XIII. PREDAVANJE

ELEKTROMAGNETSKA SVOJSTVA ELEKTRONA I NUKLEONA

- **QED u UR i u NR GRANICI**
- **PROCES 1. REDA – KULONSKO RASPRŠ.**
- **USPOREDBA MAGNETIZMA ELEKTRONA I NUKLEONA**

OSNOVNI PROCESI ELEKTRODINAMIKE

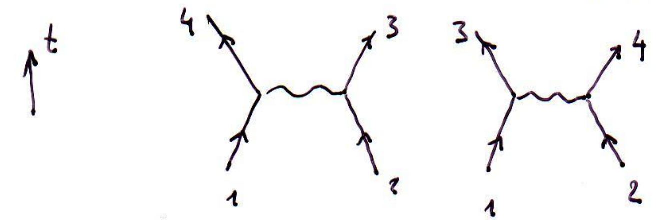
Dijagrami izmjene & anihilacije

Virtualni foton

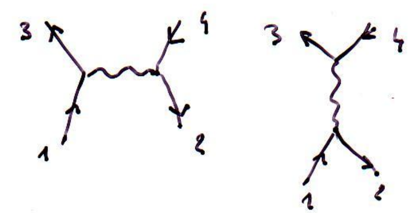
QED u 2. redu računa smetnje

FEČ § 3.3.2 | str. 155

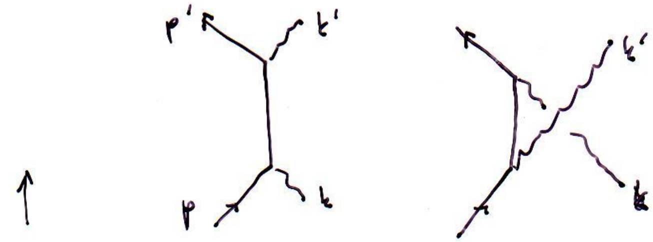
Møllerovo $e^-(p_1,r_1)+e^-(p_2,r_2)\to e^-(p_3,r_3)+e^-(p_4,r_4)$



Bhabhino $e^-(p_1)+e^+(p_2)\to e^-(p_3)+e^+(p_4)$

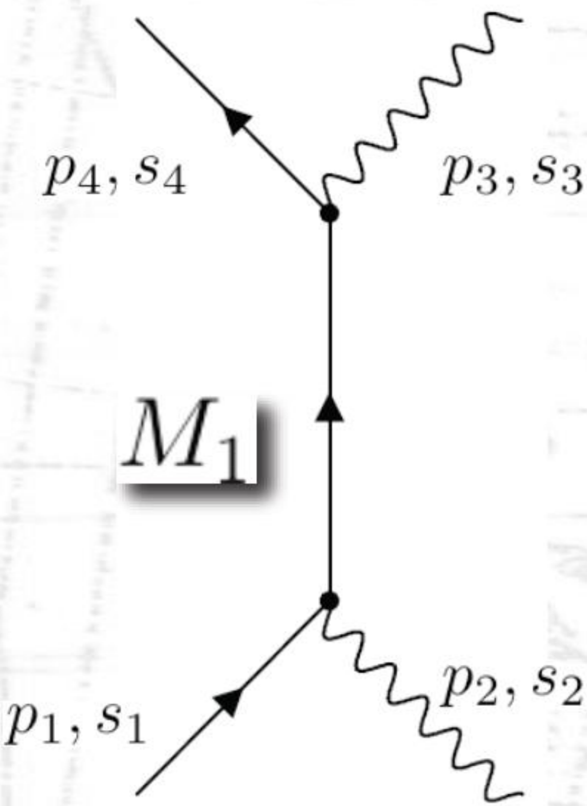


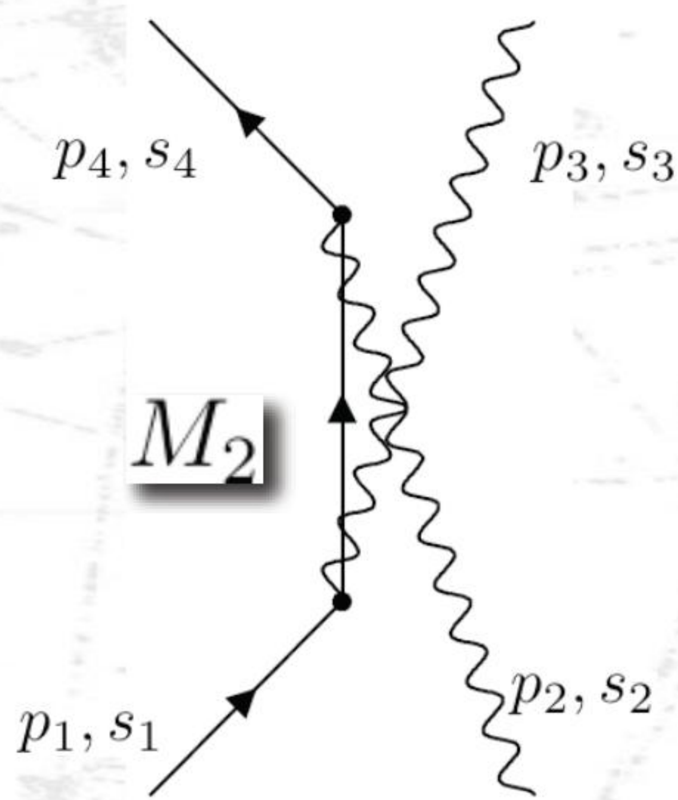
Komptonsko $\gamma(k)+e^-(p)\to\gamma(k')+e^-(p')$



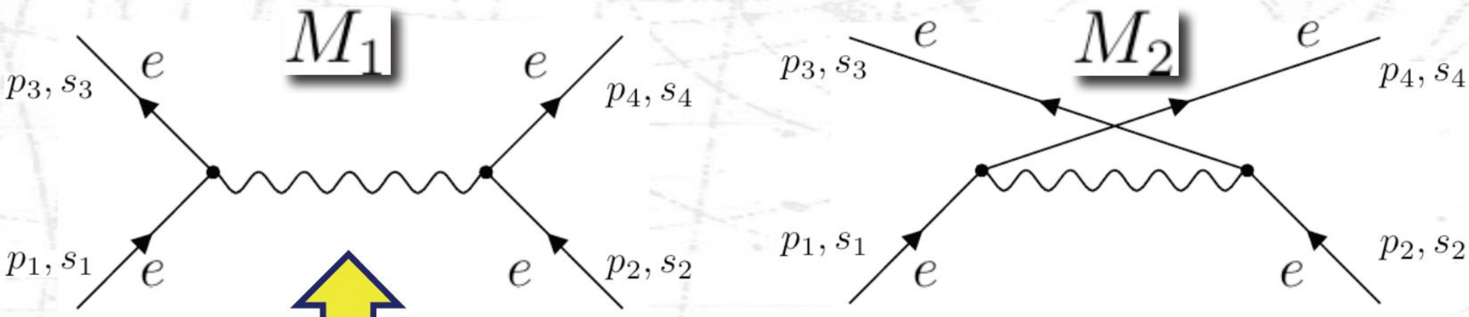
Anihilacija $e^+ + e^- \to \gamma+\gamma$

KOMPTONSKO RASPRŠENJE FEČ str. 158 & vježbe





RASPRŠENJE ELEKTRONA NA ELEKTRONU (Moellerovo -vježbe)

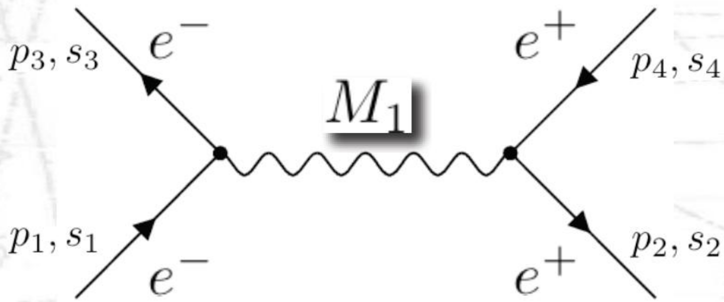


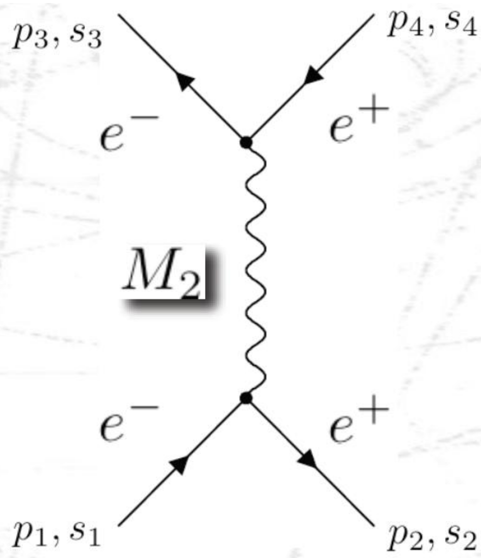
$$\mathcal{M} = -\frac{g_e^2}{(p_1-p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)]$$

Interchange: p_3,s_3 p_4,s_4

$$+\frac{g_e^2}{(p_1-p_4)^2}[\bar{u}^{(s_4)}(p_4)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_3)}(p_3)\gamma_\mu u^{(s_2)}(p_2)]$$

RASPRŠENJE ELEKTRONA NA POZITRONU (Bhabha-ino)

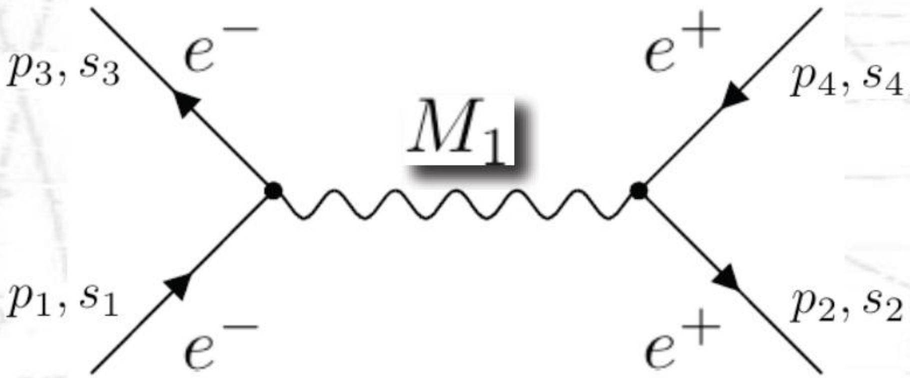


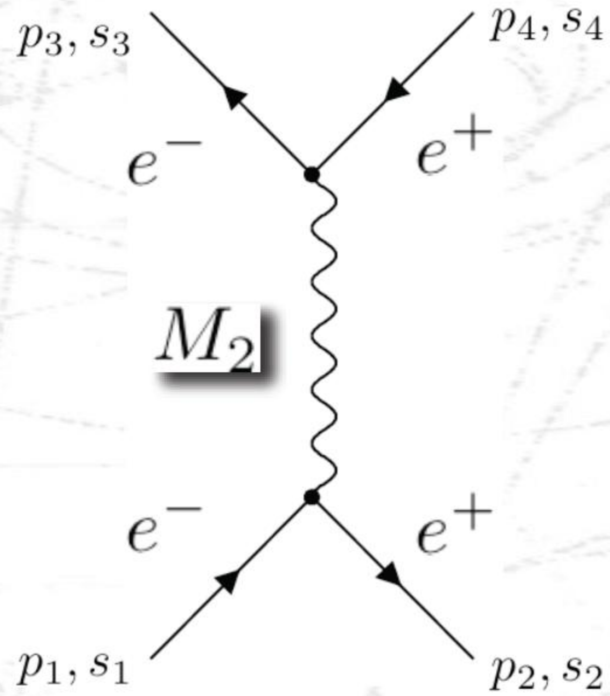


$$\mathcal{M}_1 = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{v}^{(s_2)}(p_2)\gamma_\mu v^{(s_4)}(p_4)]$$

$$\mathcal{M}_2 = -\frac{g_e^2}{(p_1 + p_2)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu v^{(s_4)}(p_4)][\bar{v}^{(s_2)}(p_2)\gamma_\mu u^{(s_1)}(p_1)]$$

Dijagrami IZMJENE i ANIHILACIJE





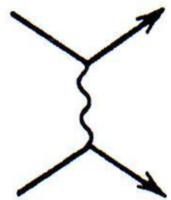
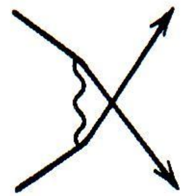
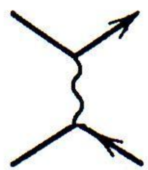
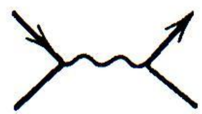
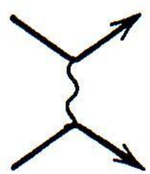
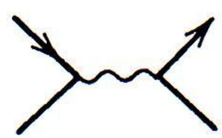
prostorni foton

$m_\gamma^2 < 0$

vremenski foton

$m_\gamma^2 > 0$

QED PROCESI 2. reda u URL

	Feynman Diagrams		$\overline{\lvert\mathcal{M}\rvert^2}/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \to e^-e^-$			$\frac{s^2+u^2}{t^2}$ +	$\frac{2s^2}{tu}$ +	$\frac{s^2+t^2}{u^2}$
				($u \leftrightarrow t$ symmetric)	
↓ (Crossing $s \leftrightarrow u$) ↓					
	Forward	"Time-like"	Forward	Interference	Time-like
Bhabha scattering $e^-e^+ \to e^-e^+$			$\frac{s^2+u^2}{t^2}$ +	$\frac{2u^2}{ts}$ +	$\frac{u^2+t^2}{s^2}$
$e^-\mu^- \to e^-\mu^-$			$\frac{s^2+u^2}{t^2}$		
(Crossing ↓ $s \leftrightarrow t$)					
$e^-e^+ \to \mu^-\mu^+$					$\frac{u^2+t^2}{s^2}$

Schroedinger-Paulijeva jedn. u NRL & g=2 za Diracove č.

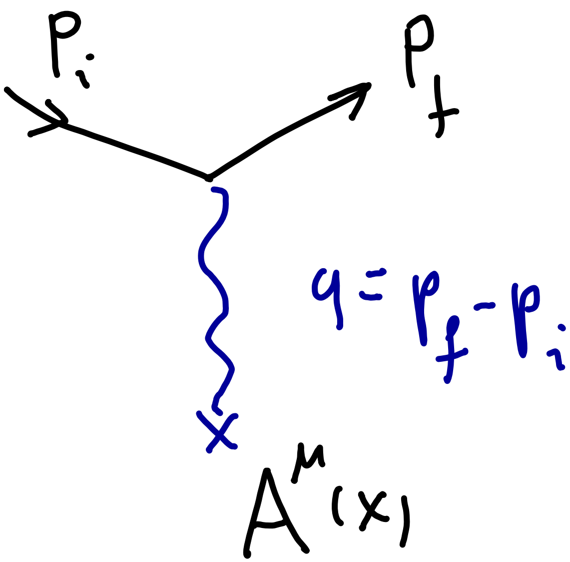
$$\left(\frac{1}{2m} (\mathbf{P} + e\mathbf{A})^2 + \frac{e}{2m} \boldsymbol{\sigma} \cdot \mathbf{B} - eA^0 \right) \psi_A = E_{NR} \psi_A$$

$$\boldsymbol{\mu} = -\frac{e}{2m} \boldsymbol{\sigma} \equiv -g \frac{e}{2m} \mathbf{S}.$$

PRIMJER PROCESA U 1. REDU

RUTHERFORDOVO RASPRŠENJE ELEKTRONA NA STATIČKOM NABOJU

$$T_{fi} = -i \int d^4x \, \underbrace{j^{fi}_{\mu}(x)}_{} A^{\mu}(x)$$



$$j^{fi}_{\mu} = -e \, \bar{u}_f \gamma_{\mu} u_i \, e^{-iq\cdot x}$$

$$A^{\mu}(q) = \int d^4x \, e^{-iq\cdot x} A^{\mu}(x)$$

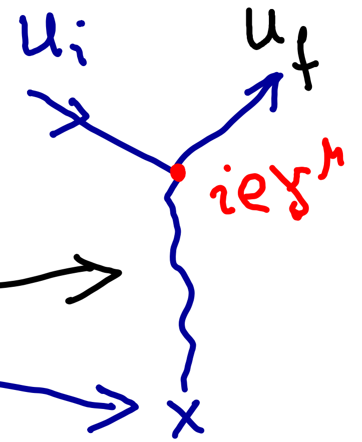
$$= 2\pi \, \delta(E_f - E_i) \int d^3x \, e^{i\vec{q}\cdot\vec{x}} A^{\mu}(x)$$

UVOĐENJE INVARIJANTNE AMPLITUDE $\mathcal{M}$

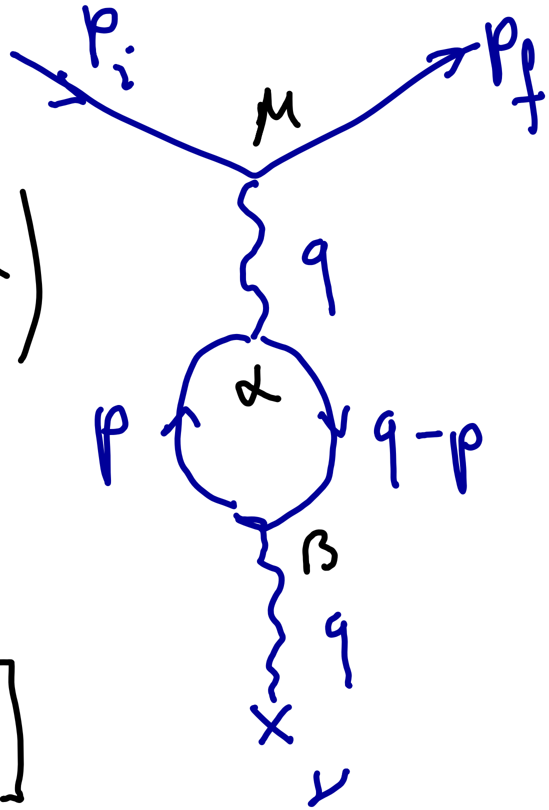
$$T_{fi} = 2\pi\,\delta(E_f - E_i)\,\underbrace{(-i)\,e\,\bar{u}_f\,\gamma_\mu\,u_i\,\frac{1}{q^2}\,j^\mu(q)}_{-i\mathcal{M}_{fi}}$$

$$-i\mathcal{M} = \left(ie\,\bar{u}_f\,\gamma^\mu\,u_i\right)\left(\frac{-i g_{\mu\nu}}{q^2}\right)\left(-i j^\nu(q)\right)$$

$$\left\{ i\mathcal{M} \longleftrightarrow -ie\gamma^\mu \qquad e = |e| \right\}$$



POOPĆENJE NA AMPLITUDE S FERMIONSKOM PETLJOM



$$iM = (-1)\left(-ie\,\bar{u}_f \gamma^{\mu} u_i\right)\left(\frac{-ig_{\mu\alpha}}{q^2}\right)$$

$$\int \frac{d^4p}{(2\pi)^4}\left[\left(-ie\gamma^{\alpha}\right)_{\varkappa\lambda} \frac{i(\not{p}+m)_{\lambda\rho}}{p^2-m^2} \left(-ie\gamma^{\beta}\right)_{\rho\sigma} \frac{i(\not{p}-\not{q}+m)_{\sigma\varkappa}}{(p-q)^2-m^2}\right]$$

$$\left(-i\frac{g_{\beta\nu}}{q^2}\right)\left(-ij^{\nu}(q)\right)$$

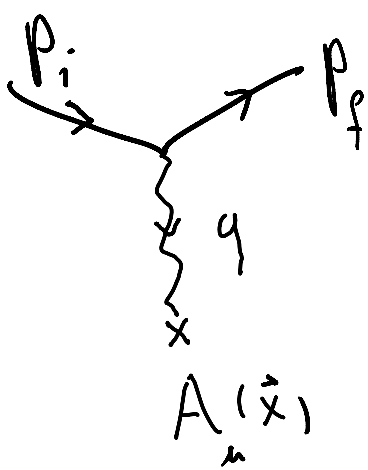
NAKON RAČUNA POMOĆU CASIMIROVOG TRIKA:

- **Mottova formula udarnog presjeka**

- u NR granici prelazi u Rutherfordovu

$$\int d^4x \, \langle e^-(p_f, s) | \bar{\Psi}(x) \gamma^\mu \Psi(x) | e^-(p_i, r) \rangle \langle 0 | A_\mu(x) | \gamma \rangle$$

$$\frac{d\sigma}{d\Omega} = \frac{(Z\alpha)^2}{4p^2\beta^2 \sin^4 \frac{\vartheta}{2}} \left[1 - \beta^2 \sin^2 \frac{\vartheta}{2} \right]$$



Gordonova dekompozicija

$$-e\bar{u}_f\gamma_\mu u_i = -\frac{e}{2m}\bar{u}_f\Big((p_f + p_i)_\mu - i\sigma_{\mu\nu}q^\nu\Big)u_i$$

- Giromagnetski faktor g=2 Diracove čestice

$$\boldsymbol{\mu} = -\frac{e}{2m}\boldsymbol{\sigma},$$

$$\boldsymbol{\mu} = -g\frac{e}{2m}\mathbf{S}$$

MAGNETIZAM ELEKTRONA

$$\vec{\mu}_s = g\left(\frac{e\hbar}{2mc}\right)\frac{\vec{s}}{\hbar}$$

$\mu_B = 0.927 \cdot \begin{cases} 10^{-23}\ JT^{-1} \\ 10^{-20}\ \frac{erg}{Gauss} \end{cases}$

(c u cgs.)

Diracova teorija predviđa g=2

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{(p'+p)^\mu}{2m} + \frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m}\right]u(p)$$

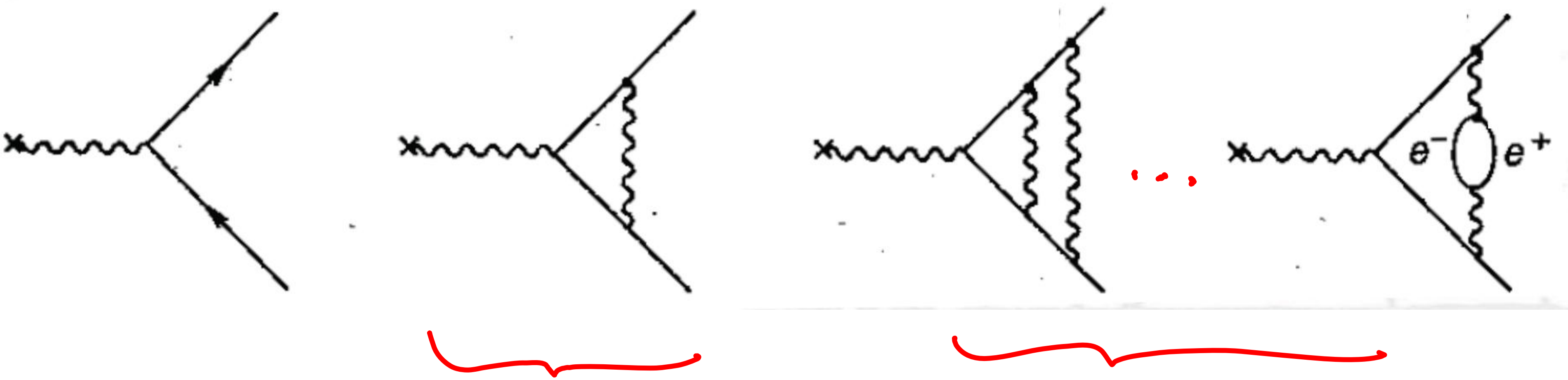
u usporedbi s

$$\bar{u}(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p,s) \xrightarrow[\text{4-imp.}]{\text{za male}}$$

$$\bar{u}(p',s')\left\{\frac{(p'+p)^\mu}{2m}\underbrace{F_1(0)}_{=1} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\underbrace{\left[F_1(0)+F_2(0)\right]}_{1+F_2(0)}\right\}u(p,s)$$

$F_2(0) \approx \alpha$

Anomalni magnetni moment leptona u 3. redu



$a = \frac{g-2}{2}$:

$\frac{1}{2}\frac{\alpha}{\pi}$

$-0.32848\left(\frac{\alpha}{\pi}\right)^2$

MJERENA VRIJEDNOST ZA MION (E821, BNL)

$a_{\mu}\ (exp) = 11\ 659\ 208\ (6) \times 10^{-10}\ (0.5\ ppm)$

- 2.7 SD ODSTUPANJA OD TEOR.
- PROTON u smislu g=2 nije Diracova čestica

$$\mu_e = g_e \frac{1}{2}\mu_B = g_e \frac{1}{2}\frac{e\hbar}{2m_e}$$

2

$$\mu_p = g_p \frac{1}{2}\mu_N = g_p \frac{1}{2}\frac{e\hbar}{2m_p}$$

2 × 2.79

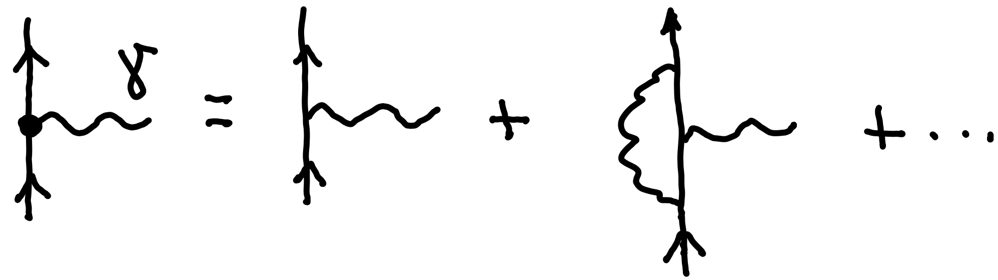
NOVA FIZIKA PUTEM PRECIZNIH TESTOVA QED

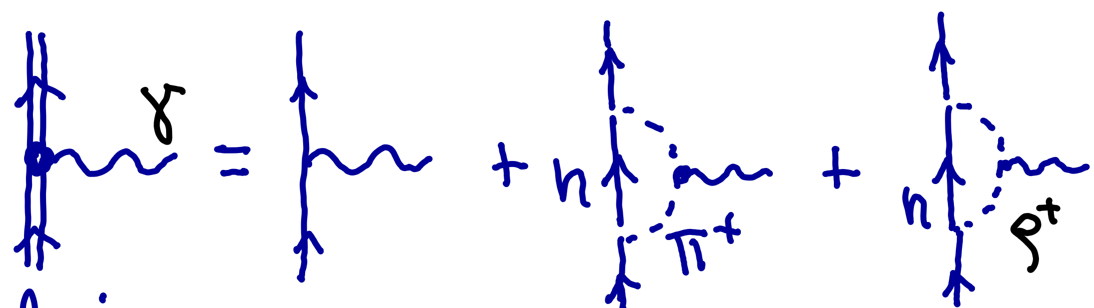
$$g_e = 2\cdot\left(1 + \frac{1}{2}\frac{\alpha}{\pi} - 0.32848\left(\frac{\alpha}{\pi}\right)^2 + \dots\right)$$

$$\mu_e = g_e \frac{1}{2}\mu_B = g_e \frac{1}{2}\frac{e\hbar}{2m_e}$$

$$\mu_p = g_p \frac{1}{2}\mu_N = g_p \frac{1}{2}\frac{e\hbar}{2m_p}$$

$2\cdot 2.79$





fizikalni
proton

ELEKTROMAGNETIZAM NUKLEONA

$$\bar{u}_p(p',s')\left[F_1^{(p)}(q^2)\gamma^\mu + i\frac{\sigma^{\mu\nu}}{2M} q_\nu F_2^{(p)}(q^2) \right.$$

$F_1^{(p)}(0) = 1$ $\qquad$ $F_2^{(p)}(0) = \varkappa_p = 1.79$

$F_1^{(n)}(0) = 0$ $\qquad$ $F_2^{(n)}(0) = \varkappa_n = -1.91$

$$\left. \cdots + \frac{F_A(q^2)}{M^2}(q^2\gamma^\mu - \not{q} q^\mu)\gamma_5 + i\frac{F_E(q^2)}{2M}\sigma^{\mu\nu} q_\nu \gamma_5 \right] u_p(p,s)$$

$\not{P}$ (anapolni) $\qquad$ $\not{P}$ & $\not{T}$ EDM