X. PREDAVANJE

KVANTIZIRANA POLJA

- KVANTIZIRANO FOTONSKO POLJE
- KVANTIZIRANO FERMIONSKO POLJE
- OPERATOR BROJA (ANTI)ČESTICA
- INTERAKCIJA FOTONA S FERMIONSKOM STRUJOM

PRIKAZ
ČESTICE KVANTNIM POLJIMA

- UGRAĐUJE IDENTIČNOST ČESTICA
- STVARANJE I PONIŠTENJE ČESTICA (elementi fizikalne realnosti)
- NEGATIVNE ENERGIJE SVEDENE NA FREKVENCIJE (ilustrirajmo na polju zračenja)

KVANTIZIRANO POLJE ZRAČENJA

Kvantizirane frekvencije kao razlike energija

Foton kao elementarna čestica

FOTON
KAO ELEMENTARNA ČESTICA

FEČ § 3.2.2 | str. 130

Einstein 1905 : svjetlost emitirana KVANTIZIRANIM OSCILATORIMA (Planck 1900) također je kvantizirana

$$H = \sum_{j,\vec{k}} \hbar\omega \left[a_j^\dagger(\vec{k}) a_j(\vec{k}) + \tfrac{1}{2} \right] \leftrightarrow H = \tfrac{1}{2} \int d^3x \, (\vec{E}^2 + \vec{B}^2)$$

Uz 2 uvjeta na $A^\mu(x)$: $A^0(x) = 0$ & $\nabla \cdot \vec{A}(x) = 0$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \; ; \; \vec{B} = \nabla \times \vec{A}$$

$$H = \tfrac{1}{2} \int d^3x \left[\dot{\hat{A}}^2 + (\nabla \times \hat{A})^2 \right]$$

kvantizirano POLJE ZRAČENJA

$$\vec{A}(x) = \text{“}\sum_{\vec{k},i}\text{”} \left[\vec{\epsilon}_{\vec{k},i} \, a_{\vec{k},i} \, e^{-ik_\mu x^\mu} + \vec{\epsilon}^{\,*}_{\vec{k},i} \, a^\dagger_{\vec{k},i} \, e^{ik_\mu x^\mu} \right]$$

Kružno polariz. $\lambda = \pm 1$ $\quad \vec{\epsilon}_\pm = (\vec{\epsilon}_1 \pm i\vec{\epsilon}_2)/\sqrt{2}$ $\quad \vec{\epsilon}$ $\quad \vec{\epsilon}^{\,*}$

1-fotonsko stanje $\quad |\vec{k},\lambda\rangle = a^\dagger_{\vec{k},\lambda} |0\rangle$

Operator broja

$$\boxed{N_{\vec{k},\lambda} = a^\dagger_{\vec{k},\lambda} a_{\vec{k},\lambda}} \Rightarrow N = \sum_{\vec{k}\lambda} N_{\vec{k}\lambda}$$

OPERATOR BROJA FOTONA

$N = a^{\dagger} a$ – kao hermitski ima realne vl. vrijednosti $(n \in \mathbb{R})$

$N|n\rangle = n|n\rangle$

– uz svojstva $[N, a^{\dagger}] = a^{\dagger}$

$[N, a] = -a$

$N a^{\dagger}|n\rangle = (n+1)\, a^{\dagger}|n\rangle$

$N a|n\rangle = (n-1)\, a|n\rangle$

$$H = \int T^{00} d^3x = \frac{1}{2} \sum_k \left(a_k^{\dagger} a_k + a_k a_k^{\dagger}\right) = = \sum_k \left(a_k^{\dagger} a_k + \frac{1}{2}\right) \omega$$

$$:H: = \sum_k a_k^{\dagger} a_k \,\omega = \sum_k N_k \,\omega$$

uz normalno uređenje

$$:a_k a_k^{\dagger}: = a_k^{\dagger} a_k$$

Pozitivno i negativno energijska rješenja Diracove j.

$\boxed{\text{KVANTIZIRANO DIRACOVO POLJE}}$

FEĆ § 3.2.3 | str. 137

čestični dio

$$p^0 = +\sqrt{\vec{p}^{\,2}+m^2} = E > 0$$

uz e^{-ip^0t}

antičestični dio

$$p^0 = -\sqrt{\vec{p}^{\,2}+m^2} = -E$$

uz e^{ip^0t}

$$\psi(x) = \text{"}\sum_{\vec{p},s}\text{"}\left[b_s(\vec{p})\,u(\vec{p},s)\,e^{-ipx} + d_s^{\dagger}(\vec{p})\,v(\vec{p},s)\,e^{ipx}\right]$$

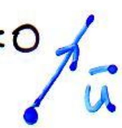
$$\bar{\psi} = \psi^{\dagger}\gamma_0 = \text{"}\cdots\text{"}\left[b_s^{\dagger}(\vec{p})\,\bar{u}(\vec{p},s)\,e^{ipx} + d_s(\vec{p})\,\bar{v}(\vec{p},s)\,e^{-ipx}\right]$$

$$\{b_s(\vec{p}),\, b_{s'}^{\dagger}(\vec{p}\,')\} = \delta_{ss'}\,\delta_{\vec{p}\vec{p}\,'} = \{d_s(\vec{p}),\, d_s^{\dagger}(\vec{p}\,')\}$$

č - dio

$$(\not{p}-m)\,u(\vec{p},s) = 0$$

$$\bar{u}(\vec{p},s)\,(\not{p}-m) = 0$$



ač - dio

$$(\not{p}+m)\,v(\vec{p},s) = 0$$

$$\bar{v}(\vec{p},s)\,(\not{p}+m) = 0$$

KVANTNO DIRACOVO POLJE I NJEMU NABOJNO KONJUGIRANO

$$\psi(x) = \sum_{\vec{p},s} \frac{1}{\sqrt{2EV}} \left[b_s(\vec{p})\, u(\vec{p},s)\, e^{-ip\cdot x} + d_s^{\dagger}(\vec{p})\, v(\vec{p},s)\, e^{ip\cdot x} \right]$$

$$\bar{\psi} = \psi^{\dagger}\gamma_0 = \sum_{\vec{p},s} \frac{1}{\sqrt{2EV}} \left[b_s^{\dagger}(\vec{p})\, \bar{u}(\vec{p},s)\, e^{ip\cdot x} + d_s(\vec{p})\, \bar{v}(\vec{p},s)\, e^{-ip\cdot x} \right]$$

$$(\psi^c)_\alpha = C\psi C^{-1} = C_{\alpha\beta}(\bar{\psi})_\beta \;; \qquad C = i\gamma^2\gamma^0$$

OPERATOR BROJA FERMIONA

$$N(p,s) = b_s^\dagger(p)\, b_s(p)$$

$$\bar{N}(p,s) = d_s^\dagger(p)\, d_s(p)$$

vac. $|0\rangle$

$$b_s(p)|0\rangle = 0$$

1-č

$$|p,s\rangle = b_s^\dagger(p)|0\rangle$$

$$N(p,s)|p,s\rangle = N(p,s)\, b_s^\dagger|0\rangle$$

$$= |p,s\rangle$$

$$[N(p,s), b_s^\dagger(p)] = b_s^\dagger(p)$$

2-č

$$b_s^\dagger(p)\, b_s^\dagger(p)|0\rangle = -b_s^\dagger(p)\, b_s^\dagger(p)|0\rangle$$

- **PRIMJER: Operator naboja fermionskog polja;**
- **VJEŽBE: Operator energije fermionskog polja;**
- **ZADAĆA: Operator impulsa fermionskog polja;**