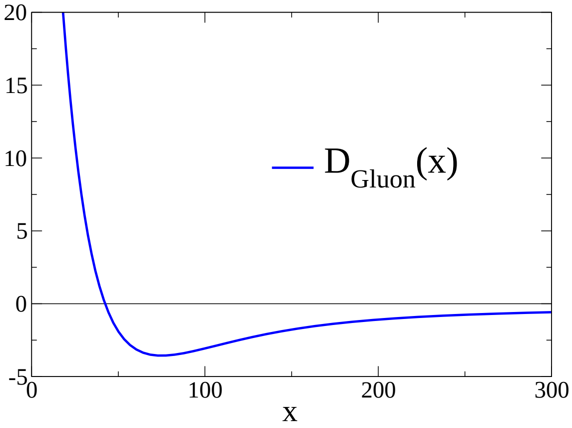
Simple argument [Zwanziger]:
IR vanishing gluon propagator implies

$$0 = D_{gluon}(k^2 = 0) = \int d^4x D_{gluon}(x)$$

$\Longrightarrow$ $D_{gluon}(x)$ has to be negative for some values of x.

Positivity violation for the gluon propagator

Fourier transform of DSE result:



Gluons unobservable $\Longrightarrow$ **Gluon Confinement!**

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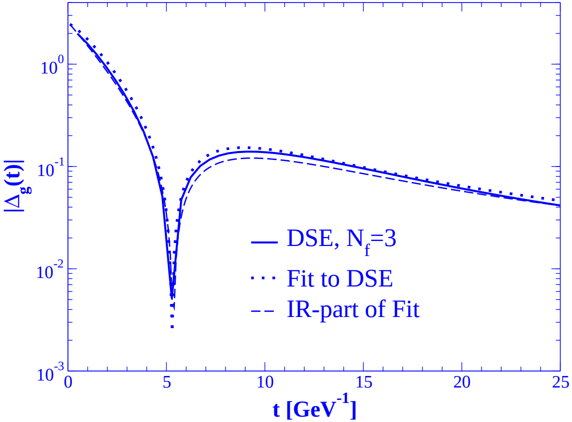

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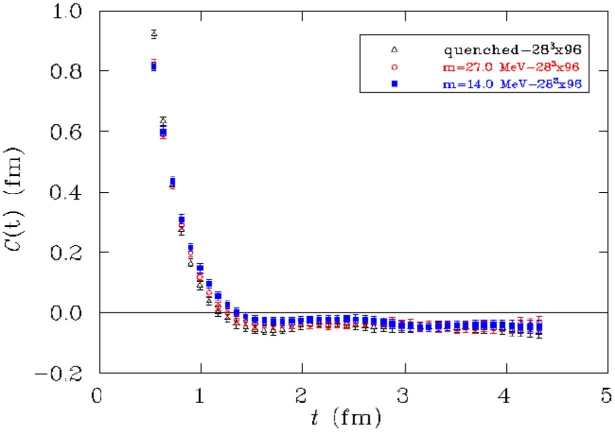
P. Bowman et al., Phys.Rev.D76 (2007) 094505



Positivity violation for the gluon propagator

Analytic structure of running coupling:

R.A., W. Detmold, C.S. Fischer and P. Maris, PR**D70** (2004) 014014

$$\alpha_{\rm fit}(p^2) = \frac{\alpha_S(0)}{1 + p^2/\Lambda^2_{\tt QCD}} + \frac{4\pi}{\beta_0} \frac{p^2}{\Lambda^2_{\tt QCD} + p^2} \left(\frac{1}{\ln(p^2/\Lambda^2_{\tt QCD})} - \frac{1}{p^2/\Lambda^2_{\tt QCD} - 1} \right)$$

with $\beta_0 = (11N_c - 2N_f)/3$

- Landau pole subtracted
- analytic in complex p^2 plane except real timelike axis
- logarithm produces cut for real $p^2 < 0$
- Cutkosky's rule obeyed

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$$D^{\rm fit}_{gluon}(p^2) = w \frac{1}{p^2} \left(\frac{p^2}{\Lambda^2_{\tt QCD} + p^2} \right)^{2\kappa} \left(\alpha_{\rm fit}(p^2) \right)^{-\gamma}$$

- IR part: cut for $-\Lambda^2_{\tt QCD} < p^2 < 0$
- $D^{\rm fit}_{gluon}$: cut along negative, i.e. timelike, half-axis!

Wick rotation possible!

- w arbitrary normalization parameter
- $\kappa = \frac{93-\sqrt{1201}}{98}$ fixed from IR analysis
- $\gamma = \frac{-13N_c+4N_f}{22N_c-4N_f}$ from perturbation theory
- **Effectively one parameter†: $\Lambda_{\tt QCD}$=520 MeV!**

---

† from fits to lattice data: $\Lambda_{\tt QCD} \approx 380$ MeV

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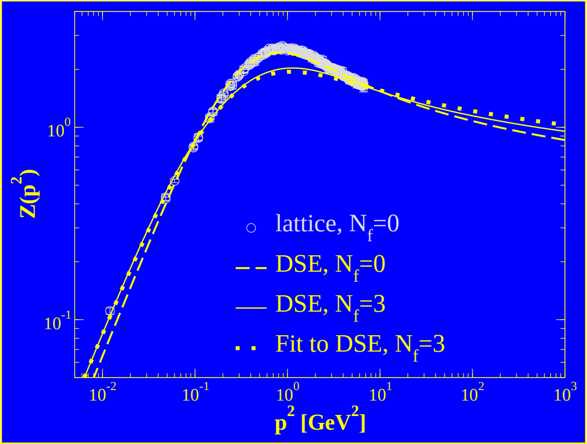

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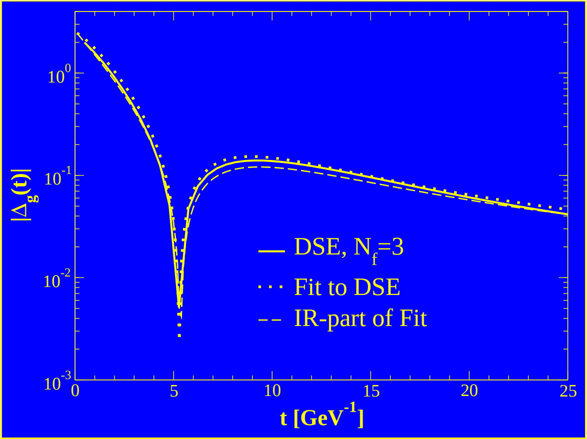

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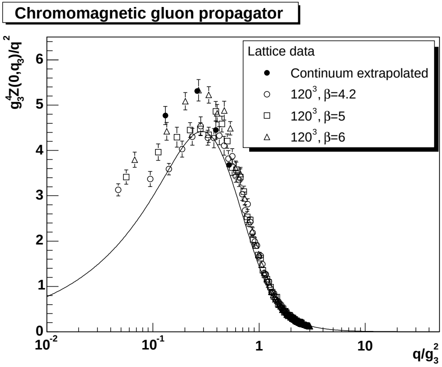
Partial gluon confinement at any T

Gluon propagator at high T:

A. Maas, J. Wambach, RA, EPJ **C37** (2004) 335; **C42** (2005) 93.

A. Cucchieri, T. Mendes and A.R. Taurines, PR **D67** (2003) 091502.

A. Cucchieri, A. Maas and T. Mendes, PR **D75** (2007) 076003.



Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation

Partial gluon confinement at any T

Gribov-Zwanziger / Kugo-Ojima scenario / positivity violation at any T:

No infrared singularities, *c.f.* Linde (1980),
because no chromomagnetic mass of type $\omega_m(\vec{k} = 0) = m_m(T)$!

D. Zwanziger, hep-ph/0610021; K. Lichtenegger, D. Zwanziger, 0805.3804 [hep-th].

No surprise:

- three-dimensional YM theory confining
- area law for spatial Wilson loop
- Coulomb string tension $\neq 0$ at any T

Static chromomagnetic sector is never deconfined!

DSE scaling solution of Yang-Mills theory:

- Gluon propagator vanishes on the light cone, and
- *n*-point gluon vertex functions diverge on the light cone!

$\Rightarrow$ Attempts to kick a gluon free (*i.e.* to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ perfect color charge screening
+ positivity violation (which implies BRST quartet cancelation):

Gluon confinement!

Picturing Gluon Confinement

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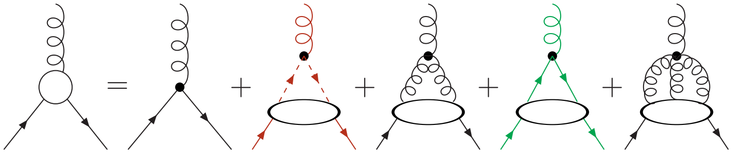
Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, Annals of Physics, in press [arXiv0804.3042[hep-ph]].

Quark-gluon vertex:

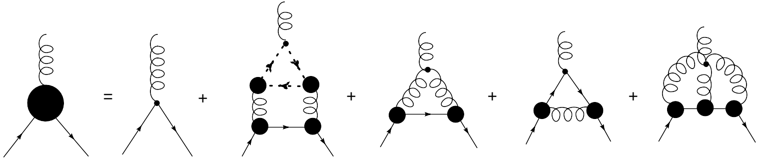


Quark diagram: Hadronic contributions ('unquenching')

Ghost diagram: Infrared leading!

Dynamically induced scalar quark confinement

Quark-gluon vertex: **lowest order** in skeleton expansion



chiral symmetry dynamically or explicitely broken:

$$S(p) = \frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not p}{M^2} + \frac{Z_f}{M}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G^i_\mu \,, \quad G^1_\mu = \gamma_\mu \,, \;\; G^2_\mu = \hat{p}_\mu \,, \;\; G^3_\mu = \hat{\not p}\, \hat{p}_\mu \,, \;\; G^4_\mu = \hat{\not p}\, \gamma_\mu$$

WITH $\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa}$

Dynamically induced scalar quark confinement

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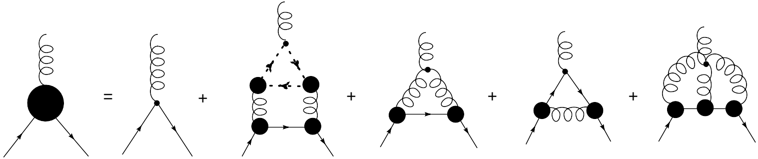

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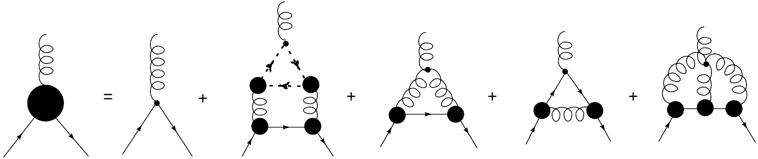

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Dynamically induced scalar quark confinement

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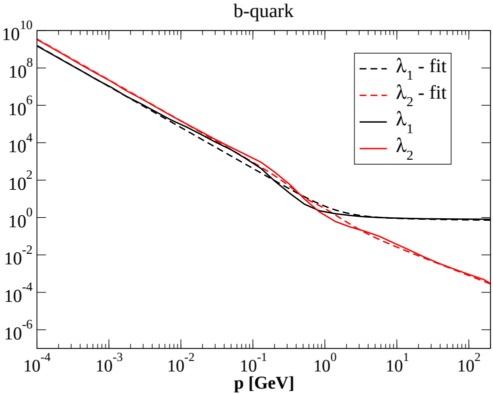
Quark-Gluon vertex IR divergent!

Scalar component λ_2 in IR even **larger** than vector component λ_1!



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As

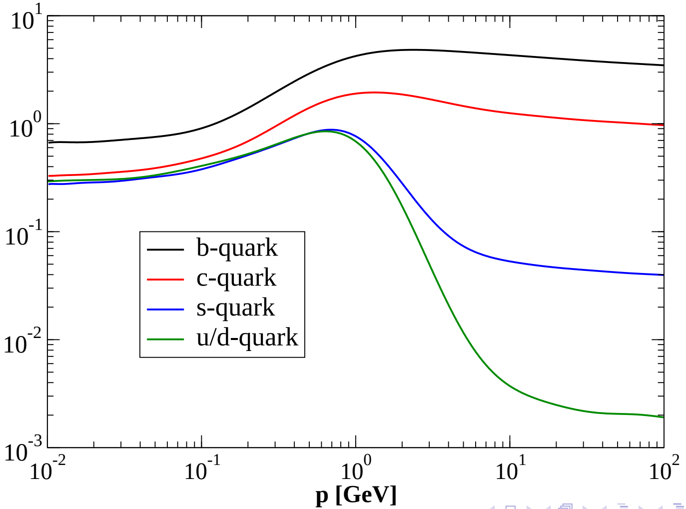
$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}, \quad Z_f(p^2) \sim const, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

running coupling from quark-gluon is IR divergent:

$$\alpha^{qg}(p^2) = \alpha_\mu [\Gamma^{qg}(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \frac{const_{qg}}{N_c} \frac{1}{p^2}$$

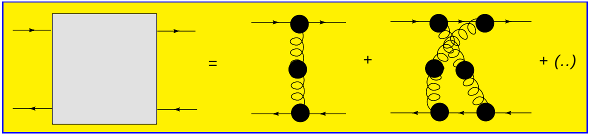
Dynamically induced scalar quark confinement

Dynamically generated quark mass function:



Dynamically induced scalar quark confinement

"Quenched´´ quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \sim |\mathbf{r}|$$

i.e. linear, dominantly scalar, quark confinement!

Dynamically induced scalar quark confinement

chiral symmetry artificially kept in Wigner-Weyl mode:

$$S(p) = \left(\frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \right)_{M\to 0} \to \frac{Z_f \not{p}}{p^2}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G^i_\mu , \quad G^1_\mu = \gamma_\mu , \; G^2_\mu = \hat{p}_\mu , \; G^3_\mu = \hat{\not{p}}\, \hat{p}_\mu , \; G^4_\mu = \hat{\not{p}}\, \gamma_\mu$$

WITH $\lambda_{1,3} \sim (p^2)^{-\kappa}$ **and** $\lambda_{2,4} = 0$.

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Running coupling: Restoration of fixed point

$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, [Z_f(p^2)]^2 \, Z(p^2) \sim const_{qg}$$

Quark-antiquark potential: No confinement

$$\Gamma^{0,0,2}(p^2) \sim const.$$

$$V(\mathbf{r}) \sim \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} e^{i\mathbf{pr}} \sim \frac{\mathbf{1}}{|\mathbf{r}|}$$

Picturing quark confinement

DSE scaling solution for quark sector:

- quark propagator IR trivial (DχSB),
- quark-gluon vertex functions including a self-consistently generated scalar quark-gluon coupling (DχSB!) diverge on the quark “mass´´ shell!

$\Rightarrow$ Attempts to kick a quark free (*i.e.* to produce a real quark) immediately results in production of infinitely many virtual soft gluons!

$\Rightarrow$ linearly rising potential
i.e., infrared slavery:

Quark confinement!

String formation? Properties of confining field configuration? ...? ...?

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Application 1: η' mass

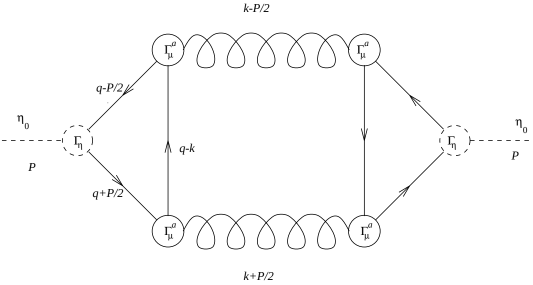
R.A., C. S. Fischer, R. Williams, arXiv:0804.3478 [hep-ph].

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

Where is this encoded in the Green functions?

J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

E.g. in:



$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$

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QCD vacuum: winding number spots as, *e.g.*, instantons, couple

to chiral quark zero modes $\Rightarrow U_A(1)$ symmetry broken!

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J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

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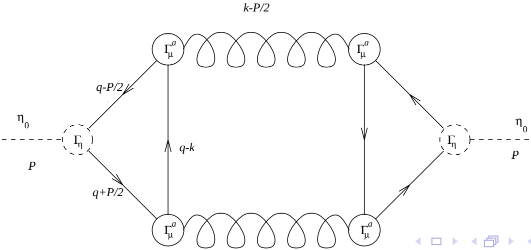

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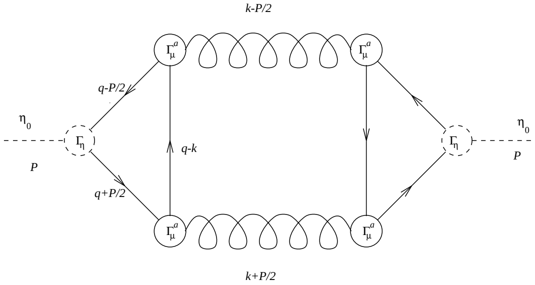


$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

Application 1: η' mass

However: Infinitely many diagrams (n-gluon exchange) contribute!

Nevertheless:
Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$\chi^2 \approx (160\mathrm{MeV})^4$ vs. phenomenological value $(180\mathrm{MeV})^4$

results in: $m_\eta = 479$MeV, $m_{\eta'} = 906$MeV, $\theta = -23^0$.

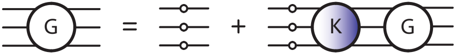
Conclusion:
(Fluct.) topologically non-trivial fields $\Leftrightarrow$ IR singularities of GF!
...another view to generate the Witten-Venezanio mechanism ...

Application 2: Nucleon properties

G. Eichmann, A. Krassnigg, M. Schwinzerl, R.A., Annals of Physics, in press [arXiv:0712.2666[hep-ph]].

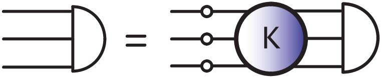
Starting point of a Poincaré-covariant Faddeev Approach:
Dyson's equation for quark 6-point function

$$G = G_0 + G_0 \, K \, G \quad \Longleftrightarrow \quad G^{-1} = G_0^{-1} - K$$



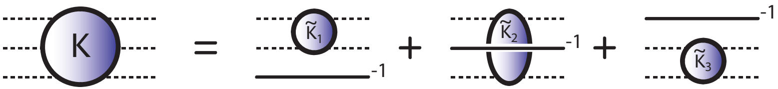
Pole approximation: bound state equation for the baryon

$$\Psi = G_0 \, K \, \Psi$$



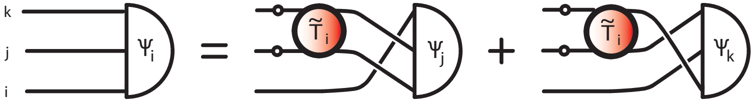
A Poincaré-covariant Faddeev Approach

Neglecting all irreducible three-particle interactions



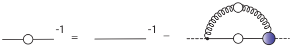
leads to the Faddeev equations

$$\Psi_i = S_j \, S_k \, \widetilde{T}_i \, (\Psi_j + \Psi_k)$$



Quark Propagator

Dressed quark propagator as solution of a model quark DSE



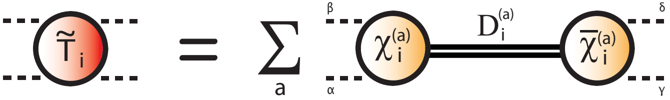
$$S(p) = Z_f(p^2)\frac{i\not{p} - M(p^2)}{p^2 + M^2(p^2)}$$

with model parameters adjusted such that solutions of coupled DSEs and/or corresponding lattice data are reproduced.

“Diquarks”

Expanding the 2-quark correlation function by employing effective diquarks:

$$\widetilde{T}_i = \sum_a \chi_i^a \, D_i^a \, \bar{\chi}_i^a$$



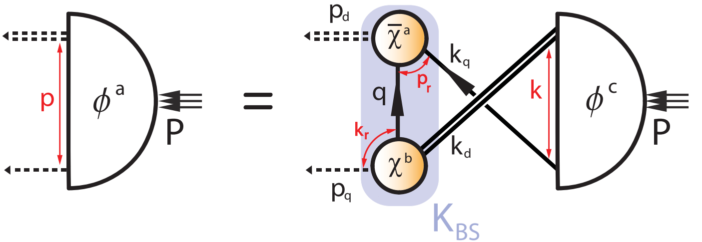
D^a	...	diquark propagator
$\chi^a, \bar{\chi}^a$	...	diquark amplitudes
		from solutions of model Bethe-Salpeter eqs.

Sum over scalar, axial-vector, . . . correlations

Quark-diquark Bethe-Salpeter eqs

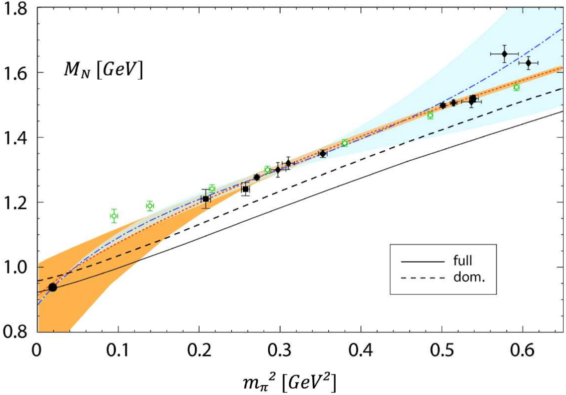
This leads to a set of coupled quark-“diquark" Bethe-Salpeter equations:

$$\phi^a(p,P) = \sum_{b,c} \int \frac{d^4k}{(2\pi)^4} \underbrace{\chi^b\, S^T(q)\ \bar{\chi}^{a\,T}}_{K_{BS}(p,k,P)} S(k_q)\, D^{bc}(k_d)\, \phi^c(k,P)$$



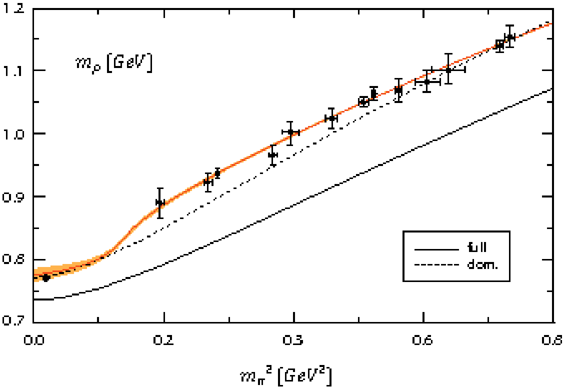
$\Rightarrow$ Interaction via quark exchange *as required by Pauli principle*

Nucleon mass



lattice data: Adelaide, Graz and CP-PACS groups
chiral extrapolations: M. Procura et al., D.B. Leinnweber et al.

Rho mass



lattice data: CP-PACS

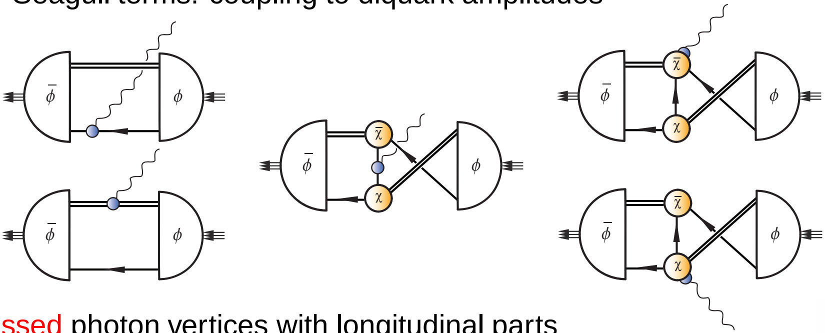
“chiral extrapolation´´: C.R. Allton et al.

Electromagnetic Current

Current conservation requires the following diagrams:

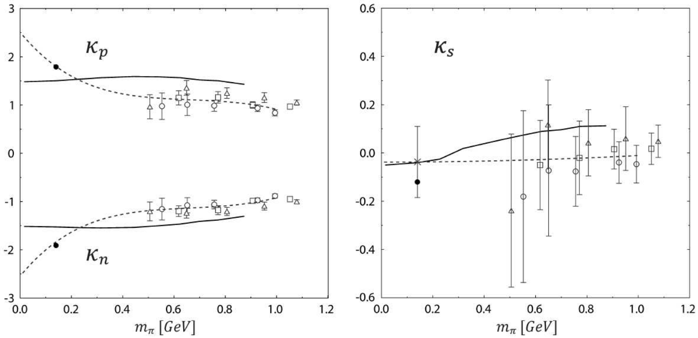
(M. Oettel, M.A. Pichowsky and L. von Smekal, Eur.Phys.J.A 8, (2000) 251-281)

- Photon-quark coupling
- Photon-diquark coupling
- Coupling to exchange quark
- Seagull terms: coupling to diquark amplitudes



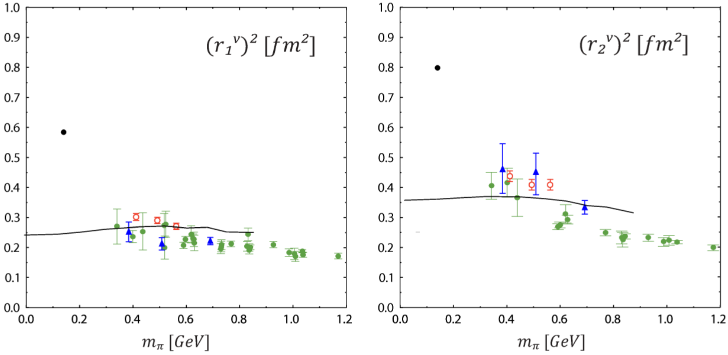
Dressed photon vertices with longitudinal parts constrained by Ward-Takahashi identities

Magnetic moments



lattice data: QCDSF
chiral extrapolation: M. Göckeler et al.

E.m. isovector radii

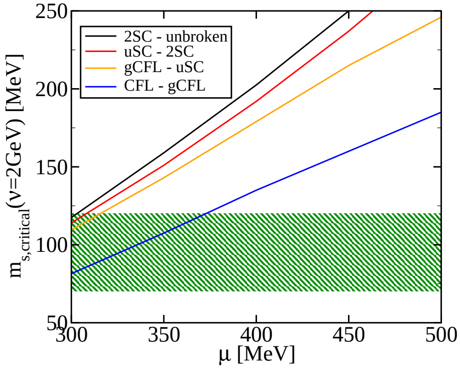


lattice data: C. Alexandrou et al., M. Göckeler et al.

Trailer: Color Superconductivity

D. Nickel, R.A., J. Wambach, Phys.Rev.**D77** (2008) 114010[arXiv:0802.3187[hep-ph]].

Quark Dyson–Schwinger eq. at finite chemical potential
include medium modifications of the gluon propagator:



Only color-flavor-locked phase for realistic strange quark masses!

Summary

Landau gauge IR QCD Green functions

- Gluons confined by ghosts: Positivity violated!
  Gluons removed from S–matrix!
- Infrared-finite strong running coupling in Yang-Mills theory!
  Conformal Nature of Infrared Yang-Mills theory!
- Analytic structure of gluon propagator:
  effectively one parameter!
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More information . . .

Homepage of the group

Strong Interactions in Continuum Quantum Field Theory:

http://physik.uni-graz.at/itp/sicqft/

Homepage of the FWF-funded Doctoral Program

Hadrons in Vacuum, Nuclei and Stars:

http://physik.uni-graz.at/itp/doktoratskolleg/

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