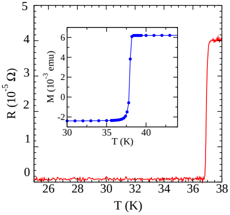
#1 Makroskopske Maxwellove jednadžbe

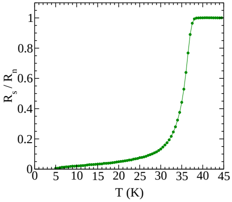
predavanja 20**

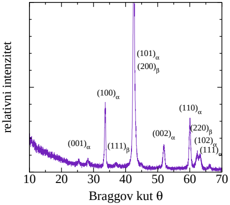
Motivacija

mjerenja na supravodiču MgB_2 ($T_c \approx 37$ K)

- otpor i magnetizacija ($\omega = 0$) [Novosel *et al.*, 2010]
- površinska impedancija ($\omega = 9.5$ GHz) [Požek *et al.*, 2003]
- raspršenje X zraka ($\hbar\omega \approx 10^4$ eV) [Skoko *et al.*, 2010]



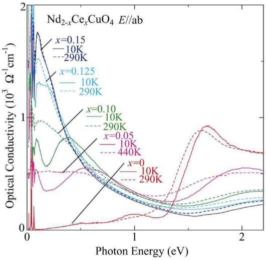


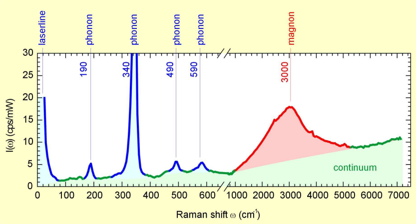


Motivacija

supravodiči $Nd_{2-x}Ce_xCuO_4$, $YB_2Cu_3O_{7-x}$ ($T_c \approx 24\,\&\,90\ K$)

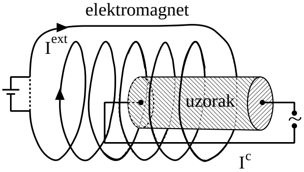
- infracrvena i optička vodljivost ($\hbar\omega < 3.5$ eV) [Onose *et al.*, 2004]
- ramansko raspršenje ($\hbar\omega < 1$ eV) [Devereaux, Hackl, 2007]





Klasična i semiklasična elektrodinamika

- Nerelativistička kvantna teorija polja u statističkoj fizici savršenih vodiča i savršenih poluvodiča (NE)
- Klasična i semiklasična elektrodinamika realnih vodiča (DA)
  - izvori klasičnih elektromagnetskih polja
  - klasična elektromagnetska polja
  - $\approx 10^{24}$ čestica u uzorku nepoznatih karakteristika, čija svojstva istražujemo



Mikroskopske Maxwellove jednadžbe

- mikroskopska Lagrangeeva funkcija

$$\mathcal{L} = \mathcal{L}_{\rm par} + \mathcal{L}_{\rm rad} + (1/c)\int {\rm d}^3x\, \mathbf{j}^{\perp}(\mathbf{x}) \cdot \mathbf{a}(\mathbf{x})$$

- jednadžbe gibanja za čestice

$$m_i \ddot{\mathbf{x}}_i = -\partial V/\partial \mathbf{x}_i + q_i \left[\mathbf{e}^{\perp}(\mathbf{x}_i) + (1/c)\dot{\mathbf{x}}_i \times \mathbf{b}(\mathbf{x}_i)\right]$$

- mikroskopske Maxwellove jednadžbe

$$\begin{aligned}
&(\mathrm{I}) & \nabla \cdot \mathbf{e}(\mathbf{x},t) &= 4\pi\eta(\mathbf{x},t) \\
&(\mathrm{II}) & \nabla \cdot \mathbf{b}(\mathbf{x},t) &= 0 \\
&(\mathrm{III}) & \nabla \times \mathbf{e}(\mathbf{x},t) &= -(1/c)\partial \mathbf{b}(\mathbf{x},t)/\partial t \\
&(\mathrm{IV}) & \nabla \times \mathbf{b}(\mathbf{x},t) &= (1/c)\partial \mathbf{e}(\mathbf{x},t)/\partial t + (4\pi/c)\mathbf{j}(\mathbf{x},t)
\end{aligned}$$

- mikroskopske gustoće

$$\eta(\mathbf{x}) = \textstyle\sum_i q_i \delta(\mathbf{x} - \mathbf{x}_i) \quad \text{i} \quad \mathbf{j}(\mathbf{x}) = \textstyle\sum_i q_i \dot{\mathbf{x}}_i \delta(\mathbf{x} - \mathbf{x}_i),$$

Minimalna supstitucija u klasičnoj elektrodinamici

- kvantno-mehaničko vezanje naboja i elektromagnetskih polja
  - hamiltonijanska formulacija

$$\mathcal{H} = \sum_i \frac{1}{2m_i}\big(\mathbf{p}_i - (q_i/c)\mathbf{A}(\mathbf{x}_i)\big)^2 + V^{(1)}(\mathbf{x}_1, \mathbf{x}_2, \ldots)$$

- doprinos vezanja u hamiltonijanu u klasičnoj ED (!)

$$\mathcal{H}_{\rm int} = -(1/c)\int \mathrm{d}^3x\, \mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})$$

- doprinos vezanja u hamiltonijanu u semiklasičnoj ED (?)

$$\mathcal{H}_{\rm int} = -(1/c)\int \mathrm{d}^3x\, \mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})$$

- u semiklasičnoj ED, u aproksimaciji čvrste veze (!)

$$\mathcal{H}_{\rm int} = -(1/c)\int \mathrm{d}^3x\, \mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})$$

- uloga kvadratnog člana u vezanju?

Prosječne mikroskopske gustoće

- karakteristična valna dužina elektromagnetskih polja $\lambda \gtrsim 1250$ Å (≈ 10 eV)
- definicija prosječne vrijednosti mikroskopske funkcije $F(\mathbf{x}, t)$

$$\langle F(\mathbf{x}, t)\rangle = \int_{L^3} \mathrm{d}^3x'\, f(\mathbf{x}')F(\mathbf{x}-\mathbf{x}', t)$$

- $f(\mathbf{x})$ (npr. $= (\pi R)^{-3/2} e^{-r^2/R^2}$) je test funkcija, $L \gg a \approx 1$ Å
- prosječna polja i gustoće

$$\mathbf{E}(\mathbf{x}, t) = \langle \mathbf{e}(\mathbf{x}, t)\rangle, \quad \mathbf{B}(\mathbf{x}, t) = \langle \mathbf{b}(\mathbf{x}, t)\rangle$$
$$\rho^{\mathrm{tot}}(\mathbf{x}, t) = \langle \eta(\mathbf{x}, t)\rangle, \quad \mathbf{J}^{\mathrm{tot}}(\mathbf{x}, t) = \langle \mathbf{j}(\mathbf{x}, t)\rangle$$

- svojstva funkcije $\langle F(\mathbf{x}, t)\rangle$

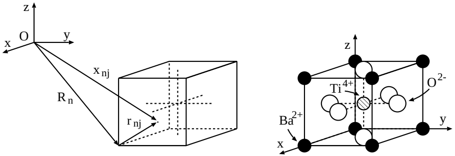
$$\frac{\partial}{\partial x_\alpha}\langle F(\mathbf{x}, t)\rangle = \left\langle \frac{\partial F(\mathbf{x}, t)}{\partial x_\alpha}\right\rangle, \quad \frac{\partial}{\partial t}\langle F(\mathbf{x}, t)\rangle = \left\langle \frac{\partial F(\mathbf{x}, t)}{\partial t}\right\rangle$$

Prosječna mikroskopska gustoća naboja

- mikroskopska gustoća naboja

$$\eta(\mathbf{x}, t) = \sum_n \sum_{j(n)} q_{nj} \delta(\mathbf{x} - \mathbf{x}_{nj})$$

- oznake vektora položaja nabijenih čestica



- prosječna mikroskopska gustoća naboja

$$\sum_n \langle \eta_n(\mathbf{x}, t) \rangle = \sum_n \sum_{j(n)} q_{nj} f(\mathbf{x} - \mathbf{R}_n - \mathbf{r}_{nj})$$

- razvoj po malom $\mathbf{r}_{nj}$ (u usporedbi sa L)

$$\sum_n \langle \eta_n(\mathbf{x}, t) \rangle \approx \sum_n \langle q_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle - \nabla \cdot \sum_n \langle \mathbf{p}_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle$$

- monopolna i dipolna gustoća naboja primitivne ćelije

$$q_n = \sum_{j(n)} q_{nj}, \quad \mathbf{p}_n = \sum_{j(n)} q_{nj} \mathbf{r}_{nj}$$

Prosječna mikroskopska gustoća naboja: rezultat

- prosječna mikroskopska gustoća naboja

$$\rho^{\mathrm{tot}}(\mathbf{x},t) \approx \sum_n \langle q_n \delta(\mathbf{x}-\mathbf{R}_n)\rangle - \nabla\cdot\sum_n\langle \mathbf{p}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle$$

- odnosno

$$\rho^{\mathrm{tot}}(\mathbf{x},t) \approx \rho(\mathbf{x},t) - \nabla\cdot\mathbf{P}(\mathbf{x},t)$$

- makroskopska gustoća naboja (?)

$$\rho(\mathbf{x},t) = \sum_n \langle q_n \delta(\mathbf{x}-\mathbf{R}_n)\rangle = \rho^{\mathrm{c}}(\mathbf{x},t) + \rho^{\mathrm{ext}}(\mathbf{x},t)$$

- makroskopska polarizacija (?)

$$\mathbf{P}(\mathbf{x},t) = \sum_n\langle \mathbf{p}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle$$

- kvantno-mehanički doprinosi (!)

Prosječna mikroskopska gustoća struja

- mikroskopska gustoća struja

$$\mathbf{j}(\mathbf{x},t) = \sum_{nj} q_{nj}\dot{\mathbf{x}}_{nj}\delta(\mathbf{x}-\mathbf{x}_{nj}(t))$$

- gustoća struje i magnetski moment primitivne ćelije

$$\mathbf{j}_n = \sum_{j(n)} q_{nj}\mathbf{v}_{nj}\,, \quad \mathbf{m}_n = \frac{1}{2c}\sum_{j(n)} q_{nj}\mathbf{r}_{nj}\times\mathbf{v}_{nj}$$

- prosječna mikroskopska gustoća struje

$$\mathbf{J}^{\mathrm{tot}}(\mathbf{x},t) \approx \mathbf{J}(\mathbf{x},t) + \frac{\partial}{\partial t}\mathbf{P}(\mathbf{x},t) + c\nabla\times\mathbf{M}(\mathbf{x},t)$$

- makroskopska gustoća struje

$$\mathbf{J}(\mathbf{x},t) = \sum_n \langle \mathbf{j}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle = \mathbf{J}^{\mathrm{c}}(\mathbf{x},t) + \mathbf{J}^{\mathrm{ext}}(\mathbf{x},t)$$

- makroskopska magnetizacija

$$\mathbf{M}(\mathbf{x},t) = \sum_n \langle \mathbf{m}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle$$

- kvantno-mehanički doprinosi (!)

Makroskopske Maxwellove jednadžbe

- prosječne mikroskopske Maxwellove jednadžbe

$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \langle \mathbf{e}(\mathbf{x}, t) \rangle &= 4\pi \langle \eta(\mathbf{x}, t) \rangle \\
\text{(II)} \quad \nabla \cdot \langle \mathbf{b}(\mathbf{x}, t) \rangle &= 0 \\
\text{(III)} \quad \nabla \times \langle \mathbf{e}(\mathbf{x}, t) \rangle &= -(1/c)\partial \langle \mathbf{b}(\mathbf{x}, t) \rangle / \partial t \\
\text{(IV)} \quad \nabla \times \langle \mathbf{b}(\mathbf{x}, t) \rangle &= (1/c)\partial \langle \mathbf{e}(\mathbf{x}, t) \rangle / \partial t + (4\pi/c) \langle \mathbf{j}(\mathbf{x}, t) \rangle
\end{aligned}
$$

- makroskopske Maxwellove jednadžbe

$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \mathbf{E}(\mathbf{x}, t) &= 4\pi \rho^{\mathrm{tot}}(\mathbf{x}, t) \\
\text{(II)} \quad \nabla \cdot \mathbf{B}(\mathbf{x}, t) &= 0 \\
\text{(III)} \quad \nabla \times \mathbf{E}(\mathbf{x}, t) &= -(1/c)\partial \mathbf{B}(\mathbf{x}, t) / \partial t \\
\text{(IV)} \quad \nabla \times \mathbf{B}(\mathbf{x}, t) &= (1/c)\partial \mathbf{E}(\mathbf{x}, t) / \partial t + (4\pi/c) \mathbf{J}^{\mathrm{tot}}(\mathbf{x}, t)
\end{aligned}
$$

Ukupne gustoće i rubni uvjeti

- prosječna mikroskopska gustoća naboja
$$\rho^{\rm tot}(\mathbf{x},t) \approx \rho^{\rm ext}(\mathbf{x},t) + \rho^{\rm c}(\mathbf{x},t) - \nabla\cdot\mathbf{P}(\mathbf{x},t)$$
- prosječna mikroskopska gustoća struje
$$\mathbf{J}^{\rm tot}(\mathbf{x},t) \approx \mathbf{J}^{\rm ext}(\mathbf{x},t) + \mathbf{J}^{\rm c}(\mathbf{x},t) + \frac{\partial}{\partial t}\mathbf{P}(\mathbf{x},t) + c\nabla\times\mathbf{M}(\mathbf{x},t)$$
- rubni uvjeti

$$\begin{aligned}
\big(\mathbf{E}^{(2)} - \mathbf{E}^{(1)} + 4\pi(\mathbf{P}^{(2)} - \mathbf{P}^{(1)})\big)\cdot\mathbf{n}_{21} &= 4\pi\sigma \\
(\mathbf{B}^{(2)} - \mathbf{B}^{(1)})\cdot\mathbf{n}_{21} &= 0 \\
\mathbf{n}_{21}\times(\mathbf{E}^{(2)} - \mathbf{E}^{(1)}) &= 0 \\
\mathbf{n}_{21}\times\big(\mathbf{B}^{(2)} - \mathbf{B}^{(1)} - 4\pi(\mathbf{M}^{(2)} - \mathbf{M}^{(1)})\big) &= \frac{4\pi}{c}\mathbf{K}
\end{aligned}$$

zaključak

- makroskopske Maxwellove jednadžbe

$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \mathbf{E}(\mathbf{x}, t) &= 4\pi\rho^{\mathrm{tot}}(\mathbf{x}, t) \\
\text{(II)} \quad \nabla \cdot \mathbf{B}(\mathbf{x}, t) &= 0 \\
\text{(III)} \quad \nabla \times \mathbf{E}(\mathbf{x}, t) &= -(1/c)\partial \mathbf{B}(\mathbf{x}, t)/\partial t \\
\text{(IV)} \quad \nabla \times \mathbf{B}(\mathbf{x}, t) &= (1/c)\partial \mathbf{E}(\mathbf{x}, t)/\partial t + (4\pi/c)\mathbf{J}^{\mathrm{tot}}(\mathbf{x}, t)
\end{aligned}
$$

su integralno-diferencijalne jednadžbe u kojima su gustoće

$\rho^{\mathrm{c}}(\mathbf{x}, t)$, $\mathbf{P}(\mathbf{x}, t)$, $\mathbf{J}^{\mathrm{c}}(\mathbf{x}, t)$ i $\mathbf{M}(\mathbf{x}, t)$

funkcije polja $\mathbf{E}(\mathbf{x}, t)$ i $\mathbf{B}(\mathbf{x}, t)$

- kako ih riješiti?
- kako uključiti vezanja elektromagnetskih polja s kvantno-mehaničkim pobuđenjima u uzorcima?