#6 Istosmjerne struje

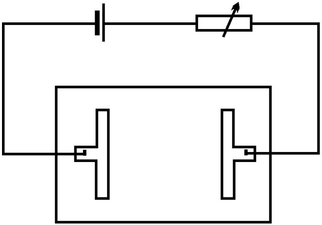
I Jednadžbe za istosmjerne struje
II Gibbsov potencijal u vodičima

predavanja 20**

Motivacija I

u kojim sve eksperimentima u FČS istosmjerne struje imaju važnu ulogu?

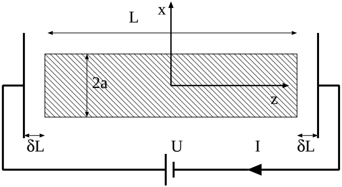
- određivanje "elektrostatskog potencijala" i pripadnog električnog polja u vježbi električno polje?
- [u elektromagnetima i] nigdje drugdje
- "istosmjerne struje" $\rightarrow \omega \approx 0$ (npr. $\omega = 22$ Hz)

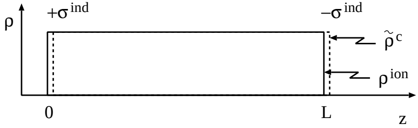


Motivacija I

usporedba elektrostatike i istosmjernih struja u vodičima

- geometrija problema
  - elektrostatika $\delta L \neq 0$
  - istosmjerne struje $\delta L = 0$
- model želea za $\delta L \neq 0$





Drudeov model za vodljive elektrone

Lorentzova sila u sustavu od jedne i od 10^{24} čestica

- jednadžba gibanja jedne nabijene čestice $(q = +e, -e, \ldots)$

$$m\big(d/dt + 1/\tau\big)\mathbf{v} = \mathbf{F} = q\big(\mathbf{E} + (1/c)\mathbf{v} \times \mathbf{H}\big)$$

- jednadžbe gibanje za 10^{24} nabijenih čestica $(\omega = 0)$

$$nq\mathbf{v} = \frac{nq^2\tau}{m}\mathbf{E} + \frac{nq^2\tau}{m}\,\frac{1}{nqc}\,nq\mathbf{v} \times \mathbf{H}$$

- gustoća struje, dc vodljivost i Hallova konstanta

$$\mathbf{J} = nq\mathbf{v}, \quad \sigma^{\mathrm{dc}} = nq^2\tau/m, \quad R_{\mathrm{H}} = (1/nqc)$$

- poopćenje Ohmovog zakona

$$\begin{aligned} \mathbf{J} &= \sigma^{\mathrm{dc}}\mathbf{E} + \sigma^{\mathrm{dc}}R_{\mathrm{H}}\,\mathbf{J} \times \mathbf{H} \\ \mathbf{E} &= \rho^{\mathrm{dc}}\mathbf{J} - R_{\mathrm{H}}\,\mathbf{J} \times \mathbf{H} \end{aligned}$$

Drudeov model za vodljive elektrone

tenzor vodljivosti $\sigma_{\alpha\beta}(\omega)$

- rješenje Drudeovih jednadžbi za $\omega = 0$

$$J_x = \tilde{\sigma}^{\mathrm{dc}} E_x + \frac{q}{|q|} \tau \omega_c \tilde{\sigma}^{\mathrm{dc}} E_y \equiv \sigma_{xx} E_x + \sigma_{xy} E_y$$

$$J_y = \tilde{\sigma}^{\mathrm{dc}} E_y - \frac{q}{|q|} \tau \omega_c \tilde{\sigma}^{\mathrm{dc}} E_x \equiv \sigma_{yy} E_y + \sigma_{yx} E_x$$

- ciklotronska frekvencija

$$\omega_c = (|q| H_z / mc)$$

- renormalizirana dc vodljivost

$$\tilde{\sigma}^{\mathrm{dc}} = \sigma^{\mathrm{dc}} / (1 + (\tau \omega_c)^2)$$

- rješenje za $H_z = 0$ i $\omega \neq 0$

$$\sigma_{xx}(\omega) = \frac{\sigma^{\mathrm{dc}}}{1 - i\omega\tau}$$

Jouleov zakon

zašto u vodičima iščezava elektrostatsko polje?

- Maxwellova jednadžba IV za $\rho^{\rm ext} = \mathbf{J}^{\rm ext} = 0$, $\mathbf{H} \approx 0$ te $\mathbf{D}_0 = (1 + 4\pi\hat{\chi}^{\rm b})\mathbf{E}$

$$\frac{4\pi}{c}\mathbf{J}^{\rm c}(\mathbf{x}) = -\frac{1}{c}\frac{\partial \mathbf{D}_0(\mathbf{x},t)}{\partial t}$$

- skalarni produkt sa $\mathbf{E}(\mathbf{x})$ i integracija preko volumena vodiča

$$\int {\rm d}^3x\, \mathbf{J}^{\rm c}\cdot\mathbf{E} = -\frac{1}{4\pi}\int {\rm d}^3x\, \mathbf{E}\cdot\frac{\partial \mathbf{D}_0}{\partial t}$$

- Jouleov zakon (susceptibilnost $\chi^{\rm b}$ neovisna o vremenu)

$$\int {\rm d}^3x\, \mathbf{J}^{\rm c}\cdot\mathbf{E} \;=\; -\frac{\partial}{\partial t}\left\{\frac{1}{8\pi}\int {\rm d}^3x\, \mathbf{E}\cdot\mathbf{D}_0\right\} = -\frac{\partial \mathcal{W}}{\partial t}$$

Makroskopske jednadžbe za istosmjerne struje

kako riješiti makroskopske jednadžbe za istosmjerne struje?

- (homogene) jednadžbe i rubni uvjeti

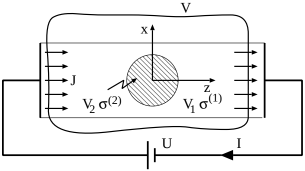
$$\nabla \cdot \mathbf{J}^{c,(i)}(\mathbf{x}) = 0, \qquad (\mathbf{J}^{c,(2)} - \mathbf{J}^{c,(1)}) \cdot \mathbf{n}_{21} = 0$$
$$\nabla \times \mathbf{E}^{(i)}(\mathbf{x}) = 0, \qquad \mathbf{n}_{21} \times (\mathbf{E}^{(2)} - \mathbf{E}^{(1)}) = 0$$

- rješavanje jednadžbi pomoću skalarnog potencijala

$$\nabla \times \mathbf{E}^{(i)}(\mathbf{x}) = 0 \Rightarrow \mathbf{E}^{(i)}(\mathbf{x}) = -\nabla \Phi^{(i)}(\mathbf{x})$$

- rješavanje jednadžbi pomoću vektorskog potencijala

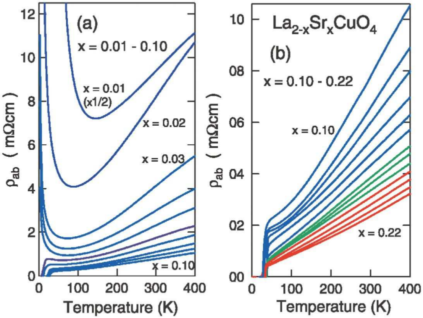
$$\nabla \cdot \mathbf{J}^{c,(i)}(\mathbf{x}) = 0 \Rightarrow \mathbf{J}^{c,(i)}(\mathbf{x}) = (c/4\pi)\nabla \times \mathbf{H}^{(i)}(\mathbf{x})$$

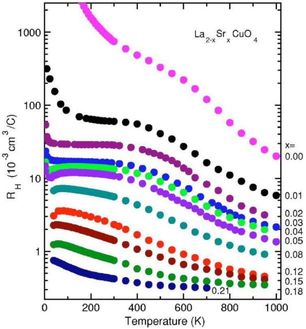


Motivacija II

na koji način ovise otpornost i Hallov koeficijent o temperaturu?

- na primjer, u poddopiranim supravodičima $La_{2-x}Sr_xCuO_4$? [Ando *et al.*, 2004] i [Ono *et al.*, 2007]
- kako objasniti rezultate pomoću dva parametra, n i τ, u Drudeovom modelu?





Gibbsov potencijal za $\omega \approx 0$

postoji li mogućnost da se elementarna pobuđenja u sustavu vezanih elektrona i u sustavu vodljivih elektrona formalno tretiraju na isti način?

- jednadžba kontinuiteta za $\mathbf{J}^{\mathrm{c}}(\mathbf{k}, \omega)$

$$\omega \rho^{\mathrm{c}}(\mathbf{k}, \omega) = \mathbf{k} \cdot \mathbf{J}^{\mathrm{c}}(\mathbf{k}, \omega), \quad -i\mathbf{k} \cdot \mathbf{P}^{\mathrm{c}}(\mathbf{k}, \omega) = \rho^{\mathrm{c}}(\mathbf{k}, \omega)$$

- imaginarne polarizacije (i imaginarne struje)

$$\mathbf{P}^{\mathrm{c}}(\mathbf{k}, \omega) = (i/\omega)\mathbf{J}^{\mathrm{c}}(\mathbf{k}, \omega)$$

- zapis termodinamičkih potnecijala u $\{\mathbf{k}\}$ reprezentaciji (Z1.2$^{\mathrm{VJ}}$)

$$-\int \mathrm{d}^3x \, \mathbf{P}^{\mathrm{c}}(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}) = -\sum_{\mathbf{k}'} \mathbf{P}^{\mathrm{c}}(\mathbf{k}') \cdot \mathbf{E}(-\mathbf{k}')$$

- Gibbsov potencijal u vodičima s jednom vrpcom?

$$\Delta\widetilde{\mathcal{F}}(\mathbf{P}^{\mathrm{c}}, \mathbf{E}) = \sum_{\mathbf{k}'} \left\{ \frac{\left|\mathbf{P}^{\mathrm{c}}(\mathbf{k}')\right|^2}{2\chi^{\mathrm{c}}(\mathbf{k}')} - \mathbf{P}^{\mathrm{c}}(\mathbf{k}') \cdot \mathbf{E}(-\mathbf{k}') - \frac{1}{8\pi}\left|\mathbf{E}(\mathbf{k}')\right|^2 \right\}$$

Gibbsov potencijal za $\omega \approx 0$

- provjera I ($\partial\widetilde{\mathcal{F}}/\partial\mathbf{P}^{\mathrm{c}}(-\mathbf{k}) = 0$)

$$\mathbf{J}^{\mathrm{c}}(\mathbf{k},\omega) = -i\omega\chi^{\mathrm{c}}(\mathbf{k},\omega)\mathbf{E}(\mathbf{k},\omega) = \sigma^{\mathrm{c}}(\mathbf{k},\omega)\mathbf{E}(\mathbf{k},\omega)$$

- provjera II ($-4\pi\partial\widetilde{\mathcal{F}}/\partial\mathbf{E}^{\mathrm{c}}(-\mathbf{k}) = \mathbf{D}^{\mathrm{c}}(\mathbf{k})$)

$$\mathbf{D}^{\mathrm{c}}(\mathbf{k},\omega) = \big(1 + (4\pi i/\omega)\sigma^{\mathrm{c}}(\mathbf{k},\omega)\big)\mathbf{E}(\mathbf{k},\omega)$$

- rezultat?

$$\begin{aligned}
\Delta\widetilde{\mathcal{F}}(\mathbf{J}^{\mathrm{c}},\mathbf{E}) &= -\frac{1}{8\pi}\sum_{\mathbf{k}'}\mathbf{D}^{\mathrm{c}}(\mathbf{k}')\cdot\mathbf{E}(-\mathbf{k}') \\
&= \sum_{\mathbf{k}'}\left\{-\frac{i}{2\omega}\mathbf{J}^{\mathrm{c}}(\mathbf{k}')\cdot\mathbf{E}(-\mathbf{k}') - \frac{1}{8\pi}\big|\mathbf{E}(\mathbf{k}')\big|^2\right\}
\end{aligned}$$

Aproksimacija slučajnih faza

veza prethodnog izraza s mikroskopskom fizikom čvrstog stanja

- prijelaz u $\{\rho, \Phi\}$ reprezentaciju

$$P^{\mathrm{c}}_\alpha(\mathbf{k},\omega) = (i/k_\alpha)\rho^{\mathrm{c}}(\mathbf{k},\omega), \quad E_\alpha(\mathbf{k},\omega) = -ik_\alpha\Phi^{\mathrm{tot}}(\mathbf{k},\omega)$$

- Gibbsov potencijal $[\tilde{\chi} = -(k'_\alpha)^2\chi]$

$$\Delta\widetilde{\mathcal{F}}(\rho^{\mathrm{c}}, \Phi^{\mathrm{tot}}) = \sum_{\mathbf{k}'}\left\{\frac{\left|\rho^{\mathrm{c}}(\mathbf{k}')\right|^2}{-2\tilde{\chi}(\mathbf{k}')} + \rho^{\mathrm{c}}(\mathbf{k}')\Phi^{\mathrm{tot}}(-\mathbf{k}') - \frac{k_\alpha^2}{8\pi}\left|\Phi^{\mathrm{tot}}(\mathbf{k}')\right|^2\right\}$$

- provjere $[V_{11}(\mathbf{k}') = 4\pi/(\mathbf{k}')^2]$

$$\partial\widetilde{\mathcal{F}}/\partial\rho^{\mathrm{c}} = -\rho^{\mathrm{c}}/\tilde{\chi} + \Phi^{\mathrm{tot}} = 0, \quad \partial\widetilde{\mathcal{F}}/\partial\Phi^{\mathrm{tot}} = \rho^{\mathrm{c}} - \Phi^{\mathrm{tot}}/V_{11} = -\rho^{\mathrm{ext}}$$

- RPA rezultat

$$\begin{aligned}\rho^{\mathrm{ind}}(\mathbf{k},\omega) &= \tilde{\chi}(\mathbf{k},\omega)\Phi^{\mathrm{tot}}(\mathbf{k},\omega) \\ \Phi^{\mathrm{tot}}(\mathbf{k},\omega) &= V_{11}(\mathbf{k})\left[\rho^{\mathrm{ext}}(\mathbf{k},\omega) + \rho^{\mathrm{ind}}(\mathbf{k},\omega)\right] = \frac{\Phi^{\mathrm{ext}}(\mathbf{k},\omega)}{\varepsilon(\mathbf{k},\omega)}\end{aligned}$$

Boltzmannove jednadžbe

vodljivi elektroni opisani vektorom položaja **r** *i kvaziimpulsom* **p**

- semiklasična neravnotežna funkcija raspodjele ($f_0(\mathbf{p}) \equiv f_0[\varepsilon(\mathbf{p})]$)

$$\frac{\mathrm{d}f(\mathbf{r},t,\mathbf{p})}{\mathrm{d}t} = -\frac{f(\mathbf{r},t,\mathbf{p}) - f_0(\mathbf{p})}{\tau}$$

- Boltzmannove jednadžbe ($I(f)$ - integral kolizije)

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\partial \mathbf{r}}{\partial t} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\partial \mathbf{p}}{\partial t} \cdot \frac{\partial f}{\partial \mathbf{p}} \equiv I(f)$$

- ili (u najgrubljoj aproksimaciji)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + q\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}\right) \cdot \frac{\partial f}{\partial \mathbf{p}} = -\frac{f - f_0}{\tau}$$

Boltzmannove jednadžbe

rješavanje jednadžbi za τ neovisan o $\mathbf{p}$

- longitudinalno monokromatsko električno polje
$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{k}, \omega)\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}$$
- pretpostavka rješenja
$$f(\mathbf{r}, t, \mathbf{p}) = f(\mathbf{p})\mathrm{e}^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \quad f(\mathbf{p}) = f_0(\mathbf{p}) + g(\mathbf{p})$$
- Boltzmannove jednadžbe za veličinu $g(\mathbf{p}) \equiv g[\mathbf{v}(\mathbf{p})]$

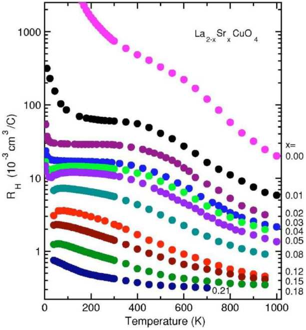
$$i(\omega - \mathbf{k}\cdot\mathbf{v}(\mathbf{p}) + i\Gamma)g(\mathbf{p}) = \sum_{\alpha} qE_{\alpha} v_{\alpha}(\mathbf{p}) \frac{\partial f_0(\mathbf{p})}{\partial \varepsilon(\mathbf{p})} + \sum_{\alpha\beta} \frac{q}{cm_{\alpha\beta}(\mathbf{p})} (\mathbf{v}(\mathbf{p}) \times \mathbf{H})_{\alpha} \frac{\partial g(\mathbf{p})}{\partial v_{\beta}(\mathbf{p})}$$

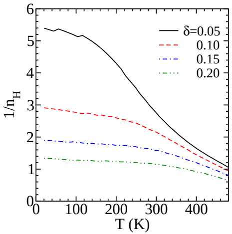
- uz $v_{\alpha}(\mathbf{p}) = \partial\varepsilon(\mathbf{p})/\partial p_{\alpha}$, $1/m_{\alpha\beta}(\mathbf{p}) = \partial^2\varepsilon(\mathbf{p})/\partial p_{\alpha}\partial p_{\beta}$ te $\Gamma = 1/\tau$

Boltzmannove jednadžbe

primjer: Hallov koeficijent u $La_{2-x}Sr_xCuO_4$

- [Ono *et al.*, 2007]
- problem 6.1 [F. Kos, Seminar iz . . . , 2010]; $R_\mathrm{H} = (1/n_\mathrm{H} qc)$





Tenzor vodljivosti

rješenje Boltzmannovih jednadžbi za $\mathbf{B} = 0$

- gustoća struje

$$\mathbf{J}(\mathbf{k}) = \frac{1}{V} \sum_{\mathbf{p}\sigma} q\mathbf{v}(\mathbf{p}) f(\mathbf{p}) = \frac{1}{V} \sum_{\mathbf{p}\sigma} q\mathbf{v}(\mathbf{p}) g(\mathbf{p})$$

- frekventno ovisna unutarvrpčana vodljivost izvan Drudeovog modela

$$\sigma_{\alpha\alpha}(\omega) = \frac{q^2 \tau n^*_{\alpha\alpha}}{m} \frac{1}{1 - i\omega\tau}, \qquad n^*_{\alpha\alpha} = \frac{1}{V} \sum_{\mathbf{p}\sigma} m v^2_\alpha(\mathbf{p})(-) \frac{\partial f_0(\mathbf{p})}{\partial \varepsilon(\mathbf{p})}$$

- istosmjerna vodljivost

$$\sigma^{\mathrm{dc}}_{\alpha\alpha} = \frac{q^2 \tau n^*_{\alpha\alpha}}{m} = \frac{q^2 \tau}{m} \frac{1}{V} \sum_{\mathbf{p}\sigma} \frac{m}{m_{\alpha\alpha}(\mathbf{p})} f_0(\mathbf{p})$$