VII. PREDAVANJE

RELATIVISTIČKA DIRACOVA QM

- RJEŠENJE SLOBODNE JEDNADŽBE
- DIRACOVO POLJE i DIRACOV SPINOR
- ADJUNGIRANO DIRACOVO POLJE
- NABOJNO KONJUGIRANO POLJE

DVA VIĐENJA DIRACOVE JEDN.

- SCHROEDINGEROVA JEDN. S DIRACOVIM HAMILTONIANOM
- EULER-LAGRANGEOVA JEDN. DIRACOVOG LAGRANGIANA

$$i\frac{\partial\Psi}{\partial t} = H_D\Psi \quad ; \quad H_D = -i\alpha_i\frac{\partial}{\partial x_i} + \beta m$$

$$(i\gamma^\mu\partial_\mu - m)\Psi = 0 \quad ; \quad \mathcal{L}_D = \bar{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi$$

adjungirano polje $\quad \bar{\Psi} = \Psi^\dagger\gamma^0$

TRANSFORMACIJA SPINORA

- **NA TRANSFORMACIJE HLG**

$\Lambda \in L_+^\uparrow$

$x \to x' = \Lambda x \quad \Longrightarrow \quad \psi(x) \to \psi'(x') = U(\Lambda)\psi(x)$

$\epsilon^{kij} M^{ij} = J^k$

$U(\Lambda) = e^{-\frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}}$

$M^{0i} = K^i$

Za spinorno polje $\to S(\Lambda)$, $\quad S^{\mu\nu} = \frac{\sigma^{\mu\nu}}{2}$

$S^{-1}(\Lambda)\gamma^\mu S(\Lambda) = \Lambda^\mu{}_\nu \gamma^\nu \quad \to \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

Vježbe: invarijantni bilinear

Weyl spinors

$(\frac{1}{2},0) \quad (0,\frac{1}{2})$

$\psi_L \qquad \psi_R$

2-component spinors of SU(2)

Rotations and Boosts $\quad \psi_{L(R)} \to S_{L(R)}\,\psi_{L(R)}$

$$S_{L(R)} = e^{i\frac{\alpha}{2}.\sigma} \;:\; \text{Rotations}$$

$$S_{L(R)} = e^{\pm\frac{\nu}{2}.\sigma} \;:\; \text{Boosts}$$

Dirac spinor

Can combine ψ_L, ψ_R to form a 4-component "Dirac" spinor $\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$

Lorentz transformations $\quad \psi \to e^{i\omega\sigma}\psi, \qquad \omega\sigma = \omega^{\mu\nu}\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu,\gamma_\nu\right]\omega^{\mu\nu}$

where $\gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$

Weyl basis

DIRACOVA VALNA FUNKCIJA - KLASIČNO POLJE

$$i\frac{\partial \psi}{\partial t} = (-i\vec{\alpha}\cdot\nabla + \beta m)\psi$$

$$\vec{\alpha} = \gamma^0\vec{\gamma} = \left(\begin{array}{c|c} 0 & \vec{\sigma} \\ \hline \vec{\sigma} & 0 \end{array}\right)$$

$$\beta = \gamma^0 = \left(\begin{array}{c|c} I & 0 \\ \hline 0 & -I \end{array}\right)$$

Ravni val

$$\psi \sim \begin{pmatrix} \varphi \\ \chi \end{pmatrix} e^{-ipx} \quad , \quad p^\mu = (p^0, \vec{p})$$

$$p^0 \begin{pmatrix} \varphi \\ \chi \end{pmatrix} = \begin{pmatrix} mI & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -mI \end{pmatrix} \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

$\boxed{p^0 > 0}$

$$\chi = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi \quad ; \quad \varphi = \begin{cases} \varphi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \varphi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

$\boxed{p^0 < 0}$

$$\begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \\ \chi \end{pmatrix} \quad ; \quad \chi^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ; \quad \chi^2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

NORMIRAN DIRACOV SPINOR

$\rho = \psi^\dagger \psi$ normirano na 2E čestica u volumenu kvantizacije

$$\int_V \rho \, d^3r = 2E$$

Pozitivno energijski spinori

$$u(p,s) = \sqrt{E+m} \begin{pmatrix} \varphi^s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \varphi^s \end{pmatrix}$$

Negativno energ. spinori

$$v(p,s) = \sqrt{E+m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \chi^s \\ \chi^s \end{pmatrix}$$

- **VJEŽBE: Izvod (iz kovarijantnosti Diracove jedn.) transformacijskih svojstava za spinore i bilineare;**
- **ZADAĆA: Izvod relacija potrebnih za nalaženje transformacijskih svojstava bilineara**

DIRACOVI SPINORI

ČESTIČNI za E>0

$$\Psi = u(E,\vec{p})\, e^{-ip\cdot x} \Rightarrow (\not{p} - m)\, u(E,\vec{p}) = 0$$

$$\varphi^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \uparrow \qquad \varphi^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \downarrow$$

$$u_1 = \sqrt{E+m} \begin{pmatrix} \varphi^1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi^1 \end{pmatrix}, \quad u_2 = \sqrt{E+m} \begin{pmatrix} \varphi^2 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\varphi^2 \end{pmatrix}$$

- **ANTIČESTIČNI za E<0**

$$(\not{p} + m)\, v(E,\vec{p}) = 0$$

$$(E,\vec{p}) \rightarrow (-E,-\vec{p})$$

$$v_1 = u_4(-E,-\vec{p}) = \sqrt{E+m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^1 \\ \chi^1 \end{pmatrix}, \quad v_2 = \sqrt{E+m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^2 \\ \chi^2 \end{pmatrix}$$

$$\left.\begin{array}{l} \vec{L} = \vec{r}\times\vec{p} \rightarrow -\vec{L} \\ \vec{S} \rightarrow -\vec{S} \end{array}\right\} \Rightarrow \quad \chi^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \chi^2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

NABOJNO KONJUGIRANO POLJE

$$-i\gamma^2 \Big\backslash\ \gamma^\mu(\partial_\mu + ieA_\mu)\Psi + im\Psi = 0 \ \Big/^{*}$$

$$\gamma^\mu(\partial_\mu - \underbrace{ie}A_\mu)\underbrace{i\gamma^2\Psi^*} + im\underbrace{i\gamma^2\Psi^*} = 0$$

$$\Psi^c = i\gamma^2\Psi^*$$

$$e \to -e$$

$$\Psi = u_1 e^{i(\vec{p}\cdot\vec{r} - Et)} \xrightarrow{C} \Psi^c = v_1 e^{-i(\vec{p}\cdot\vec{r} - Et)}$$

$$\Psi = u_2 e^{i(\vec{p}\cdot\vec{r} - Et)} \xrightarrow{C} \Psi^c = v_2 e^{-i(\vec{p}\cdot\vec{r} - Et)}$$

$$\Psi^c = i\gamma^2\gamma^0(\bar{\Psi})^T \equiv C\bar{\Psi}^T$$

Pozitivno i negativno energijska rješenja Diracove j.

KVANTIZIRANO DIRACOVO POLJE

FEĆ § 3.2.3 | str. 137

$$\Psi(x) = \text{"}\sum_{\hat{p},s}\text{"} \left[b_s(\hat{p})\, u(\hat{p},s)\, e^{-ipx} + d_s^\dagger(\hat{p})\, v(\hat{p},s)\, e^{ipx} \right]$$

$$\bar{\Psi} = \Psi^\dagger \gamma_0 = \text{"}\cdots\text{"} \left[b_s^\dagger(\hat{p})\, \bar{u}(\hat{p},s)\, e^{ipx} + d_s(\hat{p})\, \bar{v}(\hat{p},s)\, e^{-ipx} \right]$$

$$\{ b_s(\hat{p}), b^\dagger_{s'}(\hat{p}') \} = \delta_{ss'}\, \delta_{\hat{p}\hat{p}'} = \{ d_s(\hat{p}), d^\dagger_s(\hat{p}') \}$$

čestični dio

$$p^0 = +\sqrt{\hat{p}^2 + m^2} = E > 0$$

uz $e^{-ip^0 t}$

antičestični dio

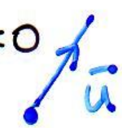
$$p^0 = -\sqrt{\vec{p}^2 + m^2} = -E$$

uz $e^{ip^0 t}$

č - dio

$$(\not{p} - m)\, u(\hat{p},s) = 0$$

$$\bar{u}(\hat{p},s)\, (\not{p} - m) = 0$$



ač - dio

$$(\not{p} + m)\, v(\hat{p},s) = 0$$

$$\bar{v}(\hat{p},s)\, (\not{p} + m) = 0$$

KVANTNO DIRACOVO POLJE I NJEMU NABOJNO KONJUGIRANO

$$\psi(x) = \sum_{\vec{p},s} \frac{1}{\sqrt{2EV}} \left[b_s(\vec{p})\, u(\vec{p},s)\, e^{-ip\cdot x} + d_s^{\dagger}(\vec{p})\, v(\vec{p},s)\, e^{ip\cdot x} \right]$$

$$\bar{\psi} = \psi^{\dagger}\gamma_0 = \sum_{\vec{p},s} \frac{1}{\sqrt{2EV}} \left[b_s^{\dagger}(\vec{p})\, \bar{u}(\vec{p},s)\, e^{ip\cdot x} + d_s(\vec{p})\, \bar{v}(\vec{p},s)\, e^{-ip\cdot x} \right]$$

$$(\psi^c)_\alpha = C\psi C^{-1} = C_{\alpha\beta}(\bar{\psi})_\beta \;; \qquad C = i\gamma^2\gamma^0$$