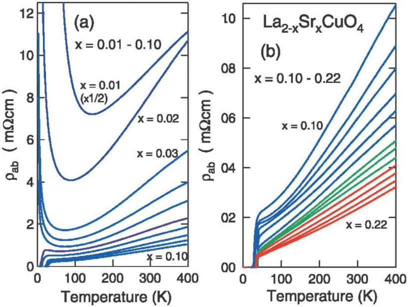
#12 Širenje elektromagnetskih valova kroz višekomponentne medije

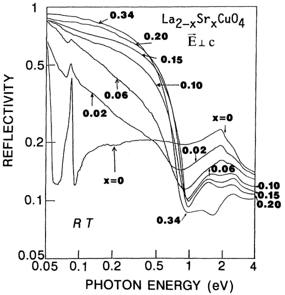
I Mjerenja koeficijenta refleksije
II Mikrovalna mjerenja

predavanja 20**

Motivacija I

- koja je glavna razlika između $\omega = 0$ opservabli i visokoenergetskih opservabli, s teorijske točke gledanja?
- [Ando *et al*,, 2004]
- [Uchida *et al*., 1991]





Mjerenje koeficijenta refleksije

ograničenja na valne vektore u em poljima na granici između dva medija

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- upadni, reflektirani i lomljeni elektromagnetski val
- moduli valnih vektora

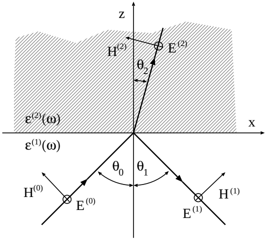
$$k_0^2 = k_1^2 = (\omega^2/c^2)\varepsilon^{(1)}(\omega), \quad k_2^2 = (\omega^2/c^2)\varepsilon^{(2)}(\omega)$$

- z komponente

$$k_{0z} = k_0 \cos\theta_0 = -k_{1z}, \;\; k_{2z} = (\omega/c)\sqrt{\varepsilon^{(2)}(\omega) - \varepsilon^{(1)}(\omega)\sin^2\theta_0}$$

$$k_{0x} = k_{1x} = k_{2x}$$

$$k_{0y} = k_{1y} = k_{2y} = 0$$



Mjerenje koeficijenta refleksije

Fresnelove formule

- rubni uvjeti

$$\begin{aligned} \hat{e}_z \times \left[\mathbf{E}^{(0)} + \mathbf{E}^{(1)} - \mathbf{E}^{(2)}\right] &= 0 \\ \hat{e}_z \times \left[\mathbf{k}_0 \times \mathbf{E}^{(0)} + \mathbf{k}_1 \times \mathbf{E}^{(1)} - \mathbf{k}_2 \times \mathbf{E}^{(2)}\right] &= 0 \end{aligned}$$

- vode do

$$E^{(0)} + E^{(1)} = E^{(2)}, \quad k_{0z}(E^{(0)} - E^{(1)}) = k_{2z}E^{(2)}$$

- odnosno

$$(k_{0z} + k_{2z})E^{(1)} = (k_{0z} - k_{2z})E^{(0)}$$

- pri mjerenju koeficijenta refleksije je $\theta_0 \approx 0$

$$\frac{E^{(1)}}{E^{(0)}} = \frac{\sqrt{\varepsilon^{(1)}(\omega)} - \sqrt{\varepsilon^{(2)}(\omega)}}{\sqrt{\varepsilon^{(1)}(\omega)} + \sqrt{\varepsilon^{(2)}(\omega)}}$$

Mjerenje koeficijenta refleksije

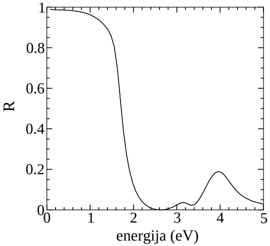
koeficijent refleksije

- definicija

$$R(\omega) = \frac{\sqrt{\varepsilon^{(1)}}\cos\theta_0 |E^{(1)}(\omega)|^2}{\sqrt{\varepsilon^{(1)}}\cos\theta_0 |E^{(0)}(\omega)|^2} = \left|\frac{E^{(1)}(\omega)}{E^{(0)}(\omega)}\right|^2$$

- $\varepsilon^{(1)}(\omega) = 1$ i $\varepsilon^{(2)}(\omega) \equiv \varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ daju

$$R(\omega) = \left|\frac{1 - \sqrt{\varepsilon_1(\omega) + i\varepsilon_2(\omega)}}{1 + \sqrt{\varepsilon_1(\omega) + i\varepsilon_2(\omega)}}\right|^2$$



Površinski plazmoni

koje je fizikalno značenje signala u "electron-loss" spektroskopiji pri $\omega = \omega_{pl}/\sqrt{2}$?

- elektromagnetski val opisan sa

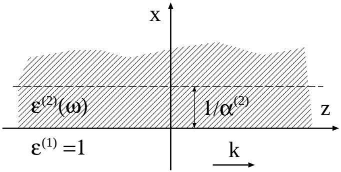
$$\mathbf{H}^{(i)}(\mathbf{x}, t) = \hat{e}_y H_0^{(i)} \mathrm{e}^{-\alpha^{(i)}(\omega)|x| + ikz - i\omega t}$$

- Maxwellova jednadžba IV daje

$$\mathbf{E}^{(i)}(\mathbf{x}, t) = \frac{-ic}{\omega\varepsilon^{(i)}(\omega)} H_0 \mathrm{e}^{-\alpha^{(i)}(\omega)|x| + ikz - i\omega t}\left[ik\hat{e}_x + \mathrm{sign}(x)\alpha^{(i)}(\omega)\hat{e}_z\right]$$

- Maxwellova jednadžba I i rubni uvjeti vode do uvjeta

$$\varepsilon(\omega) = -\frac{\alpha^{(2)}(\omega)}{\alpha^{(1)}(\omega)}$$



Površinski plazmoni

značenje signala u "electron-loss" spektroskopiji pri $\omega = \omega_{pl}/\sqrt{2}$*?*

- Maxwellove jednadžbe III i IV (tj. valna jednadžba za $\mathbf{H}^{(i)}(\mathbf{x},\omega)$)

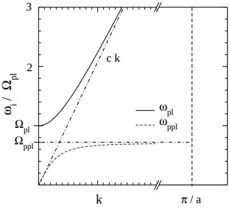
$$\alpha^{(i)}(\omega) = \sqrt{k^2 - (\omega^2/c^2)\varepsilon^{(i)}(\omega)}$$

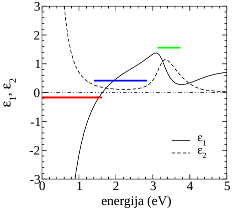
- implicitna disperzivna relacija

$$\varepsilon(\omega) = -\frac{\sqrt{k^2 - (\omega^2/c^2)\varepsilon(\omega)}}{\sqrt{k^2 - (\omega^2/c^2)}} < 0 \Rightarrow \omega^2 = \frac{\varepsilon(\omega)+1}{\varepsilon(\omega)} c^2 k^2$$

- površinski plazmoni

$$\omega_{ppl}^2(k) = \frac{\Omega_{pl}^2 c^2 k^2}{\Omega_{pl}^2 + 2c^2k^2}$$





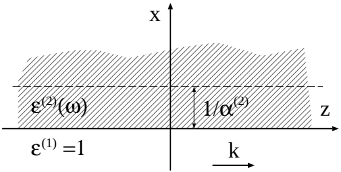
Površinski plazmoni

veza površinskih plazmona i transverzalnim magnetskih polja?

- električna komponenta polja

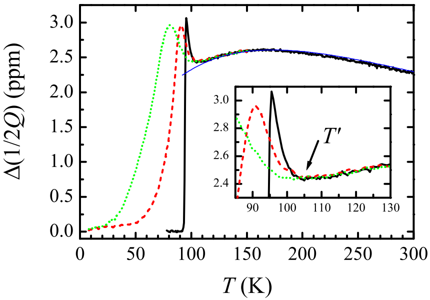
$$\mathbf{E}^{(i)}(\mathbf{x},t) = E_z^{(i)}(\mathbf{x},t)\left[\frac{ik}{\mathrm{sign}(x)\alpha^{(i)}(\omega)}\hat{e}_x + \hat{e}_z\right]$$

- na koji način se problem komplicira ako površina vodiča ima oblik cilindra?



Motivacija II

- koja je veza između dc vodljivosti i površinske impedancije?
- visokotemperaturni supravodič $HgBa_2CuO_{4+x}$, $\mu_0 H_z = 0, 3$ i 8 T [Grbić *et al*,, 2009]



Valovodi

- valna jednadžba za cilindrični valovod $[\nabla_t^2 = \nabla^2 - \partial^2/\partial z^2]$

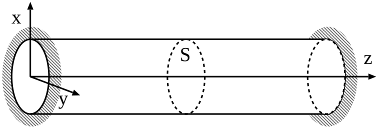
$$\left[\nabla_t^2 + \mu\varepsilon(\omega^2/c^2) - k^2\right]\mathbf{E}(\mathbf{x},t) = 0$$

- strategija računa

$$\nabla_t \cdot \mathbf{E}_t = -\frac{\partial E_z}{\partial z}, \qquad \nabla_t \cdot \mathbf{H}_t = -\frac{\partial H_z}{\partial z}$$

$$\frac{\partial \mathbf{E}_t}{\partial z} + \frac{i\omega}{c}\mu\hat{e}_z \times \mathbf{H}_t = \nabla_t E_z, \qquad \hat{e}_z \cdot \left(\nabla_t \times \mathbf{E}_t\right) = \frac{i\omega}{c}\mu H_z$$

$$\frac{\partial \mathbf{H}_t}{\partial z} - \frac{i\omega}{c}\varepsilon\hat{e}_z \times \mathbf{E}_t = \nabla_t H_z, \qquad \hat{e}_z \cdot \left(\nabla_t \times \mathbf{H}_t\right) = -\frac{i\omega}{c}\varepsilon E_z$$



Valovodi

rješenje

- opći zapis

$$\mathbf{E}_t(\mathbf{x},t) = \frac{1}{\gamma^2}\left[\frac{\partial}{\partial z}\nabla_t E_z - \frac{i\omega}{c}\mu\hat{e}_z \times \nabla_t H_z\right]$$

$$\mathbf{H}_t(\mathbf{x},t) = \frac{1}{\gamma^2}\left[\frac{\partial}{\partial z}\nabla_t H_z + \frac{i\omega}{c}\varepsilon\hat{e}_z \times \nabla_t E_z\right]$$

- parametar γ

$$\gamma^2 = \mu\varepsilon(\omega^2/c^2) - k^2 \equiv -[\alpha^{(1)}]^2$$

- (i) transverzalni magnetski (TM) valovi (površinski plazmoni):

$$H_z^{(1)} = 0 \text{ i } E_z^{(1)}|_s = 0$$

- (ii) transverzalni električni (TE) valovi:

$$E_z^{(1)} = 0, \;\; \partial H_z^{(1)}/\partial n|_s = 0$$

Rezonantne šupljine

- pretpostavka rješenja $[p = 0, 1, 2, \dots]$

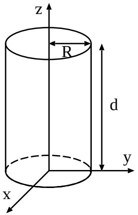
$$E_z(\mathbf{x}) = \psi(x, y) \cos \frac{p\pi z}{d} \Rightarrow \left(\nabla_t^2 + \gamma^2\right)\psi(x, y) = 0$$

- rješenje [Besselove funkcije $J_m(x)$; $x_{mn} = \gamma_{mn} R$ n-ta nul-točka]

$$E_z(\mathbf{x}) = E_0 J_m(\gamma_{mn}\rho) \mathrm{e}^{\pm im\phi} \cos \frac{p\pi z}{d}$$

- vlastite frekvencije

$$\omega^0_{mnp} = \frac{c}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{x_{mn}}{R}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$



Mikrovalna mjerenja

koju opservablu mjerimo u mikrovalnim eksperimentima?

- ukupna energija pohranjena u mikrovalnom rezonatoru

$$\mathcal{U} = \int \mathrm{d}^3x\, U(\mathbf{x}) \approx \int_V \mathrm{d}^3x\, U(\mathbf{x}) + \int_{\delta V} \mathrm{d}^3x\, U(\mathbf{x})$$

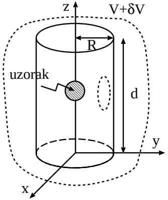
- gustoće unutarnje energije

$$U = \frac{1}{8\pi}\left(\varepsilon_1 \mathbf{E}^2 + \mu_1 \mathbf{H}^2\right), \qquad \frac{\partial U}{\partial t} = \frac{\omega_0}{4\pi}\left(4\pi\chi_2^e \mathbf{E}^2 + 4\pi\chi_2^m \mathbf{H}^2\right)$$

- faktor dobrote Q

$$\frac{\partial \mathcal{U}}{\partial t} = -\frac{\omega_0}{Q}\mathcal{U}$$

$$\Rightarrow \mathcal{U}(t) = \mathcal{U}_0 \mathrm{e}^{-(\omega_0/Q)t}$$



Mikrovalna mjerenja

koju opservablu mjerimo u mikrovalnim eksperimentima?

- električna komponenta polja

$$\mathbf{E}(t) = \mathbf{E}_0 \mathrm{e}^{-(\omega_0/2Q)t} \mathrm{e}^{-i(\omega_0+\Delta\omega)t}$$

- Fourierov transformat

$$\mathbf{E}(\omega) = \int_0^\infty \mathrm{d}t\, \mathbf{E}(t) \mathrm{e}^{-i\omega t} \Rightarrow |\mathbf{E}(\omega)|^2 \propto \frac{1}{(\omega - \omega_0 - \Delta\omega)^2 + (\omega_0/2Q)^2}$$

- odgovor: $\Delta(1/2Q)$

$$\Delta\left(\frac{1}{2Q}\right) \approx -\frac{4\pi \int \mathrm{d}^3x\, (\alpha_2^e \mathbf{E}^2 + \alpha_2^m \mathbf{H}^2)}{\int \mathrm{d}^3x\, \left(\varepsilon_1 \mathbf{E}^2 + \mu_1 \mathbf{H}^2\right)}$$