Konačne grupe

Sage putem sučelja prema GAP sustavu za računalnu algebru ima implementiran niz operacija s konačnim i Lievim grupama.

```
C2 = CyclicPermutationGroup(2)
C3 = CyclicPermutationGroup(3)
D3 = DihedralGroup(3)
```

```
print C3
D3
```

```
Cyclic group of order 3 as a permutation group
Dihedral group of order 6 as a permutation group
```

Broj elemenata grupe:

```
print C3.order()
D3.order()
```

```
3
6
```

```
print C3.is_abelian()
D3.is_abelian()
```

```
True
False
```

Popis svih elemenata grupe kao permutacija. Za notaciju vidi ovdje.

```
print C3.list()
D3.list()
```

```
[(), (1,2,3), (1,3,2)]
[(), (2,3), (1,2), (1,2,3), (1,3,2), (1,3)]
```

Rad s pojedinim elementima grupe:

```
c = D3("(1,2,3)")
b = D3("(2,3)")
b2 = D3("(1,3)")
```

Treba uočiti da se kompozicija u GAP-u/Sage-u izvrijednjuje s lijeva na desno, a ne obratno kao na predavanjima!

```
c^-1*b*c == b2   # relacija s predavanja
```

```
True
```

```
(b*c)^2  # definiciona relacija za dihedralne grupe
```

```
()
```

Provjera ove iste relacije za grupu D_7:

```
D7 = DihedralGroup(7); D7
```

```
Dihedral group of order 14 as a permutation group
```

```
cc = D7('(1,2,3,4,5,6,7)')
bb = D7('(2,7)(3,6)(4,5)')
(bb*cc)^2 == D7('()')
```

```
True
```

Reprezentanti klasa konjugacije:

```
print C3.conjugacy_classes_representatives()
D3.conjugacy_classes_representatives()
```

```
[(), (1,2,3), (1,3,2)]
[(), (1,2), (1,2,3)]
```

... i cijele klase:

```
for el in D3.conjugacy_classes_representatives():
    print "\nclass of %s:" % str(el)
    print list(set([g^-1*el*g for g in D3.list()]))
```

```
class of ():
[()]

class of (1,2):
[(2,3), (1,2), (1,3)]

class of (1,2,3):
[(1,2,3), (1,3,2)]
```

Grupna tablica množenja:

```
TC3 = C3.cayley_table(); TC3
```

```
*   a b c
 +------
a|  a b c
b|  b c a
c|  c a b
```

Legenda:

```
C3.cayley_table().row_keys()
```

```
((), (1,2,3), (1,3,2))
```

To odgovara elementima $\{e, c, c^2\}$ iz predavanja.

```
TC3.change_names(['e', 'c', 'c^2']); TC3
```

```
   *     e   c c^2
    +------------
   e|    e   c c^2
   c|    c c^2   e
 c^2|  c^2   e   c
```

```
latex.eval(TC3._latex_(),{},"")
```

```
''
```

$\times$	e	c	c^2
e	e	c	c^2
c	c	c^2	e
c^2	c^2	e	c

Skup svih podgrupa grupe D_3:

```
D3.subgroups()
```

```
[Permutation Group with generators [()], Permutation Group with
generators [(2,3)], Permutation Group with generators [(1,2)],
Permutation Group with generators [(1,3)], Permutation Group with
generators [(1,2,3)], Permutation Group with generators [(1,3,2),
(1,3)]]
```

To su trivijalna podgrupa ({e}), zatim tri C2 podgrupe, C3 podgrupa i sama grupa D3

```
D3.subgroups()[-2] == C3
```

```
True
```

Lijeve susjedne klase podgrupe C_3 u D_3:

```
D3.cosets(C3, side='left')
```

```
[[(), (1,2,3), (1,3,2)], [(2,3), (1,2), (1,3)]]
```

Kvocijentni skup D_3/C_3 je jednak grupi C_2:

```
print D3.quotient(C3) == C2
print C3.is_normal(D3)   # C3 je normalna podgrupa od D3 ...
C2.is_normal(D3)       #... ali C2 nije
```

```
True
True
False
```

Tablica karaktera:

```
D3.character_table()
```

```
[ 1 -1  1]
[ 2  0 -1]
[ 1  1  1]
```

Ljepši ispis može se dobiti izravnom komunikacijom sa GAP paketom:

```
print gap.eval("Display(%s)" % gap(D3).CharacterTable().name())
```

```
CT2

     2  1  1  .
     3  1  .  1

       1a 2a 3a
    2P 1a 1a 3a
    3P 1a 2a 1a

X.1     1 -1  1
X.2     2  . -1
X.3     1  1  1
```

Treba uočiti da je i redoslijed klasa i redoslijed reprezentacija u tablici drugačiji nego na predavanjima. Redoslijed klasa odgovara redoslijedu u listi koju daje metoda `conjugacy_classes_representatives()`

```
C3.character_table()
```

```
[         1          1          1]
[         1      zeta3 -zeta3 - 1]
[         1 -zeta3 - 1      zeta3]
```

```
print gap.eval("Display(%s)" % gap(C3).CharacterTable().name())
```

```
CT1

      3  1  1  1

        1a 3a 3b

X.1      1  1  1
X.2      1  A /A
X.3      1 /A  A

A = E(3)
  = (-1+ER(-3))/2 = b3
```

Ovdje je `zeta3 = b3 = E(3) =` $e^{2\pi/3} = (-1 + i\sqrt{3})/2$

Literatura: Robert A. Beezer, *Group Theory and Sage*.