

#5 Makroskopska magnetostatika

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predavanja 20**

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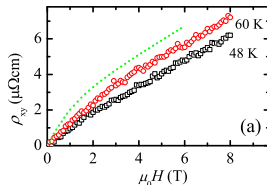
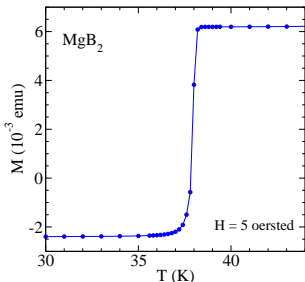
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Motivacija I

u kojim sve eksperimentima u FČS statičko magnetsko polje ima važnu ulogu?

- mjerenje magnetizacije (1 tesla = 10^4 gaussa $\equiv 10^4$ oersted)
- mjerenje magnetootpornosti i Hallvog koeficijenta u $\text{RuSr}_2\text{EuCu}_2\text{O}_8$ [Požek *et al.*, 2007]
- nuklearna magnetska rezonancija u FČS
- brojne druge metode



Jednadžbe makroskopske magnetostatike

postoje samo izvori struja koji su neovisni o vremenu

- opći izrazi za ukupnu gustoću struja

$$\begin{aligned}\mathbf{J}^{\text{tot}}(\mathbf{x}) &= \mathbf{J}(\mathbf{x}) + \partial\mathbf{P}(\mathbf{x})/\partial t + c\nabla \times \mathbf{M}(\mathbf{x}) \\ \nabla \cdot \mathbf{J}(\mathbf{x}) &= -\partial\rho(\mathbf{x})/\partial t\end{aligned}$$

- izrazi u magnetostatici

$$\mathbf{J}^{\text{tot}}(\mathbf{x}) = \mathbf{J}(\mathbf{x}) + c\nabla \times \mathbf{M}(\mathbf{x}), \quad \nabla \cdot \mathbf{J}(\mathbf{x}) = 0$$

- Maxwellove jednadžbe ($\mathbf{E}(\mathbf{x}) = 0$) i rubni uvjeti

$$\begin{aligned}\nabla \cdot \mathbf{B}^{(i)} &= 0, & (\mathbf{B}^{(2)} - \mathbf{B}^{(1)}) \cdot \mathbf{n}_{21} &= 0 \\ \nabla \times \mathbf{H}^{(i)} &= \frac{4\pi}{c}\mathbf{J}^{(i)}, & \mathbf{n}_{21} \times (\mathbf{H}^{(2)} - \mathbf{H}^{(1)}) &= \frac{4\pi}{c}\mathbf{K}\end{aligned}$$

Magnetske odzivne funkcije u slučaju $\mathbf{M}^0(\mathbf{x}) = 0$

magnetska susceptibilnost paramagneta ($\chi > 0$)

i dijamagneta ($\chi < 0$)

- "pomoćno" magnetsko polje

$$\mathbf{H}(\mathbf{x}) = \mathbf{B}(\mathbf{x}) - 4\pi\mathbf{M}(\mathbf{x})$$

- magnetizacija u općem slučaju

$$\mathbf{M}(\mathbf{x}, t) = \mathbf{M}^0(\mathbf{x}) + \mathbf{M}(\mathbf{x}, t, \mathbf{H})$$

- sustavi s permanentnom magnetizacijom $\mathbf{M}^0(\mathbf{x}) \neq 0$:
feromagnetni (poglavlje VII) i antiferomagnetni (FČS)
- $\mathbf{M}^0(\mathbf{x}) = 0$: tenzor magnetske susceptibilnosti

$$M_\alpha(\mathbf{x}, t) = \sum_\beta \int d^3x' \int dt' \chi_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t') H_\beta(\mathbf{x}', t')$$

Magnetske odzivne funkcije u slučaju $\mathbf{M}^0(\mathbf{x}) = 0$

- tenzor magnetske permeabilnosti

$$B_\alpha(\mathbf{x}, t) = \sum_\beta \int d^3x' \int dt' \mu_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t') H_\beta(\mathbf{x}', t')$$

- izotropni homogeni sustavi

$$\chi_{\alpha\beta}(\mathbf{x} - \mathbf{x}', t - t') \approx \chi \delta_{\alpha,\beta} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

$$\mu_{\alpha\beta}(\mathbf{x} - \mathbf{x}', t - t') \approx \mu \delta_{\alpha,\beta} \delta(\mathbf{x} - \mathbf{x}') \delta(t - t')$$

- te

$$\mathbf{M}(\mathbf{x}, t) = \chi \mathbf{H}(\mathbf{x}, t), \quad \mathbf{B}(\mathbf{x}, t) = \mu \mathbf{H}(\mathbf{x}, t) = (1 + 4\pi\chi) \mathbf{H}(\mathbf{x}, t)$$

- bakar $\Rightarrow 4\pi\chi = -9.7 \times 10^{-6}$; aluminij $\Rightarrow 4\pi\chi = +2.1 \times 10^{-5}$

Rješavanje jednadžbi magnetostatike za $\mathbf{M}^0(\mathbf{x}) = 0$

vektorski potencijal u Coulombovom baždarenju

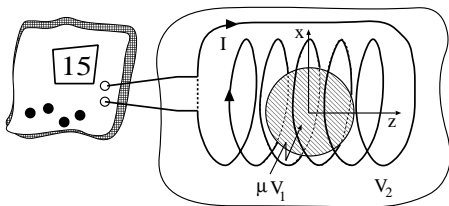
- karakteristična geometrija problema

- Maxwellove jednadžbe

$$\nabla \cdot \mathbf{B}^{(i)} = 0, \quad \nabla \times \mathbf{B}^{(i)} = (4\pi/c)(\mathbf{J}^{(i)} + c\nabla \times \mathbf{M}^{(i)})$$

- rubni uvjeti

$$(\mathbf{B}^{(2)} - \mathbf{B}^{(1)}) \cdot \mathbf{n}_{21} = 0, \quad \mathbf{n}_{21} \times (\mathbf{B}^{(2)} - \mathbf{B}^{(1)}) = (4\pi/c)\mathbf{K}^{\text{ind}}$$



Rješavanje jednadžbi magnetostatik za $\mathbf{M}^0(\mathbf{x}) = 0$

Coulombovo baždarenje

- uvodimo vektorski potencijal $[\nabla \cdot \mathbf{B}^{(i)}(\mathbf{x}) = 0]$

$$\mathbf{B}^{(i)}(\mathbf{x}) = \nabla \times \mathbf{A}^{(i)}(\mathbf{x})$$

- jednadžba za vektorski potencijal u Coulombovom baždarenju $[\nabla \cdot \mathbf{A}^{(i)}(\mathbf{x}) = 0]$

$$\nabla^2 \mathbf{A}^{(i)}(\mathbf{x}) = -(4\pi/c) [\mathbf{J}^{(i)}(\mathbf{x}) + c \nabla \times \mathbf{M}^{(i)}(\mathbf{x})]$$

- izotropni homogeni sustavi

$$\nabla^2 \mathbf{A}^{(i)}(\mathbf{x}) = -\frac{4\pi}{c} \mu^{(i)} \mathbf{J}^{(i)}(\mathbf{x})$$

Primjer: demagnetizacijski efekti za paramagnetsku kuglu

paramagnetska kugla u konstantnom magnetskom polju \mathbf{H}_0

- polja su superpozicija dva doprinosa

$$\mathbf{H}^{(i)}(\mathbf{x}) = \mathbf{H}_0^{(i)} + \mathbf{H}_1^{(i)}(\mathbf{x})$$

- demagnetizacijsko polje kugle

$$\mathbf{H}_1^{(1)} = \mathbf{H}_1 = - \sum_{\alpha} N_{\alpha} M_{\alpha} \hat{e}_{\alpha}$$

- polje izvan kugle, dolazi od magnetskog momenta

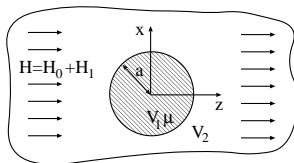
$$\mathbf{m} = (4\pi/3)a^3\mathbf{M}$$

$$\mathbf{H}_1^{(2)}(\mathbf{x}) = -\nabla\left(\frac{\mathbf{m} \cdot \mathbf{x}}{r^3}\right)$$

kugla : $N_z = 4\pi/3$

L ploča : $N_z = 4\pi$

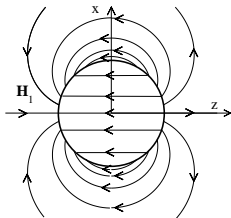
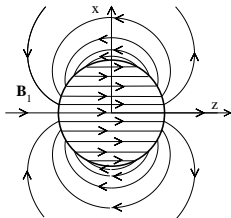
T ploča : $N_z = 0$



Primjer: demagnetizacijski efekti za paramagnetsku kuglu

- silnice magnetskog polja $\mathbf{B}_1^{(i)}(\mathbf{x})$
- i pomoćnog magnetskog polja $\mathbf{H}_1^{(i)}(\mathbf{x})$
- veza među poljima

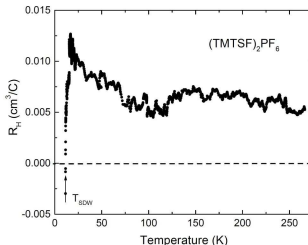
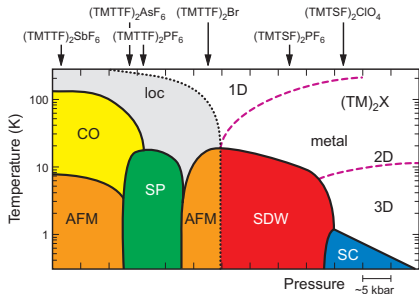
$$\mathbf{H}^{(2)}(\mathbf{x}) = \mathbf{B}^{(2)}(\mathbf{x}), \quad \mathbf{H}^{(1)}(\mathbf{x}) = \mathbf{B}^{(1)}(\mathbf{x}) - 4\pi\mathbf{M}(\mathbf{x})$$



Motivacija II

na koji način možemo unutar istog formalizma studirati postojanje različitih magnetski uređenih faza (FM, AFM, SDW, ...) i analizirati razne magnetske (ili elektromagnetske) odzivne funkcije?

- fazni dijagram Bechgaardovih soli [Dressel, 2007]
- Hallov koeficijent u (petoj) Bechgaardovoj soli [Mihaly *et al.*, 2000]



Energija magnetostatskog polja

zanima nas energija magnetostatskog polja izražena pomoću polja $\mathbf{H}(\mathbf{x})$ i $\mathbf{B}(\mathbf{x})$

- realni vodiči: Jouleov zakon

$$\delta\mathcal{W} = -\delta t \int d^3x \mathbf{J} \cdot \mathbf{E}$$

- Maxwellove jednažbe za $\omega \approx 0$

$$\nabla \times \mathbf{E} = -(1/c)\partial\mathbf{B}/\partial t, \quad \nabla \times \mathbf{H} = (4\pi/c)\mathbf{J}$$

- "parcijalne" integracije

$$\delta\mathcal{W} = \frac{c}{4\pi}\delta t \int d^3x \nabla \cdot (\mathbf{E} \times \mathbf{H}) - \frac{c}{4\pi}\delta t \int d^3x \mathbf{H} \cdot (\nabla \times \mathbf{E})$$

- rezultat

$$\delta\mathcal{W} = \frac{1}{4\pi} \int d^3x \mathbf{H} \cdot \delta t \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{4\pi} \int d^3x \mathbf{H}(\mathbf{x}) \cdot \delta \mathbf{B}(\mathbf{x})$$

Termodinamičke relacije

uvodimo dvije nove varijable: $\mathbf{H}(\mathbf{x})$ i $\mathbf{B}(\mathbf{x})$

- termodinamičke relacije za gustoće unutarnje i slobodne energije za fiksni broj čestica imaju oblik

$$\begin{aligned}dU &= TdS + (1/4\pi)\mathbf{H} \cdot d\mathbf{B} \\dF &= -SdT + (1/4\pi)\mathbf{H} \cdot d\mathbf{B}\end{aligned}$$

- također definiramo pomoćne potencijale

$$\tilde{U} = U - (1/4\pi)\mathbf{H} \cdot \mathbf{B}, \quad \tilde{F} = F - (1/4\pi)\mathbf{H} \cdot \mathbf{B}$$

- oni zadovoljavaju relacije

$$d\tilde{U} = TdS - (1/4\pi)\mathbf{B} \cdot d\mathbf{H}, \quad d\tilde{F} = -SdT - (1/4\pi)\mathbf{B} \cdot d\mathbf{H}$$

- veza između polja i potencijala

$$\begin{aligned}\mathbf{B} &= -4\pi(\partial\tilde{U}/\partial\mathbf{H})_S, & \mathbf{B} &= -4\pi(\partial\tilde{F}/\partial\mathbf{H})_T \\ \mathbf{H} &= 4\pi(\partial U/\partial\mathbf{B})_S, & \mathbf{H} &= 4\pi(\partial F/\partial\mathbf{B})_T\end{aligned}$$

Termodinamički potencijali u $\mathbf{J}(\mathbf{x})$ i $\mathbf{A}(\mathbf{x})$ reprezentaciji

kako zamijeniti $\mathbf{H}(\mathbf{x}), \mathbf{B}(\mathbf{x})$ reprezentaciju sa $\mathbf{J}(\mathbf{x}), \mathbf{A}(\mathbf{x})$ reprezentacijom

- polazimo od $\delta \mathbf{B} = \nabla \times \delta \mathbf{A}$ [$\delta \mathcal{W} = (1/4\pi) \int d^3x \mathbf{H} \cdot (\nabla \times \delta \mathbf{A})$]

$$\delta \mathcal{W} = -(1/4\pi) \int d^3x \nabla \cdot (\mathbf{H} \times \delta \mathbf{A}) + (1/4\pi) \int d^3x \delta \mathbf{A} \cdot (\nabla \times \mathbf{H})$$

- rezultat

$$(\delta \mathcal{F})_T = \frac{1}{c} \int d^3x \mathbf{J}(\mathbf{x}) \cdot \delta \mathbf{A}(\mathbf{x}), \quad (\delta \tilde{\mathcal{F}})_T = -\frac{1}{c} \int d^3x \mathbf{A}(\mathbf{x}) \cdot \delta \mathbf{J}(\mathbf{x})$$

- veza između polja i potencijala

$$\begin{aligned} \mathbf{A} &= -c(\partial \tilde{U} / \partial \mathbf{J})_S, & \mathbf{A} &= -c(\partial \tilde{\mathcal{F}} / \partial \mathbf{J})_T \\ \mathbf{J} &= c(\partial U / \partial \mathbf{A})_S, & \mathbf{J} &= c(\partial \mathcal{F} / \partial \mathbf{A})_T \end{aligned}$$

Primjer: Gibbsov potencijal

paramagneti s konstantnom magnetskom permeabilnošću

- pretpostavka

$$\tilde{F}(T, \mathbf{M}, \mathbf{H}) - F_0(T) = \frac{\mathbf{M}^2}{2\chi} - \mathbf{M} \cdot \mathbf{H} - \frac{1}{8\pi} \mathbf{H}^2$$

- provjere I i II

$$\begin{aligned} \frac{\partial \tilde{F}}{\partial \mathbf{M}} = 0 &\Rightarrow \mathbf{M}(\mathbf{x}) = \chi \mathbf{H}(\mathbf{x}) \\ \mathbf{B} = -4\pi \left(\frac{\partial \tilde{F}}{\partial \mathbf{H}} \right)_T &\Rightarrow \mathbf{B}(\mathbf{x}) = \mathbf{H}(\mathbf{x}) + 4\pi \mathbf{M}(\mathbf{x}) \end{aligned}$$

- rezultat

$$\tilde{F}(T, \mathbf{M}, \mathbf{H}) - F_0(T) = -\frac{1}{8\pi} \mathbf{B} \cdot \mathbf{H} = -\frac{1}{8\pi} \mu \mathbf{H}^2$$