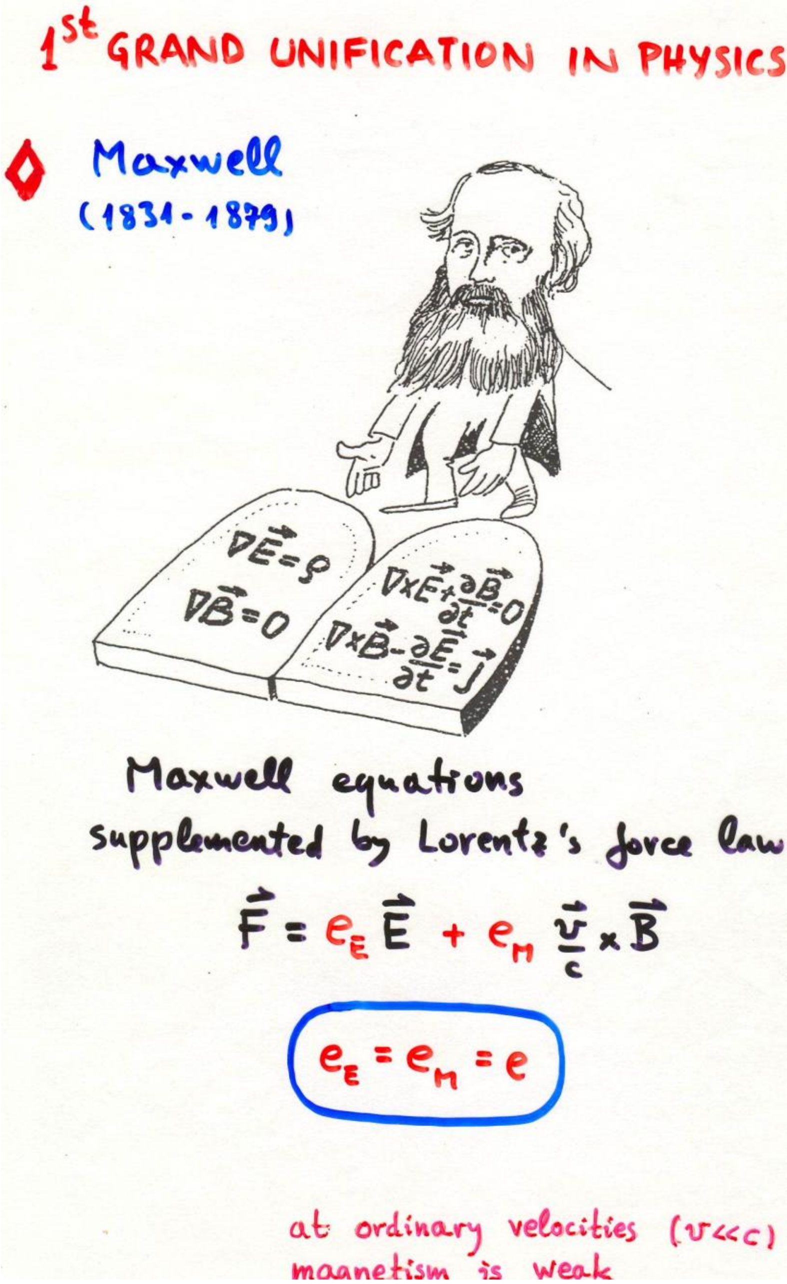
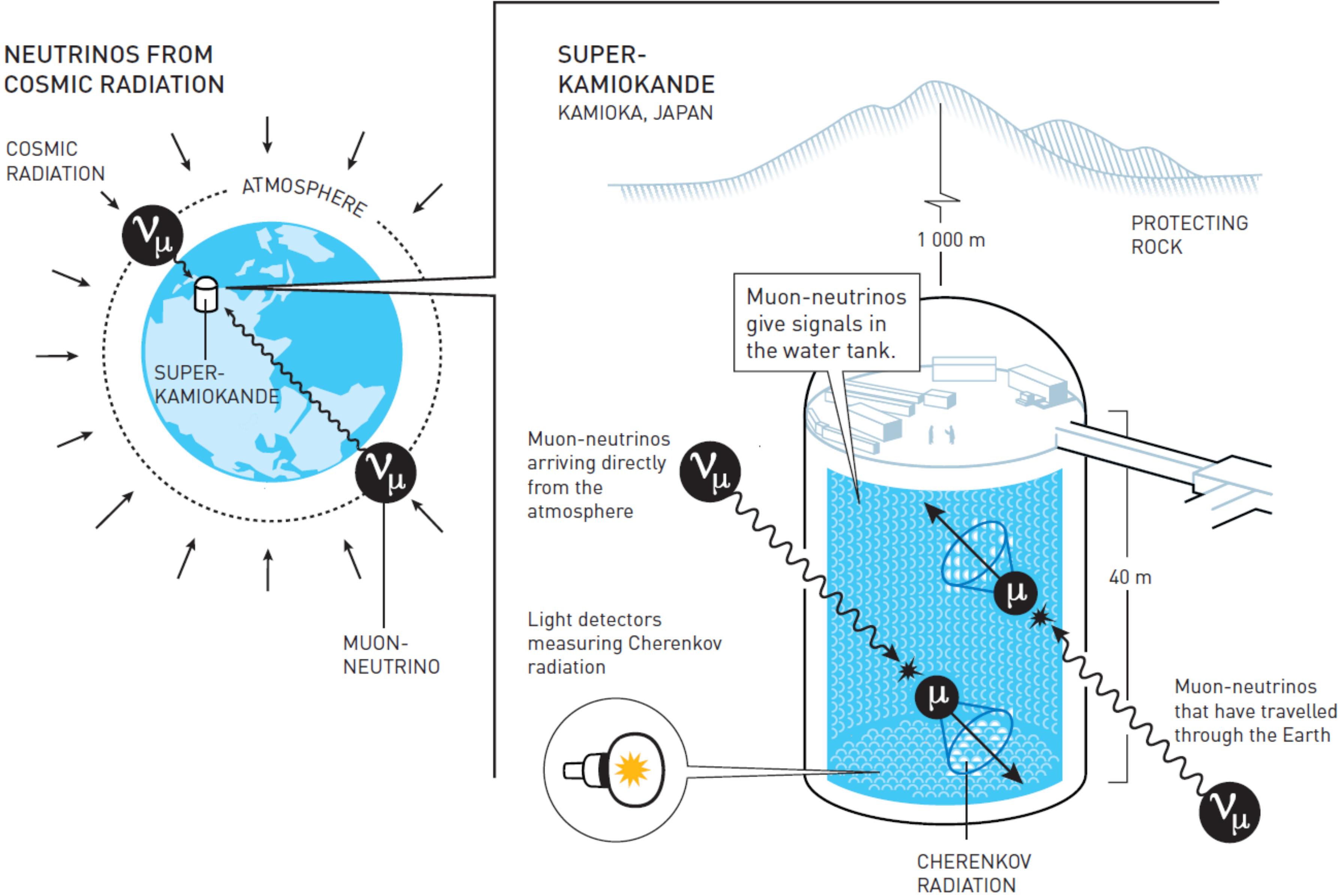
III. SIMETRIJE ELEKTROMAGNETIZMA KAO ORIJENTIR u FEČ

- NEUTRINSKE OSCILACIJE (neutrini posjeduju masu)
- BAŽDARNA SIMETRIJA (ptice na vodovima visokog napona i bezmaseni fotoni)
- RELATIVISTIČKI ZAPIS MAXWELLOVIH JEDNADŽBI

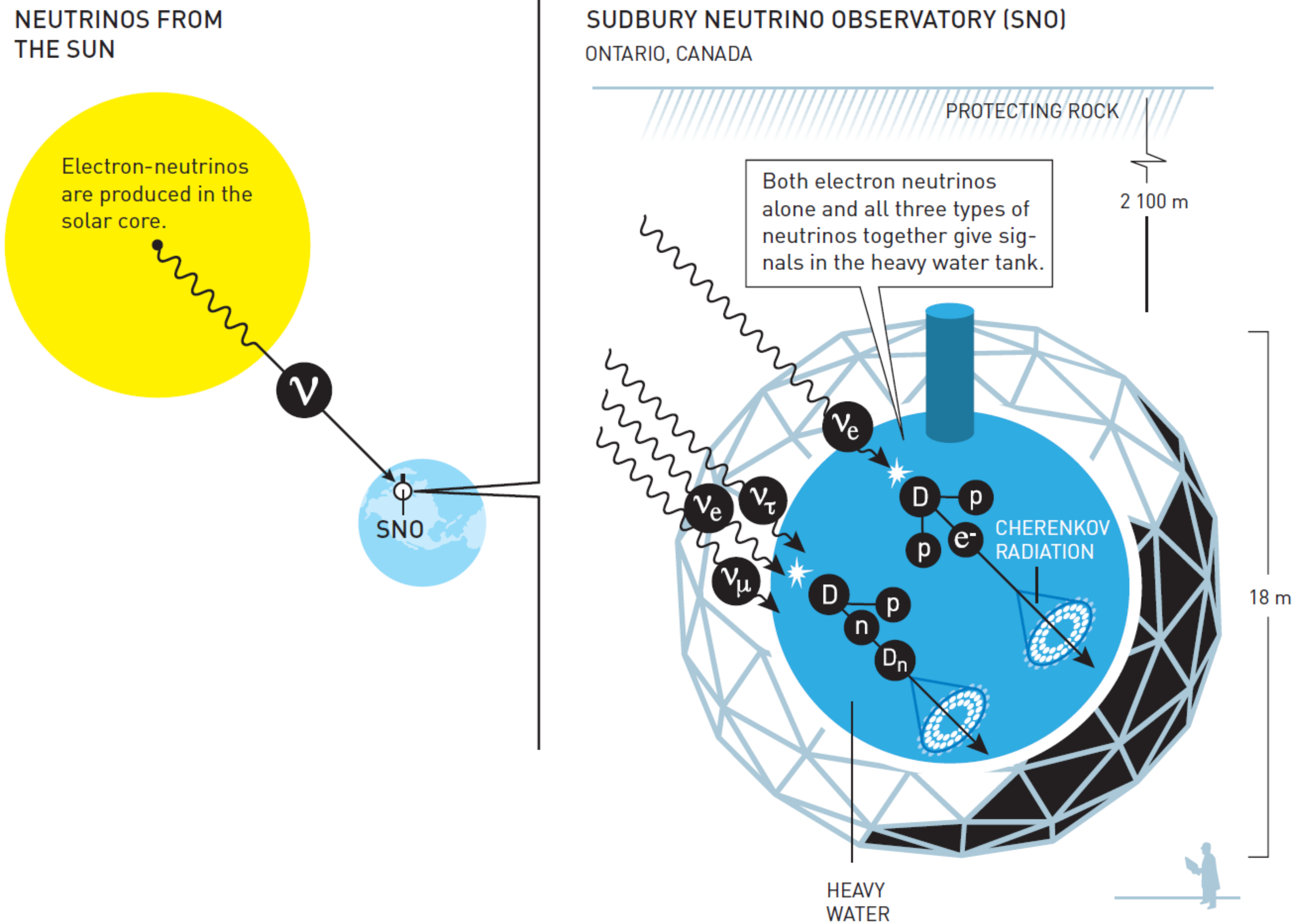


Najnovija Nobelova nagrada (lista u FEČ_01)

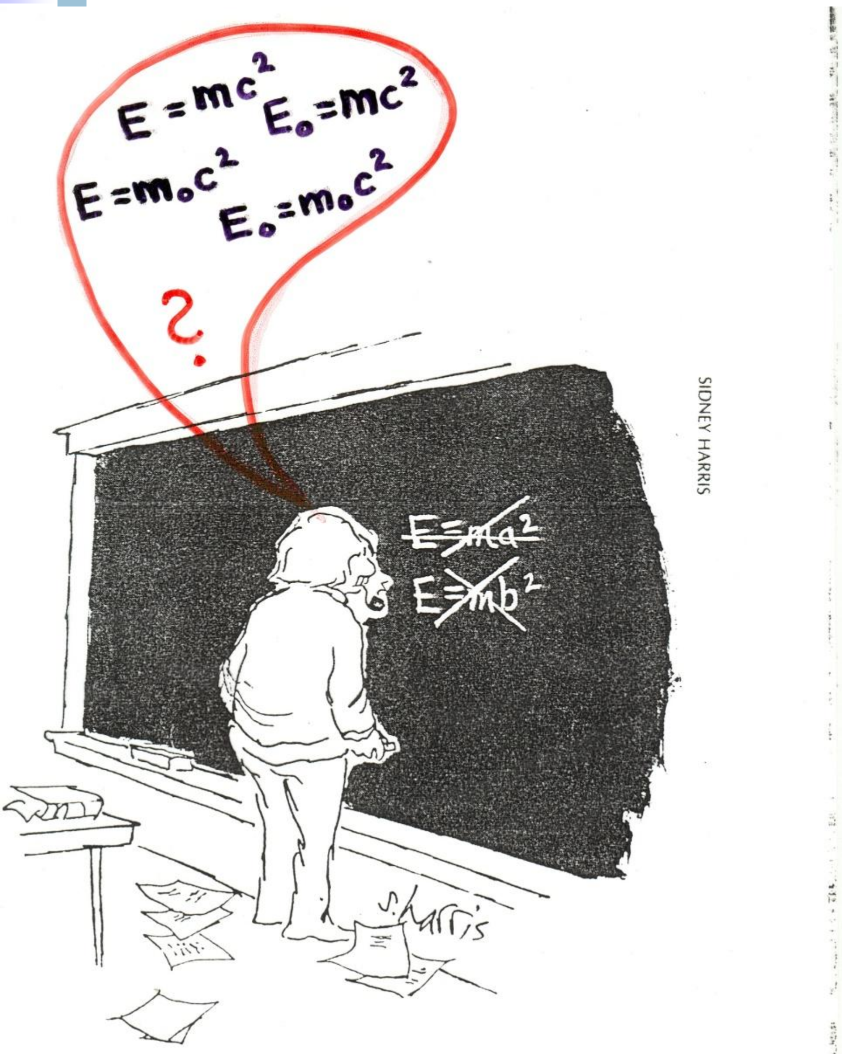
- Takaaki Kajita & Artur B.McDonald 2015.
  “za otkriće neutrinskih oscilacija, koje pokazuju da neutrini posjeduju masu“



Super-Kamiokande detects atmospheric neutrinos. When a neutrino collides with a water molecule in the tank, a rapid, electrically charged particle is created. This generates Cherenkov radiation that is measured by the light sensors. The shape and intensity of the Cherenkov radiation reveals the type of neutrino that caused it and from where it came. The muon-neutrinos that arrived at Super-Kamiokande from above were more numerous than those that travelled through the entire globe. This indicated that the muon-neutrinos that travelled longer had time to change into another identity on their way.



Sudbury Neutrino Observatory detects neutrinos from the Sun, where only electron-neutrinos are produced. The reactions between neutrinos and the heavy water in the tank yielded the possibility to measure both electron-neutrinos and all three types of neutrinos combined. It was discovered that the electron-neutrinos were fewer than expected, while the total number of all three types of neutrinos combined still corresponded to expectations. The conclusion was that some of the electron-neutrinos had changed into another identity.



Koja je formula ispravna?

SIMETRIJA SPECIJALNE RELATIVNOSTI (FEČ, § 2.2)

- RAVNOPRAVNO POJAVLJIVANJE PROSTORA I VREMENA
- OSTVARENO U SVIJETU ELEMENTARNIH ČESTICA

Neobična svojstva prostora-vremena STR

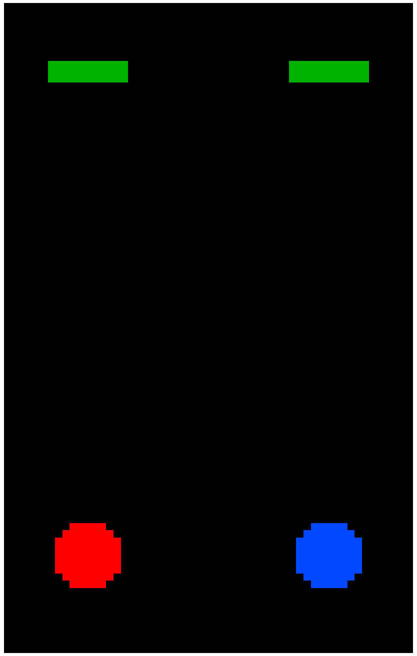
- opažanje ***dilatacije vremena***
- opažanje ***kontrakcije dužina***
- ***ekvivalentnost*** mase i energije

relativistički učinci određeni Lorentzovim faktorom, gama:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

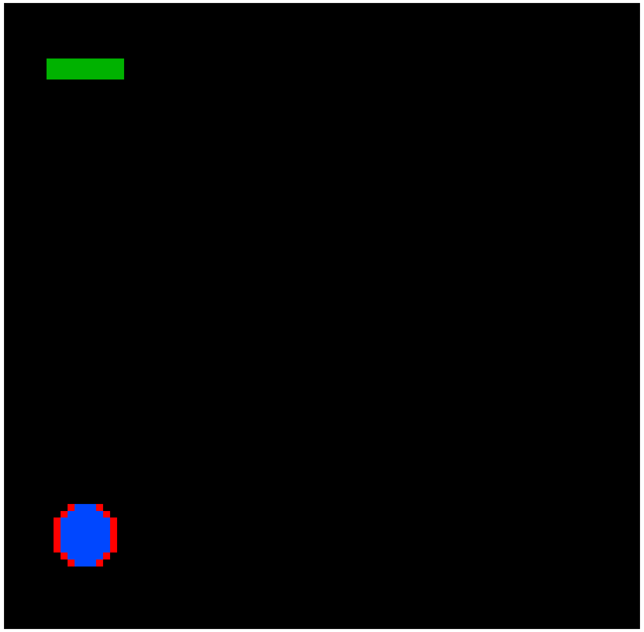
Dilatacija vremena

Dok su u relativnom mirovanju **Mirko** i **Žurko** se slažu da njihovi satovi tik-takaju istom mjerom



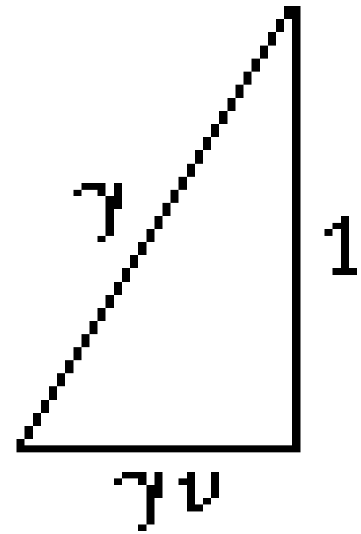
Žurkov sat, potisnut okomito na zraku sata,
tik-taka jednako kao prije

S gledišta Mirka, svjetlost Žurkova sata mora prijeći dulji put. S obzirom na konstantnost brzine svjetlosti, to zahtijeva dulji tik-tak (Žurkov sat djeluje usporeno u odn. na Mirkov)



Nađimo faktor γ za koji je usporen Žurkov sat

Na početku se oba slože da su zrcala udaljena 1 “tik” (okomita stranica u jedinicama gdje je c=1);



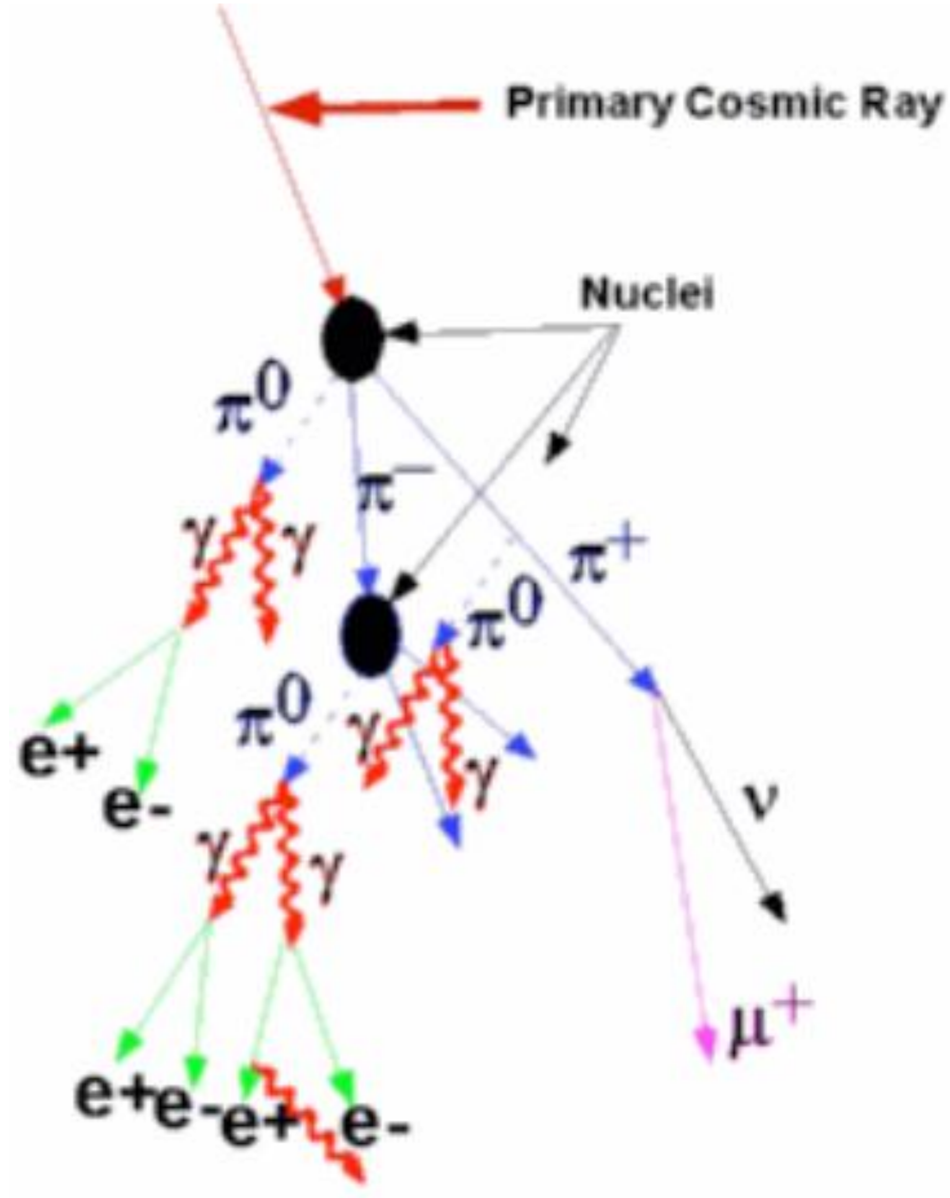
Mirko je uvjeren da svjetlost prevali udaljenost od Žurka do Žurkova zrcala koja iznosi γ “tika” (tijekom kojeg se Žurko brzine v pomakne za γv).

Pitagorin teorem vodi na čuvenu Lorentzovu formulu za γ !

RELATIVISTIČKA INVARIJANTNOST

opažanje
dilatacije vremena
opažanje
kontrakcije dužina
ekvivalentnost
mase i energije

Primjer: vrijeme života miona

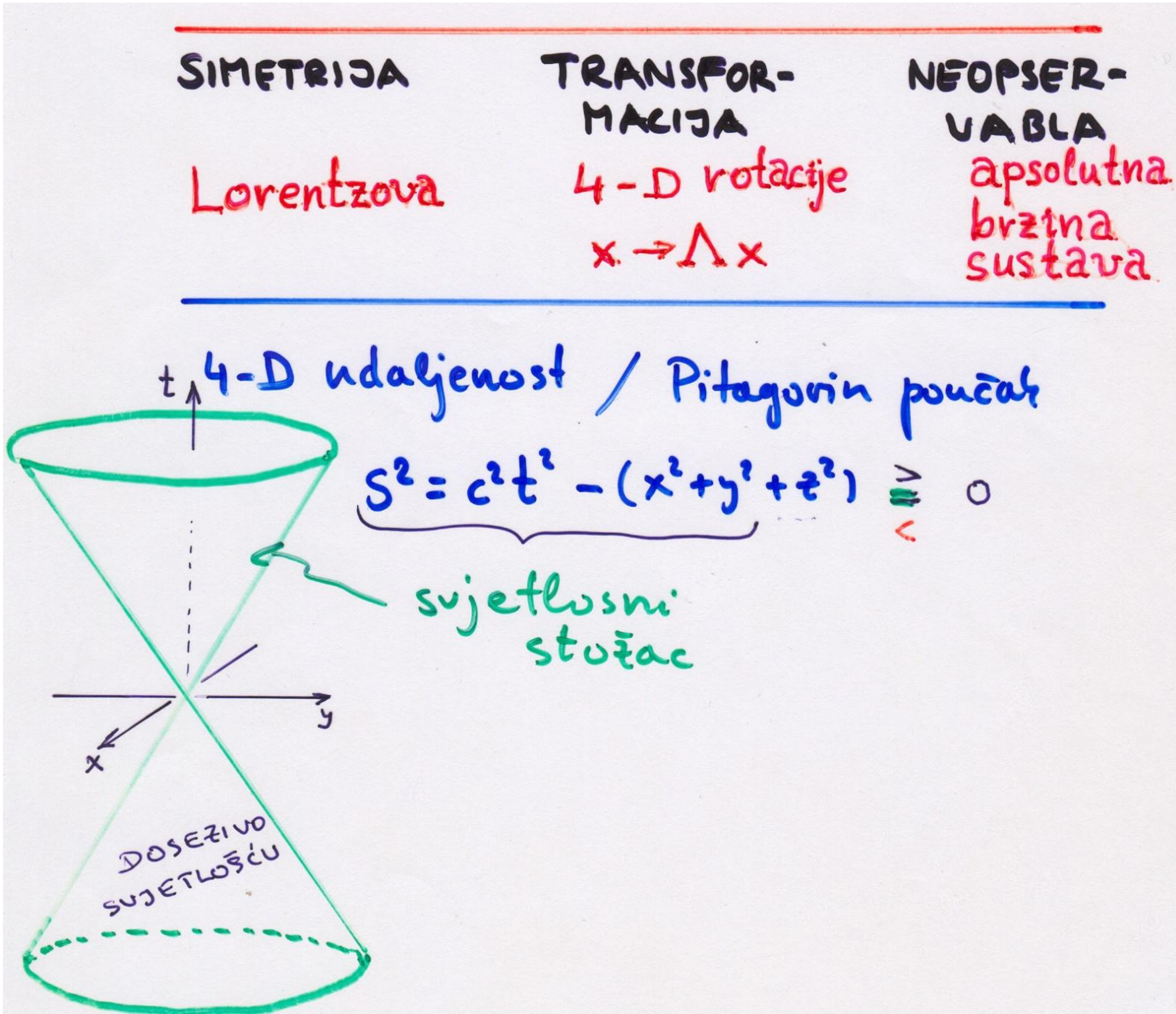


Primjeri iz relativističke kinematike

- Pr. 1: Naći brzinu miona pri raspadu mirujućeg piona;
- Pr. 2: Procijenite brzine miona koji unatoč kratkom životu od 2.2 mikro sec prolijeću do detektora u podnožju planine koji mjeri njihov fluks

  422 miona/sat, dok detektor 1220 m poviše njega mjeri 550 miona/sat.

RELATIVISTIČKA SIMETRIJA



PODSJETNIK NA RELATIVISTIČKE INVARIJANTE

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x^2 = x^\mu x_\mu = g_{\mu\nu} x^\mu x^\nu = (x^0)^2 - (\vec{x})^2$$

$$\frac{\partial}{\partial x^\mu} \equiv \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right),$$

$$\frac{\partial}{\partial x_\mu} \equiv \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right)$$

$$\Box = \partial_\mu \partial^\mu = \frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2 - \left(\frac{\partial}{\partial \vec{x}}\right)^2$$

$$p^\mu = i\hbar \frac{\partial}{\partial x_\mu} = \left(i\frac{\hbar}{c}\frac{\partial}{\partial t}, -i\hbar\nabla\right)$$

$$p_\mu p^\mu = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2 c^2$$

LORENTZOVE TRANSFORMACIJE KAO ROTACIJE U 4-PROSTORU

- **PASIVNA I AKTIVNA TRANSFORMACIJA**

$$
\begin{aligned}
x'^0 &= \gamma(x^0 - \beta x^1) \\
x'^1 &= \gamma(x^1 - \beta x^0) \\
x'^2 &= x^2 \\
x'^3 &= x^3 \,.
\end{aligned}
$$

$$x'^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \equiv \Lambda^\mu_\nu x^\nu$$

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Lambda_1 = \begin{pmatrix} \gamma & \gamma\beta_1 & 0 & 0 \\ \gamma\beta_1 & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LORENTZOVE TRANSFORMACIJE KAO ROTACIJE U 4-PROSTORU

- **PASIVNA I AKTIVNA TRANSFORMACIJA**

$$\begin{aligned} x'^0 &= \gamma(x^0 - \beta x^1) \\ x'^1 &= \gamma(x^1 - \beta x^0) \\ x'^2 &= x^2 \\ x'^3 &= x^3 \,. \end{aligned}$$

$$x'^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \equiv \Lambda^\mu_\nu x^\nu$$

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Lambda_1 = \begin{pmatrix} \gamma & \gamma\beta_1 & 0 & 0 \\ \gamma\beta_1 & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

VLASTITE I ORTOKRONE TRANSFORMACIJE

$$x^2 = x'^2 = g_{\mu\nu} x'^\mu x'^\nu \qquad g_{\mu\nu} \Lambda^\mu_\rho \Lambda^\nu_\sigma = g_{\rho\sigma}$$

$$\det \Lambda = \pm 1 \qquad \mathbf{i} \qquad |\Lambda^0_0| \geq +1$$

- **ROTACIJE** $\qquad \vec{r} \longrightarrow \vec{r}\,' = R_{(\hat{n},\varphi)} \vec{r}$

$$\psi(\vec{r}) \rightarrow \psi'(\vec{r}) \quad = \quad U_{(\hat{n}\varphi)} \psi(\vec{r}) = \psi\left(R^{-1} \vec{r}\right)$$

$$U_{(\hat{n}\varphi)} = e^{-i\varphi \hat{n} \cdot \vec{L}}$$

- **POTISCI**

$$\beta_j'' = \frac{\beta_j' + \beta_j}{1 + \beta_j' \beta_j}$$

UVOĐENJEM RAPIDITETA

$$\beta_j = \operatorname{th} \zeta_j \qquad \gamma = \operatorname{ch} \zeta_j$$

$$\zeta_j'' = \operatorname{Arth} \beta_j'' = \operatorname{Arth} \beta_j' + \operatorname{Arth} \beta_j = \zeta_j' + \zeta_j$$

$$\begin{aligned} x_j' &= x_j \operatorname{ch} \zeta_j + t \operatorname{sh} \zeta_j , \\ x_k' &= x_k \qquad \text{za } k \neq j \\ t' &= x_j \operatorname{sh} \zeta_j + t \operatorname{ch} \zeta_j \ . \end{aligned}$$

$$\Lambda_3 = \begin{pmatrix} \operatorname{ch} \zeta & 0 & 0 & \operatorname{sh} \zeta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \operatorname{sh} \zeta & 0 & 0 & \operatorname{ch} \zeta \end{pmatrix}$$

GENERATORI POTISAKA I ROTACIJA

Potisci u koordinatnom prostoru

$$x^\mu \to x'^\mu = \left[\Lambda_{(\zeta,\hat{n})}\right]^\mu_{\ \nu} x^\nu$$

praćeni su transformacijom u vektorskom prostoru stanja

$$\psi'(x^\mu) = U_{(\zeta,\hat{\eta})}\psi(x^\mu) = \psi\left\{\left[\Lambda^{-1}_{(\zeta,\hat{n})}\right]^\mu_{\ \nu} x^\nu\right\},$$

gdje je unitarna transformacija U izražena generatorom potiska $\vec{K}$,

$$U_{(\zeta,\hat{n})} = e^{-i\zeta\hat{n}\cdot\vec{K}} \ .$$

$$K_i = \frac{1}{i}\frac{\partial\Lambda_i}{\partial\zeta}\bigg|_{\zeta=0}, \qquad J_i = \frac{1}{i}\frac{\partial R_i}{\partial\varphi}\bigg|_{\varphi=0} \qquad K_3 = -i\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

RELATIVISTIČKA KOVARIJANTNOST

Primjer kovarijantnog zapisa zakona prirode

Maxwellov zapis elektromagnetizma   FEČ I str. 79

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 4\pi \rho \qquad \nabla \cdot \vec{E} = 4\pi\rho$$

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\nu$$

$$\left.\begin{array}{l} \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = \frac{4\pi}{c} j_z + \frac{1}{c}\dot{E}_z \\ \frac{\partial B_y}{\partial z} - \\ \frac{\partial B_z}{\partial x} - \end{array}\right\} \quad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\dot{\vec{E}}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\left.\begin{array}{l} \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} = -\frac{1}{c}\dot{B}_z \\ \frac{\partial E_y}{\partial z} - \\ \frac{\partial E_z}{\partial x} - \end{array}\right\} \quad \nabla \times \vec{E} = -\frac{1}{c}\dot{\vec{B}}$$

Einsteinove jednadžbe OTR $G_{\mu\nu} = \varkappa T_{\mu\nu}$

RELATIVISTIČKI OBLIK MAXWELL-ove TEORIJE

u racionaliziranim Gaussovim (Heaviside-Lorentzovim) jedinicama 6 polja & 4 izvora

$(\vec{E}, \vec{B})$ $(\rho, \vec{j}) = j^\mu$

zadov.

$$\nabla \cdot \vec{E} = \rho \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t}$$

– prijelazom na potencijale

$$A^\mu = (A^0, \vec{A}) \quad \begin{cases} \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}, & A^0 = \phi \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

i antisim. tenzor (sa 6 komp.!), $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Uz "dualni" tenzor

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\mathcal{L}_{Maxw.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$$

Uočimo da su prijelazom na $A^\mu = (\phi, \vec{A})$

- – dvije Maxw. j-be $\nabla(\nabla \times \vec{A}) \equiv 0 \Rightarrow \nabla \vec{B} = 0$

  automatski zadov. $\nabla \times \nabla\phi \equiv 0 \Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

- – preostale dvije $\partial_\mu F^{\mu\nu} = j^\nu$

$$\Box A^\nu - \partial^\nu(\partial_\mu A^\mu) = j^\nu$$

$\partial_\mu A^\mu = 0$ odabirom *) Lorentzovog uvjeta ("baždarenja")

$$\Box A^\nu = 0 \Rightarrow \text{K.G. j-ba za slobodno polje } (j^\nu = 0)$$

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*) sloboda baždarne transf.

$$\left.\begin{aligned} \phi &\to \phi + \frac{\partial \chi(x)}{\partial t} \\ \vec{A} &\to \vec{A} - \nabla\chi(x) \end{aligned}\right\} A^\mu \to A^\mu + \partial^\mu \chi(x)$$

ostavlja $\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$

$\vec{B} = \nabla \times \vec{A}$

nepromijenjenima!

Elektromagn. val:

- energija
- impuls (zalet)
- impuls vrtnje (zamah)