#8 Kvazistatička elektromagnetska polja u vodičima

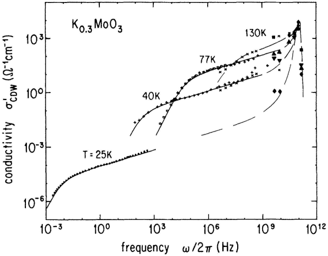
I Maxwellove jednadžbe
II Skin efekt

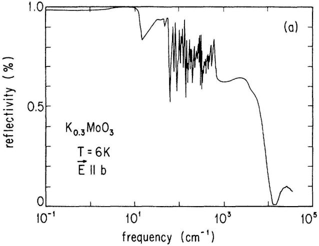
predavanja 20**

Motivacija I

na koji način određujemo strukturu dielektričnih odzivnih funkcija u vodičima u širokom području frekvencija [Degiorgi et al., 1991]?

- kako definiramo kvazistatička elektromagnetska polja?
- što se događa u sustavima s metal-izolator faznim prijelazima?
- što se događa u sustavima s prijelazom iz normalne u supravodljivu metalnu fazu?

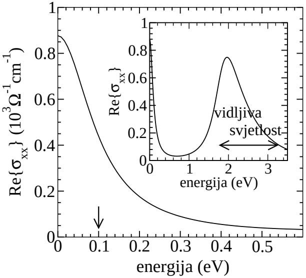


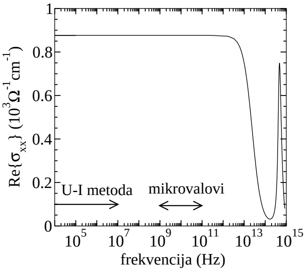


$\omega \gg 0$ i $\omega \approx 0$ elektromagnetska polja u FČS

na koji način se eksperimentalne metode u FČS razvrstavaju prema frekvenciji korištenih elektromagnetskih polja?

- vidljiva (i ultravioletna) svjetlost: $\lambda = 380 - 750$ nm
- infracrvena elektromagnetska polja
- mikrovalovi: $\lambda = 0.3 - 30$ cm ($\omega \approx 0$)
- kontaktne metode: $\lambda > 3 \times 10^4$ cm ($\omega \approx 0$)





Kvazistatička elektromagnetska polja

kako izgledaju pomoćna polja i odzivne funkcije u $\omega \approx 0$ *režimu* ($\omega_{\mathrm{maks}} = 10^{12}$ *Hz, ili* ≈ 1 *meV)?*

- podsustav vezanih naboja $[\omega_{TO} \gg \omega_{\mathrm{maks}}]$

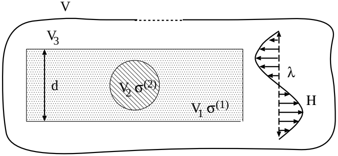
$$\chi^{\mathrm{b}}(\omega) \approx \chi^{\mathrm{b}}(0) \to \varepsilon_0^{\mathrm{b}} \equiv 1 + 4\pi\chi^{\mathrm{b}}(0)$$

- dielektrični pomak

$$\mathbf{D}^{\mathrm{c}}(\mathbf{x},\omega) \approx \big(\varepsilon_0^{\mathrm{b}} + (4\pi i/\omega)\sigma^{\mathrm{c}}(\omega)\big)\mathbf{E}^{\mathrm{c}}(\mathbf{x},\omega)$$

- pomoćno magnetsko polje $[\mu(\omega) \approx \mu$, najčešće$]$

$$\mathbf{B}^{\mathrm{c}}(\mathbf{x},\omega) \approx \mu(\omega)\mathbf{H}^{\mathrm{c}}(\mathbf{x},\omega)$$



Kvazistatička elektromagnetska polja

aproksimativni izraz za dielektričnu funkciju za $\omega \approx 0$

- dielektrični pomak $[\sigma^{\rm tot}(\omega) \equiv \sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)]$

$$\mathbf{D}^{\rm c}(\mathbf{x},\omega) \approx \big(1 + (4\pi i/\omega)\sigma^{\rm tot}(\omega)\big)\mathbf{E}^{\rm c}(\mathbf{x},\omega)$$

- Drudeov model vodljivosti

$$\sigma^{\rm c}(\omega) = \frac{\sigma^{\rm dc}}{1+(\omega\tau)^2} + \frac{i\omega\tau\sigma^{\rm dc}}{1+(\omega\tau)^2} \equiv \sigma_1^{\rm c}(\omega) + i\sigma_2^{\rm c}(\omega)$$

- $\omega \approx 0$ dielektrična funkcija

$$\varepsilon(\omega) \approx \varepsilon_0^{\rm b} - 4\pi\tau\sigma_1^{\rm c}(\omega) + \frac{4\pi i}{\omega}\sigma_1^{\rm c}(\omega) \approx i\varepsilon_2(\omega) = \frac{4\pi i\sigma_1^{\rm c}}{\omega}$$

Maxwellove jednadžbe u vodičima za $\omega \approx 0$

- jednadžbe [za $\varepsilon_{\alpha\beta}(\mathbf{x}-\mathbf{x}',\omega) = \delta_{\alpha,\beta}\delta(\mathbf{x}-\mathbf{x}')\varepsilon(\omega)$]

$$\nabla \cdot \mathbf{E}^{(i)}(\mathbf{x},\omega) = 0, \qquad \nabla \times \mathbf{E}^{(i)}(\mathbf{x},\omega) = (i\omega/c)\mu^{(i)}(\omega)\mathbf{H}^{(i)}(\mathbf{x},\omega)$$
$$\nabla \cdot \mathbf{H}^{(i)}(\mathbf{x},\omega) = 0, \qquad \nabla \times \mathbf{H}^{(i)}(\mathbf{x},\omega) = -(i\omega/c)\mathbf{D}^{(i)}(\mathbf{x},\omega)$$

- uz

$$\mathbf{D}^{\mathrm{c}}(\mathbf{x},\omega) \approx i\varepsilon_2(\omega)\mathbf{E}^{\mathrm{c}}(\mathbf{x},\omega)$$

- rubni uvjeti [$\mathbf{H}^{(2)} = \mathbf{H}^{(1)}$, za $\mu(\omega) = 1$]

$$\left(\mathbf{D}^{(2)} - \mathbf{D}^{(1)}\right) \cdot \mathbf{n}_{21} = 0, \qquad \mathbf{n}_{21} \times \left(\mathbf{E}^{(2)} - \mathbf{E}^{(1)}\right) = 0$$
$$\left(\mathbf{B}^{(2)} - \mathbf{B}^{(1)}\right) \cdot \mathbf{n}_{21} = 0, \qquad \mathbf{n}_{21} \times \left(\mathbf{H}^{(2)} - \mathbf{H}^{(1)}\right) = 0$$

- u jednokomponentnim vodičima

$$\left(\mathbf{E} - i\varepsilon_2(\omega)\mathbf{E}^{\mathrm{c}}\right) \cdot \mathbf{n} = 0$$

Dubina prodiranja δ

valna jednadžba? ili jednadžba difuzije?

- iz Maxwellovih jednadžbi III i IV slijedi

$$\nabla^2 \mathbf{H}^{(i)}(\mathbf{x},\omega) = -[\omega^2 \mu^{(i)}(\omega)\varepsilon^{(i)}(\omega)/c^2]\mathbf{H}^{(i)}(\mathbf{x},\omega)$$

- aproksimativna forma u vodičima, za $\omega \approx 0$

$$\nabla^2 \mathbf{H}^{\mathrm{c}}(\mathbf{x},t) \;\approx\; -(4\pi i\omega\mu\sigma_1/c^2)\mathbf{H}^{\mathrm{c}}(\mathbf{x},t)$$

- dubina prodiranje elektromagnetskih polja δ

$$k = \sqrt{4\pi i\omega\mu\sigma_1}/c = (1+i)\sqrt{2\pi\omega\mu\sigma_1}/c = (1+i)/\delta$$

- jednadžba difuzije

$$\nabla^2 \mathbf{H}^{\mathrm{c}}(\mathbf{x},t) \;\approx\; (4\pi\mu\sigma_1/c^2)\partial\mathbf{H}^{\mathrm{c}}(\mathbf{x},t)/\partial t$$

Skin efekt u polubeskonačnom vodiču

- "Laplaceova" jednadžba [$\nabla_{\mathrm{n}}^2 = \nabla^2 - \partial^2/\partial z^2$]

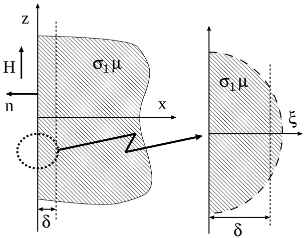
$$\nabla_{\mathrm{n}}^2 \mathbf{H}^{\mathrm{c}}(\mathbf{x},\omega) + k^2 \mathbf{H}^{\mathrm{c}}(\mathbf{x},\omega) = 0$$

- pretpostavka rješenja [$\mathbf{H}^{\mathrm{c}}(\mathbf{x},t) = h(x)\hat{e}_z \mathrm{e}^{-i\omega t}$]

$$\frac{\partial^2 h(x)}{\partial x^2} + k^2 h(x) = 0 \rightarrow h(x) = H_{\parallel} \mathrm{e}^{ikx} = H_{\parallel} \mathrm{e}^{ix/\delta - x/\delta}$$

- rješenje

$$\mathrm{Re}\{\mathbf{H}^{\mathrm{c}}(x,t)\} = H_{\parallel} \hat{e}_z \mathrm{e}^{-x/\delta} \cos(x/\delta - \omega t)$$



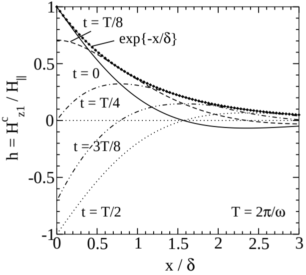
Skin efekt u polubeskonačnom vodiču

- magnetska komponenta polja [u $t = n(\pi/4\omega)$, $n = 0, 1, 2, 3, 4$]
- električna komponenta polja

$$\mathbf{E}^{\mathrm{c}}(x,t) = (ikc/4\pi\sigma_1)\mathbf{H}^{\mathrm{c}} \times \mathbf{n} = (1-i)\sqrt{(\omega\mu/8\pi\sigma_1)}\mathbf{n} \times \mathbf{H}^{\mathrm{c}}$$

- te

$$\mathrm{Re}\{\mathbf{E}^{\mathrm{c}}(x,t)\} = (\delta\omega\mu/\sqrt{2}c)H_{\|}\hat{e}_y \mathrm{e}^{-x/\delta}\cos(x/\delta - \omega t - \pi/4)$$



Površinska impedancija

ima li Ohmov zakon površinsku inačicu?

- integrirana gustoća inducirane struje

$$\mathbf{K}_{\mathrm{ef}}(t) = \int_0^\infty \mathrm{d}x\, \mathbf{J}^{\mathrm{c}}(x,t) = \int_0^\infty \mathrm{d}x\, \sigma_1(\omega)\mathbf{E}^{\mathrm{c}}(x,t)$$

- rezultat

$$\mathbf{K}_{\mathrm{ef}}(t) = \frac{c}{4\pi}\mathbf{n} \times \mathbf{H}^{\mathrm{c}}(0,t)$$

- površinska impedancija Z_{s}

$$Z_{\mathrm{s}}(\omega)\mathbf{K}_{\mathrm{ef}}(\omega) = \mathbf{E}^{\mathrm{c}}(0,\omega) \rightarrow Z_{\mathrm{s}}(\omega) = \frac{1-i}{\delta\sigma_1(\omega)}$$

- podsjetnik

$$\delta = c/\sqrt{2\pi\omega\mu\sigma_1(\omega)}$$

Skin efekt u cilindričnom vodiču

- "Laplaceova" jednadžba [$\nabla_{\mathrm{n}}^2 = \nabla^2 - \partial^2/\partial z^2$]

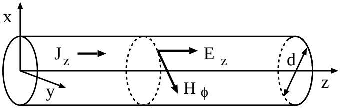
$$\nabla_{\mathrm{n}}^2 \mathbf{E}^{\mathrm{c}}(\mathbf{x},\omega) + k^2 \mathbf{E}^{\mathrm{c}}(\mathbf{x},\omega) = 0$$

- pretpostavka rješenja [$\mathbf{E}^{\mathrm{c}}(\rho, t) = E_{\|}(\rho)\hat{e}_z \mathrm{e}^{-i\omega t}$]

$$\frac{\partial^2 E_{\|}(x)}{\partial x^2} + \frac{1}{x}\frac{\partial E_{\|}(x)}{\partial x} - \left(1 + \frac{\nu^2}{x^2}\right) E_{\|}(x) = 0$$

- rješenje [modificirana Besselova funkcija, $I_0(x)$, uz $x = -ik\rho$]

$$\mathbf{E}^{\mathrm{c}}(\rho, t) = E_{\|}\hat{e}_z I_0(-ik\rho)\mathrm{e}^{-i\omega t}$$



Skin efekt u cilindričnom vodiču

na koji način rješenje ovisi o omjeru δ/a?

- asimptotski razvoj modificiranih Besselovih funkcija

$$
\begin{aligned}
I_0(x) &\approx 1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} + \cdots, \text{ za } x < 1, \\
I_0(x) &\approx \frac{\mathrm{e}^x}{\sqrt{2\pi x}}, \text{ za } x \gg 1,
\end{aligned}
$$

- "prozirni" loši vodiči [kada je $\delta > a$]

$$\mathbf{E}^{\mathrm{c}}(\rho, t) = E_{\|}\hat{e}_z\left(1 - (i/2)\left(\rho/\delta\right)^2 - \left(\rho/2\delta\right)^4 + \cdots\right)\mathrm{e}^{-i\omega t}$$

- normalni skin efekt u dobrim vodičima [$\delta \ll a$, $x = a - \rho$]

$$\mathbf{E}^{\mathrm{c}}(\rho, t) = E_{\|}\hat{e}_z \mathrm{e}^{ix/\delta - x/\delta}\mathrm{e}^{-i\omega t}$$

Anomalni skin efekt

što se događa kada δ postane usporediv sa srednjim slobodnim putom l (tj. $\delta < l$)?

- treba riješiti sustav integralno-diferencijalnih jednadžbi

$$\begin{aligned}
\mathbf{J}(\mathbf{r}) &= \frac{1}{V}\sum_{\mathbf{p}\sigma} q\mathbf{v}(\mathbf{p})g(\mathbf{p},\mathbf{r}) \\
l\hat{\mathbf{v}}\cdot\frac{\partial g(\mathbf{p},\mathbf{r})}{\partial \mathbf{r}} + g(\mathbf{p},\mathbf{r}) &= q\tau\mathbf{E}(\mathbf{r})\cdot\mathbf{v}(\mathbf{p})(-)\frac{\partial f_0}{\partial \varepsilon} \\
\nabla^2\mathbf{E}(\mathbf{r}) &= \frac{4\pi i\omega}{c^2}\mathbf{J}(\mathbf{r})
\end{aligned}$$

- skin efekt ove vrste naziva se anomalni skin efekt