The Vertex Function of Fundamentally Charged Scalars in Landau QCD

Diploma Thesis

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QuantumChromoDynamics (QCD)

- QCD is a quantum field theory describing the strong interaction between quarks and gluons
- QCD has different properties in different regions, UV and IR sector have to be handled differently.
- UV sector can be described by perturbative QCD, because QCD is asymptotically free.
- IR sector must be investigated by non-perturbative methods like Dyson-Schwinger equations (DSEs), Lattice QCD, the Renormalization Group or others.
- IR: *confinement ...* long distance IR phenomenon, whose underlying mechanism is still not fully understood.

QCD - Results in Landau Gauge

Yang-Mills theory:

- IR divergent ghost propagator, finite/vanishing gluon propagator (confined gluons[1]) (Kugo-Ojima/Gribov-Zwanziger scenario)
- bare ghost-gluon vertex
- 3-gluon and 4-gluon vertex divergent[2] (ghost-dominance)

[1]Alkofer,Hauck,von Smekal, Phys. Rev. Lett. 79, 3591(1997)
[2]Alkofer,Fischer,Llanes-Estrade, Phys. Lett. B611, 279(2005)

Fundamentally Charged Scalars - Test for QCD?

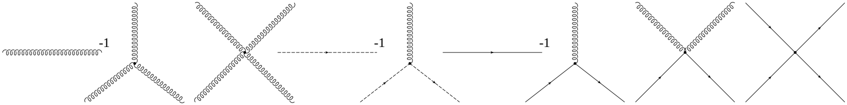
- QCD: 12 tensor structures for the quark-gluon vertex.
  For fundamental scalar charges there are only two different tensor structures for the scalar-gluon vertex.
- On the lattice fermions are hard to implement. Scalars are bosons and thus easier to describe in lattice QCD *(future work by Axel Maas)*.
- Theory with fundamentally charged scalars could serve as a test for QCD in the IR, if it had the same qualitative behavior.

Lagrangian for Fundamentally Charged Scalars

$$\mathcal{L} = \underbrace{\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\zeta}(\partial_\mu A^a_\mu)^2 + \bar{c}^a\partial_\mu D^{ab}_\mu c^b}_{\text{Yang-Mills sector (Landau gauge } \zeta\to 0)} +$$

$$+ \underbrace{\frac{1}{2}(D_{\mu,ij}\phi^*_j)(D_{\mu,ik}\phi_k) - \frac{m^2}{2}\phi^*_i\phi_i - \frac{\lambda}{4!}(\phi^*_i\phi_i)^2}_{\text{interactions of the scalar particle}}$$

$\rightarrow$ primitively divergent n-point functions



"Phases" of the System

three different cases have to be investigated:

- massless scalar particle
- massive scalar particle,
  - no scalar condensate $\langle \phi^2 \rangle = 0$, unbroken "phase"
  - scalar condensate $\langle \phi^2 \rangle \neq 0$,
    i.e. Higgs phase (spontaneously broken "phase")

Only the two cases without a condensate are analyzed within my thesis.

DSEs - Derivation

Dyson-Schwinger equations: non-perturbative functional method, able to analyze the IR sector of QCD.
DSEs are the equations of motion of Green's functions.

$$0 = \int \mathcal{D}\phi \frac{\delta}{\delta\phi} e^{-S[\phi]+j_i\phi_i}$$

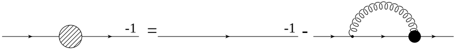
several possibilities for the derivation of higher *n*-point functions:

- algebraic method
- graphical method[3]

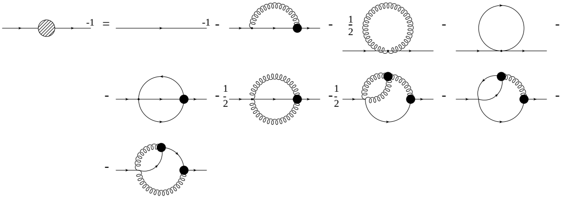
[3] R. Alkofer, M. Q. Huber, K. Schwenzer: *hep-th/0808.2939*

DSEs - One-Loop Truncation

DSE for the quark propagator

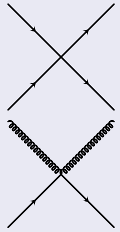


DSE for the scalar particle propagator



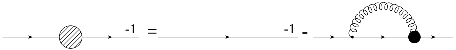
(internal propagators are to be seen as dressed)

complications due to two primitively divergent vertices

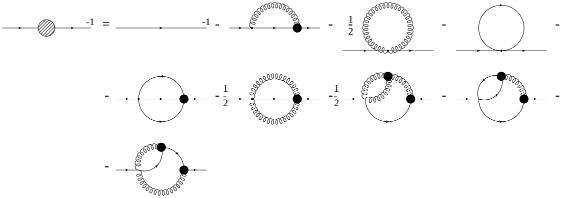


DSEs - One-Loop Truncation

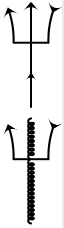
DSE for the quark propagator



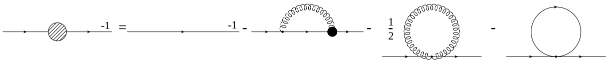
DSE for the scalar particle propagator

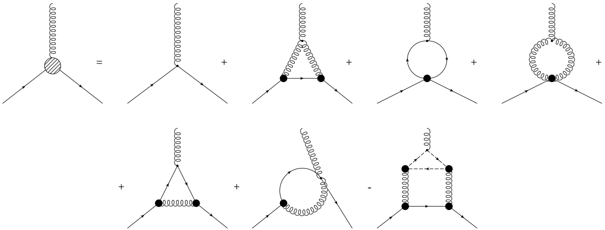


(internal propagators are to be seen as dressed)



Examples: Scalar Propagator and Scalar-Gluon Vertex in One-Loop Truncation





DSEs are by construction infinitely coupled integral equations.

$\longrightarrow$ truncation needed!

Skeleton Expansion

skeleton expansion

A loop expansion using dressed propagators and vertices, based on the assumption of valid multiplicative renormalization up to all loop-orders.

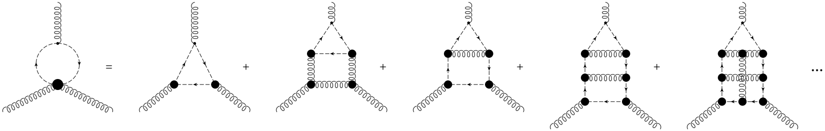


Figure: Exemplary graphs in a skeleton expansion for the ghost-loop in the 3-gluon vertex.

Ghost Contributions for One-Loop Approximation

$\nexists$ a direct scalar-ghost interaction $\rightarrow$ two loop-order terms including ghost-scalar interactions have to be considered[4]

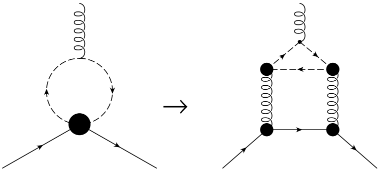


Figure: Ghost contribution to the scalar-gluon vertex DSE

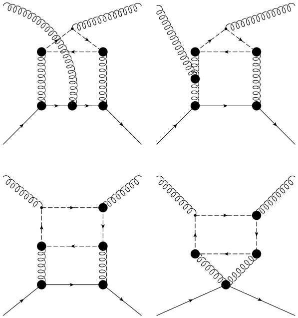


Figure: Ghost contribution to the 2-scalar-2-gluon vertex DSE

[4] R. Alkofer, C. S. Fischer, F. J. Llanes-Estrada, K. Schwenzer: *hep-ph/0804.3042*

Infrared Analysis - Power Law Ansatz

General Vertex can be decomposed in several tensor structures:

$$\Gamma_{\mu\nu\rho\dots}(p_1,\dots,p_n) = \sum_i G_i(p_1,\dots,p_n)\, T^i_{\mu\nu\rho\dots}(p_1,\dots,p_n)$$

$T^i \dots$ tensor structures, $G_i \dots$ dressing functions

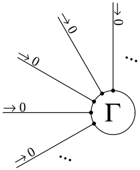
$$G_i(p_1,\dots,p_n) = \sum_j (q_j^2(p_1^2,\dots,p_n^2))^{\delta_{i,j}}\, c_{i,j}\left(\frac{p_1^2}{q_j^2},\dots,\frac{p_n^2}{q_j^2}\right)$$

The IR dominating tensor has the highest order of singularity $\Rightarrow$ power counting yields lowest exponent $\delta_j = min_{(i)}(\delta_{i,j})$

Uniform Scaling

uniform scaling: Possibly singular dressing functions for the case that all external momenta vanish.

$$q^2(p_1^2, \dots, p_n^2) \to 0 \Leftrightarrow p_1, \dots, p_n \to 0 \wedge \frac{p_1^2}{q^2}, \dots, \frac{p_n^2}{q^2} \text{ constant}$$



soft singularities: In general singularities can emerge if only a subset of $p_i \to 0$

Propagators - Parametrization

Propagators and vertices contribute with the canonical and the anomalous dimension, both are counted in the analysis giving a solvable system of equations for the infrared exponents[5].
Ansatz for the dressed propagators:

$$\text{Scalar:} \quad S_{ij}(p^2) = -\delta_{ij}\frac{Z_s(p^2)}{p^2}\ , \rightarrow \text{count as } (p^2)^{-1+\delta_s}$$

$$\text{Ghost:} \quad D^G(p^2) = -\frac{G(p^2)}{p^2}\ , \rightarrow \text{count as } (p^2)^{-1+\delta_{gh}}$$

$$\text{Gluon:} \quad D_{\mu\nu}(p^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}\ , \rightarrow \text{count as } (p^2)^{-1+\delta_g}$$

... similar ansaetze for vertices

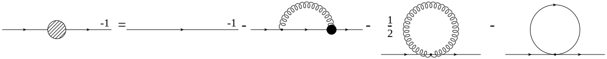
[5] Alkofer,Hauck,von Smekal, Phys. Rev. Lett. 79, 3591(1997)

Power Counting - Massless Scalar Particle

$m^2 = 0$ yields an IR divergent bare propagator for the scalar charge.

$$S^{0}_{ij}(p^2) = -\delta_{ij}\frac{1}{p^2} \;,\; \text{count as } (p^2)^{-1}$$

The power counting in the DSE for the propagator of the scalar particle yields:

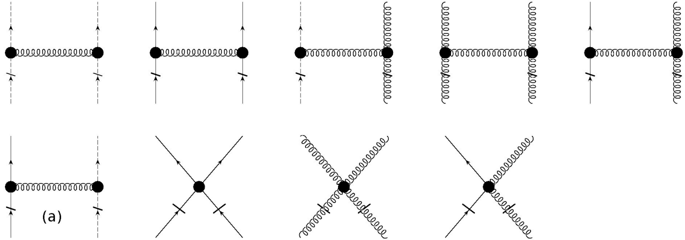


$$1 - \delta_s = \min\{1,\ 1 + \delta_s + \delta_g + \delta_{sg},\ 1 + \delta_g,\ 1 + \delta_s\}$$

$$\Rightarrow\ -\delta_s = \min\{0, \delta_s + \delta_g + \delta_{sg},\ \delta_g,\ \delta_s\}$$

Skeleton Expansion Constraints for Anomalous Dimensions

Higher loop-orders in the skeleton expansion must not raise the order of divergence. Thus the sum of the constituents in each insertion is $\geq 0 \Rightarrow$ yields constraints for the infrared exponents:

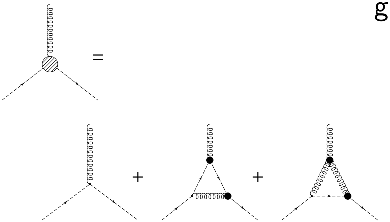


$$\text{(a)}: \; 2 \underbrace{-1-1-1+\frac{1}{2}+\frac{1}{2}}_{\text{canonical dimensions}} +\delta_s+\delta_g+\delta_{gh}+\delta_{sg}+\delta_{ggh} \geq 0$$

$\Rightarrow$ constraint for (a): $\boxed{\delta_s+\delta_g+\delta_{gh}+\delta_{sg}+\delta_{ggh} \geq 0}$

Power Counting - Example:

ghost-gluon vertex:



power counting for the DSE for the ghost-gluon vertex:

$$\delta_{ggh} = min(\quad 0, \underbrace{\delta_g + 2\delta_{gh} + 2\delta_{ggh}}_{\geq 0 \text{ (constraint)}}, \underbrace{2\delta_g + \delta_{gh} + \delta_{ggh} + \delta_{3g}}_{\geq 0 \text{(constraint)}})$$

A demanded stable skeleton expansions forbids a scaling ghost-gluon vertex [6], it stays bare in all orders:

$$\delta_{ggh} = 0$$

[6] Alkofer, Fischer, Llanes-Estrade, Phys. Lett. B611, 279(2005)

δ_{gh} and δ_{ggh}

ghost propagator: bare propagator is renormalized.

$$\delta_{gh} = -\frac{1}{2}\left(\delta_g + \delta_{ggh}\right) = -\frac{1}{2}\delta_g$$

parametrization: $\boxed{\frac{1}{2}\delta_g = \kappa = -\delta_{gh}}$

general constraints $\rightarrow 0 \leq \kappa < 1$, numerical calculation yields $\kappa \approx 0.595$, see [7], [8], [9]

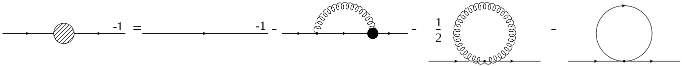
infrared exponents for the gluon propagator δ_g, ghost propagator δ_{gh} and the ghost-gluon vertex δ_{ggh}

$$\delta_g = 2\kappa, \quad \delta_{gh} = -\kappa, \quad \delta_{ggh} = 0$$

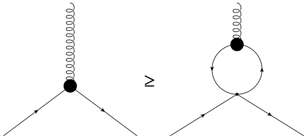
[7] Lerche,von Smekal, Phys. Rev. D65, 125006 (2002)
[8] Pawlowski,Litim,Nedelko,von Smekal, Phys. Rev. Lett. 93, 152002 (2004)
[9] Zwanziger, Phys. Rev. D65, 094039 (2002)

Scalar Propagator and Scalar-Gluon Vertex

Figure: DSE for the scalar propagator in one-loop approximation



constraints for δ_s ?



primitively divergent 4-scalar vertex implies:

$$\delta_{sg} \leq 2\delta_s + \delta_{sg}$$

$$\delta_s \geq 0$$

... same for δ_g (Alkofer,Huber,Schwenzer:*hep-th/0801.2762*)

δ_s and δ_{sg}

- δ_s stays bare up to all orders,
  there is no consistent solution with $\delta_s > 0$

infrared exponent for the scalar propagator

$$\delta_s = 0$$

- Consistency with the skeleton expansion yields 3 different possibilities for the scaling behavior of the scalar-gluon vertex δ_{sg}:
  - decoupling scalar sector:
  $$\delta_{sg} = 0$$
  - scaling scalar sector:
  $$\delta_{sg} = -\kappa$$
  - third solution drops by another consistency check

Anomalous Dimensions for a Massless Scalar Particle

δ_s	δ_g	δ_{gh}	δ_{sg}	δ_{ggh}	δ_{3g}	δ_{4g}	δ_{4s}	δ_{ssgg}
0	0	0	0	0	0	0	0	0
0	2κ	$-\kappa$	0	0	-3κ	-4κ	0	0
0	2κ	$-\kappa$	$-\kappa$	0	-3κ	-4κ	0	-2κ

Table: Infrared exponents of the Green's functions for a quantum field theory including massless fundamentally charged scalar particles

- δ_s must be 0, due to stable skeleton expansion.
- 4-point functions are not dominating lower n-point functions.

trivial solution: all vertices stay bare, $\kappa = 0$

decoupling solution: scalar charges decouple from the YM sector.

scaling solution: scalars are strongly coupled to YM sector.

Massive Particle - Difference

Difference in power counting:

- $m^2 \neq 0$: bare propagator has no divergency in the infrared, it approaches a finite value for $p^2 \to 0$

$$S_{ij}(p^2 \to 0) \to -\delta_{ij}\frac{1}{m^2}$$

- this not necessarily true for dressed propagator $\to$ for the dressed propagator the canonical and anomalous dimension have to be counted.
- Only change is the bare propagator in the scalar DSE $\Rightarrow$ causes major changes in the structure of solutions, because the canonical dimensions do not sum up to zero.

Anomalous Dimensions for the Massive Case

δ_s	δ_g	δ_{gh}	δ_{sg}	δ_{ggh}	δ_{3g}	δ_{4g}	δ_{4s}	δ_{ssgg}
1	0	0	0	0	0	0	0	0
1	2κ	$-\kappa$	0	0	-3κ	-4κ	0	0
1	2κ	$-\kappa$	$-1-\kappa$	0	-3κ	-4κ	~~$-1+\kappa$~~	~~$-1-2\kappa \leq \delta_{ssgg} \leq 0$~~

Table: Infrared exponents of the Green's functions for a quantum field theory including massive fundamentally charged scalar particles, without a scalar condensate

trivial solution: all vertices stay bare

decoupling solution: scalar particles decouple from YM-sector

confining solution: scalar particles have scaling behavior in the IR

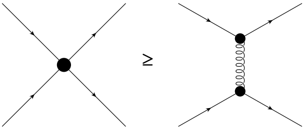
Problem

The only solution with a scaling scalar sector is not consistent.

Problems with the Approximation for the Massive Case

inconsistency of scaling solution :

First order graph must scale with the highest order of singularity of all possible diagrams!



$$\underbrace{-1+2\kappa}_{\text{gluon prop.}} + \underbrace{2(-\frac{1}{2}-\kappa)}_{\text{scalar-gluon v.}} = -2 < \delta_{4s}$$

- $-2 \rightarrow \frac{1}{p^4}$: order of singularity is high enough to yield confinement.
- The calculation in two-loop approximation in the uniform limit was checked, but yields the same infrared behaviour.
- $\Rightarrow$ Problem is caused by another mechanism

SOFT SINGULARITIES

Soft Singularities

- Singularities can emerge, if not all momenta vanish uniformly, but rather only a subset scales $\to 0$, whereas the others stay finite. These Green's functions possibly have a different IR scaling behavior.
- $\to$ Decompose momentum integral into different regions for the particular scales.

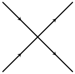
Example: scalar-gluon vertex

$\to$ additional infrared exponents

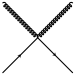
δ_{sg}^{gl}, δ_{sg}^{s}: different scaling behavior for only one vanishing external momentum

Kinematic Divergencies for Fundamentally Charged Scalars

Great variety of different kinematic combinations, each adding a new anomalous dimension and equation to be determined consistently. $\rightarrow$ Beyond the scope of this diploma thesis.



4-scalar vertex: possibly different anomalous dimensions for δ^{u}_{4s}, δ^{s}_{4s} and δ^{2s}_{4s}



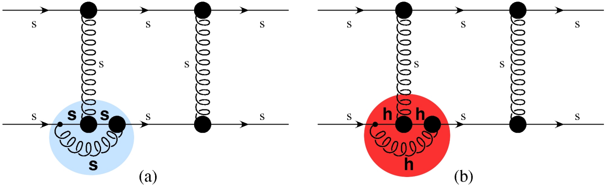
2-scalar-2-gluon vertex: possibly different anomalous dimensions for δ^{u}_{ssgg}, δ^{s}_{ssgg} ,δ^{2s}_{ssgg},$\delta^{g}_{ssgg}$and δ^{2g}_{ssgg}

Compare with QCD

DSE for the quark-antiquark is topologically equal to the four-scalar DSE, $\rightarrow$ comparison with QCD and a heavy quark scattering kernel is possible, if the canonical dimensions for fundamental scalar charges cancel in such a way, that the QCD equations are reproduced.

Future Work - Comparison with QCD

quark-antiquark scattering kernel



(a) The uniform infrared limit is not the leading behavior for massive particles.

(b) Kinematic divergencies with a soft-gluon dominate the IR sector. $\rightarrow$ degree of divergence high enough to yield confinement.

Summary

massless particle

- no confining solution, as in chiral QCD
- decoupling solution

quenched massive particle

- confining solution possible
- In numerical calculations the confining solution in quenched QCD is realized.

$\Rightarrow$ The theory of fundamentally charged scalars may provide a suitable test for further analysis of the origin of confinement in QCD.

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$\Rightarrow$ The theory of fundamentally charged scalars may provide a suitable test for further analysis of the origin of confinement in QCD.

Thank you for your attention!