IX. PREDAVANJE

PREMA KVANTIZIRANOM ELEKTROMAGNETSKOM POLJU

- **NOETHERIČIN TEOREM**

  za prost-vrem simetrije

  za interne simetrije

- **KVANTIZIRANO POLJE ZRAČENJA**
- **FOTON KAO ČESTICA**

Odziv djelovanja na promjenu koordinata i polja –prost-vrem.

◊ **Noether** - ičin tm, za $\delta\phi = (\delta_0 + \delta x^\nu \partial_\nu)\phi(x)$

• prost-vrem

$$\delta x^\mu = \left(\frac{\delta x^\mu}{\delta \omega^a}\right)\delta\omega^a \qquad \left(\frac{\delta x^\mu}{\delta \omega^a}\right) \equiv X^\mu_a$$

$$\delta\phi = \left(\frac{\delta\phi}{\delta\omega^a}\right)\delta\omega^a \qquad \left(\frac{\delta\phi}{\delta\omega^a}\right) \equiv \Phi_a$$

transl. očuvani tenzor energije impulsa (kanonski)

$$\theta^\mu{}_\nu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial_\nu\phi - \mathcal{L} \quad ; \quad \partial_\mu\Theta^\mu_\nu = 0$$

$$\Theta^\mu{}_\nu \rightarrow T^{\mu\nu} = \Theta^{\mu\nu} + \partial_\lambda f^{\lambda\mu\nu} \quad ; \quad \partial_\mu T^{\mu\nu} = 0$$

Očuvana Noetheričina struja

- **NOETHERIČINA STRUJA za prostorno vremenske transformacije**

$$\delta x^\mu = \left(\frac{\delta x^\mu}{\delta \omega^a}\right)\delta\omega^a \equiv X^\mu_a\,\delta\omega^a \; ; \quad \delta\phi = \underbrace{\left(\frac{\delta\phi}{\delta\omega^a}\right)}_{\equiv\,\Phi_a}\delta\omega^a$$

$$j^\mu_a = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\,\Phi_a - \Theta^\mu{}_\varkappa\, X^\varkappa_a$$

$$\partial_\mu j^\mu_a = 0$$

Očuvana struja za prost-vremenske translacije

◊ Noetheričina struja za translacije - u "smjeru" ν

$j^{\mu}{}_{\nu}$ FEČ (2.271) | str. 96 $\longrightarrow$ $\Theta^{\mu}{}_{\nu}(x)$; $\partial_\mu \Theta^{\mu\nu} = 0$

OČUVANI KANONSKI TENZOR ENERGIJE-IMPULSA

OČUVANI "NABOJ"

$$P^{\nu} = \int d^3x \; \Theta^{0\nu}$$

$\nu = 0$: $\partial_0 \Theta^{00} + \partial_i \Theta^{i0} = 0$

$\nu = j$: $\partial_0 \Theta^{0j} + \partial_i \Theta^{ij} = 0$

$$\Theta^{\mu\nu} = \left(\begin{array}{c|ccc} \Theta^{00} & \Theta^{01} & \Theta^{02} & \Theta^{03} \\ \hline \Theta^{10} & & & \\ \Theta^{20} & & \Theta^{ij} & \\ \Theta^{30} & & & \end{array} \right)$$

→ simetrični: $T^{\mu\nu}$

struja energije

napetost energije (stress-energy tensor)

$$P^0 = \int d^3x \left[\pi(x)\dot{\phi}(x) - \mathcal{L}(x) \right] = \int d^3x \, \mathcal{H}(x)$$

$$P^j = \int d^3x \; \pi(x) \, \partial^j \phi(x)$$

Očuvana struja za rotacije 4-dim prostora

◊ Noetherična struja za rotacije – antisim. u $\{\alpha,\beta\}$

$$j^{\mu\alpha\beta} \longrightarrow M^{\mu\alpha\beta} \quad ; \quad \partial_\mu M^{\mu\alpha\beta} = 0$$

OČUVANI KANONSKI TENZOR IMPULSA VRTNJE

$$K^j = \int d^3x \, M^{00j}(x) = \int d^3x \left[x^0 T^{0j}(x) - x^j T^{00}(x) \right]$$

$$J^k = \frac{1}{2} \epsilon^{ijk} \int d^3x \, M^{0ij}(x) = \frac{1}{2} \epsilon^{ijk} \int d^3x \left[x^i T^{0j} - x^j T^{0i} \right.$$
$$\left. + \text{dio spina} \right]$$

... i interne simetrije

rotac. očuvani tenzor impulsa vrtnje

$$M^{\mu\rho\sigma} = T^{\mu\rho}x^{\sigma} - T^{\mu\sigma}x^{\rho} \quad ; \quad \partial_\rho M^{\rho\mu\nu} = 0$$

• interne sim. / Gell-Mann-Levy

$$\delta\phi_r = -i\,\varepsilon_\alpha(x)\,F_r^\alpha(\phi)$$

$$\delta\mathcal{L} = \varepsilon_\alpha(x)\,\partial_\mu j^{\alpha\mu}(x) + j^{\alpha\mu}(x)\,\partial_\mu\varepsilon_\alpha(x)$$

$$\Rightarrow \quad j^{\alpha\mu}(x) = \frac{\partial(\delta\mathcal{L})}{\partial(\partial_\mu\varepsilon_\alpha(x))} \quad ; \quad \partial_\mu j^{\alpha\mu} = \frac{\partial(\delta\mathcal{L})}{\partial\varepsilon_\alpha(x)}$$

Očuvana struja za prostorne rotacije i potiske (4-dim rotacije)

◊ Noetheričina struja za rotacije – antisim. u $\{\alpha, \beta\}$

$$j^{\mu\alpha\beta} \longrightarrow M^{\mu\alpha\beta} \quad ; \quad \partial_\mu M^{\mu\alpha\beta} = 0$$

OČUVANI KANONSKI TENZOR IMPULSA VRTNJE

$$K^j = \int d^3x \, M^{00j}(x) = \int d^3x \left[x^0 T^{0j}(x) - x^j T^{00}(x) \right]$$

$$J^k = \frac{1}{2} \epsilon^{ijk} \int d^3x \, M^{0ij}(x) = \frac{1}{2} \epsilon^{ijk} \int d^3x \left[x^i T^{0j} - x^j T^{0i} + \text{dio spina} \right]$$

PRIJELAZ S KLASIČNIH NA KVANTNA POLJA

Lagrangian $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

Tenzor en-impulsa $\Theta^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\rho)} (\partial^\nu A_\rho) - g^{\mu\nu} \mathcal{L}$

→ simetrični $T^{\mu\nu} = \Theta^{\mu\nu} + \partial_\rho (F^{\mu\rho} A^\nu)$

za polje zračenja
$A^\mu = (A^0, \vec{A})$ $A^0 = 0 \,, \quad \nabla \cdot \vec{A} = 0$

$\Rightarrow \quad \epsilon^0 = 0 \,, \quad \vec{k} \cdot \vec{\epsilon} = 0$

KVANTIZIRANO POLJE ZRAČENJA

Kvantizirane frekvencije kao razlike energija

Foton kao elementarna čestica

$\boxed{\text{FOTON KAO ELEMENTARNA ČESTICA}}$

FEČ §3.2.2 | str. 130

Einstein 1905: svjetlost emitirana KVANTIZIRANIM OSCILATORIMA (Planck 1900) također je kvantizirana

$$H = \sum_{j,\vec{k}} \hbar\omega \left[a_j^\dagger(\vec{k})\, a_j(\vec{k}) + \frac{1}{2} \right] \leftrightarrow H = \frac{1}{2}\int d^3x \,(\vec{E}^2 + \vec{B}^2)$$

Uz 2 uvjeta na $A^\mu(x)$: $A^0(x) = 0$ & $\nabla\cdot\vec{A}(x) = 0$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \; ; \; \vec{B} = \nabla\times\vec{A}$$

$$H = \frac{1}{2}\int d^3x \left[\dot{\hat{A}}^2 + (\nabla\times\hat{A})^2 \right]$$

kvantizirano POLJE ZRAČENJA

$$\vec{A}(x) = “\sum_{\vec{k},i}” \left[\vec{\epsilon}_{\vec{k},i}\, a_{\vec{k},i}\, e^{-ik_\mu x^\mu} + \vec{\epsilon}^{\,*}_{\vec{k},i}\, a^\dagger_{\vec{k},i}\, e^{ik_\mu x^\mu} \right]$$

Kružno polariz. $\lambda = \pm 1$ $\quad \vec{\epsilon}_\pm = (\vec{\epsilon}_1 \pm i\vec{\epsilon}_2)/\sqrt{2}$

$\vec{\epsilon}$ $\quad$ $\vec{\epsilon}^{\,*}$

1-fotonsko stanje $\quad |\vec{k},\lambda\rangle = a^\dagger_{\vec{k},\lambda}|0\rangle$

Operator broja

$$\boxed{N_{\vec{k},\lambda} = a^\dagger_{\vec{k},\lambda}\, a_{\vec{k},\lambda}} \Rightarrow N = \sum_{\vec{k}\lambda} N_{\vec{k}\lambda}$$

KLASIČNO POLJE – KVANTNO (FOTON)

$$T^{00} = \frac{1}{2}(\vec{E}^2 + \vec{B}^2) \qquad H = \int T^{00} d^3x = \frac{1}{2}\sum_{k\lambda}(a^\dagger_{k\lambda} a_{k\lambda} + a_{k\lambda} a^\dagger_{k\lambda})\,\omega$$

$$T^{0i} = (\vec{E} \times \vec{B})_i \qquad P_i = \int T^{0i} d^3x = \sum_{k\lambda} \underbrace{a^\dagger_{k\lambda} a_{k\lambda}}_{N_{k\lambda}} k_i$$

$$\vec{S} = \vec{E} \times \vec{A} \qquad \vec{S} = \sum_{k\lambda} N_{k\lambda} \lambda \hat{k}$$

1-fotonsko stanje $|k\lambda\rangle = a^\dagger_{k\lambda}|0\rangle$

heliciteta $\lambda = \pm 1$

$N|k\lambda\rangle = |k\lambda\rangle$

$H|k\lambda\rangle = \omega|k\lambda\rangle$

$\vec{P}|k\lambda\rangle = \vec{k}|k\lambda\rangle$

$\vec{S}|k\lambda\rangle = \lambda\hat{k}|k\lambda\rangle$

paritet

C-paritet

Primjer e.m. polja:

- energija
- impuls (zalet)
- impuls vrtnje (zamah)