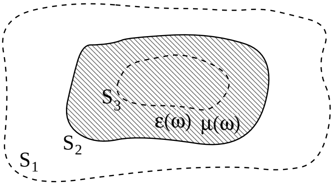
#11 Energija elektromagnetskih polja u medijima s disperzijom

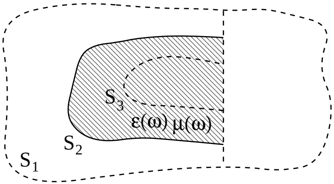
I Disipacija energije
II Energija elektromagnetskog polja

predavanja 20**

Motivacija I i II

- kako izgledaju aproksimativna rješenja generalizirane valne jednadžbe u medijima u području daleko od upadne površine?
- do koje mjere se koncept valnog vektora može koristiti u opisu tih rješenja?
- o čemu ovisi energija koja se apsorbira u mediju, a o čemu energija elektromagnetskog polja?





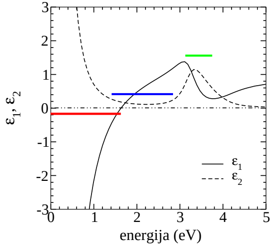
em valna jednadžba u sustavima s disperzijom

Maxwellove jednadžbe bez izvora polja u volumenu V_3

- jednadžbe III i IV (monokromatski slučaj)
  $c\nabla \times \mathbf{E}(\mathbf{x},\omega) = i\omega\mu(\omega)\mathbf{H}(\mathbf{x},\omega), \quad c\nabla \times \mathbf{H}(\mathbf{x},\omega) = -i\omega\varepsilon(\omega)\mathbf{E}(\mathbf{x},\omega)$
- poopćena elektromagnetska valna jednadžba
  $$\nabla^2\mathbf{E}(\mathbf{x},\omega) + \varepsilon(\omega)\mu(\omega)(\omega^2/c^2)\mathbf{E}(\mathbf{x},\omega) = 0$$
- postoji li rješenje oblika $\exp\{i\mathbf{k}\cdot\mathbf{x}\}$, gdje je $\mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2$?
  $c\,\mathbf{k} \times \mathbf{E}(\mathbf{x},\omega) = \omega\mu(\omega)\mathbf{H}(\mathbf{x},\omega), \quad c\,\mathbf{k} \times \mathbf{H}(\mathbf{x},\omega) = -\omega\varepsilon(\omega)\mathbf{E}(\mathbf{x},\omega)$

$$\mathbf{k}^2 = k_1^2 - k_2^2 + 2i\mathbf{k}_1 \cdot \mathbf{k}_2$$

$$\mathbf{k}^2(\omega) = \varepsilon(\omega)\mu(\omega)\omega^2/c^2$$



Indeks loma

na koliko jednako vrijednih načina možemo prikazati dielektrične odzivne funkcije?

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(1) koeficijent refleksije $R(\omega)$ i Kramers-Kronigove relacije

(2) – (5) $\{\varepsilon_1(\omega), \sigma_1(\omega)\}$, $\{\varepsilon_1, \varepsilon_2\}$, $\{\sigma_1, \sigma_2\}$, $\{\varepsilon_2, \sigma_2\}$

$$\varepsilon_1(\omega) = 1 - (4\pi/\omega)\sigma_2(\omega), \quad \varepsilon_2(\omega) = (4\pi/\omega)\sigma_1(\omega)$$

(6) pomoću indeksa loma $n(\omega)$ i koeficijenta ekstinkcije $\kappa(\omega)$

$$k(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)}\,\omega/c = \left[n(\omega) + i\kappa(\omega)\right]\omega/c$$

- ako smo izmjerili $n(\omega)$, kako odrediti $\varepsilon(\omega)$?

$$\varepsilon_1(\omega) + i\varepsilon_2(\omega) = n^2(\omega) - \kappa^2(\omega) + 2in(\omega)\kappa(\omega)$$

- nikako

Zakon o sačuvanju energije

u jednostavnim dielektricima

- neprekidnost Poyntingovog vektora na granici medija

$$\mathbf{S}(\mathbf{x},t) = (c/4\pi)\mathbf{E}(\mathbf{x},t) \times \mathbf{H}(\mathbf{x},t)$$

- gustoća toka energije

$$-\nabla \cdot \mathbf{S} = \frac{c}{4\pi}\left[\mathbf{E} \cdot (\nabla \times \mathbf{H}) - \mathbf{H} \cdot (\nabla \times \mathbf{E})\right] = \frac{1}{4\pi}\left(\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t}\right)$$

- homogeni izotropni dielektrici $[\varepsilon(\omega) = \varepsilon$ i $\mu(\omega) = \mu]$

$$U = \frac{1}{8\pi}\left[\varepsilon \mathbf{E}^2(\mathbf{x},t) + \mu \mathbf{H}^2(\mathbf{x},t)\right] \Rightarrow -\nabla \cdot \mathbf{S} = \frac{\partial U}{\partial t}$$

- jednadžba kontinuiteta za energiju (slučaj bez disipacija)

Disipacija energije u medijima s disperzijom

koje su posljedice ovisnosti odzivnih funkcija o ω? (I)

- kompleksna reprezentacija monokromatskih polja

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}(\mathbf{x},\omega)\mathrm{e}^{-i\omega t}, \;\; \partial\mathbf{D}(\mathbf{x},t)/\partial t = -i\omega\mathbf{D}(\mathbf{x},t)$$

- promjena unutarnje energije

$$\frac{\partial U}{\partial t} = \frac{1}{4\pi}\left[\frac{1}{2}(\mathbf{E}+\mathbf{E}^*)\cdot\frac{1}{2}\frac{\partial(\mathbf{D}+\mathbf{D}^*)}{\partial t} + \frac{1}{2}(\mathbf{H}+\mathbf{H}^*)\cdot\frac{1}{2}\frac{\partial(\mathbf{B}+\mathbf{B}^*)}{\partial t}\right]$$

- vremensko usrednjenje preko perioda $2\pi/\omega$

$$\overline{\mathbf{E}^2}(\mathbf{x},\omega) = \int_0^{2\pi/\omega} \mathrm{d}t\, \mathbf{E}^2(\mathbf{x},\omega)\cos^2\omega t = \frac{1}{2}\mathbf{E}^2(\mathbf{x},\omega)$$

- kvadrat amplitude polja $\left|\mathbf{E}(\mathbf{x},t)\right|^2 = \mathbf{E}^2(\mathbf{x},\omega)$

Disipacija energije u medijima s disperzijom

rezultat

- doprinosi oblika $\mathbf{E}\cdot\mathbf{E}$, $\mathbf{E}^*\cdot\mathbf{E}^*$, ... iščezavaju
- usrednjenje po vremenu daje prosječnu apsorbiranu energiju

$$Q = \overline{\frac{\partial U}{\partial t}} = \frac{1}{16\pi}\left[i\omega(\varepsilon^* - \varepsilon)\left|\mathbf{E}\right|^2 + i\omega(\mu^* - \mu)\left|\mathbf{H}\right|^2\right]$$

- ili

$$\overline{\frac{\partial U}{\partial t}} = \frac{\omega}{8\pi}\left(\varepsilon_2\left|\mathbf{E}\right|^2 + \mu_2\left|\mathbf{H}\right|^2\right) = \frac{\omega}{4\pi}\left[\varepsilon_2(\omega)\overline{\mathbf{E}^2}(\mathbf{x},\omega) + \mu_2(\omega)\overline{\mathbf{H}^2}(\mathbf{x},\omega)\right]$$

- slično, u nemonokromatskom slučaju

$$\int_{-\infty}^{\infty} Q(t)\,\mathrm{d}t = \int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\omega}{4\pi}\left[\varepsilon_2(\omega)\left|\mathbf{E}(\mathbf{x},\omega)\right|^2 + \mu_2(\omega)\left|\mathbf{H}(\mathbf{x},\omega)\right|^2\right]$$

Energija *em* polja u medijima s disperzijom

koje su posljedice ovisnosti odzivnih funkcija o ω? (II)

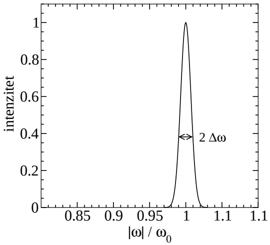
- uskopojasna *em* polja
- jednadžba kontinuiteta za energiju

$$\mathbf{E}\cdot\frac{\partial \mathbf{D}}{\partial t} = \int\frac{\mathrm{d}\omega}{2\pi}\int\frac{\mathrm{d}\omega'}{2\pi}\,\mathrm{e}^{-i(\omega-\omega')t}\mathbf{E}(\omega)\cdot\mathbf{E}^*(\omega')[i\omega'\varepsilon^*(\omega')]$$

$$= \int\frac{\mathrm{d}\omega}{2\pi}\int\frac{\mathrm{d}\omega'}{2\pi}\,\mathrm{e}^{-i(\omega-\omega')t}\mathbf{E}(\omega)\cdot\mathbf{E}^*(\omega')\frac{1}{2}[-i\omega\varepsilon(\omega)+i\omega'\varepsilon^*(\omega')]$$

$$\mathbf{E}(\mathbf{x},t) = \int\frac{\mathrm{d}\omega}{2\pi}\,\mathrm{e}^{-i\omega t}\,\mathbf{E}(\mathbf{x},\omega)$$

$$\mathbf{D}(\mathbf{x},\omega) = \varepsilon(\omega)\mathbf{E}(\mathbf{x},\omega)$$



Energija *em* polja u medijima s disperzijom

- simetrizacija izraza u [. . .] zagradama

$$\frac{1}{2}(i\omega + i\omega')[-\varepsilon(\omega) + \varepsilon^*(\omega')] + \frac{1}{2}(-i\omega + i\omega')[\varepsilon(\omega) + \varepsilon^*(\omega')]$$
$$\approx 2\omega_0\varepsilon_2(\omega_0) + i(\omega' - \omega)\frac{\partial}{\partial\omega}\left[\frac{\omega}{2}\left[\varepsilon(\omega) + \varepsilon^*(\omega)\right]\right]_{\omega=\omega_0}$$

- vodi do izraza za $\mathbf{E}\cdot\partial\mathbf{D}/\partial t$ $[\omega_0 \to \omega]$

$$\left(\omega\varepsilon_2(\omega) + \frac{\partial\left[\omega\varepsilon_1(\omega)\right]}{2\partial\omega}\frac{\partial}{\partial t}\right)\int\frac{\mathrm{d}\omega'}{2\pi}\int\frac{\mathrm{d}\omega''}{2\pi}\mathbf{E}(\omega')\cdot\mathbf{E}^*(\omega'')\mathrm{e}^{-i(\omega'-\omega'')t}$$

- te

$$\overline{\mathbf{E}\cdot\frac{\partial\mathbf{D}}{\partial t}} = \omega\varepsilon_2(\omega)\overline{\mathbf{E}(t)\cdot\mathbf{E}(t)} + \frac{1}{2}\frac{\partial\overline{\mathbf{E}(t)\cdot\mathbf{E}(t)}}{\partial t}\frac{\partial\left[\omega\varepsilon_1(\omega)\right]}{\partial\omega}$$

Energija *em* polja u medijima s disperzijom

rezultat

- apsorbirana energija

$$\overline{\frac{\partial U}{\partial t}} = \frac{\omega}{4\pi}\left[\varepsilon_2(\omega)\overline{\mathbf{E}(t)\cdot\mathbf{E}(t)} + \mu_2(\omega)\overline{\mathbf{H}(t)\cdot\mathbf{H}(t)}\right]$$

- energija polja u transparentnom režimu

$$\overline{U} = \frac{1}{8\pi}\left[\frac{\partial\left[\omega\varepsilon(\omega)\right]}{\partial\omega}\overline{\mathbf{E}(t)\cdot\mathbf{E}(t)} + \frac{\partial\left[\omega\mu(\omega)\right]}{\partial\omega}\overline{\mathbf{H}(t)\cdot\mathbf{H}(t)}\right]$$

- u $\Delta\omega \to 0$ granici $[\varepsilon(\omega)|\mathbf{E}|^2 = \mu(\omega)|\mathbf{H}|^2]$

$$\overline{U} = \frac{1}{16\pi\omega\mu(\omega)}\frac{\partial\left[\omega^2\varepsilon(\omega)\mu(\omega)\right]}{\partial\omega}|\mathbf{E}|^2$$

Grupna brzina

- plavi, crveni i zeleni dio frekventnog područja
- u kojem području je izraz za $\overline{U}$ valjan

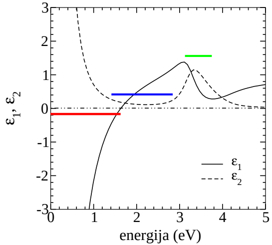
$$\overline{U} = \frac{1}{16\pi\omega\mu(\omega)} \frac{\partial\left[\omega^2\varepsilon(\omega)\mu(\omega)\right]}{\partial\omega}|\mathbf{E}|^2 = \frac{c^2}{16\pi\omega\mu(\omega)} \frac{\partial k^2}{\partial\omega}|\mathbf{E}|^2$$

- odgovor: $\varepsilon_2(\omega) \approx 0$, $\varepsilon_1(\omega) > 0$ i $\partial k/\partial\omega > 0$ [$u = \partial\omega/\partial k$]

$$\varepsilon_2(\omega) \approx 0$$

$$\varepsilon_1(\omega) < 0$$

$$\partial\varepsilon_1/\partial\omega < 0$$



Grupna brzina

prikaz grupne brzine u terminima energije em polja

- transparentni monokromatski režim

$$k = \frac{\omega\sqrt{\varepsilon(\omega)\mu(\omega)}}{c} \approx \frac{\omega n(\omega)}{c}, \quad \mathbf{H} = \sqrt{\varepsilon(\omega)/\mu(\omega)}\,\mathbf{n} \times \mathbf{E}$$

- energija polja i modul Poyntingova vektora

$$\overline{U} = \frac{c}{8\pi}\sqrt{\frac{\varepsilon(\omega)}{\mu(\omega)}}\frac{\partial k}{\partial \omega}|\mathbf{E}|^2, \quad \overline{S} = \frac{c}{8\pi}\left|\mathbf{E} \times \mathbf{H}^*\right| = \frac{c}{8\pi}\sqrt{\frac{\varepsilon(\omega)}{\mu(\omega)}}|\mathbf{E}|^2$$

- grupna brzina

$$u = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{c}{\mathrm{d}[n(\omega)\omega]/\mathrm{d}\omega} \Rightarrow u = \frac{\overline{S}}{\overline{U}}$$

Grupna brzina

primjer: polaritoni u ionskim izolatorima

- transparentni režim i $\hbar\gamma \approx 0$
- bi-kvadratna jednadžba $[\Omega_\infty^2 = c^2k^2/\varepsilon_\infty]$

$$\omega^4 - \left(\Omega_\infty^2 + \omega_{\mathrm{LO}}^2\right)\omega^2 + \Omega_\infty^2\omega_{\mathrm{TO}}^2 = 0$$

- rješenje

$$\omega_\pm^2 = (1/2)\left(\Omega_\infty^2 + \omega_{\mathrm{LO}}^2\right) \pm (1/2)\sqrt{\left(\Omega_\infty^2 + \omega_{\mathrm{LO}}^2\right)^2 - 4\Omega_\infty^2\omega_{\mathrm{TO}}^2}$$

$$k^2c^2 = \omega^2\varepsilon_1(\omega)$$

$$\varepsilon_1(\omega) = \varepsilon_\infty \frac{\omega_{\mathrm{LO}}^2 - \omega^2}{\omega_{\mathrm{TO}}^2 - \omega^2}$$