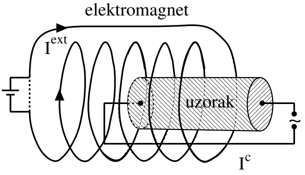
P#1: Uvod; Sažeci iz *Klasične elektrodinamike*

Fizika čvrstog stanja 2

predavanja 2022

1.1 Mikroskopske Maxwellove jednadžbe



- mikroskopske Maxwellove jednadžbe

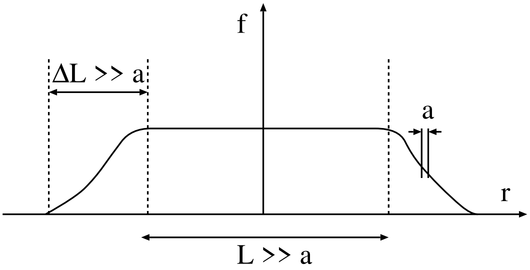
$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \mathbf{e}(\mathbf{x}, t) &= 4\pi\eta(\mathbf{x}, t) \\
\text{(II)} \quad \nabla \cdot \mathbf{b}(\mathbf{x}, t) &= 0 \\
\text{(III)} \quad \nabla \times \mathbf{e}(\mathbf{x}, t) &= -(1/c)\partial \mathbf{b}(\mathbf{x}, t)/\partial t \\
\text{(IV)} \quad \nabla \times \mathbf{b}(\mathbf{x}, t) &= (1/c)\partial \mathbf{e}(\mathbf{x}, t)/\partial t + (4\pi/c)\mathbf{j}(\mathbf{x}, t)
\end{aligned}
$$

- mikroskopske gustoće

$$\eta(\mathbf{x}) = \sum_i q_i \delta(\mathbf{x} - \mathbf{x}_i) \quad \text{i} \quad \mathbf{j}(\mathbf{x}) = \sum_i q_i \dot{\mathbf{x}}_i \delta(\mathbf{x} - \mathbf{x}_i),$$

- jednadžbe gibanja za čestice

$$m_i \ddot{\mathbf{x}}_i = -\partial V/\partial \mathbf{x}_i + q_i \left[\mathbf{e}^{\perp}(\mathbf{x}_i) + (1/c)\dot{\mathbf{x}}_i \times \mathbf{b}(\mathbf{x}_i)\right]$$



- definicija prosječne vrijednosti mikroskopske funkcije $F(\mathbf{x}, t)$

$$\langle F(\mathbf{x}, t)\rangle = \int_{L^3} \mathrm{d}^3 x' \, f(\mathbf{x}') F(\mathbf{x} - \mathbf{x}', t)$$

- $f(\mathbf{x})$ (npr. $= (\pi R)^{-3/2} e^{-r^2/R^2}$) je test funkcija, $L \gg a \approx 1$ Å
- karakteristična valna duljina elektromagnetskih polja $\lambda \geq 1250$ Å (≈ 10 eV)
- prosječna polja i gustoće

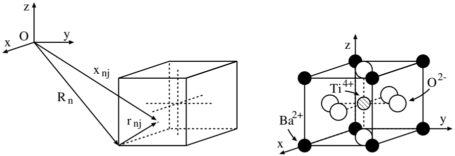
$$\mathbf{E}(\mathbf{x}, t) = \langle \mathbf{e}(\mathbf{x}, t)\rangle, \quad \mathbf{B}(\mathbf{x}, t) = \langle \mathbf{b}(\mathbf{x}, t)\rangle$$
$$\rho^{\rm tot}(\mathbf{x}, t) = \langle \eta(\mathbf{x}, t)\rangle, \quad \mathbf{J}^{\rm tot}(\mathbf{x}, t) = \langle \mathbf{j}(\mathbf{x}, t)\rangle$$

Prosječna mikroskopska gustoća naboja

- mikroskopska gustoća naboja

$$\eta(\mathbf{x}, t) = \sum_n \sum_{j(n)} q_{nj} \delta(\mathbf{x} - \mathbf{x}_{nj})$$

- oznake vektora položaja nabijenih čestica



- prosječna mikroskopska gustoća naboja

$$\sum_n \left\langle \eta_n(\mathbf{x}, t) \right\rangle = \sum_n \sum_{j(n)} q_{nj} f(\mathbf{x} - \mathbf{R}_n - \mathbf{r}_{nj})$$

- razvoj po malom $\mathbf{r}_{nj}$ (u usporedbi sa L)

$$\sum_n \left\langle \eta_n(\mathbf{x}, t) \right\rangle \approx \sum_n \langle q_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle - \nabla \cdot \sum_n \langle \mathbf{p}_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle$$

- monopolna i dipolna gustoća naboja primitivne ćelije

$$q_n = \sum_{j(n)} q_{nj}, \quad \mathbf{p}_n = \sum_{j(n)} q_{nj} \mathbf{r}_{nj}$$

Rezultat

- prosječna mikroskopska gustoća naboja

$$\rho^{\mathrm{tot}}(\mathbf{x}, t) \approx \sum_n \langle q_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle - \nabla \cdot \sum_n \langle \mathbf{p}_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle$$

- odnosno

$$\rho^{\mathrm{tot}}(\mathbf{x}, t) \approx \rho(\mathbf{x}, t) - \nabla \cdot \mathbf{P}(\mathbf{x}, t)$$

- makroskopska gustoća naboja

$$\rho(\mathbf{x}, t) = \sum_n \langle q_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle = \rho^{\mathrm{c}}(\mathbf{x}, t) + \rho^{\mathrm{ext}}(\mathbf{x}, t)$$

- makroskopska polarizacija

$$\mathbf{P}(\mathbf{x}, t) = \sum_n \langle \mathbf{p}_n \delta(\mathbf{x} - \mathbf{R}_n) \rangle$$

- kvantno-mehanički doprinosi (!)

Prosječna mikroskopska gustoća struja

- mikroskopska gustoća struja

$$\mathbf{j}(\mathbf{x},t) = \sum_{nj} q_{nj}\dot{\mathbf{x}}_{nj}\delta(\mathbf{x}-\mathbf{x}_{nj}(t))$$

- gustoća struje i magnetski moment primitivne ćelije

$$\mathbf{j}_n = \sum_{j(n)} q_{nj}\mathbf{v}_{nj}, \quad \mathbf{m}_n = \frac{1}{2c}\sum_{j(n)} q_{nj}\mathbf{r}_{nj}\times\mathbf{v}_{nj}$$

- prosječna mikroskopska gustoća struje

$$\mathbf{J}^{\mathrm{tot}}(\mathbf{x},t) \approx \mathbf{J}(\mathbf{x},t) + \frac{\partial}{\partial t}\mathbf{P}(\mathbf{x},t) + c\nabla\times\mathbf{M}(\mathbf{x},t)$$

- makroskopska gustoća struje

$$\mathbf{J}(\mathbf{x},t) = \sum_n \langle \mathbf{j}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle = \mathbf{J}^{\mathrm{c}}(\mathbf{x},t) + \mathbf{J}^{\mathrm{ext}}(\mathbf{x},t)$$

- makroskopska magnetizacija

$$\mathbf{M}(\mathbf{x},t) = \sum_n \langle \mathbf{m}_n\delta(\mathbf{x}-\mathbf{R}_n)\rangle$$

- kvantno-mehanički doprinosi (!)

1.2.1 Makroskopske Maxwellove jednadžbe

- prosječne mikroskopske Maxwellove jednadžbe

$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \langle \mathbf{e}(\mathbf{x}, t) \rangle &= 4\pi \langle \eta(\mathbf{x}, t) \rangle \\
\text{(II)} \quad \nabla \cdot \langle \mathbf{b}(\mathbf{x}, t) \rangle &= 0 \\
\text{(III)} \quad \nabla \times \langle \mathbf{e}(\mathbf{x}, t) \rangle &= -(1/c)\partial \langle \mathbf{b}(\mathbf{x}, t) \rangle / \partial t \\
\text{(IV)} \quad \nabla \times \langle \mathbf{b}(\mathbf{x}, t) \rangle &= (1/c)\partial \langle \mathbf{e}(\mathbf{x}, t) \rangle / \partial t + (4\pi/c) \langle \mathbf{j}(\mathbf{x}, t) \rangle
\end{aligned}
$$

- makroskopske Maxwellove jednadžbe

$$
\begin{aligned}
\text{(I)} \quad \nabla \cdot \mathbf{E}(\mathbf{x}, t) &= 4\pi \rho^{\mathrm{tot}}(\mathbf{x}, t) \\
\text{(II)} \quad \nabla \cdot \mathbf{B}(\mathbf{x}, t) &= 0 \\
\text{(III)} \quad \nabla \times \mathbf{E}(\mathbf{x}, t) &= -(1/c)\partial \mathbf{B}(\mathbf{x}, t) / \partial t \\
\text{(IV)} \quad \nabla \times \mathbf{B}(\mathbf{x}, t) &= (1/c)\partial \mathbf{E}(\mathbf{x}, t) / \partial t + (4\pi/c) \mathbf{J}^{\mathrm{tot}}(\mathbf{x}, t)
\end{aligned}
$$

1.3 Pomoćna polja

- pomoćna dielektrična polja ili kratice u jednadžbama?

$$c\nabla \times \mathbf{B}(\mathbf{x},t) = \frac{\partial}{\partial t}[\mathbf{E}(\mathbf{x},t) + 4\pi\mathbf{P}(\mathbf{x},t)] + 4\pi[\mathbf{J}^{\mathrm{ext}}(\mathbf{x},t) + \mathbf{J}^{\mathrm{c}}(\mathbf{x},t)] + 4\pi c\nabla \times \mathbf{M}(\mathbf{x},t)$$

- pomoćna magnetska polja

$$c\nabla \times [\mathbf{B}(\mathbf{x},t) - 4\pi\mathbf{M}(\mathbf{x},t)] = \frac{\partial}{\partial t}[\mathbf{E}(\mathbf{x},t) + 4\pi\mathbf{P}(\mathbf{x},t)] + 4\pi[\mathbf{J}^{\mathrm{ext}}(\mathbf{x},t) + \mathbf{J}^{\mathrm{c}}(\mathbf{x},t)]$$

- dielektrični pomak u dielektricima (!)

$$\mathbf{D}_0(\mathbf{x},t) = \mathbf{E}(\mathbf{x},t) + 4\pi\mathbf{P}(\mathbf{x},t)$$

Problemi invarijantni na translacije u vremenu

- Jednadžbe kontinuiteta

$$\nabla \cdot \mathbf{J}^{\rm tot}(\mathbf{x}, t) + \partial \rho^{\rm tot}(\mathbf{x}, t)/\partial t = 0$$

-

$$\nabla \cdot \mathbf{J}^{\rm c}(\mathbf{x}, t) + \frac{\partial}{\partial t} \rho^{\rm c}(\mathbf{x}, t) = 0,$$

- generalni izraz za dielektrični pomak za $\omega \neq 0$

$$\mathbf{D}(\mathbf{x}, \omega) = \mathbf{E}(\mathbf{x}, \omega) + 4\pi \mathbf{P}(\mathbf{x}, \omega) + (4\pi i/\omega)\mathbf{J}^{\rm c}(\mathbf{x}, \omega)$$

- I i IV MJ

$$\begin{aligned} \nabla \cdot \mathbf{D}(\mathbf{x}, \omega) &= 4\pi \rho^{\rm ext}(\mathbf{x}, \omega) \\ \nabla \times \mathbf{H}(\mathbf{x}, \omega) + (i\omega/c)\mathbf{D}(\mathbf{x}, \omega) &= (4\pi/c)\mathbf{J}^{\rm ext}(\mathbf{x}, \omega) \end{aligned}$$

1.4 Makroskopske odzivne funkcije

- dielektrična funkcija

$$D_\alpha(\mathbf{x}, t) = \sum_\beta \int \mathrm{d}^3 x' \int \mathrm{d}t' \, \varepsilon_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t') E_\beta(\mathbf{x}', t')$$

- makroskopska susceptibilnost

$$P_\alpha(\mathbf{x}, t) = \sum_\beta \int \mathrm{d}^3 x' \int \mathrm{d}t' \, \tilde{\chi}_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t') E_\beta(\mathbf{x}', t')$$

- električna vodljivosti

$$J_\alpha(\mathbf{x}, t) = \sum_\beta \int \mathrm{d}^3 x' \int \mathrm{d}t' \, \sigma_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t') E_\beta(\mathbf{x}', t')$$

Translacijski invarijantni sustavi

- ukupna dielektrična funkcija, dielektrična susceptibilnost i frekventno ovisna vodljivost (kompleksne funkcije od $\mathbf{k}, \omega$)

$$\begin{aligned}
\varepsilon_{\alpha\beta}(\mathbf{k},\omega) &= \delta_{\alpha,\beta} + 4\pi\tilde{\chi}^{\mathrm{b}}_{\alpha\beta}(\mathbf{k},\omega) + (4\pi i/\omega)\sigma^{\mathrm{c}}_{\alpha\beta}(\mathbf{k},\omega) \\
&\equiv \delta_{\alpha,\beta} + 4\pi\tilde{\chi}^{\mathrm{tot}}_{\alpha\beta}(\mathbf{k},\omega) \equiv \delta_{\alpha,\beta} + (4\pi i/\omega)\sigma^{\mathrm{tot}}_{\alpha\beta}(\mathbf{k},\omega)
\end{aligned}$$

Literatura

1) Jackson, *Classical Electrodynamics*, § 6.7
2) Kupčić, *Elektrodinamika kontinuuma*, Pog. 1, 2
3) Ashcroft & Mermin, *Solid State Physics*, § 27.1