Holographic imaging of nucleon via deeply virtual Compton scattering and conformal symmetry

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Collaboration with:

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Institut "Jožef Stefan"
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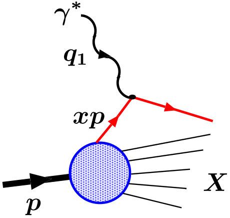
Outline

Parton distribution functions

- Deeply inelastic scattering, $-q_1^2 \to \infty,\ x_{BJ} \equiv \frac{-q_1^2}{2p\cdot q_1} \to \text{const}$



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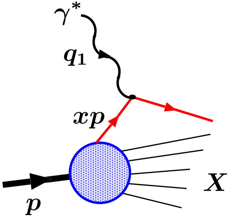

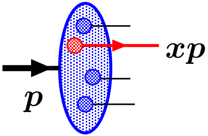

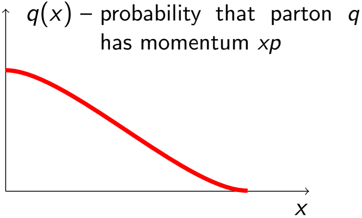

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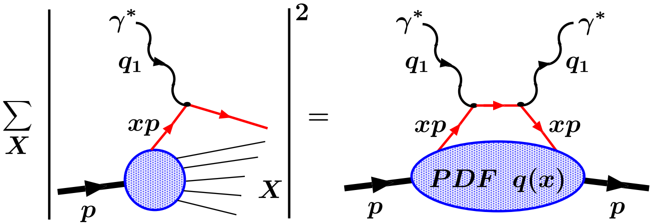

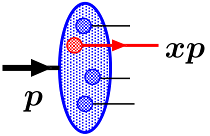

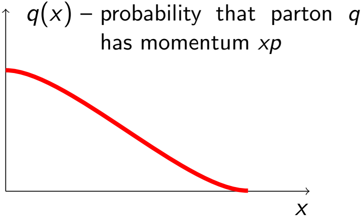

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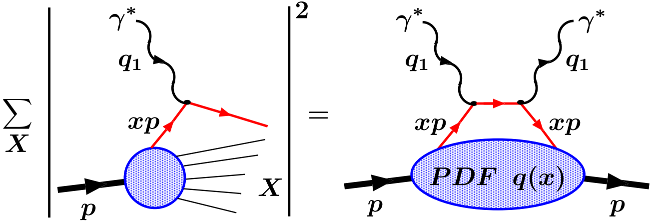
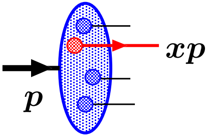
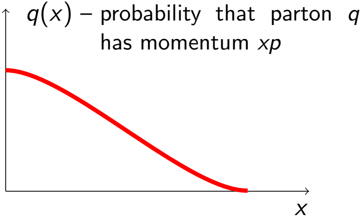
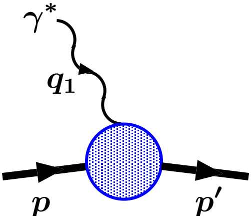






- no information on spatial distribution of partons

Electromagnetic form factors



- Dirac and Pauli form factors:

$$F_{1,2}(t = q_1^2)$$

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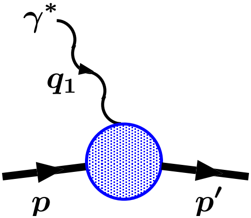

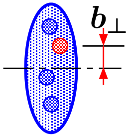

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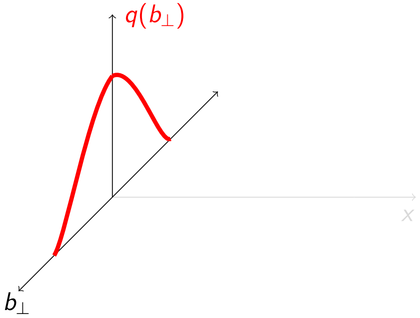
$$q(b_\perp) \sim \int \mathrm{d}b_\perp \, e^{iq_1\cdot b_\perp} F_1(t = q_1^2)$$



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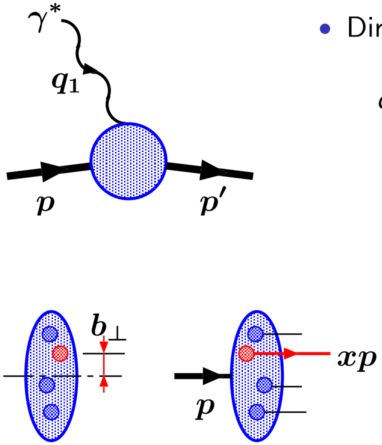

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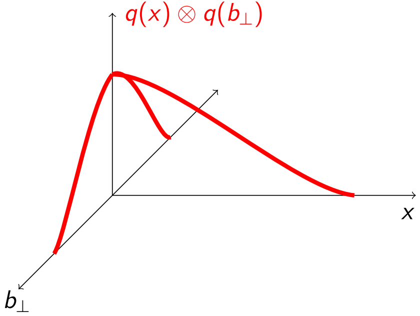

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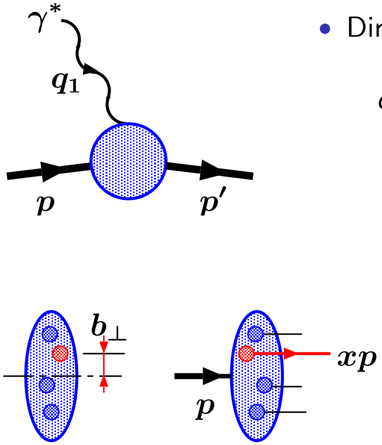

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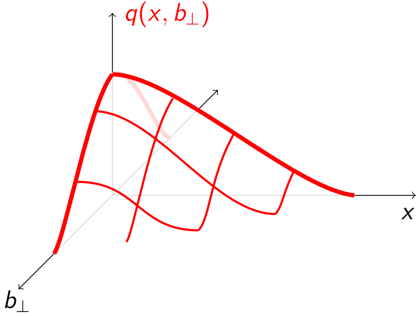

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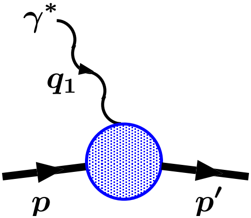

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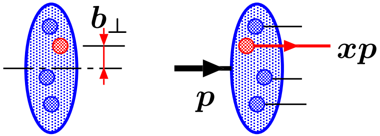
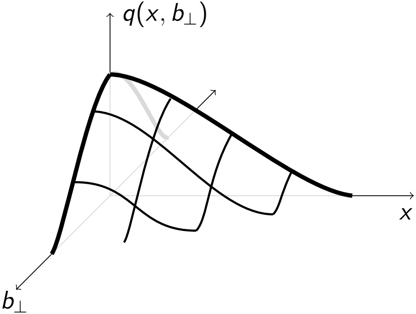
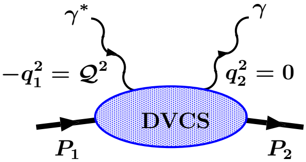




- GPD: $H^q(x, 0, t = \Delta^2) = \int \mathrm{d}b_\perp \, e^{i\Delta \cdot b_\perp} q(x, b_\perp)$

Probing the proton with two photons

- Deeply virtual Compton scattering [Müller '92, et al. '94]



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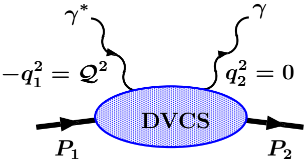


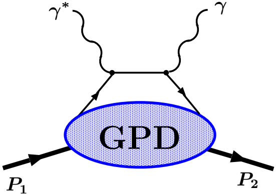
$$P = P_1 + P_2 \qquad q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P \cdot q} \to \text{const}$$

- QCD: factorization of short- and long-distance physics



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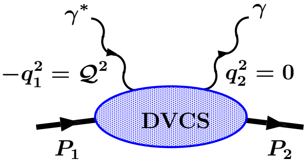

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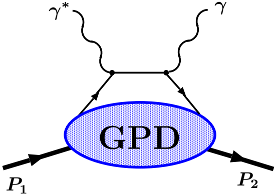
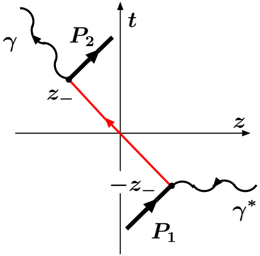




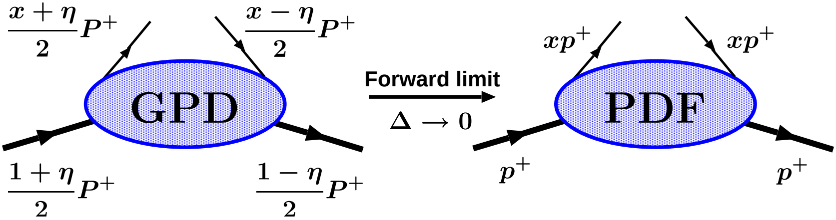
$$\langle P_2 | \bar{q}(-z_-)\gamma^+ q(z_-) | P_1 \rangle$$

Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$$P = P_1 + P_2\ ; \qquad \Delta = P_2 - P_1\ ; \qquad \eta = -\frac{\Delta^+}{P^+}\ \text{(skewedness)}$$

Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

$$F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \qquad a = q, g$$

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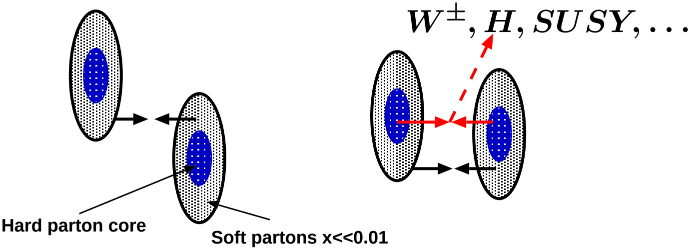
- Possibility of flavour decomposition of proton spin

$$\frac{1}{2}\int_{-1}^{1} dx\, x \left[H^q(x,\eta,\Delta^2) + E^q(x,\eta,\Delta^2) \right] = J^q(\Delta^2)$$ [Ji '96]

Relevance of GPDs for collider physics



- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions
- [Frankfurt, Strikman and Weiss '04]

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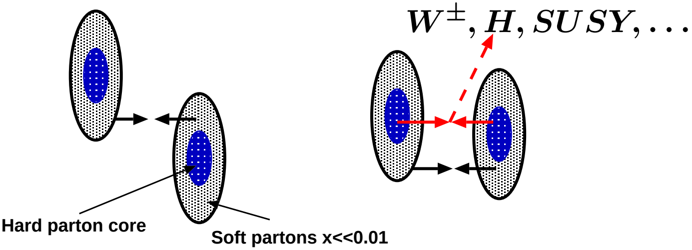
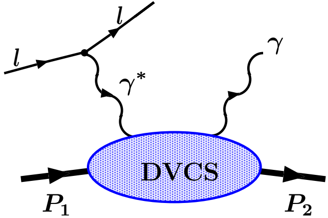


- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions
- [Frankfurt, Strikman and Weiss '04]
- No influence on total inclusive cross sections, but event structure is sensitive to transversal parton distributions.
- Relevant for LHC?

Deeply virtual Compton scattering (I)

- Measured in leptoproduction of a real photon:



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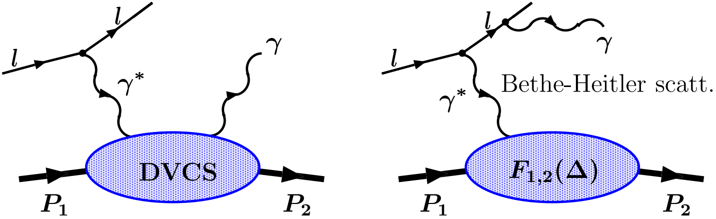

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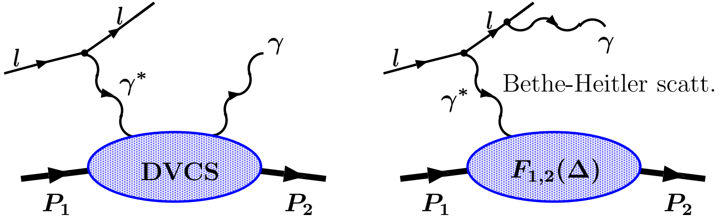
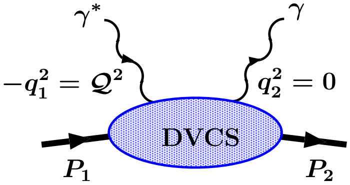


- There is a background process but it can be used to our advantage:

$$\sigma \propto |\mathcal{T}_{\rm DVCS}|^2 + |\mathcal{T}_{\rm BH}|^2 + \mathcal{T}^*_{\rm DVCS}\mathcal{T}_{\rm BH} + \mathcal{T}_{\rm DVCS}\mathcal{T}^*_{\rm BH}$$

- Using $\mathcal{T}_{\rm BH}$ as a referent "source" enables measurement of the phase of $\mathcal{T}_{\rm DVCS}$ → proton "holography" [Belitsky and Müller '02]

Deeply virtual Compton scattering (II)



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$$\mathcal{A}(\xi, \Delta^2, \mathcal{Q}^2) = \sum_i \int \mathrm{d}x \; C_i(x, \xi, \mathcal{Q}^2/\mu^2) \; \mathrm{GPD}_i(x, \eta = \xi, \Delta^2, \mu^2)$$

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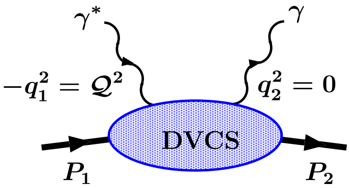

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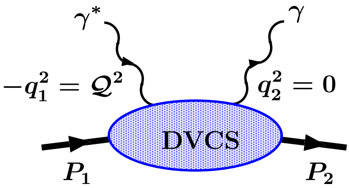

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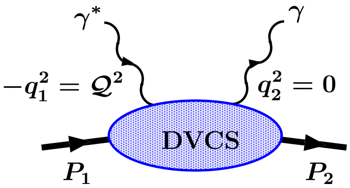

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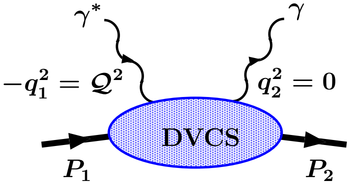

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- DIS experience at small x : gluons $\gg$ sea quarks
- $\Rightarrow$ need NNLO to stabilize perturbation series and investigate convergence $\Rightarrow$ conformal approach

Operator Product Expansion

$$J_{\rm em}(x)J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k}\, O_{n,k}$$

$$O_{n,k} \equiv (i\partial_+)^k\, \bar{\psi}\,\gamma^+(i\stackrel{\leftrightarrow}{D}_+)^n\psi$$

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- $C_{n,0}$ and γ_n of $O_{n,0}$ are well known from DIS up to NNLO.
- But $C_{n,k}$ and $\gamma_{n,k}$ are not so well known.
- $\gamma_{n,k} \neq 0 \;\Rightarrow$ operators $O_{n,k}$ mix under evolution.

Operator Product Expansion

$$J_{\rm em}(x) J_{\rm em}(0) \longrightarrow \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{1}{x^2}\right)^2 x_-^{n+k+1} C_{n,k} O_{n,k}$$

$$O_{n,k} \equiv (i\partial_+)^k \, \bar\psi \, \gamma^+ (i \overleftrightarrow{D}_+)^n \psi \qquad\qquad i\partial_+ \overset{\rm M.E.}{\longrightarrow} -\Delta_+$$

$$\overleftrightarrow{D}_+ \equiv \overrightarrow{D}_+ - \overleftarrow{D}_+$$

- $C_{n,0}$ and γ_n of $O_{n,0}$ are well known from DIS up to NNLO.
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- (At least) to LO answer is: use conformal operators.

Conformal operators

$$\mathbb{O}_{n,n+k} = (i\partial^+)^{n+k}\, \bar{\psi}\, \gamma^+\, C_n^{3/2}\left(\frac{\overleftrightarrow{D}^+}{\partial^+}\right) \psi$$

- they have well-defined conformal spin $j = n + 2$
- massless QCD is conformally symmetric at the tree level
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  - running of the coupling constant $\partial g/\partial \ln\mu \equiv \beta \neq 0$
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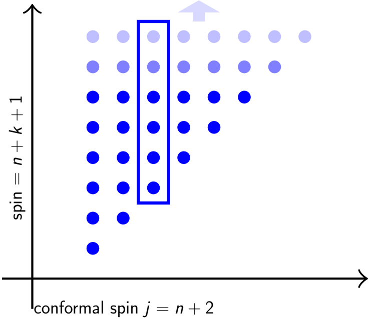
$\Rightarrow$ $\mathbb{O}_{n,n+k}$ start to mix at NLO

Conformal Towers



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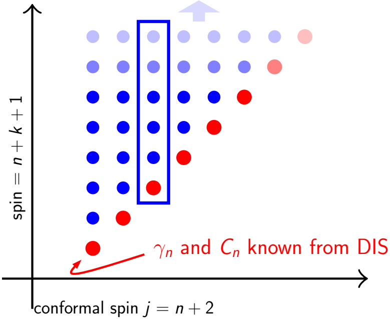

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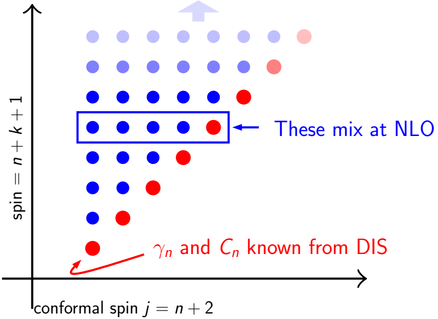

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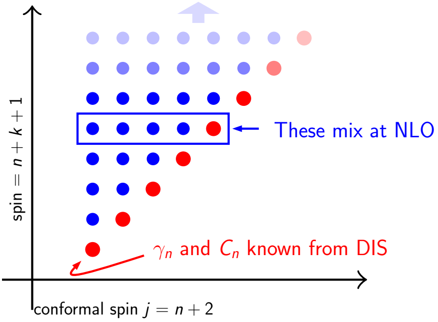

- Diagonalize in artificial $\beta = 0$ theory by changing scheme

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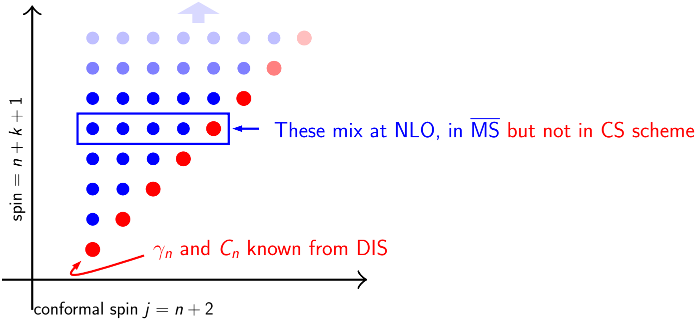

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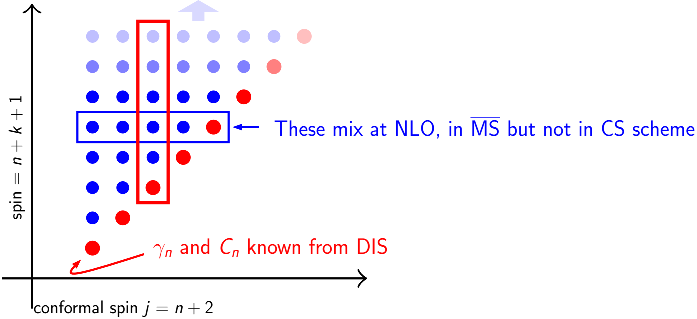

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- $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0}$ $\quad\Rightarrow$ summing complete tower

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- But how to calculate rotation matrix B? This is problem equivalent to calculation of $\gamma_{j,k}$.

$\beta \neq 0$ (II)

- The $B^{(\beta=0)}$ is constrained by conformal Ward identities ...

$$B_{jk}^{(\beta=0)\mathrm{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi}\theta(j > k)\frac{\gamma_{jk}^{\mathrm{SCT,\ LO}}}{a_{jk}}$$

(a_{jk} — known matrix)

[Müller '93]

SCT $\equiv$ special conformal transformation

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- Final result:
  n-loop DIS (diagonal) result + $(n-1)$-loop SCT anomaly =
  n-loop non-diagonal prediction

NNLO DVCS (I)

- DVCS amplitude in terms of conformal moments:

$$^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi = \eta, \Delta^2, \mu^2)$$

$$H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x\, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots)$$

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- Here, Wilson coefficients C_j read ...

$\Rightarrow$

NNLO DVCS (II)

$$C_j(Q^2/\mu^2, Q^2/\mu^{*2}, \alpha_s(\mu)) = \sum_{k=j}^{\infty} C_k(1, \alpha_s(Q))\, \mathcal{P} \exp\left\{ \int_Q^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_j(\alpha_s(\mu'))\delta_{kj} + \frac{\beta}{g} \Delta_{kj}(\alpha_s(\mu'), \mu'/\mu^*) \right] \right\}$$

with

$$C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2}\Gamma\left(\frac{5}{2}+j+\gamma_j(\alpha_s)/2\right)}{\Gamma(3/2)\Gamma\left(3+j+\gamma_j(\alpha_s)/2\right)}\, c_j^{\overline{\mathrm{MS}},\mathrm{DIS}}(\alpha_s)$$

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- Finally, evolution of conformal moments is given by . . . ⇒

NNLO DVCS (III)

$$\mu \frac{d}{d\mu} H_j(\cdots, \mu^2) = -\gamma_j(\alpha_s(\mu))\, H_j\,(\cdots, \mu^2) - \frac{\beta(\alpha_s(\mu))}{g(\mu)} \sum_{k=0}^{j-2} \eta^{j-k} \Delta_{jk}\left(\alpha_s(\mu), \frac{\mu}{\mu^*}\right) H_k(\cdots, \mu^2)$$

- Δ_{jk} — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
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- We have used these expressions to
  1. investigate size of NNLO corrections to non-singlet [Müller '05] and singlet [K.K., Müller, Passek-Kumerički and Schäfer '06] Compton form factors
  2. perform fits to DVCS (and DIS) data and extract information about GPDs [K.K., Müller and Passek-Kumerički '07]

Results on NNLO DVCS

- We use simple Regge-inspired ansatz for GPDs ...

$$\mathbf{H}_j(\xi, \Delta^2, \mathcal{Q}_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(\Delta^2)\, \mathrm{B}\big(1 + j - \alpha_\Sigma(\Delta^2), 8\big) \\ N'_G F_G(\Delta^2)\, \mathrm{B}\big(1 + j - \alpha_G(\Delta^2), 6\big) \end{pmatrix}$$

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- ... corresponding in forward case ($\Delta = 0$) to PDFs of form

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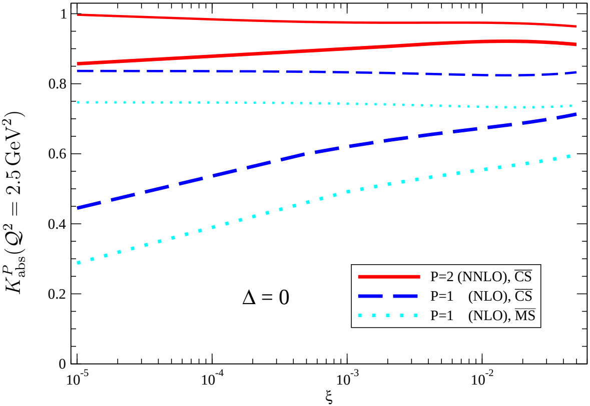
- We calculate K-factors

$$K^P_{\mathrm{abs}} = \frac{\left| {}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}} \right|}{\left| {}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}} \right|}\ ; \qquad K^P_{\mathrm{arg}} = \frac{\arg\left({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}} \right)}{\arg\left({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}} \right)}\ .$$

Size of Radiative Corrections - Modulus



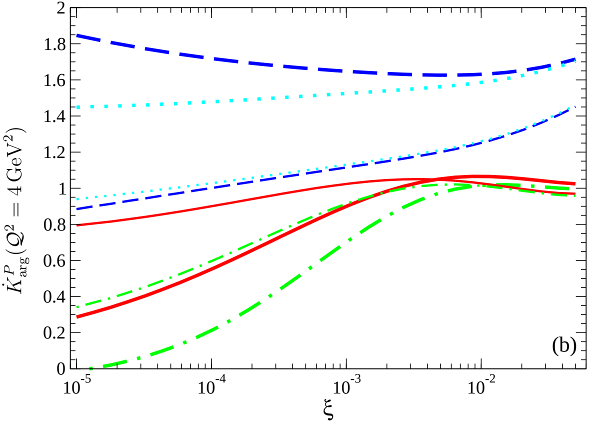
Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0) = \alpha_\Sigma(0) + 0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0) = \alpha_\Sigma(0)$

- NLO: up to 40–70% ($\overline{\rm MS}$); up to 30–55% ($\overline{\rm CS}$) ["hard"]
- NNLO: 8–14% ("hard"); 1-4% ("soft") [$\overline{\rm CS}$]

Scale Dependence

Same K-factors, but with $\mathcal{H} \to \mathrm{d}\mathcal{H}/\mathrm{d}\ln\mathcal{Q}^2$



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$N_G = 0.3$
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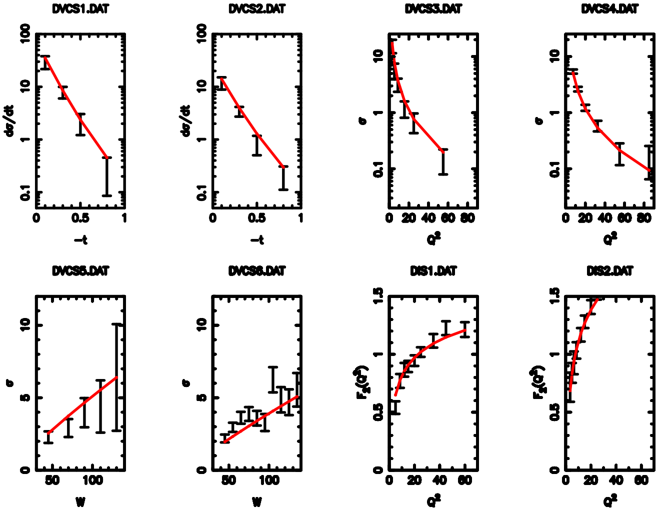
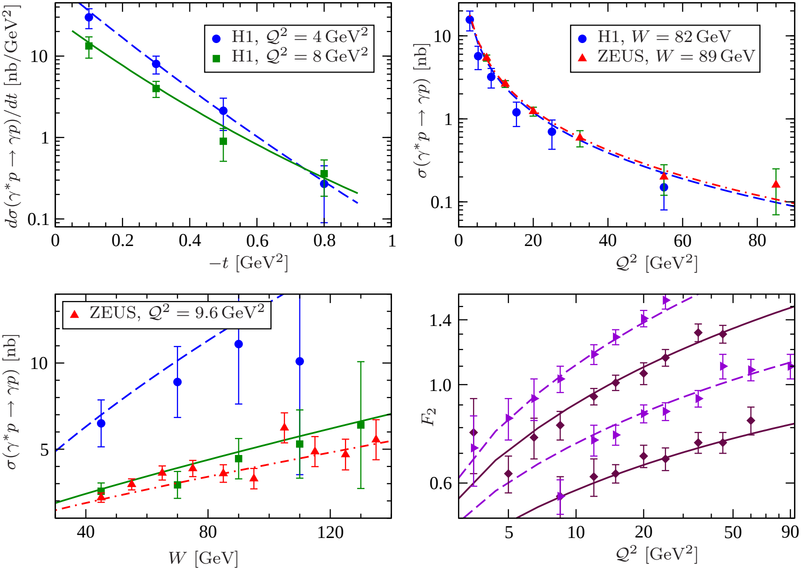
- NLO: even 100%
- NNLO: somewhat smaller, but acceptable only for larger ξ

Fast fitting routine



- $N_\Sigma = 0.143$, $\alpha_\Sigma(0) = 1.10$, $m_\Sigma = 1.26$, $N_G = 0.549$, $\alpha_G(0) = 0.915$, $m_G = 1.66$, $\mathcal{Q}_0^2 = 2.5\ \mathrm{GeV}^2$
- χ^2/(number of degrees of freedom) = 54/64

Example of final fit result



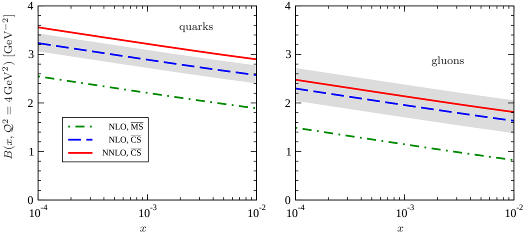
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

$$H(x, \vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)\,,$$

- Average transversal distance :

$$\langle \vec{b}^2 \rangle(x, \mathcal{Q}^2) = \frac{\int d\vec{b}\, \vec{b}^2 H(x, \vec{b}, \mathcal{Q}^2)}{\int d\vec{b}\, H(x, \vec{b}, \mathcal{Q}^2)} = 4B(x, \mathcal{Q}^2)\,,$$



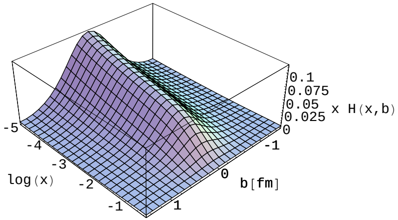
(at $\mathcal{Q}^2 = 4\,\text{GeV}^2$)

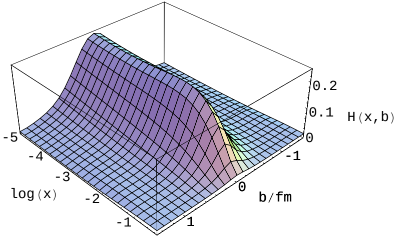
$$\langle \vec{b}^2 \rangle_{\text{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04}\ \text{fm}^2$$

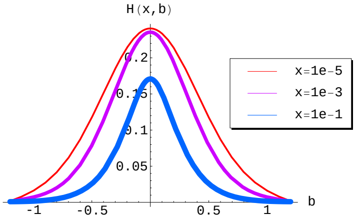
Three-dimensional image of a proton

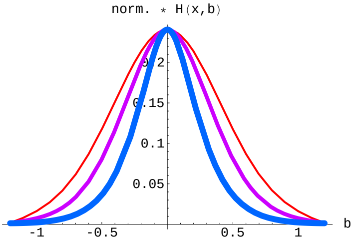
Quarks:

Gluons:









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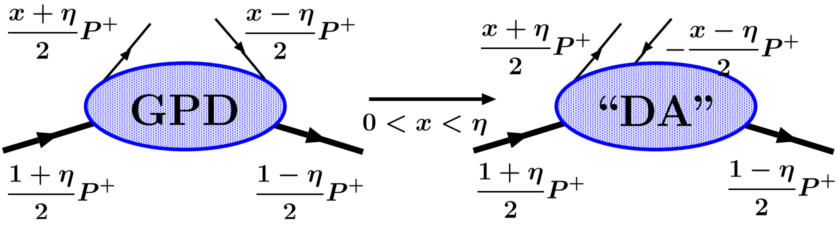
The End

Relation to distribution amplitudes

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,\Delta^2) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$F^g(x,\eta,\Delta^2) = \frac{4}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|G_a^{+\mu}(-z)G_{a\mu}{}^{+}(z)|P_1\rangle\Big|_{\dots}$$



$$P = P_1 + P_2\,; \qquad \Delta = P_2 - P_1\,; \qquad \eta = -\frac{\Delta^+}{P^+} \ \text{(skewedness)}$$

Conformal algebra

- Conformal group restricted to light-cone $\sim O(2,1)$

$$L_+ = -iP_+ \qquad [L_0, L_\mp] = \mp L_\mp$$

$$L_- = \frac{i}{2}K_- \qquad [L_-, L_+] = -2L_0$$

$$L_0 = \frac{i}{2}(D + M_{-+})$$

Casimir:

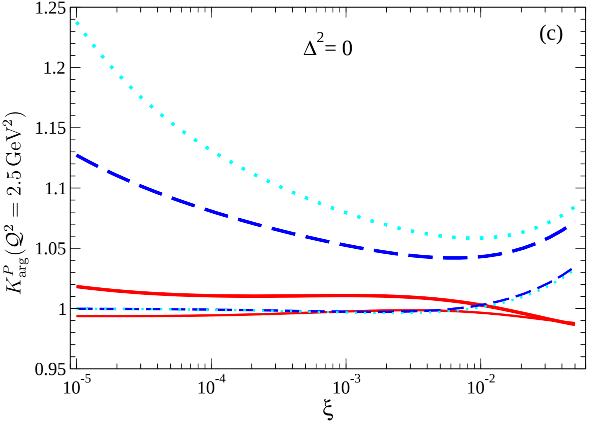
$$L^2 = L_0^2 - L_0 + L_- L_+$$

conf.spin j:

$$[L^2, \mathbb{O}_{n,n+k}] = j(j-1)\mathbb{O}_{n,k}$$

(D — dilatations, K_- — special conformal transformation (SCT))

Size of Radiative Corrections - phase

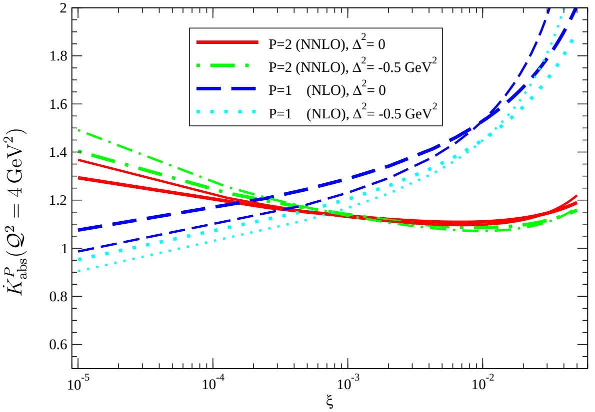


Thick lines:
"hard" gluon
$N_G = 0.4$
$\alpha_G(0)=\alpha_\Sigma(0)+0.1$

Thin lines:
"soft" gluon
$N_G = 0.3$
$\alpha_G(0)=\alpha_\Sigma(0)$

- NLO: up to 24% ($\overline{\rm MS}$); up to 13% ($\overline{\rm CS}$) ["hard"]
- NNLO and "soft" NLO — less than 5%

Scale Dependence - Modulus



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- NLO: even 100%
- NNLO: smaller (largest for “hard” gluons)