



# **X Dinamika okusa II**

## **OSCILACIJE STRANOSTI I DRUGIH OKUSA**

- **OSCILACIJA I REGENERACIJA STRANOSTI**
- **OSCILACIJA DRUGIH OKUSA**
- **MASE I OSCILACIJE NEUTRINA**

# 1. MIJEŠANJE U SUSTAVU NEUTRALNIH MEZONA

1956,  $K^0 - \bar{K}^0$ : discovery of  $K_L$  (proposal of  $C$  non-c

1987,  $B^0 - \bar{B}^0$ : discovery of mixing ( $\Rightarrow$  large  $m_t$ )

2006,  $B_s^0 - \bar{B}_s^0$ : measurement of  $\Delta m_s$

2007,  $D^0 - \bar{D}^0$ : growing evidence for  $\Delta\Gamma = \mathcal{O}(0.01)$

# SUSTAV NEUTRALNIH KAONA

The quartet of kaons

- belongs to the pseudoscalar meson octet

I S

$\frac{1}{2}$  -1

$\frac{1}{0}$   $\frac{0}{0}$

$\frac{1}{2}$  +1

$K^+$   $K^0$

$\pi^+$   $\pi^0$   $\pi^-$   
 $\eta^0$

$\bar{K}^0$   $K^-$

two isodoublets  $\begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$  &  $\begin{pmatrix} \bar{K}^0 \\ -K^- \end{pmatrix}$

# NEUTRALNI KAONI

- forms one of the most intriguing systems in nature

accelerator experiments in 50-ies :

- unknown reasons for the strangeness
- unknown inner machinery of the  $K^0$ - $\bar{K}^0$  mixing

Cronin & Fitch 1964

- revealed to us ~~CP~~
- has provided the breakthroughs in our knowledge of fundamental physics

# MIJEŠANJE NEUTRALNIH KAONA

Strongly produced  $J^P = 0^-$  states  $\begin{cases} K^0 (\bar{s}d) \\ \bar{K}^0 (d\bar{s}) \end{cases}$

e.g.  $\pi^- + p \rightarrow \Lambda^0 + X$  "associated"  $K^0$  or  $K^+ \pi^-$

◇ strong interaction / strangeness eigenstates

$|K^0\rangle$  ( $S=1$ ) ;  $|\bar{K}^0\rangle$  ( $S=-1$ ) convention

$|\bar{K}^0\rangle = -CP|K^0\rangle$  convention

weak decays

CP-conserving

$2\pi, 3\pi, \pi l \nu_e, \dots$   
 Hamilt.

◇ CP eigenstates  $|K_{1,2}\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle \mp |\bar{K}^0\rangle)$

CP  $|K_1\rangle = |K_1\rangle (+)$ , CP  $|K_2\rangle = -|K_2\rangle (-)$

CP-allowed decays  
 $\downarrow$   
 $2\pi (+)$

$\tau_1 \rightarrow \tau_{\text{short}}$   
 $0.8926(12) \cdot 10^{-10} \text{ s}$

$\downarrow$   
 $3\pi (-)$   
 Suppressed  
 $\tau_2 = \tau_{\text{Long}}$   
 $5.15(4) \cdot 10^{-8} \text{ s}$

$$\Delta m_K = m_L - m_S = 3.4782(176) \cdot 10^{-12} \text{ MeV};$$

$$\Delta \Gamma_K = \Gamma_S - \Gamma_L = 7.2823(98) \cdot 10^{-12} \text{ MeV}$$

The sign measurement involves "regeneration" expt.

means

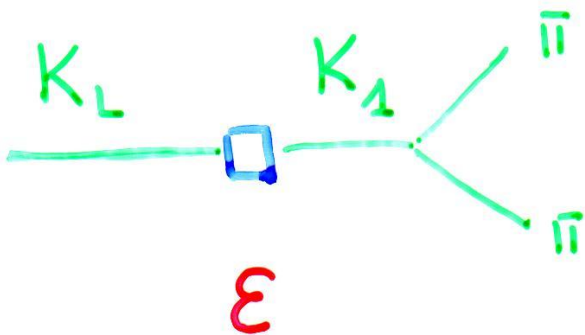
observed  
 $K_2 \rightarrow \pi^+ \pi^-$   
~~CP~~ !

# NARUŠENJE CP SIMETRIJE

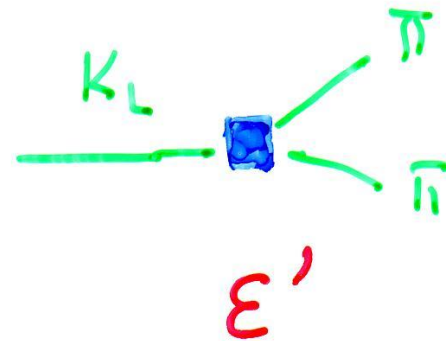
◇ Physical decay states (CP included)

$$|K_{L,S}\rangle = \frac{1}{\sqrt{1+\epsilon^2}} (|K_{2,1}\rangle + \epsilon |K_{1,2}\rangle)$$

lead to CP in  $K_L \rightarrow \pi\pi$  via two mechanisms



indirect CP (in the mass matrix)



direct CP (in transition)

•

$$\Delta S = 2$$

$$H = \begin{bmatrix} M_{-i\Gamma/2} & M_{12-i\Gamma_{12}/2} \\ M_{12}^* & M_{-i\Gamma/2} \end{bmatrix}$$

amplitude

$$\mathcal{E} = \frac{\text{Im} \frac{\Gamma_{12}}{2} + i \text{Im} M_{12}}{2 \text{Re} M_{12} - \frac{i}{2} (2 \text{Re} \Gamma_{12})} \approx \frac{e^{i\gamma_3}}{\sqrt{2}} \frac{\text{Im} M_{12}}{\Delta M_K} \equiv \mathcal{E}_K$$

•

$$\Delta S = 1$$

CP part

shifts  $\mathcal{E}_K \rightarrow \mathcal{E}_K + \xi$   
induces  $\mathcal{E}'$

$$\mathcal{E}' = \frac{\omega}{\sqrt{2}} e^{i(\frac{\pi}{2} + \delta_2 - \delta_0)} \left[ \frac{\text{Im} A_2}{\text{Re} A_1} - \xi \right]$$

$$\omega = \frac{\text{Re} A_2}{\text{Re} A_0} \approx \frac{1}{20} ; \quad \xi = \frac{\text{Im} A_0}{\text{Re} A_0}$$



# USPOREDBA MIJEŠANJA NEUTRALNIH K- i B-MEZONA

- USPOREDBA PRODUKCIJE NA REZONANCAMA
- USPOREDBA "OZNAČAVANJA"
- USPOREDBA DULJINE RASPADA  
(asimetrične B-tvornice)

# DIJAGRAMI MIJEŠANJA NA KVARKOVSKOJ RAZINI

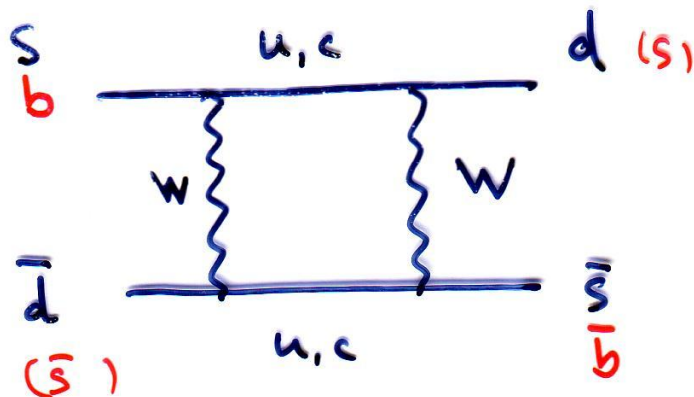
$$H_{\text{Box}} \sim G_F^2 (\bar{d} \gamma^\mu L s)_b^{\bar{s}} \left\{ \eta_1 \lambda_u^2 m_c^2 + \eta_2 \lambda_t^2 m_t^2 - 2\eta_3 \lambda_u \lambda_t m_c^2 \ln \frac{m_t^2}{m_c^2} \right\}$$

CKM factors

$$\left\{ \begin{array}{l} \lambda_q = V_{qd} V_{qs}^* \\ \lambda_q = V_{qd} V_{qb}^* \end{array} \right.$$

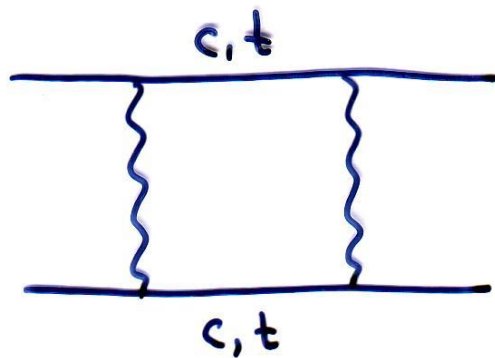
$$\left\{ \begin{array}{l} K^0 - \bar{K}^0 \\ B_d^0 - \bar{B}_d^0 \end{array} \right.$$

$$\lambda_u + \lambda_c + \lambda_t = 0$$



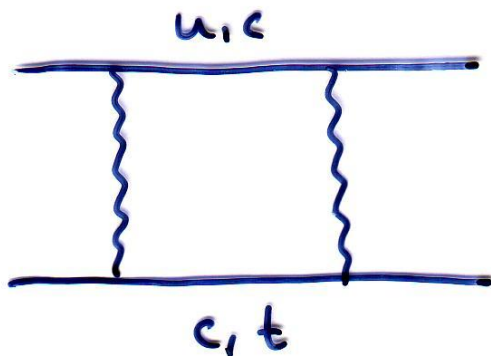
$$\sim \lambda_u^2 m_c^2 \left\{ \begin{array}{l} \text{CKM favored} \quad \Delta M_K \\ \text{CKM suppressed} \quad B - \bar{B} \end{array} \right.$$

# CKM potisnuti dijagrami kaonskog miješanja



$$\sim \lambda_t^2 m_t^2 \left\{ \begin{array}{l} \text{CKM suppr. (CP dominant} \\ \text{K-E for } m_t \gtrsim m_w) \\ \text{CKM fav. B-B} \end{array} \right.$$

"top-meter"  $\Delta M \sim m_t^2 |V_{td} V_{tb}^*|^2$



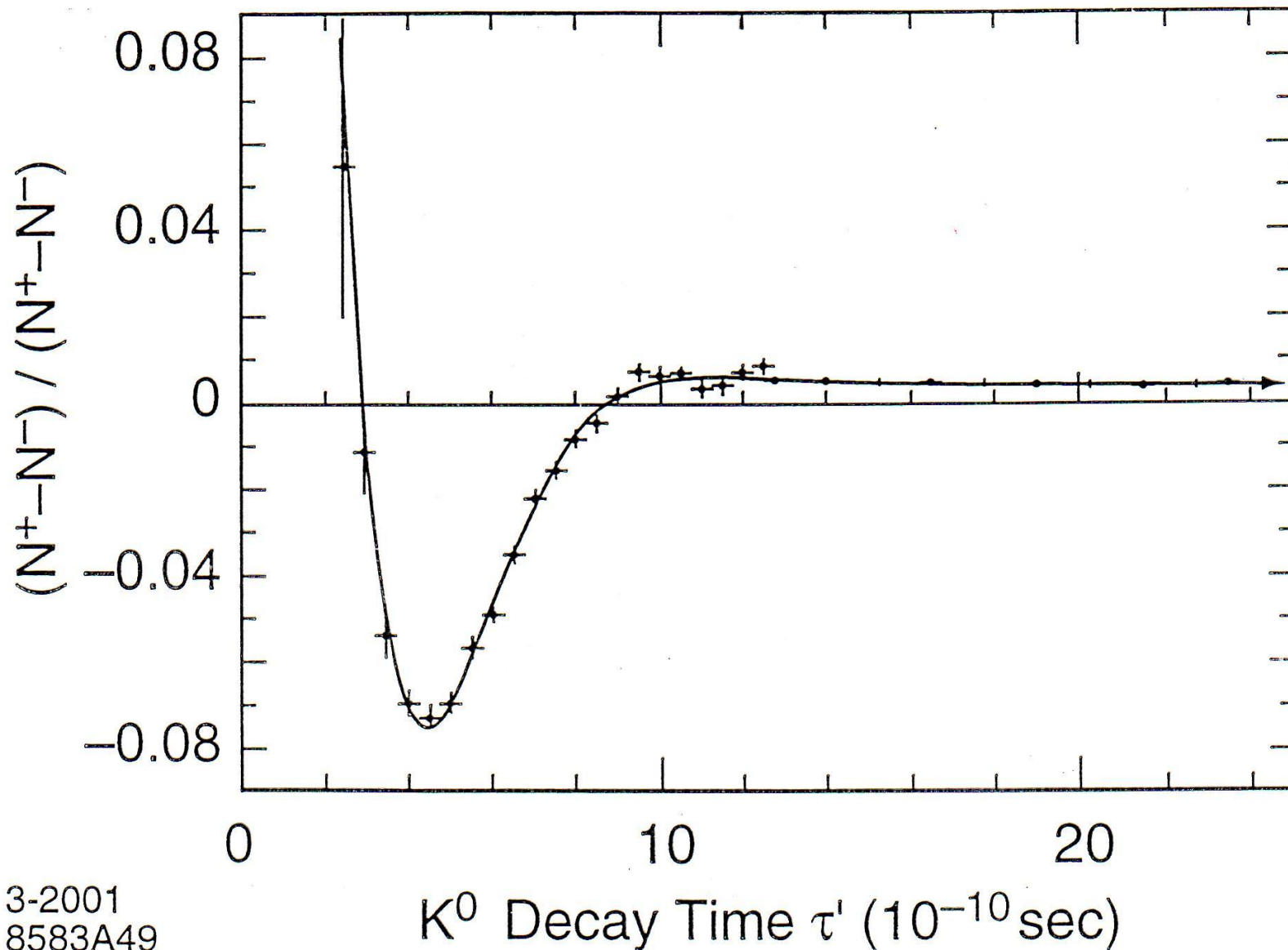
$$\sim \lambda_u \lambda_t m_c^2 \lambda_u \frac{m_t^2}{m_c^2}$$

- CKM suppr. "interference term"  
(CP dominant K-E for  $m_t^2 \ll m_w^2$ )

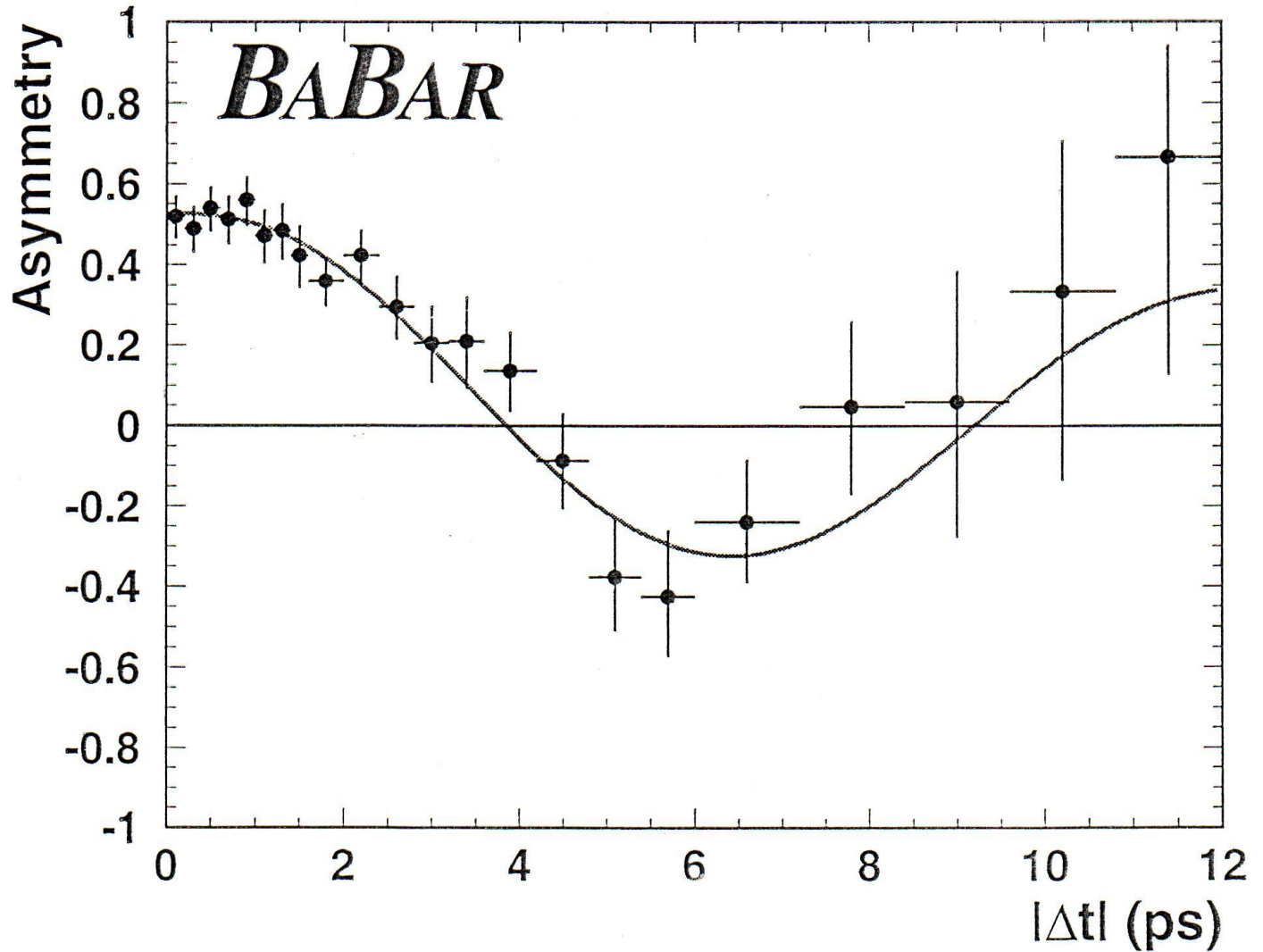


$K^0/\bar{K}^0$

Charge Asymmetry in the Decays  $K^0 \rightarrow \pi^{\mp} e^{\pm} \nu$



$B/\bar{B}$



The asymmetry in # of  $K^0$  ( $B_d$ ) mesons w.r.t.  $\bar{K}^0$  ( $\bar{B}_d$ ) as a function of time



# Oscilacije i mase neutrina

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# NOBELOVA NAGRADA IZ FIZIKE 2015



Takaaki Kajita

Super-Kamiokande Collaboration  
University of Tokyo, Kashiwa, Japan



Arthur B. McDonald

SNO Collaboration  
Queen's University, Kingston, Canada

*“for the discovery of neutrino oscillations,  
which shows that neutrinos have mass”*

# HOMESTAKE EKSPERIMENT



## Raymond Davis Jr. - Facts



Raymond Davis Jr.

**Born:** 14 October 1914, Washington, DC, USA

**Died:** 31 May 2006, Blue Point, NY, USA

**Affiliation at the time of the award:** University of Pennsylvania, Philadelphia, PA, USA

**Prize motivation:** "for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

**Field:** neutrino astrophysics

**Prize share:** 1/4



# PROBLEM SOLARNIH NEUTRINA

	$^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$ (SNU)	$^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ (SNU)
Homestake [6]	$2.56 \pm 0.16 \pm 0.16$	–
GALLEX [10]	–	$77.5 \pm 6.2^{+4.3}_{-4.7}$
GALLEX- Reanalysis [109]	–	$73.4^{+6.1+3.7}_{-6.0-4.1}$
GNO [11]	–	$62.9^{+5.5}_{-5.3} \pm 2.5$
GNO+GALLEX [11]	–	$69.3 \pm 4.1 \pm 3.6$
GNO+GALLEX- Reanalysis [109]	–	$67.6^{+4.0+3.2}_{-4.0-3.2}$
SAGE [8]	–	$65.4^{+3.1+2.6}_{-3.0-2.8}$
SSM [BPS08(GS)] [104]	$8.46^{+0.87}_{-0.88}$	$127.9^{+8.1}_{-8.2}$

# MASE NEUTRINA

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \end{pmatrix} \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\nu_\alpha = \sum_{k=1}^3 U_{\alpha k} \nu_k \quad (\alpha = e, \mu, \tau)$$

$$\mathcal{L}_{\text{mass}} \sim \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \sum_{k=1}^3 m_k \nu_k \nu_k$$

# MIJEŠANJE NEUTRINA

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

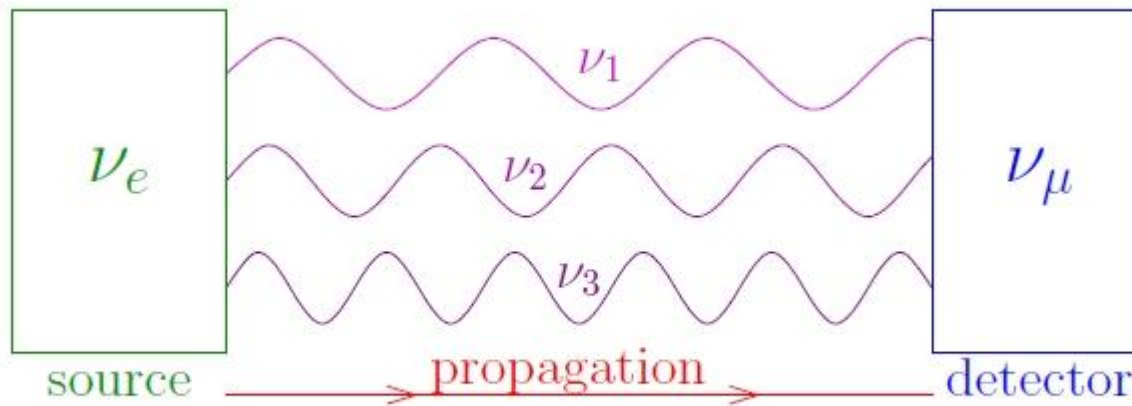
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ab} \equiv \cos \vartheta_{ab} \quad s_{ab} \equiv \sin \vartheta_{ab} \quad 0 \leq \vartheta_{ab} \leq \frac{\pi}{2} \quad 0 \leq \delta_{13} < 2\pi$$

# OSCILACIJE NEUTRINA

$$|\nu(t=0)\rangle = |\nu_e\rangle = U_{e1} |\nu_1\rangle + U_{e2} |\nu_2\rangle + U_{e3} |\nu_3\rangle$$



$$|\nu(t > 0)\rangle = U_{e1} e^{-iE_1 t} |\nu_1\rangle + U_{e2} e^{-iE_2 t} |\nu_2\rangle + U_{e3} e^{-iE_3 t} |\nu_3\rangle \neq |\nu_e\rangle$$

# PRAG DETEKCIJE NEUTRINA

$$\begin{aligned} \nu + A &\rightarrow B + C \\ &\downarrow \\ s = 2Em_A + m_A^2 &\geq (m_B + m_C)^2 \\ &\downarrow \\ E_{\text{th}} &= \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2} \end{aligned}$$

$$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^- \quad E_{\text{th}} = 0.233 \text{ MeV}$$

$$\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^- \quad E_{\text{th}} = 0.81 \text{ MeV}$$

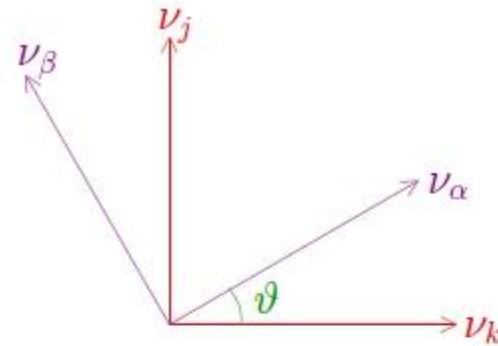
$$\bar{\nu}_e + p \rightarrow n + e^+ \quad E_{\text{th}} = 1.8 \text{ MeV}$$

$$\nu_\mu + n \rightarrow p + \mu^- \quad E_{\text{th}} = 110 \text{ MeV}$$

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \quad E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9 \text{ GeV}$$

# OSCILACIJE NEUTRINA

$$\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta |\nu_k\rangle + \sin\vartheta |\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta |\nu_k\rangle + \cos\vartheta |\nu_j\rangle \end{aligned}$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv \Delta m_{kj}^2 \equiv m_k^2 - m_j^2$$

Transition Probability:  $P_{\nu_\alpha \rightarrow \nu_\beta} = P_{\nu_\beta \rightarrow \nu_\alpha} = \sin^2 2\vartheta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$

Survival Probabilities:  $P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\nu_\beta \rightarrow \nu_\beta} = 1 - P_{\nu_\alpha \rightarrow \nu_\beta}$

# OSCILACIJE NEUTRINA

Fields  $\overline{\nu_{\alpha L}} = \sum_k U_{\alpha k}^* \overline{\nu_{kL}} \implies |\nu_{\alpha}\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$  States

$$\mathcal{H}|\nu_k\rangle = E_k|\nu_k\rangle \implies |\nu_k(t)\rangle = e^{-iE_k t} |\nu_k\rangle \implies |\nu_{\alpha}(t)\rangle = \sum_k U_{\alpha k}^* e^{-iE_k t} |\nu_k\rangle$$

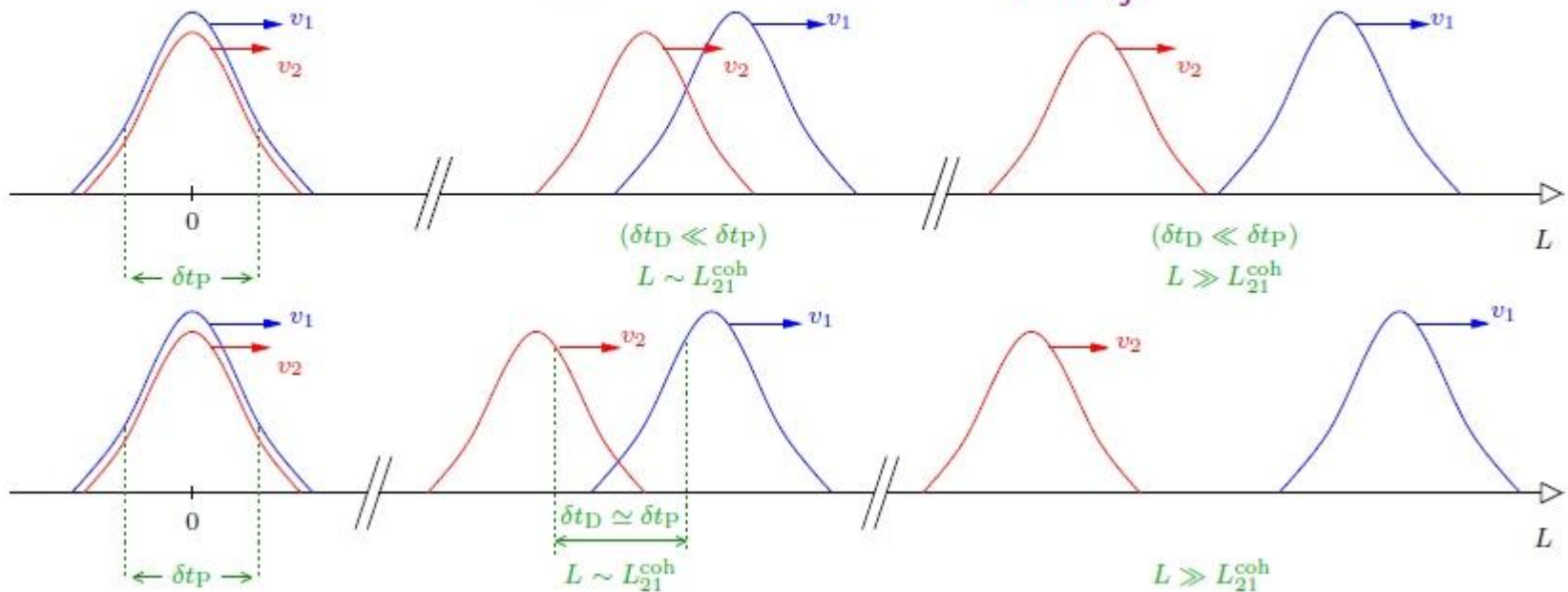
$$|\nu_k\rangle = \sum_{\beta=e,\mu,\tau} U_{\beta k} |\nu_{\beta}\rangle \implies |\nu_{\alpha}(t)\rangle = \sum_{\beta=e,\mu,\tau} \underbrace{\left( \sum_k U_{\alpha k}^* e^{-iE_k t} U_{\beta k} \right)}_{\mathcal{A}_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t)} |\nu_{\beta}\rangle$$

Transition Probability

$$P_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = |\mathcal{A}_{\nu_{\alpha} \rightarrow \nu_{\beta}}(t)|^2 = \left| \sum_k U_{\alpha k}^* e^{-iE_k t} U_{\beta k} \right|^2$$

# VALNI PAKETI

$$|\delta t_{kj}| \simeq |v_k - v_j| T \simeq \frac{|\Delta m_{kj}^2|}{2E^2} L \implies L_{kj}^{\text{coh}} \sim \frac{2E^2}{|\Delta m_{kj}^2|} \sqrt{(\delta t_P)^2 + (\delta t_D)^2}$$





# VALNI PAKETI

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* \int dp \psi_k^P(p) |\nu_k(p)\rangle \quad |\nu_\beta\rangle = \sum_k U_{\beta k}^* \int dp \psi_k^D(p) |\nu_k(p)\rangle$$

$$\begin{aligned} A_{\alpha\beta}(x, t) &= \langle \nu_\beta | e^{-i\hat{E}t + i\hat{P}x} | \nu_\alpha \rangle \\ &= \sum_k U_{\alpha k}^* U_{\beta k} \int dp \psi_k^P(p) \psi_k^{D*}(p) e^{-iE_k(p)t + ipx} \end{aligned}$$

## Gaussian Approximation of Wave Packets

$$\psi_k^P(p) = (2\pi\sigma_{pP}^2)^{-1/4} \exp\left[-\frac{(p - p_k)^2}{4\sigma_{pP}^2}\right]$$

$$\psi_k^D(p) = (2\pi\sigma_{pD}^2)^{-1/4} \exp\left[-\frac{(p - p_k)^2}{4\sigma_{pD}^2}\right]$$

$$A_{\alpha\beta}(x, t) \propto \sum_k U_{\alpha k}^* U_{\beta k} \exp\left[-iE_k t + ip_k x - \underbrace{\frac{(x - v_k t)^2}{4\sigma_x^2}}\right]$$

# OSCILACIJE NEUTRINA

$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp \left[ -i \frac{\Delta m_{kj}^2 L}{2E} \right] \\ \times \exp \left[ - \left( \frac{\Delta m_{kj}^2 L}{4\sqrt{2}E^2\sigma_x} \right)^2 - 2\xi^2 \left( \frac{\Delta m_{kj}^2 \sigma_x}{4E} \right)^2 \right]$$

Oscillation  
Lengths

$$L_{kj}^{\text{osc}} = \frac{4\pi E}{\Delta m_{kj}^2}$$

Coherence  
Lengths

$$L_{kj}^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m_{kj}^2|} \sigma_x$$

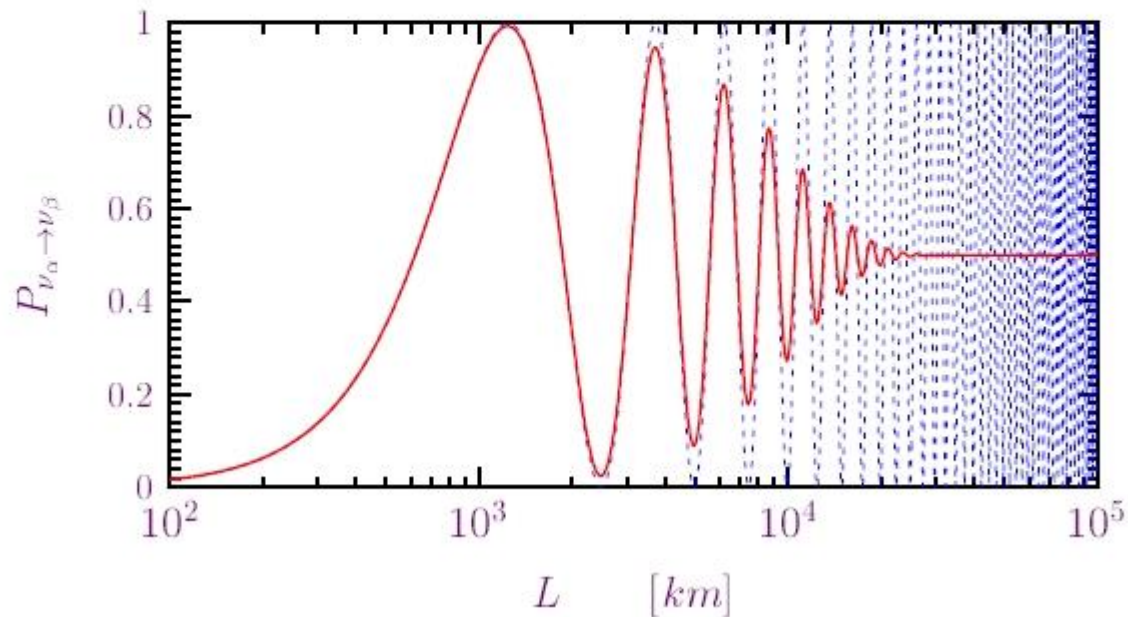
$$P_{\alpha\beta}(L) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp \left[ -2\pi i \frac{L}{L_{kj}^{\text{osc}}} \right] \\ \times \exp \left[ - \left( \frac{L}{L_{kj}^{\text{coh}}} \right)^2 - 2\pi^2 \xi^2 \left( \frac{\sigma_x}{L_{kj}^{\text{osc}}} \right)^2 \right]$$

# OSCILACIJE NEUTRINA

$$\Delta m^2 = 10^{-3} \text{ eV}^2 \quad \sin^2 2\vartheta = 1 \quad E = 1 \text{ GeV} \quad \sigma_p = 50 \text{ MeV}$$

$$L^{\text{osc}} = \frac{4\pi E}{\Delta m^2} = 2480 \text{ km}$$

$$L^{\text{coh}} = \frac{4\sqrt{2}E^2}{|\Delta m^2|} \sigma_x = 11163 \text{ km}$$



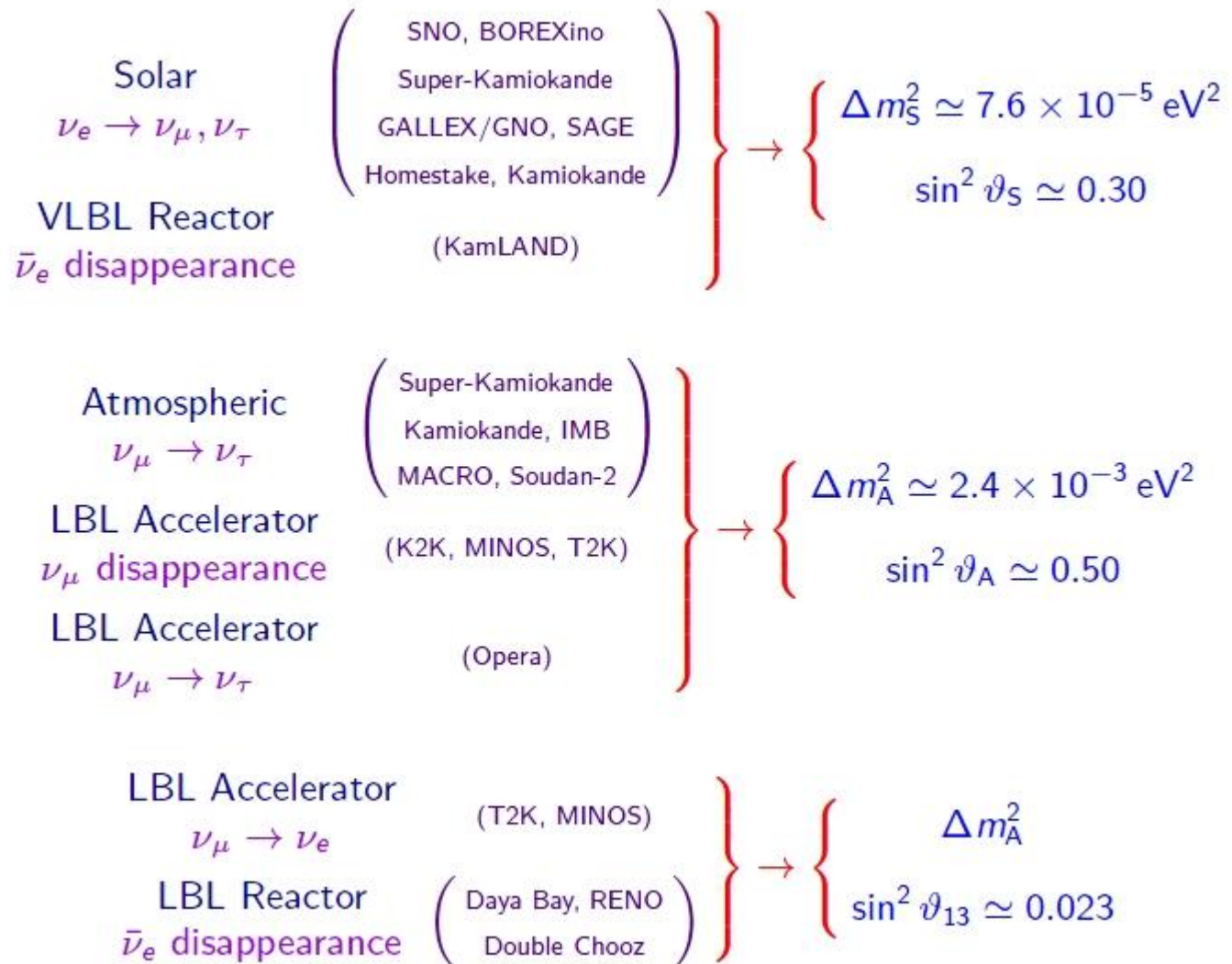
# PROBLEM SOLARNIH NEUTRINA

	Reaction	${}^8\text{B } \nu$ flux ( $10^6 \text{cm}^{-2}\text{s}^{-1}$ )
Kamiokande [7]	$\nu e$	$2.80 \pm 0.19 \pm 0.33$
Super-K I [114,116]	$\nu e$	$2.38 \pm 0.02 \pm 0.08$
Super-K II [115,116]	$\nu e$	$2.41 \pm 0.05^{+0.16}_{-0.15}$
Super-K III [116]	$\nu e$	$2.32 \pm 0.04 \pm 0.05$
SNO Phase I [14]	CC	$1.76^{+0.06}_{-0.05} \pm 0.09$
(pure D <sub>2</sub> O)	$\nu e$	$2.39^{+0.24}_{-0.23} \pm 0.12$
	NC	$5.09^{+0.44+0.46}_{-0.43-0.43}$
SNO Phase II [118]	CC	$1.68 \pm 0.06^{+0.08}_{-0.09}$
(NaCl in D <sub>2</sub> O)	$\nu e$	$2.35 \pm 0.22 \pm 0.15$
	NC	$4.94 \pm 0.21^{+0.38}_{-0.34}$
SNO Phase III [119]	CC	$1.67^{+0.05+0.07}_{-0.04-0.08}$
( <sup>3</sup> He counters)	$\nu e$	$1.77^{+0.24+0.09}_{-0.21-0.10}$
	NC	$5.54^{+0.33+0.36}_{-0.31-0.34}$
SNO Phase I+II [120]	NC	$5.140^{+0.160+0.132}_{-0.158-0.117}$
	$\Phi_B$ from fit to all reactions	$5.046^{+0.159+0.107}_{-0.152-0.123}$
SNO Phase I+II+III [121]	$\Phi_B$ from fit to all reactions	$5.25 \pm 0.16^{+0.11}_{-0.13}$
Borexino [126]	$\nu e$	$2.4 \pm 0.4 \pm 0.1$
SSM [BPS08(GS)] [104]	–	$5.94(1 \pm 0.11)$
SSM [SHP11(GS)] [106]	–	$5.58(1 \pm 0.14)$

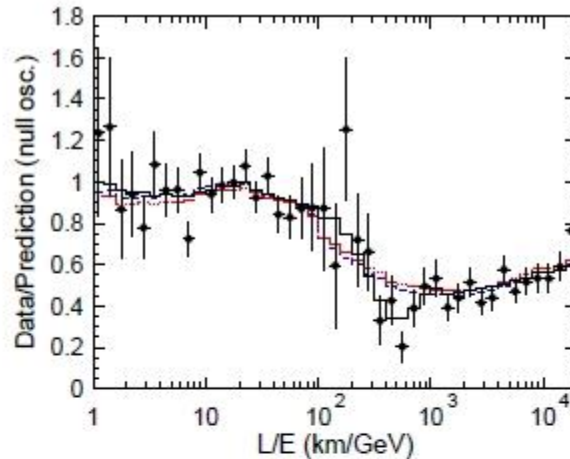
# OSCILACIJE NEUTRINA

Source	Type of $\nu$	$\bar{E}$ [MeV]	$L$ [km]	$\min(\Delta m^2)$ [eV <sup>2</sup> ]
Reactor	$\bar{\nu}_e$	$\sim 1$	1	$\sim 10^{-3}$
Reactor	$\bar{\nu}_e$	$\sim 1$	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1	$\sim 1$
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric $\nu$ 's	$\nu_{\mu,e}, \bar{\nu}_{\mu,e}$	$\sim 10^3$	$10^4$	$\sim 10^{-4}$
Sun	$\nu_e$	$\sim 1$	$1.5 \times 10^8$	$\sim 10^{-11}$

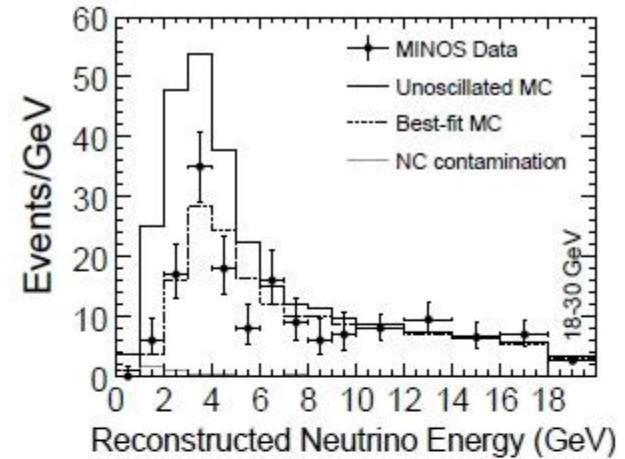
# OSCILACIJE NEUTRINA



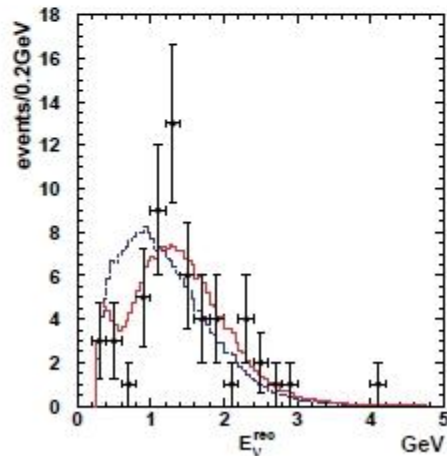
# OSCILACIJE NEUTRINA



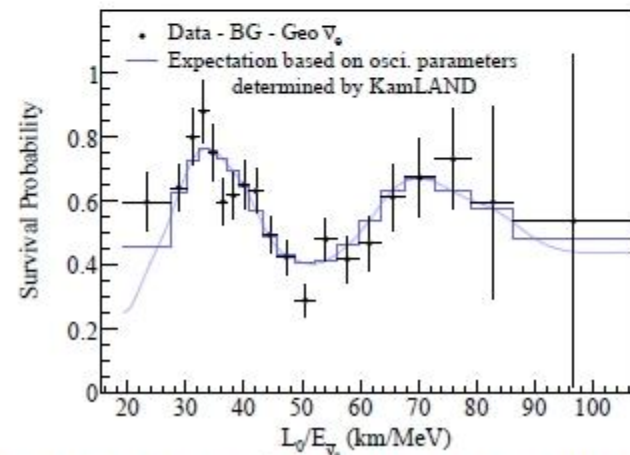
[Super-Kamiokande, PRL 93 (2004) 101801, hep-ex/0404034]



[MINOS, PRD 77 (2008) 072002, arXiv:0711.0769]

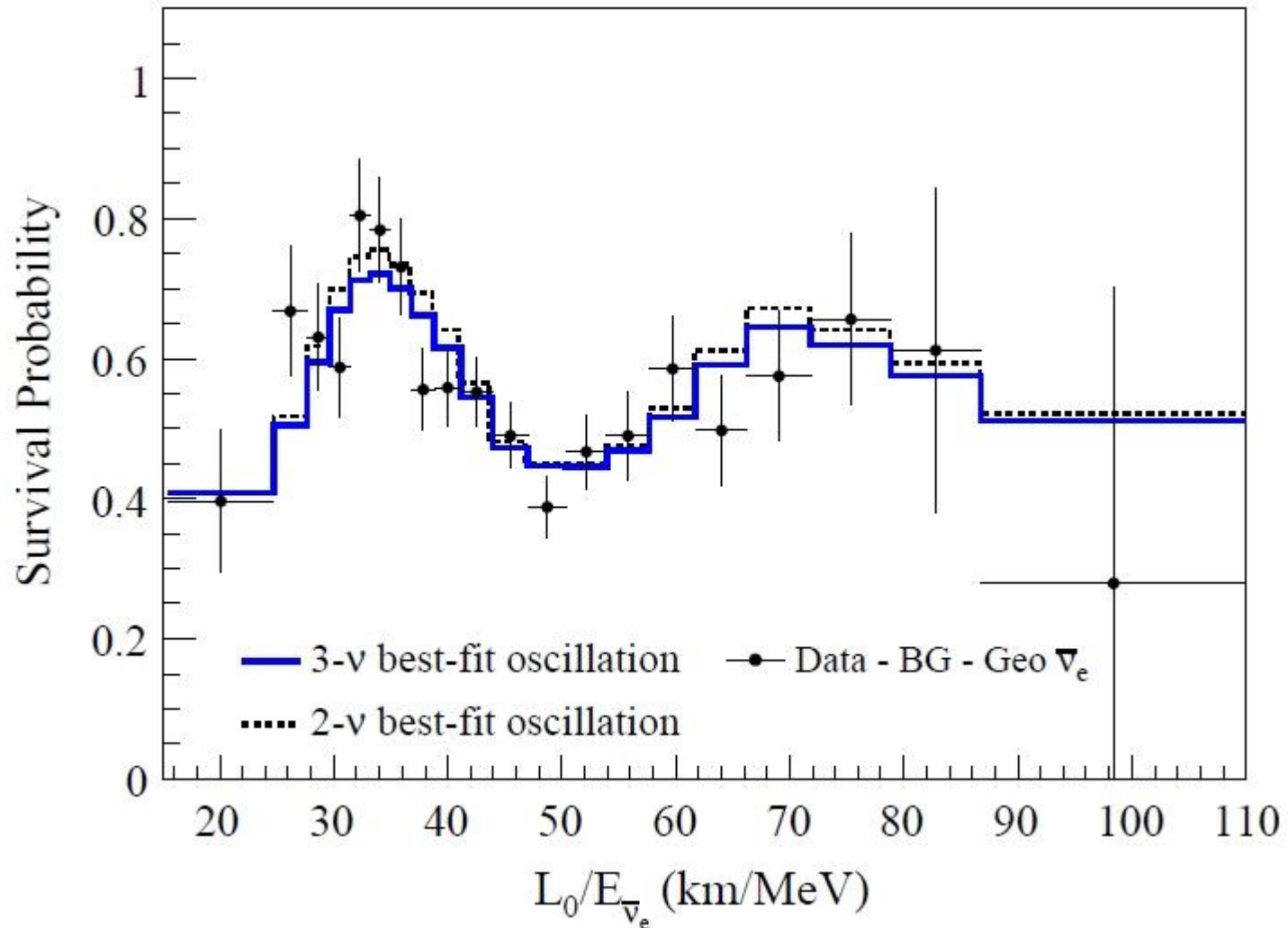


[K2K, PRD 74 (2006) 072003, hep-ex/0606032v3]



[KamLAND, PRL 100 (2008) 221803, arXiv:0801.4589]

# KAMLAND



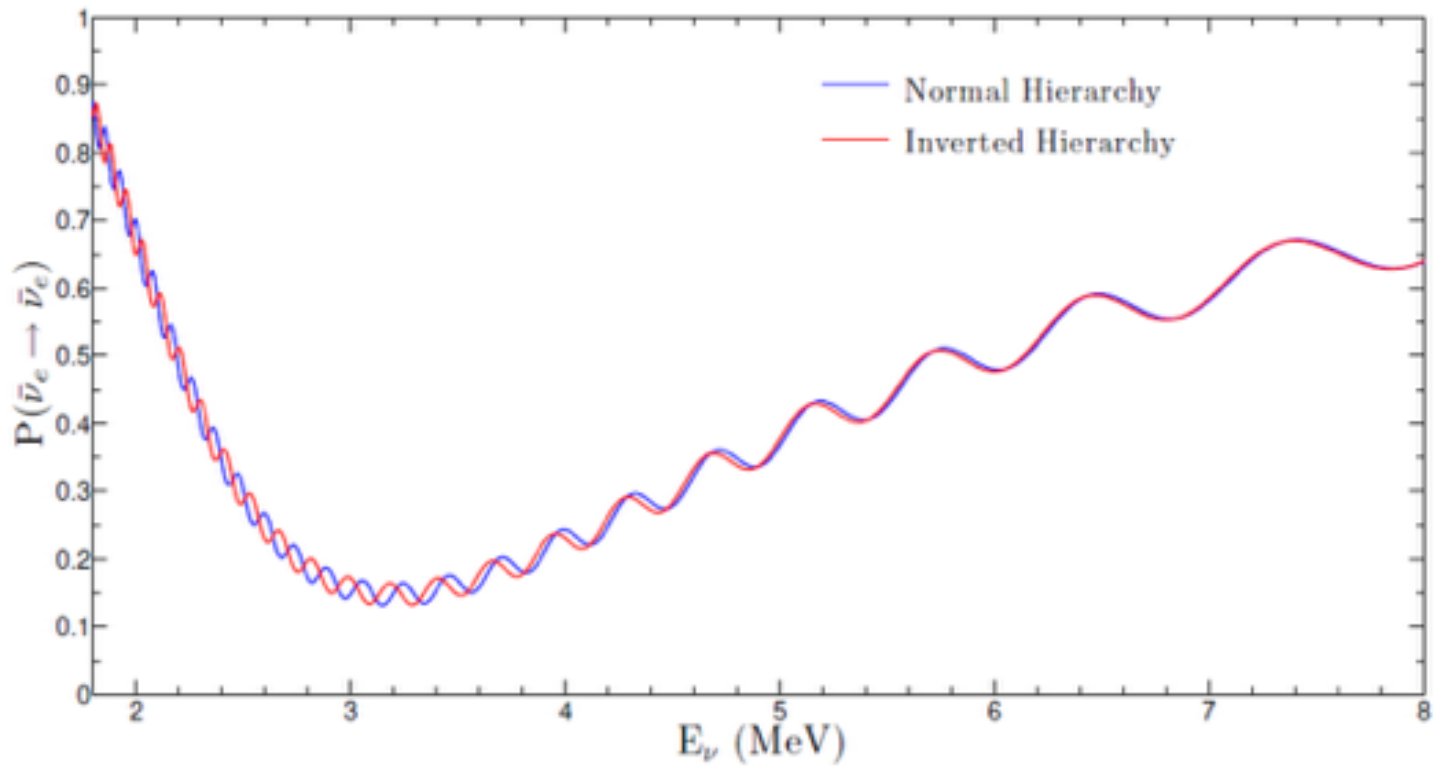


# GLOBALNA PRILAGODBA

	Normal Ordering ( $\Delta\chi^2 = 0.97$ )		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	0.270 $\rightarrow$ 0.344	$0.304^{+0.013}_{-0.012}$	0.270 $\rightarrow$ 0.344	0.270 $\rightarrow$ 0.344
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	31.29 $\rightarrow$ 35.91	$33.48^{+0.78}_{-0.75}$	31.29 $\rightarrow$ 35.91	31.29 $\rightarrow$ 35.91
$\sin^2 \theta_{23}$	$0.452^{+0.052}_{-0.028}$	0.382 $\rightarrow$ 0.643	$0.579^{+0.025}_{-0.037}$	0.389 $\rightarrow$ 0.644	0.385 $\rightarrow$ 0.644
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	38.2 $\rightarrow$ 53.3	$49.5^{+1.5}_{-2.2}$	38.6 $\rightarrow$ 53.3	38.3 $\rightarrow$ 53.3
$\sin^2 \theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	0.0186 $\rightarrow$ 0.0250	$0.0219^{+0.0011}_{-0.0010}$	0.0188 $\rightarrow$ 0.0251	0.0188 $\rightarrow$ 0.0251
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	7.85 $\rightarrow$ 9.10	$8.51^{+0.20}_{-0.21}$	7.87 $\rightarrow$ 9.11	7.87 $\rightarrow$ 9.11
$\delta_{CP}/^\circ$	$306^{+39}_{-70}$	0 $\rightarrow$ 360	$254^{+63}_{-62}$	0 $\rightarrow$ 360	0 $\rightarrow$ 360
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.50^{+0.19}_{-0.17}$	7.02 $\rightarrow$ 8.09	$7.50^{+0.19}_{-0.17}$	7.02 $\rightarrow$ 8.09	7.02 $\rightarrow$ 8.09
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.457^{+0.047}_{-0.047}$	+2.317 $\rightarrow$ +2.607	$-2.449^{+0.048}_{-0.047}$	-2.590 $\rightarrow$ -2.307	$\left[ \begin{array}{l} +2.325 \rightarrow +2.599 \\ -2.590 \rightarrow -2.307 \end{array} \right]$

Gonzalez-Garcia, Maltoni, Schwetz arXiv:1409.5439

# OSCILACIJE NEUTRINA



# OSCILACIJE U MATERIJU

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + \sqrt{2} G_F N_e & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

# MASE NEUTRINA

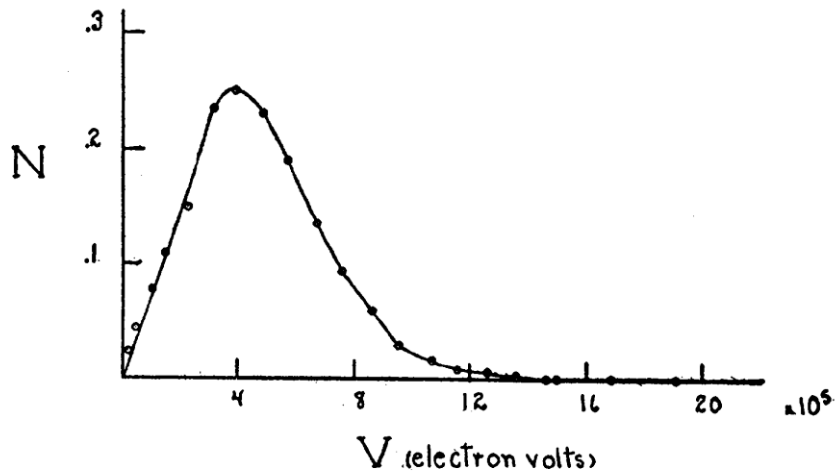


FIG. 5. Energy distribution curve of the beta-rays.

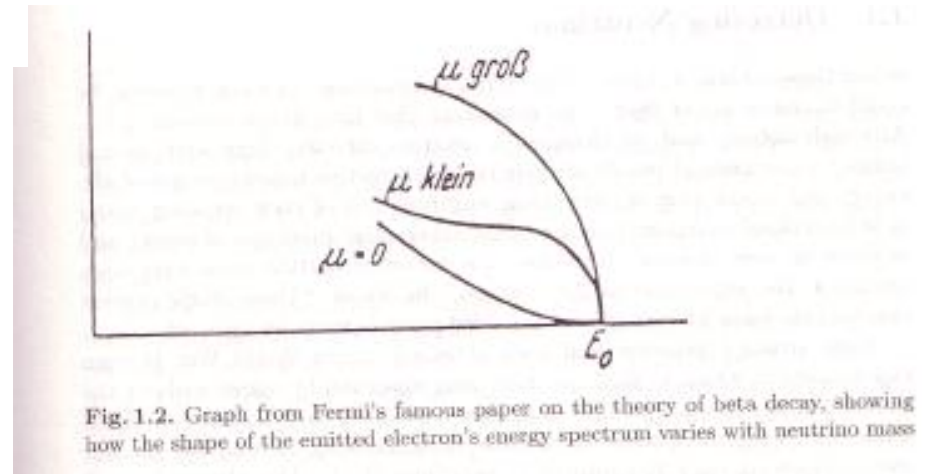


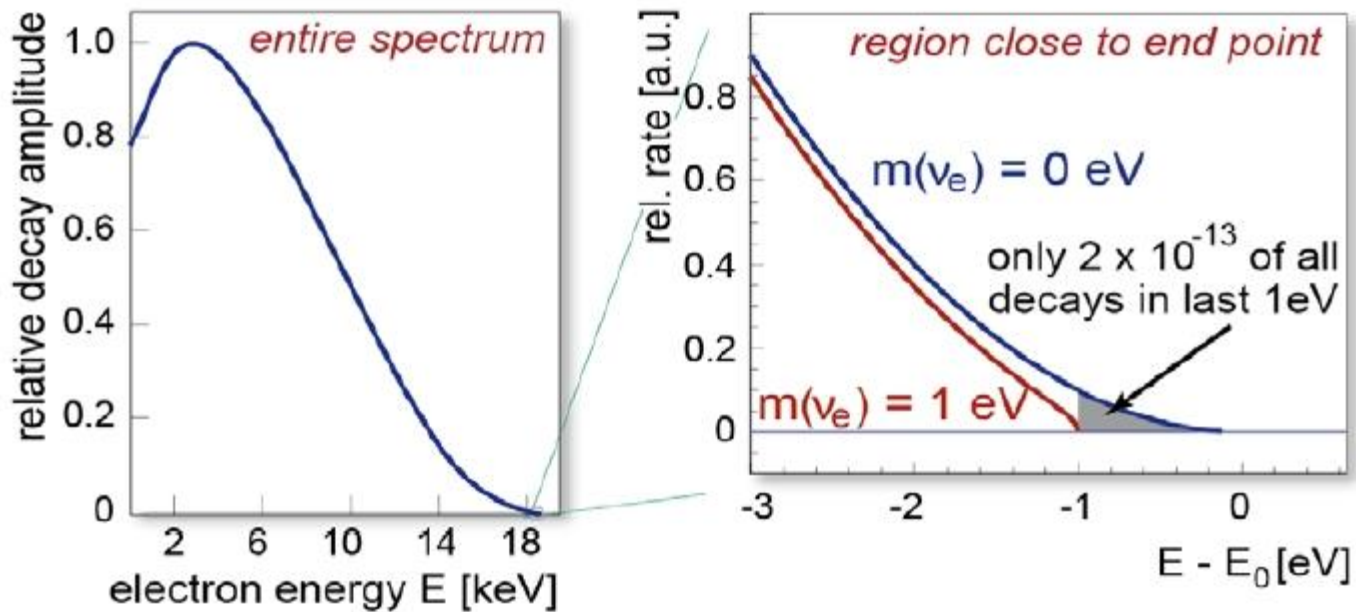
Fig. 1.2. Graph from Fermi's famous paper on the theory of beta decay, showing how the shape of the emitted electron's energy spectrum varies with neutrino mass

# KATRIN – MASE NEUTRINA



# KATRIN – MASE NEUTRINA

$\beta$ -spectrum for tritium:



# SAŽETAK

- Promjene okusa neutrina prilikom propagacije (oscilacije neutrina) potvrđene u nizu eksperimenata
- Oscilacije neutrina vode na masivne neutrine
- $P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \sin^2(\Delta m^2 L / 4E)$
- Zašto su mase neutrina tako male?