XI. PREDAVANJE

FEYNMANOV OPIS ELEKTRODINAMIKE

- LOKALNA BAŽDARNA SIMETRIJA
- SLIKA INTERAKCIJE
- S-MATRICA i OSNOVE QED

PRINCIP LOKALNE SIMETRIJE - za QED, FEČ 3.2, STR.127

POLAZEĆI OD DIRACOVE TEORIJE

U(1) baždarna invarijantnost

$$\mathcal{L}_0 = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$$

$$\boxed{\begin{aligned} \Psi(x) &\to \Psi'(x) = \underbrace{e^{-i\alpha(x)}}_{\equiv S(x)}\Psi(x) \\ \bar{\Psi}(x) &\to \bar{\Psi}'(x) = \bar{\Psi}(x)\, S^\dagger(x) \end{aligned}} \quad (*)$$

$$\Rightarrow \quad \delta\mathcal{L}_0 = \bar{\Psi}\gamma^\mu\Psi\, \cancel{\partial_\mu \alpha(x)}$$

kompenziran dodavanjem interaktivnog člana

$$\mathcal{L}_0 \to \mathcal{L}_{tot} = \mathcal{L}_0 - q\bar{\Psi}\gamma^\mu\Psi A_\mu$$

$$\boxed{\begin{aligned} A_\mu(x) \to A'_\mu(x) &= A_\mu(x) + \cancel{\tfrac{1}{q}\partial_\mu \alpha(x)} \\ &\equiv A_\mu(x) + \tfrac{i}{q}(\partial_\mu S(x))\, S^\dagger(x) \end{aligned}} \quad (**)$$

Kinetički član e.m. (baždarnog) polja

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

invarijantan je na (**)

LOKALNA BAŽDARNA SIMETRIJA

$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$ simetričan na $\begin{cases} \Psi \to \Psi' = e^{iQ\vartheta(x)}\Psi \\ \bar{\Psi} \to \bar{\Psi}' = \bar{\Psi}e^{-iQ\vartheta(x)} \end{cases}$

$$\partial_\mu \Psi(x) \to e^{iQ\vartheta(x)}\left(\partial_\mu + iQ\,\partial_\mu\vartheta(x)\right)\Psi(x)$$

$$\Rightarrow \delta\mathcal{L} = -Q\bar{\Psi}\gamma^\mu\Psi\,\partial_\mu\vartheta(x)$$

— simetričan ako $\exists$ kompenzirajući član:

$$\mathcal{L} \to \mathcal{L}_{tot} = \mathcal{L} - eQ\bar{\Psi}\gamma^\mu\Psi \underbrace{A_\mu}_{\Rightarrow\, -\delta\mathcal{L}\ \checkmark}, \text{ gdje } A_\mu(x) \to A'_\mu(x) = A_\mu - \frac{1}{e}\partial_\mu\vartheta(x)$$

postiže se s:

$$\partial_\mu\Psi \to D_\mu\Psi \equiv \left[\partial_\mu + ieQA_\mu(x)\right]\Psi \quad \& \quad \mathcal{L}_{kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

ČLAN INTERAKCIJE STRUJE I POLJA

$$\mathcal{L}_{Tot} = \mathcal{L}_0 + \mathcal{L}_{int} \;, \quad \mathcal{L}_{int} = -\underset{\overset{\shortparallel}{eQ}}{q}\,\bar{\psi}\gamma^\mu\psi A_\mu$$

- POSTIŽE SE MINIMALNOM SUPSTITUCIJOM

$$\mathcal{L}_0 - q\bar{\psi}\gamma^\mu\psi A_\mu = \bar{\psi}\left[\gamma^\mu i(\underbrace{\partial_\mu + iq}A_\mu) - m\right]\psi$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ieQA_\mu$$

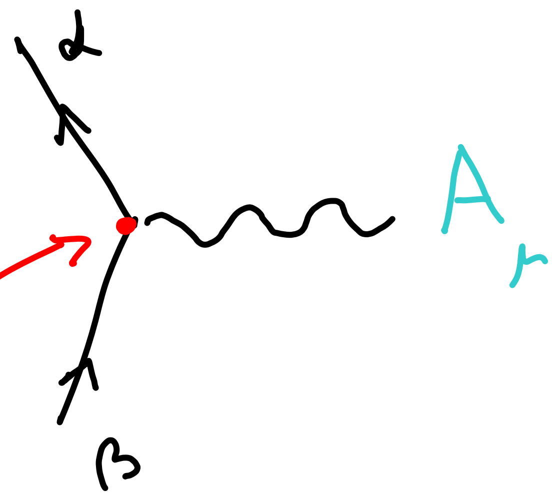
$$= \partial_\mu - ieA_\mu \quad (\text{za elektron } Q = -1)$$

I VODI NA TEMELJNI “VRH” U QED

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - e\, j^\mu_Q A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$j^\mu_Q = Q \bar{\Psi} \gamma^\mu \Psi$$

$$-ieQ(\gamma^\mu)_{\alpha\beta}$$



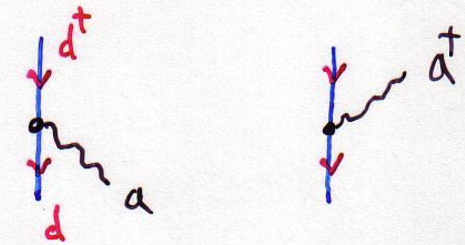
Temelj QED

$$H_I(t) = -\int d^3x\, \mathcal{L}_{int}(\vec{x},t) = \int d^3x\, j^\mu(\vec{x},t)\, A_\mu(\vec{x},t)$$

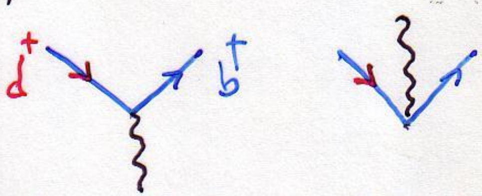
$$= -e \int d^3x \, :\bar{\psi}(\vec{x},t)\,\gamma^\mu \psi(\vec{x},t): \, A_\mu(\vec{x},t)$$

$$\sim \; :(b^\dagger + \overset{\text{ač}}{d})(b + \overset{\text{ač}}{d^\dagger}): \, (a + a^\dagger)$$

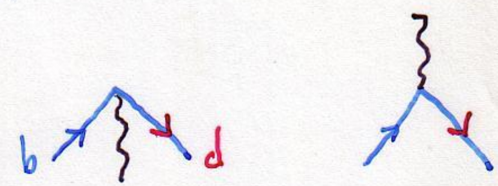
$= -d^\dagger d\,(a+a^\dagger)$



$+\, b^\dagger d^\dagger (a+a^\dagger)$



$+\, d\, b\, (a+a^\dagger)$



$+\, b^\dagger b\, (a+a^\dagger)$

SLIKA INTERAKCIJE i S-MATRICA

KVANTNO-MEHANIČKE SLIKE

Schrödingerova

Operatori $\Psi_s(\vec{x})$, $A_s(\vec{x})$ vrem. neovisni

$H \rightarrow$ op. evolucije

vektora stanja / val. f-je $\quad i\frac{\partial}{\partial t}|a,t\rangle_s = H\,|a,t\rangle_s$

Heisenbergova

$\rightarrow$ vremenski OVISNI operatori

Relacija translacije u vremenu

$$\underbrace{A(\vec{x},t)}_{``A_H(\vec{x},t)} = e^{iHt}\,\underbrace{A(\vec{x},0)}_{A_s}\,e^{-iHt}$$

povezuje dvije slike

$|a,t\rangle_s = e^{-iHt}|a\rangle_H$

zadovoljava Schröd. j-bu

Diracova / SLIKA INTERAKCIJE

$\rightarrow$ u kojoj se samo DIO vrem. ovisnosti vekt. st. prenosi na operatore

$H = H_0 + H_I$

$$A_I(\vec{x},t) = e^{iH_0t}\,\underbrace{A_s(\vec{x})}_{e^{-iHt}A_H(\vec{x},t)e^{iHt}}\,e^{-iH_0t}$$

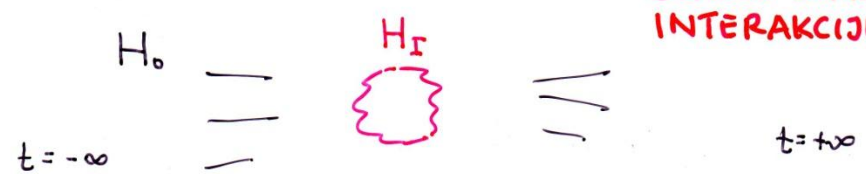
$$A_I(\vec{x},t) = U(t,0)\,A_H(\vec{x},t)\,U^{-1}(t,0)$$

$|a,t\rangle_I = e^{iH_0t}|a,t\rangle_s$

$$i\frac{\partial}{\partial t}|a,t\rangle_I = \underbrace{e^{iH_0t}H_I e^{-iH_0t}}_{\equiv H_I(t)}|a,t\rangle_I$$

op. interakcije u slici interakcije

ODABIR SLIKE INTERAKCIJE



H_0 određuje evoluciju operatora

$$i\frac{\partial}{\partial t}A_I(t) = [A_I(t), H_0]$$

H_I određuje evoluciju stanja / op. U_I

$$i\frac{\partial}{\partial t}|a,t\rangle_I = H_I(t)\,|a,t\rangle_I$$

iz $\quad |a,t\rangle_I = U_I(t,t_0)\,|a,t_0\rangle_I$

$$i\frac{\partial}{\partial t}U_I(t,t_0) = H_I(t)\,U_I(t,t_0)$$

Op. interakcije u slici interakcije

$$H_I(t) = e^{iH_0t}H_I e^{-iH_0t} = \sum_{n=0}^{\infty}\frac{i^n}{n!}[H_0,H_I]_n t^n$$

VREMENSKI OVISNI OPERATORI

- **Kao u QM Heisenbergovoj slici**

$$\Psi(\vec{x},t)\,,\quad A(\vec{x},t)$$

Prilagođavamo poljima u interakciji

$$\mathcal{H}_I = -\mathcal{L}_I = e\,\bar{\Psi}(\vec{x},t)\gamma^\mu\Psi(\vec{x},t)A_\mu(\vec{x},t)$$

$$H_I \equiv H_I^{\text{Heis.}} = \int d^3x\,\mathcal{H}_I(\vec{x},t)$$

DIRACOVA SLIKA (INTERAKCIJE): RASTAV NA SLOBODNI I INTERAKCIJSKI DIO

$H = H_0 + H_I$ u čas $t=0$

- **OPERATORI POLJA ZADOVOLJAVAJU SLOBODNE JEDN.**

$i\frac{\partial}{\partial t} A_I(t) = [A_I(t), H_0]$

određuje evoluciju op.

u usp. s Heis. j. g. b.

$i\frac{\partial}{\partial t} A_H(t) = [A_H(t), H]$

- **VEKTORI STANJA ZADOVOLJAVAJU SCHROEDINGEROVU JEDN.**

$$i\frac{\partial}{\partial t}|\alpha,t\rangle_I = H_I(t)\,|\alpha,t\rangle_I$$

određuje evoluciju vekt. st.

- operator interakcije u slici interakcije

$$H_I(t) = e^{iH_0t}\,H_I\,e^{-iH_0t} = \sum_{n=0}^{\infty}\frac{i^n}{n!}[H_0,H_I]_n\,t^n$$

OPERATOR EVOLUCIJE – podliježe iteracijskom rješavanju

$$i\frac{\partial}{\partial t}U_I(t,t_0) = H_I(t)\,U_I(t,t_0)$$

$$U_I(t_0,t_0) = I$$

$$S = \lim_{\substack{t\to\infty \\ t_0\to-\infty}} U_I(t,t_0) = T\exp\left\{-i\int d^4x\, \mathcal{H}_I(x)\right\}$$

$$S_{fi} = \langle f|\,S\,|i\rangle = \delta_{fi} + i\,T_{fi}$$

$$|T_{fi}|^2 = \frac{\left[(2\pi)^4\delta^4(p_f-p_i)\right]^2}{\prod_i(2E_iV)\prod_f(2E_fV)}\,|\mathcal{M}_{fi}|^2$$

Feynmanova pravila

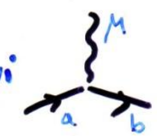
I Ulazne crte $|i\rangle$ — **Izlazne crte $|f\rangle$**

fermiona $\xrightarrow{p,s}\bullet_a$ $= u_a(p,s)$; $\;{}_a\bullet\xrightarrow{p,s}$ $= \bar{u}_a(p,s)$

anti-fermiona $\longleftarrow\bullet$ $= \bar{v}_b(p,s)$; $\bullet\longleftarrow$ $= v_b(p,s)$

fotona $\overset{k,\lambda}{\rightsquigarrow}\bullet^{\mu}$ $= \epsilon^{\mu}(k,\lambda)$; ${}^{\mu}\bullet\overset{k,\lambda}{\rightsquigarrow}$ $= \epsilon^{\mu *}(k,\lambda)$

II Vrhovi



$= -ie(\gamma^{\mu})_{ba}$

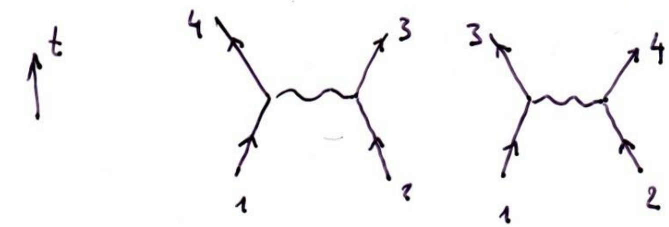
III Propagatori

fermiona ${}^a\bullet\!\longrightarrow\!\bullet^b$ $= \dfrac{i(\not p+m)_{ba}}{p^2-m^2+i\epsilon}$; fotona ${}^{\mu}\bullet\overset{k}{\sim\sim}\bullet^{\nu}$ $\dfrac{-i g_{\mu\nu}}{k^2+i\epsilon}$

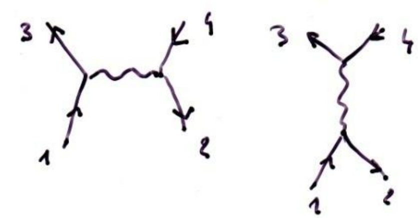
QED u 2. redu računa smetnje

FEČ § 3.3.2 | str. 155

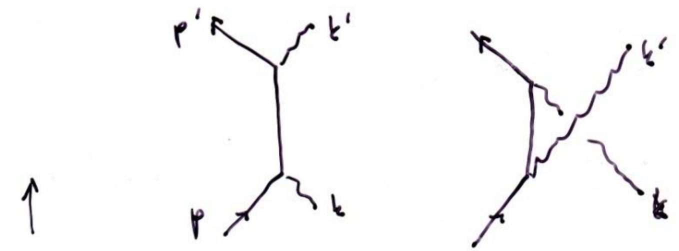
Møllerovo $e^-(p_1,r_1)+e^-(p_2,r_2)\to e^-(p_3,r_3)+e^-(p_4,r_4)$



Bhabhino $e^-(p_1)+e^+(p_2)\to e^-(p_3)+e^+(p_4)$



Komptonsko $\gamma(k)+e^-(p)\to\gamma(k')+e^-(p')$



$\overrightarrow{\text{Anihilacija}}$ $e^+ + e^- \to \gamma+\gamma$