



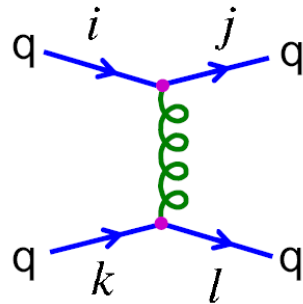
IV. Kvantna kromodinamika i hadroni

KLIZNO VEZANJE I REŽIMI QCD-a

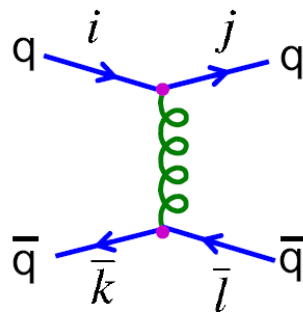
- **REŽIM ZATOČENJA**
- **REŽIM ASIMPTOTSKE SLOBODE**
- **BOJNO PRIVLAČNI KRATKODOSEŽNI KANAL**

BOJNI FAKTORI ZA GRANASTE PROCESE

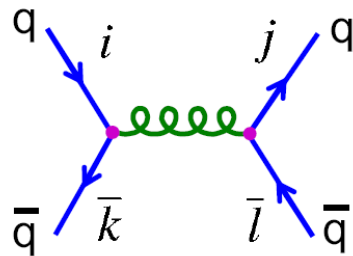
- Consequently the colour factors for the different diagrams are:



$$C(ik \rightarrow jl) \equiv \frac{1}{4} \sum_{a=1}^8 \lambda_{ji}^a \lambda_{lk}^a$$



$$C(i\bar{k} \rightarrow j\bar{l}) \equiv \frac{1}{4} \sum_{a=1}^8 \lambda_{ji}^a \lambda_{kl}^a$$



$$C(i\bar{k} \rightarrow j\bar{l}) \equiv \frac{1}{4} \sum_{a=1}^8 \lambda_{ki}^a \lambda_{jl}^a$$

e.g.

$$C(rr \rightarrow rr) = \frac{1}{3}$$

$$C(rg \rightarrow rg) = -\frac{1}{6}$$

$$C(rg \rightarrow gr) = \frac{1}{2}$$

$$C(r\bar{r} \rightarrow r\bar{r}) = \frac{1}{3}$$

$$C(r\bar{g} \rightarrow r\bar{g}) = -\frac{1}{6}$$

$$C(r\bar{r} \rightarrow g\bar{g}) = \frac{1}{2}$$

$$C(r\bar{r} \rightarrow r\bar{r}) = \frac{1}{3}$$

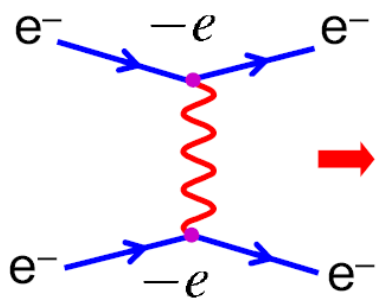
$$C(r\bar{g} \rightarrow r\bar{g}) = \frac{1}{2}$$

$$C(r\bar{r} \rightarrow g\bar{g}) = -\frac{1}{6}$$

Colour index of adjoint spinor comes first

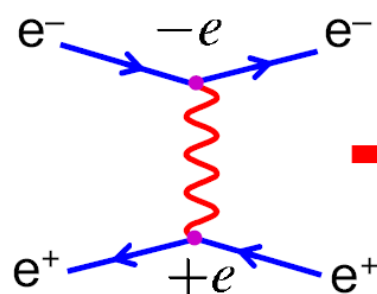
TRAŽENJE PRIVLAČNIH KONFIGURACIJA

QED



$$V(r) = +\frac{\alpha}{r}$$

Repulsive Potential

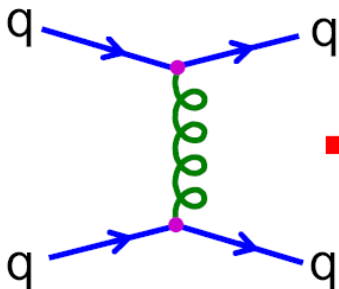


$$V(r) = -\frac{\alpha}{r}$$

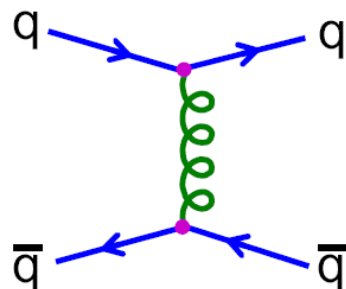
Attractive Potential

QCD

★ by analogy with QED expect potentials of form



$$V(r) = +C\frac{\alpha_S}{r}$$



$$V(r) = -C\frac{\alpha_S}{r}$$

★ Whether it is a attractive or repulsive potential depends on **sign of colour factor**

PRIVLAČENJE U SINGLETU

- ★ Consider the colour factor for a qq system in the colour singlet state:

$$\psi = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

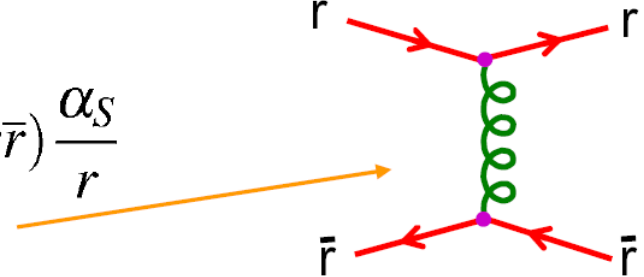
with colour potential $\langle V_{q\bar{q}} \rangle = \langle \psi | V_{\text{QCD}} | \psi \rangle$

$$\rightarrow \langle V_{q\bar{q}} \rangle = \frac{1}{3} (\langle r\bar{r} | V_{\text{QCD}} | r\bar{r} \rangle + \dots + \langle r\bar{r} | V_{\text{QCD}} | b\bar{b} \rangle + \dots)$$

- Following the QED analogy:

$$\langle r\bar{r} | V_{\text{QCD}} | r\bar{r} \rangle = -C(r\bar{r} \rightarrow r\bar{r}) \frac{\alpha_S}{r}$$

which is the term arising from $r\bar{r} \rightarrow r\bar{r}$



- Have 3 terms like $r\bar{r} \rightarrow r\bar{r}$, $b\bar{b} \rightarrow b\bar{b}$, ... and 6 like $r\bar{r} \rightarrow g\bar{g}$, $r\bar{r} \rightarrow b\bar{b}$, ...

$$\langle V_{q\bar{q}} \rangle = -\frac{1}{3} \frac{\alpha_S}{r} [3 \times C(r\bar{r} \rightarrow r\bar{r}) + 6 \times C(r\bar{r} \rightarrow g\bar{g})] = -\frac{1}{3} \frac{\alpha_S}{r} \left[3 \times \frac{1}{3} + 6 \times \frac{1}{2} \right]$$

$$\rightarrow \langle V_{q\bar{q}} \rangle = -\frac{4}{3} \frac{\alpha_S}{r}$$

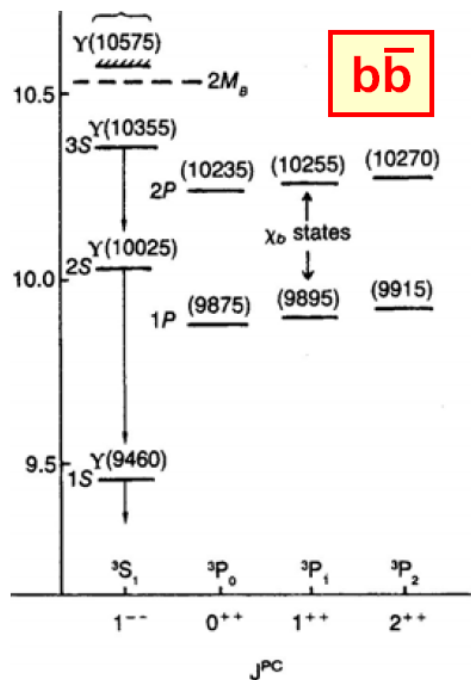
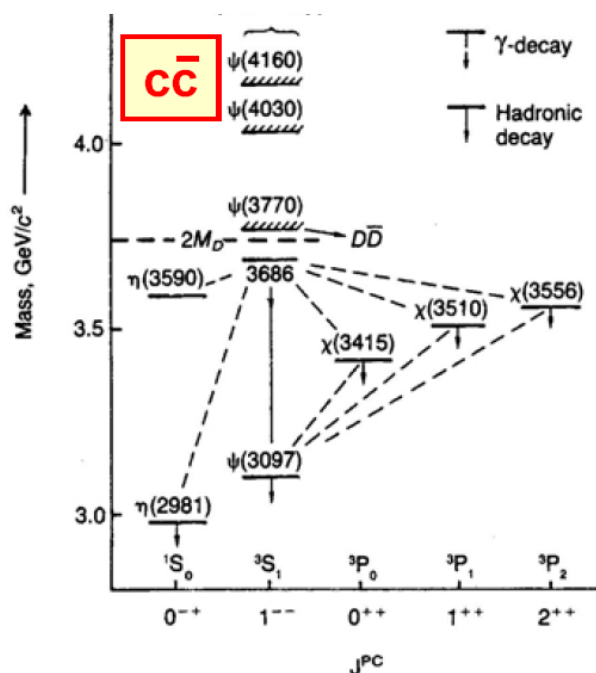
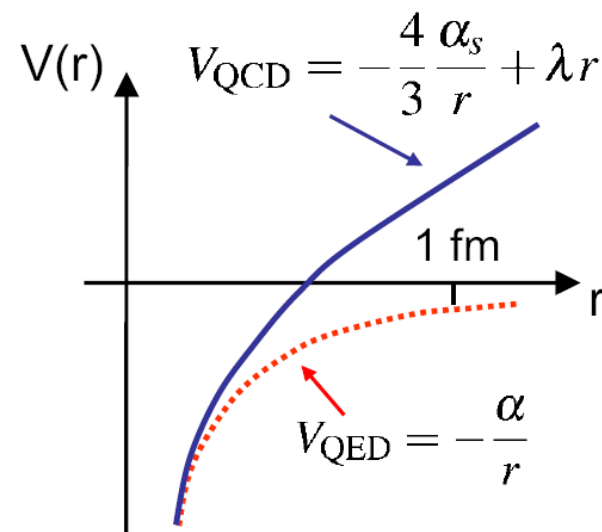
NEGATIVE → ATTRACTIVE

- The same calculation for a colour octet state, e.g. $r\bar{g}$ gives a positive repulsive potential: $C(r\bar{g} \rightarrow r\bar{g}) = -\frac{1}{6}$

- ★ Combining the short-range QCD potential with the linear long-range term discussed previously:

$$V_{\text{QCD}} = -\frac{4}{3} \frac{\alpha_s}{r} + \lambda r$$

- ★ This potential is found to give a good description of the observed charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) bound states.

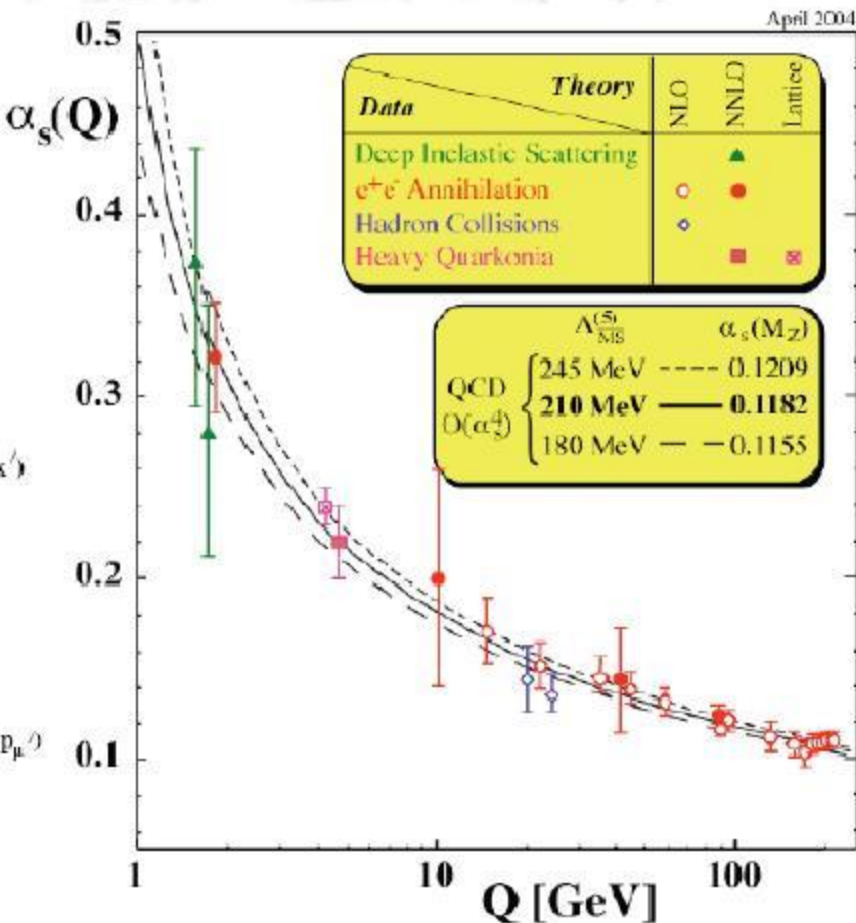


NOTE:

- c, b are heavy quarks
- approx. non-relativistic
- orbit close together
- probe $1/r$ part of V_{QCD}

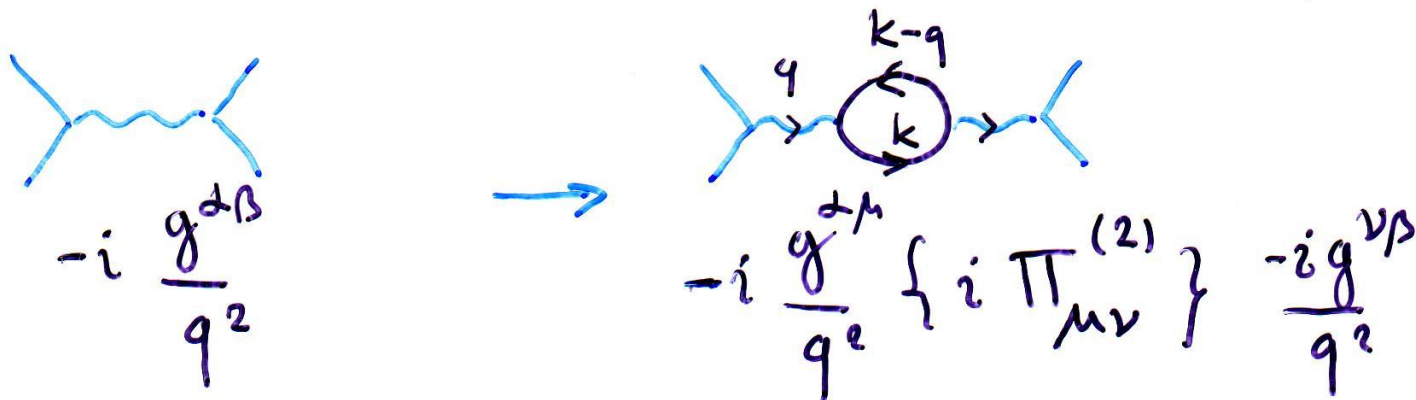
Agreement of data with prediction provides strong evidence that V_{QCD} has the Expected form

PROMJENLJIVA ("RUNING") ili KLIZNA JAKOST VEZANJA



VAKUUMSKA POLARIZACIJA U QED (FEČ str. 161)

- pr. QED u Feynmanovom bazidarenju



$$\begin{aligned}
 i \Pi_{\mu\nu}^{(2)} &= (-1) (ie)^2 \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \{ \gamma_\mu i S(k) \gamma_\nu i S(k-q) \} \\
 &= -e^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\text{Tr} [\gamma_\mu (\not{k} + m) \gamma_\nu (\not{k} - \not{q} + m)]}{(k^2 - m^2 + i\epsilon) [(k-q)^2 - m^2 + i\epsilon]}
 \end{aligned}$$

KLIZNA JAKOST VEZANJA U QED

- MIJENJA COULOMBOV ZAKON
- RENORMALIZIRA NABOJ: za $-q^2 \gg m_e^2$

$$\alpha(-q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln\left(\frac{-q^2}{m_e^2}\right)} \quad \left\{ \begin{array}{l} \alpha(m_e^2) \equiv \alpha \approx \frac{1}{137} \\ \alpha(M_Z^2) \approx \frac{1}{128} \end{array} \right.$$

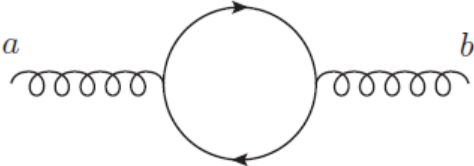
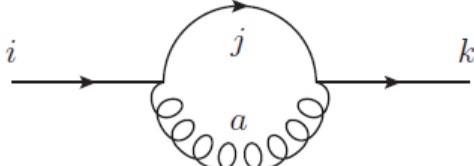
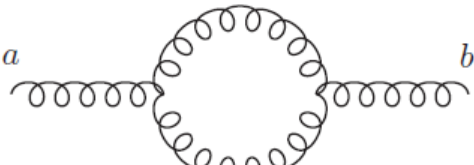
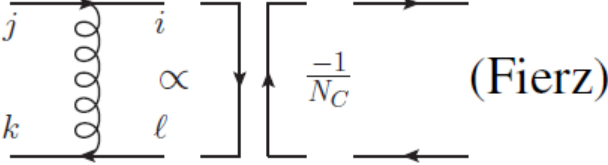
POSJEDUJE LANDAUOV POL

$$-q^2 = m_e^2 e^{\frac{3\pi}{\alpha}} \approx m_e^2 10^{560}$$

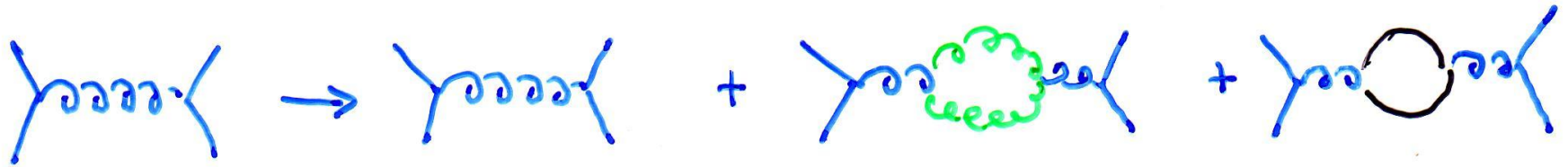
Tahionska
čestica

$$M^2 = -m_e^2 \cdot 10^{560} \Rightarrow v^2 = 1 - \frac{M^2}{E^2} > 1(c)$$

BOJNI FAKTORI ZA PROCESE QCD PETLJI

Trace Relation	Indices	Occurs in Diagram Squared
$\text{Tr}\{t^a t^b\} = T_R \delta^{ab}$	$a, b \in [1, \dots, 8]$	
$\sum_a t_{ij}^a t_{jk}^a = C_F \delta_{ik}$	$a \in [1, \dots, 8]$ $i, j, k \in [1, \dots, 3]$	
$\sum_{c,d} f^{acd} f^{bcd} = C_A \delta^{ab}$	$a, b, c, d \in [1, \dots, 8]$	
$t_{ij}^a t_{kl}^a = T_R \left(\delta_{jk} \delta_{il} - \frac{1}{N_C} \delta_{ij} \delta_{kl} \right)$	$i, j, k, l \in [1, \dots, 3]$	 (Fierz)

VAKUUMSKA POLARIZACIJA ZA QCD (FEČ str. 234)



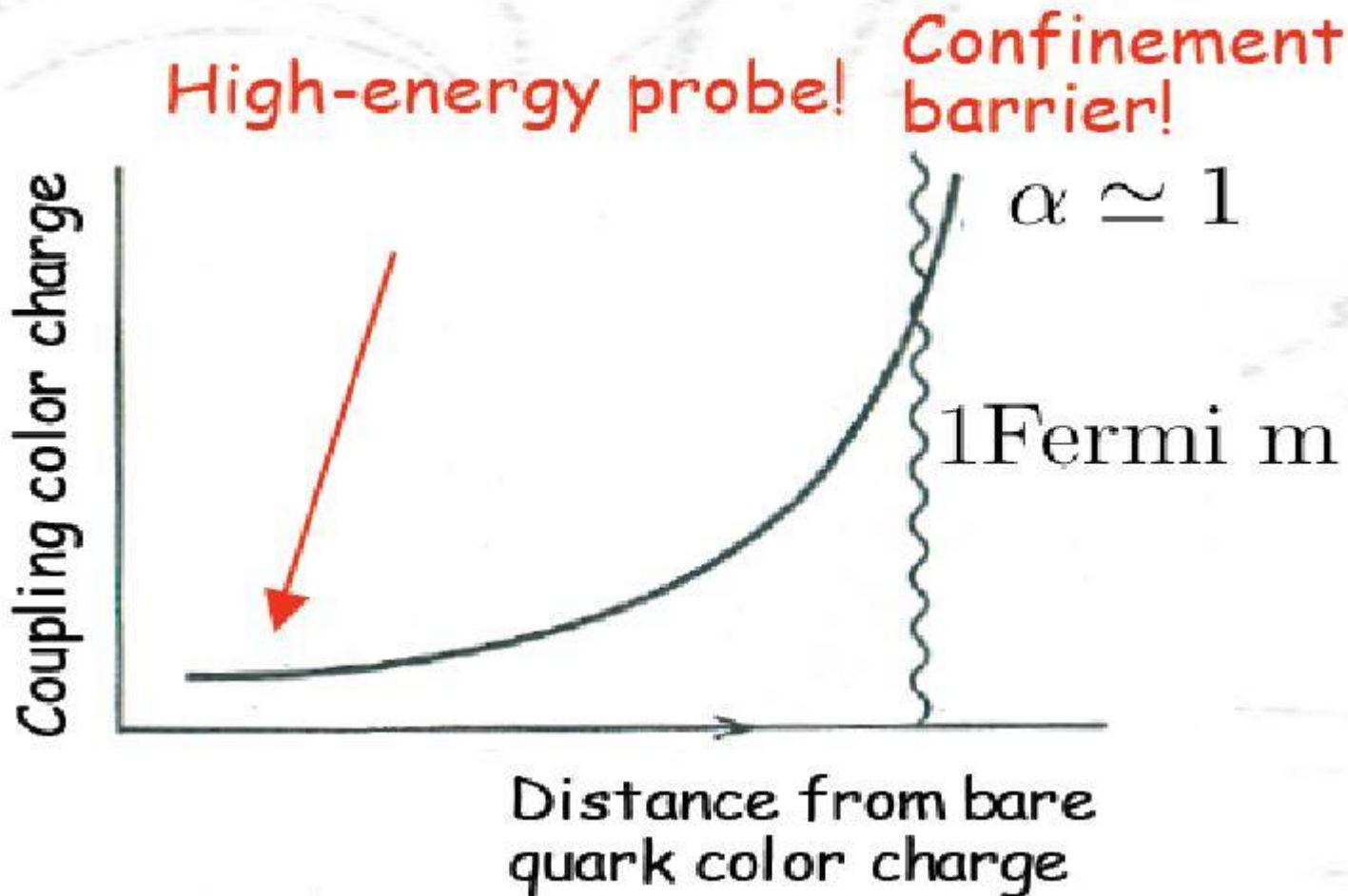
$$\alpha_s \rightarrow \alpha_s(-q^2) = \alpha_s(\mu^2) \left[1 + \underbrace{\left(11 - \frac{2}{3} N_F \right)}_{\equiv b_0} \frac{\alpha_s(\mu^2)}{4\pi} \ln\left(\frac{-q^2}{\mu^2}\right) \right]^{-1}$$

◇ slobodna RENORMALIZACIJSKA TOČKA na kojoj mjerenje ustanovljava $\alpha_s^0 \equiv \alpha_s(\mu^2)$

$$\Rightarrow \text{za } M^2 = -q^2 > \mu^2 : \frac{1}{\alpha_s(M^2)} = \frac{1}{\alpha_s(\mu^2)} + \frac{b_0}{4\pi} \ln \frac{M^2}{\mu^2}$$

$$\alpha_s(134 \text{ GeV})^2 = 0.148 \pm 0.018$$

$$\alpha_s(M_Z^2) = 0.118 \pm 0.003$$



◇ Λ_{QCD} kao Landauovljevu pol QCD-u

uvaden tako da

$$\text{za } M^2 = -q^2 \rightarrow \Lambda_{\text{QCD}}^2 \quad \Rightarrow \quad \alpha_s(\Lambda_{\text{QCD}}) \rightarrow \infty$$

$$\alpha_s(M^2) = \frac{\alpha_s^0}{1 + b_0 \frac{\alpha_s^0}{4\pi} \ln \frac{M^2}{\mu^2}} \stackrel{\text{def.}}{=} \frac{1}{\frac{b_0}{4\pi} \ln \frac{M^2}{\Lambda_{\text{QCD}}^2}}$$

$$\Rightarrow \Lambda_{\text{QCD}}^2 = \mu^2 \exp\left(-\frac{4\pi}{\alpha_s^0 b_0}\right)$$

ANTIZASJENJENJE I ASIMPTOTSKA SLOBODA (Mat.Fiz.List 2/218 ('04/05) 91

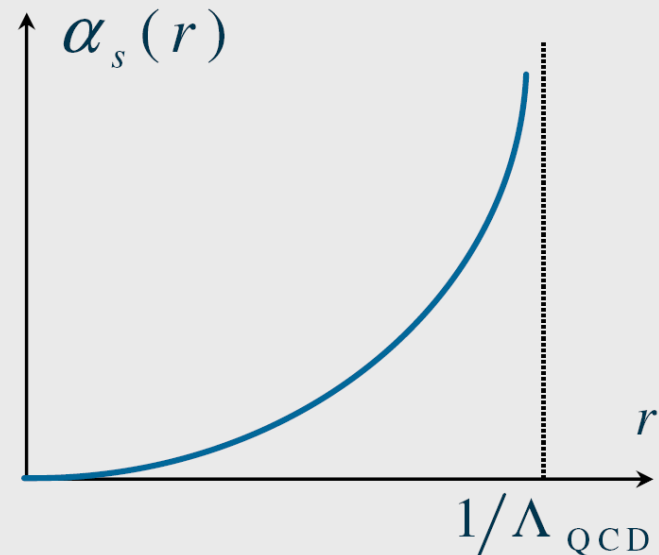
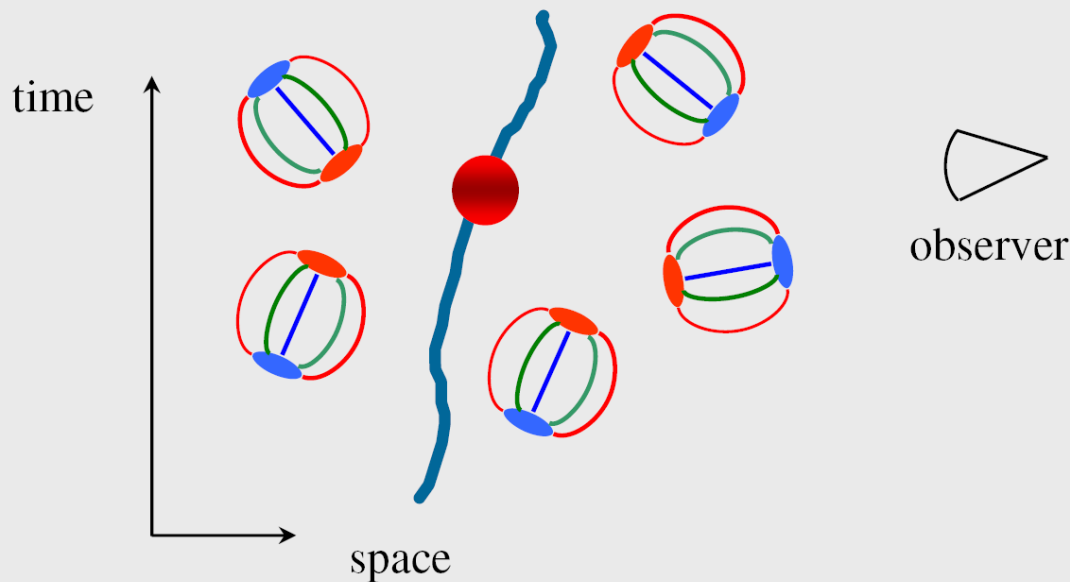
Asymptotic freedom at short distances:

D. J. Gross, H. D. Politzer, F. Wilczek,
2004 Nobel Prize

$$\alpha_s(r) = \frac{1}{\beta_0 \ln(1/r\Lambda_{\text{QCD}})}$$

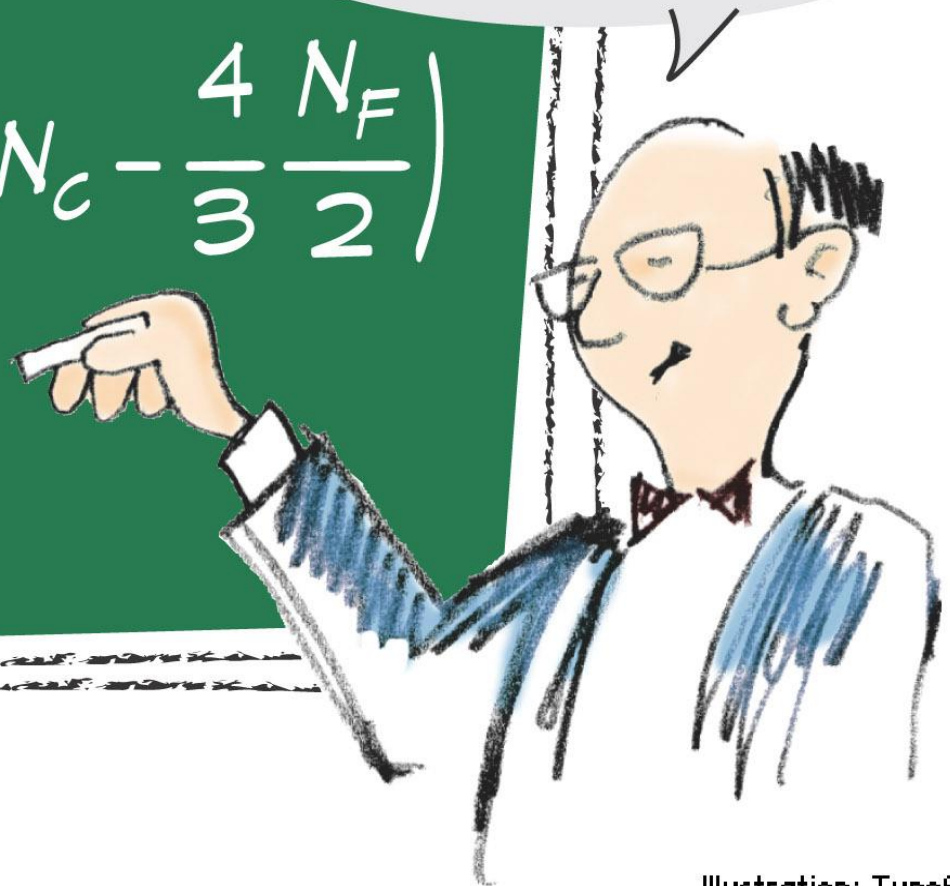
$$1/\Lambda_{\text{QCD}} \approx 10^{-15} \text{ m}$$

charge anti-screening



In QCD and the Standard Model
the beta function is indeed
negative!

$$\beta(g) = \frac{-g^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{4}{3} \frac{N_F}{2} \right)$$



THE ROYAL SWEDISH ACADEMY OF SCIENCES HAS DECIDED TO AWARD THE NOBEL PRIZE IN PHYSICS FOR 2004 "FOR THE DISCOVERY OF ASYMPTOTIC FREEDOM IN THE THEORY OF THE STRONG INTERACTION" JOINTLY TO DAVID J. GROSS, H. DAVID POLITZER AND FRANK WILCZEK

A colourful connection

The scientists awarded this year's Nobel Prize in Physics have solved a mystery surrounding the strongest of nature's four fundamental forces. The three quarks within the proton can sometimes appear to be free, although no free quarks have ever been observed. The quarks have a quantum mechanical property called colour and interact with each other through the exchange of gluons – nature's glue.



A high-energy electron on collision course with ...

... a quark, confined in the proton.

The Standard Model and the four forces

The quarks and gluons of the strong (or colour) force are the third piece in the puzzle of nature's four forces. The first piece, the electromagnetic force, is similar to the strong force but instead of gluons, particles of light, photons, are the force carriers. The gluons carry colour charge while the photons are electrically neutral. The second piece in the puzzle is the weak force, which controls some radioactive decays and energy production in the sun. This force differs from the other two because the force-carrying particles are very heavy. The fourth force, gravity, is the least understood even though it is experienced by us all. Gravitons are thought to be the force-carrying particles, but they have yet to be discovered. The Standard Model provides a description of all the forces apart from gravity.

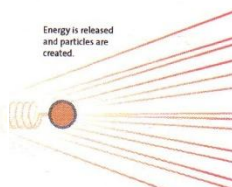


Inside the proton

The three quarks within the proton are held together by the powerful force mediated by the gluons, depicted here as coiled springs. As the distance between the quarks increases so does the force between them.



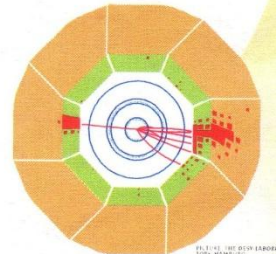
If a quark is knocked out of the proton in a high energy collision, it appears to behave as a free particle for an instant.



Energy is released and particles are created.

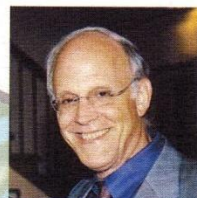
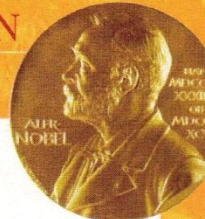
THE THEORY SHOWS ITS TRUE COLOURS

The aftermath of a high energy collision between a proton and an electron, as seen by the H_x experiment at the DESY laboratory in Hamburg. The experiment is shown in cross-section, perpendicular to colliding beams of protons and electrons. The electron has struck one of the quarks in a proton. An impressive shower of particles – providing information about the struck quark – is spontaneously produced from the energy stored in the gluon force-field. The charged particles in the shower bend in the experiment's strong magnetic field.



DESY, HAMBURG

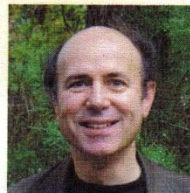
THE NOBEL PRIZE IN PHYSICS 2004



David J. Gross
Kavli Institute for Theoretical Physics,
University of California, Santa Barbara, USA



H. David Politzer
California Institute of Technology
(Caltech), Pasadena, USA



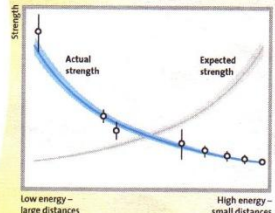
Frank Wilczek
Massachusetts Institute of Technology
(MIT), Cambridge, USA

A good start ...

Frank Wilczek and David Politzer were barely 20 years old and still PhD students when their discovery of asymptotic freedom was published. These were their very first scientific publications!

Many tried, but failed, to find a theory in which the strength of the strong force decreases as the energy increases. This year's Nobel laureates found a theory with the required minus sign. When the quarks are very close to each other, i.e. when the distance between them is asymptotically approaching zero, the force is so weak that they behave almost as free particles.

$$\beta(g) = -\frac{g^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{4}{3} \frac{N_f}{2} \right)$$



Low energy – large distances
High energy – small distances

A unified theory for all forces?

This year's prize paves the way for a more fundamental future description of the forces in nature. The electromagnetic, weak and strong forces have much in common and are perhaps different aspects of a single force. They also appear to have the same strength at very high energies, especially if 'supersymmetric' particles exist. It may even be possible to include gravity if theories which treat matter as small vibrating strings are correct.



EARLIER NOBEL LAUREATES WHOSE WORK WAS OF GREAT CONSEQUENCE FOR THIS YEAR'S AWARD:

- 1949 HIROKI YUKAWA, The theory of nuclear forces
- 1957 CHEN NING YANG AND TING-DAO LEI, Parity violation in particle physics
- 1965 SHIN-ICHI TOMONAGA, JULIAN SCHWINGER AND RICHARD P. FEYNMAN, QED – the quantum theory of electromagnetic interactions

- 1969 MURRAY GELL-MANN, Symmetry properties of elementary particles

- 1979 SHELDON LEE GLASHOW, ABUS SALAM AND STEVEN WEINBERG, The theory of electroweak interactions

- 1982 KENNETH G WILSON, The theory of phase transformations

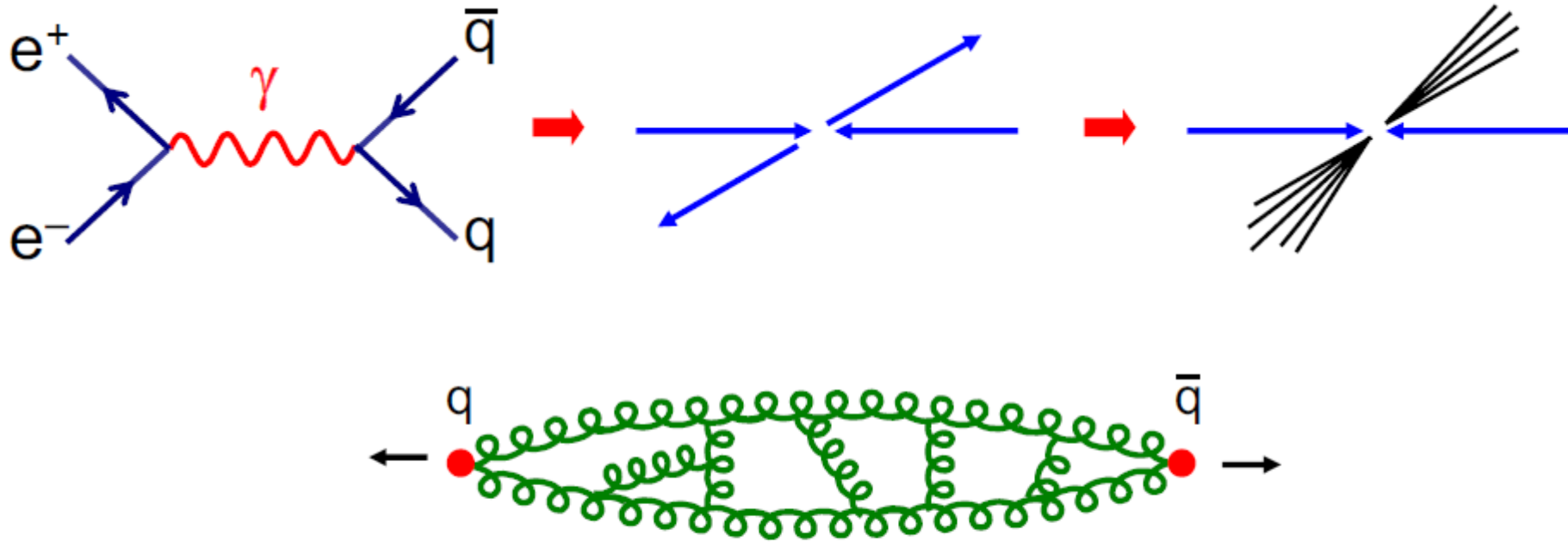
- 1990 JEROME I. FRIEDMAN, HENRY W. KENDALL AND RICHARD E. TAYLOR, The discovery of quarks through electron-scattering experiments

- 1999 GERARDUS 'T HOOFT AND MARTINUS J.G. VELTMAN, The quantum structure of the electroweak interaction

FURTHER READING

- Information on the Nobel Prize in Physics 2004: www.nobelprize.org
- CERN: www.cern.ch
- DESY: www.desy.de
- Hands-on CERN: <http://handson.cern.ch>
- The particle adventure: <http://particleadventure.org/particleadventure/>
- QCD Made Simple, *Physics Today* August 2000, p. 22
- Joining up the dots with the strong force, by C. Davies, *CERN Courier* June 2004, p. 23
- The W and Z at LEP, by C. Sutton and P. Zerwas, *CERN Courier* May 2004, p. 21
- Lattice Quantum Chromodynamics Comes of Age, by C. De Tar and S. Gottlieb, *Physics Today* February 2004, p. 45
- In search of the ultimate building blocks, by G. 't Hooft, *Cambridge University Press* 1997

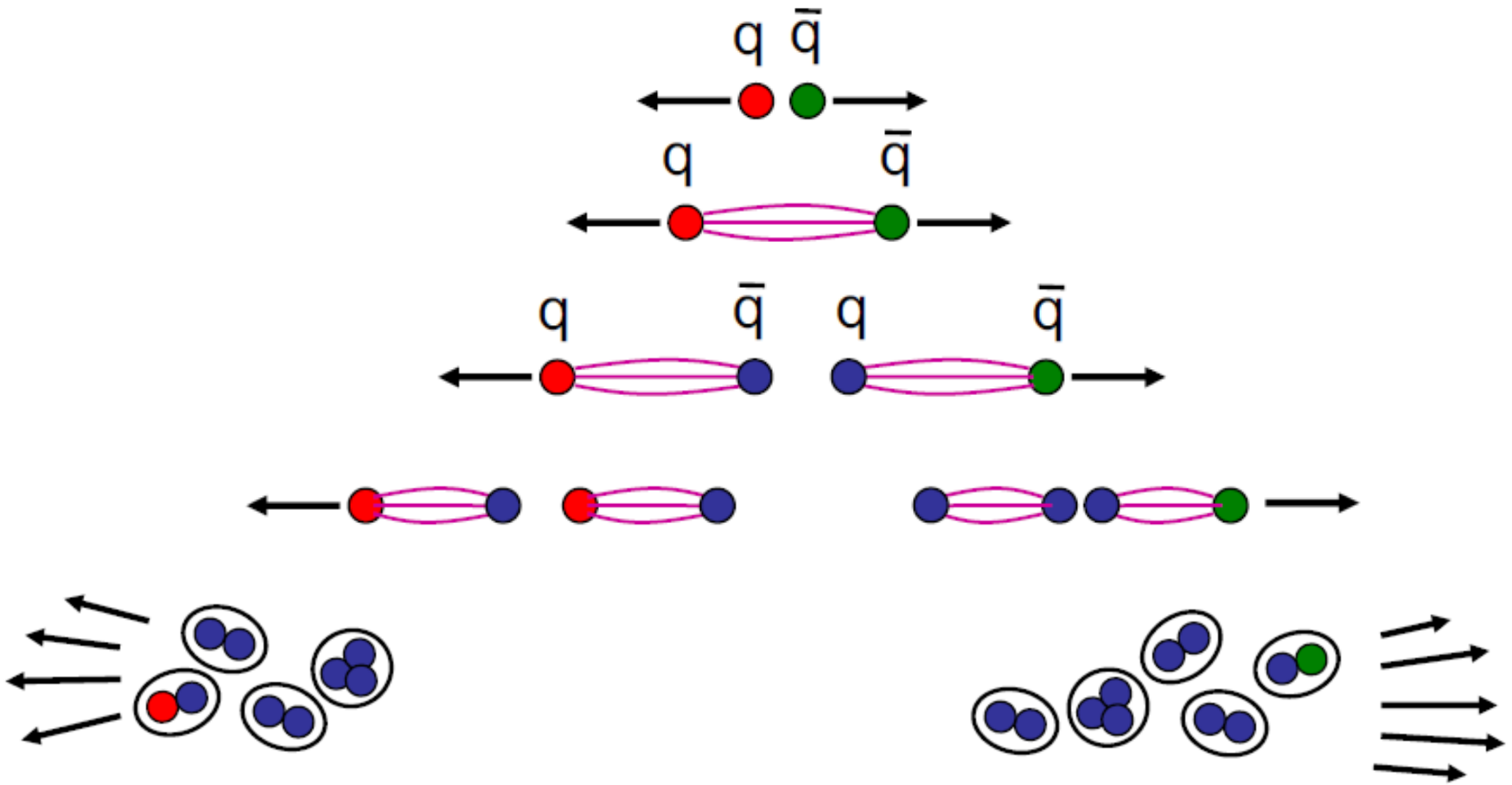
HADRONSKI MLAZOVI



Form a flux tube of interacting gluons of approximately constant energy density $\sim 1 \text{ GeV/fm}$

$\Rightarrow V(r) \sim \lambda r$

HADRONIZACIJA U MLAZOVE



BOJNI FAKTOR PRI ANIHILACIJI ELEKTRONA I POZITRONA

Colour is conserved and quarks are produced as $r\bar{r}$, $g\bar{g}$, $b\bar{b}$
For a **single quark flavour** and **single colour**

$$\sigma(e^+e^- \rightarrow q_i\bar{q}_i) = \frac{4\pi\alpha^2}{3s} Q_q^2$$

- Experimentally observe jets of hadrons:

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = 3 \sum_{u,d,s,\dots} \frac{4\pi\alpha^2}{3s} Q_q^2$$

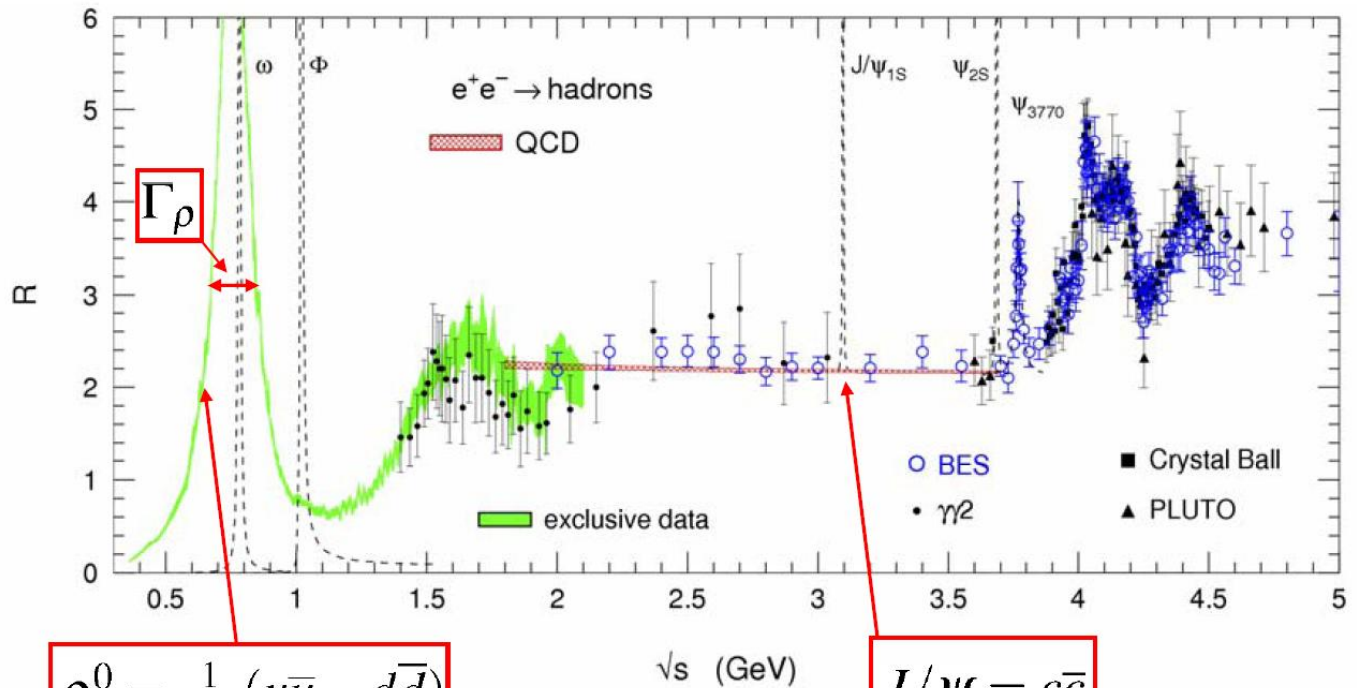
Factor 3 comes from colours

- Usual to express as ratio compared to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

$$R_\mu = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_{u,d,s,\dots} Q_q^2$$

NISKOENERGIJSKE REZONANTNE PRODUKCIJE

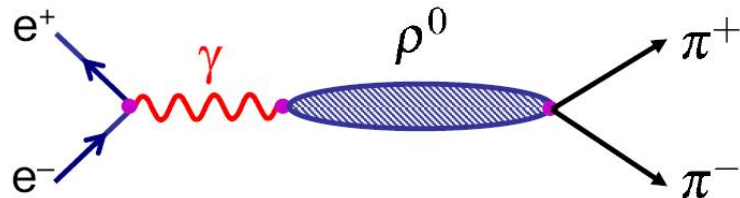
- Low energy region complicated by resonant production of decaying meson states



$$\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$J/\psi = c\bar{c}$$

e.g.



FWHM Width of resonance:

$$\Gamma_\rho = 146 \text{ MeV}$$

PRODUKCIJE GLUONA

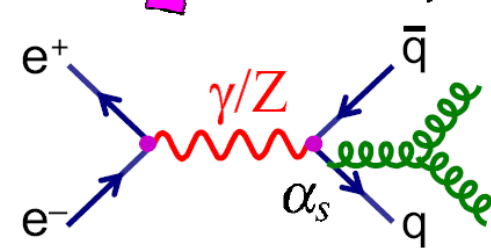
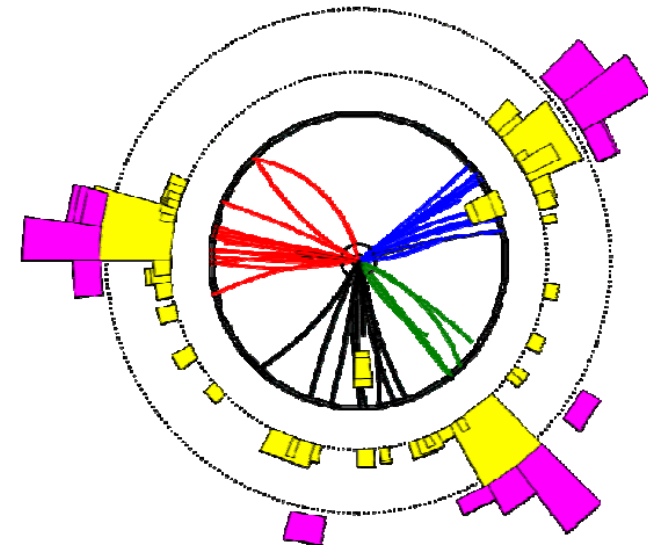
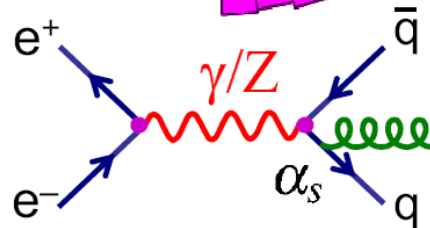
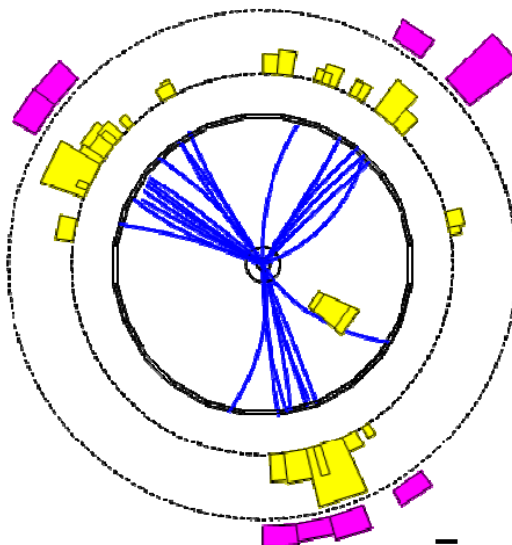
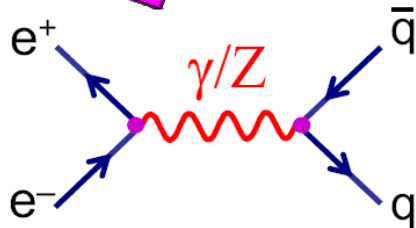
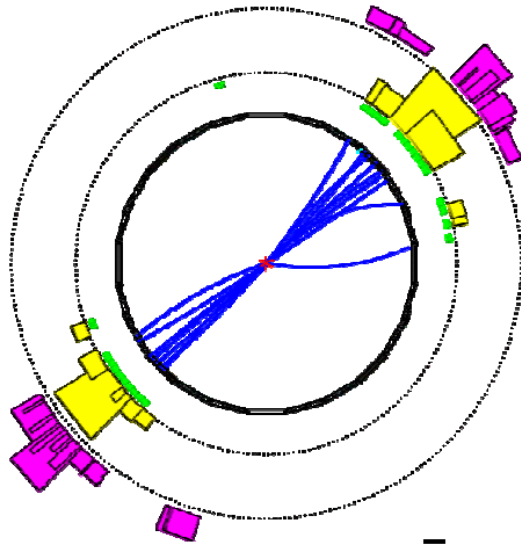
★ e^+e^- colliders are also a good place to study gluons

$$e^+e^- \rightarrow q\bar{q} \rightarrow 2\text{jets}$$

$$e^+e^- \rightarrow q\bar{q}g \rightarrow 3\text{jets}$$

$$e^+e^- \rightarrow q\bar{q}gg \rightarrow 4\text{jets}$$

OPAL at LEP (1989-2000)

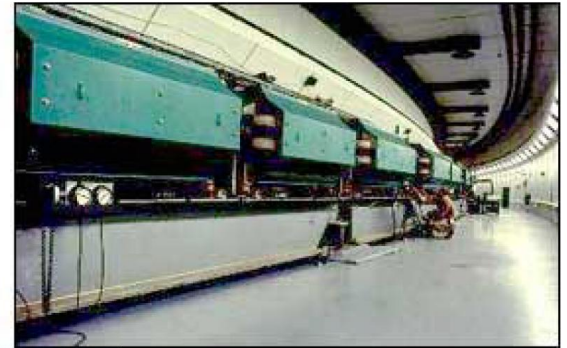


Experimentally:

- Three jet rate \rightarrow measurement of α_s
- Angular distributions \rightarrow gluons are spin-1
- Four-jet rate and distributions \rightarrow QCD has an underlying SU(3) symmetry

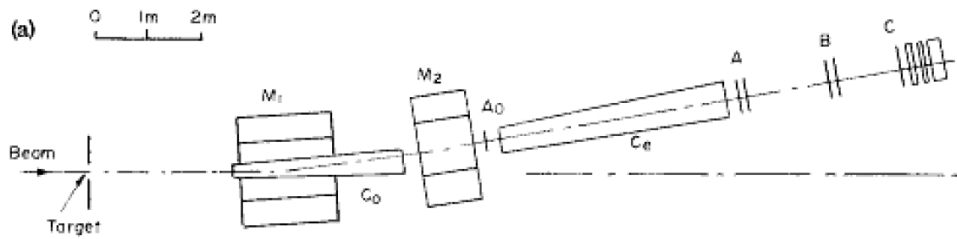
DRELL-YAN-ove PRODUKCIJE

**Brookhaven National Lab
Alternating Gradient Synchrotron**



The Process: $p + \text{Be} \rightarrow e^+ e^- X$

very narrow width
 \Rightarrow long lifetime



at BNL AGS

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Experimental Observation of a Heavy Particle J^\dagger

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen
 J. Leong, T. McCorrison, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan
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(Received 12 November 1974)

We report the observation of a heavy particle J , with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \text{Be} \rightarrow e^+ + e^- + X$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.

This experiment is part of a large program to

study the production of heavy particles daily with a thin Al foil. The beam spot

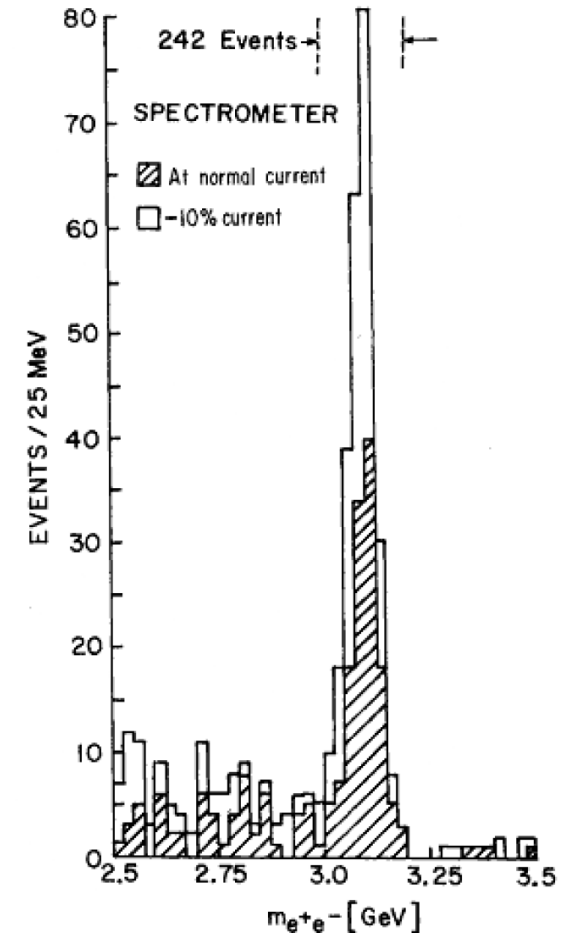
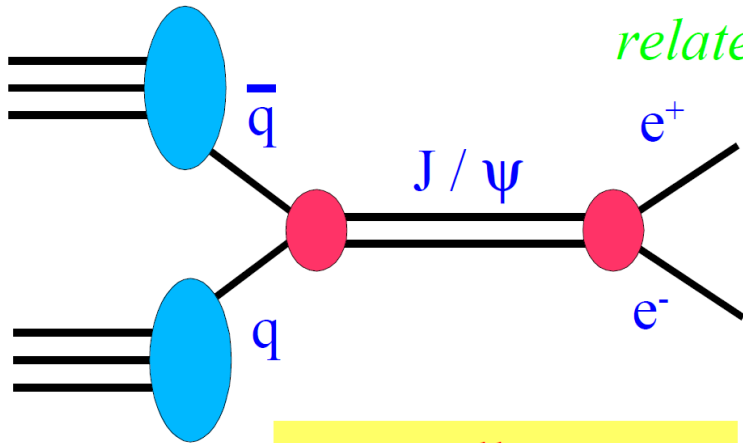
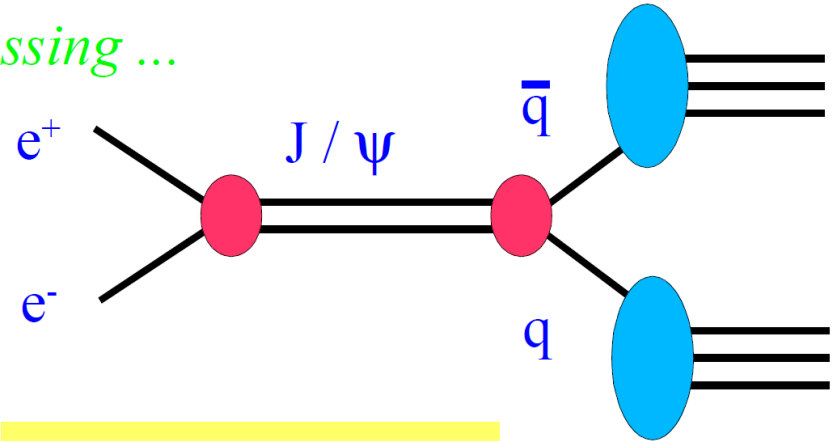


FIG. 2. Mass spectrum showing the existence of J . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



related by crossing ...



Drell-Yan
Brookhaven AGS

e⁺e⁻ Production
SLAC SPEAR
Frascati ADONE

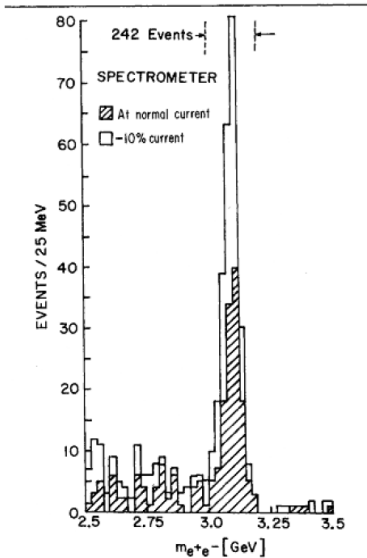
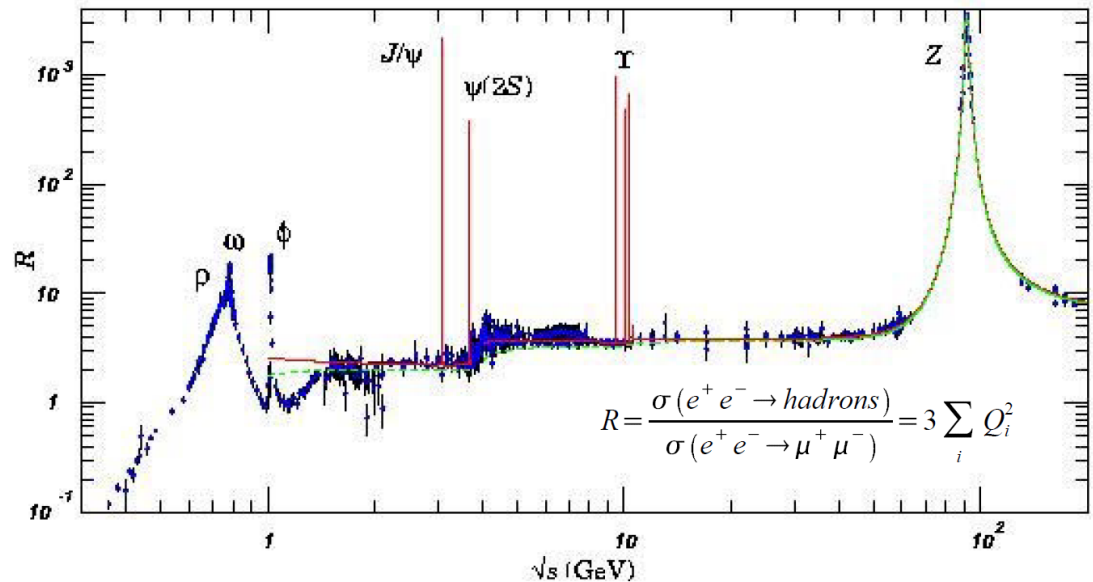


FIG. 2. Mass spectrum showing the existence of J . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.



PRODUKCIJE NOVIH STANJA

1974: The J/Psi (charm) discovery

$$p+N \rightarrow J/\psi$$

... 1976 Nobel Prize

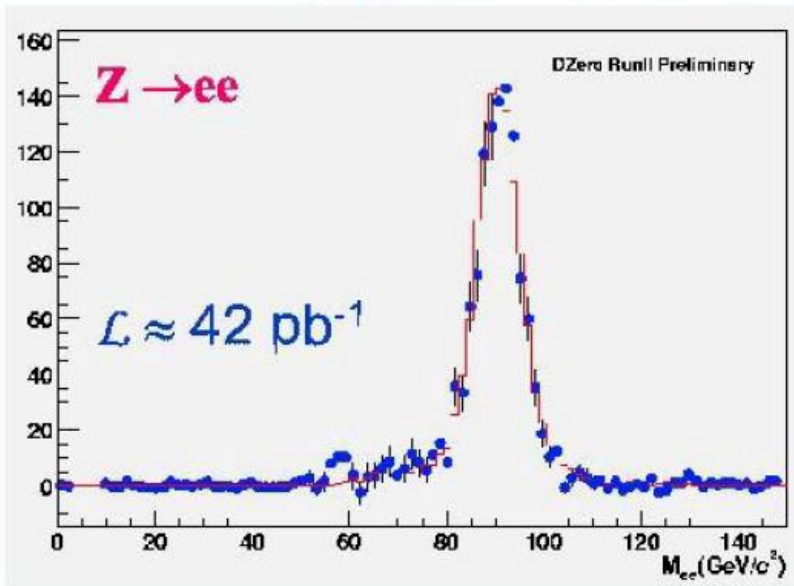
1977: The Upsilon (bottom) discovery

$$p+N \rightarrow \Upsilon$$

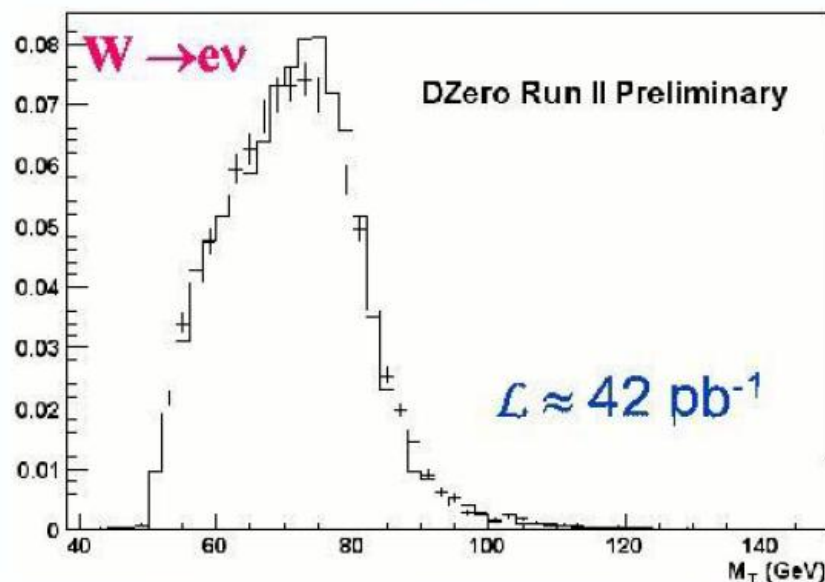
1983: The W and Z discovery

$$p + \bar{p} \rightarrow W/Z$$

... 1984 Nobel Prize



- 1139 Z $\rightarrow ee$ candidates
 - $|\eta^e| < 1.1$, $E_T > 25 \text{ GeV}$, no track match required
- $\epsilon(Z) \approx 8\%$, bkgd $\sim 18\%$

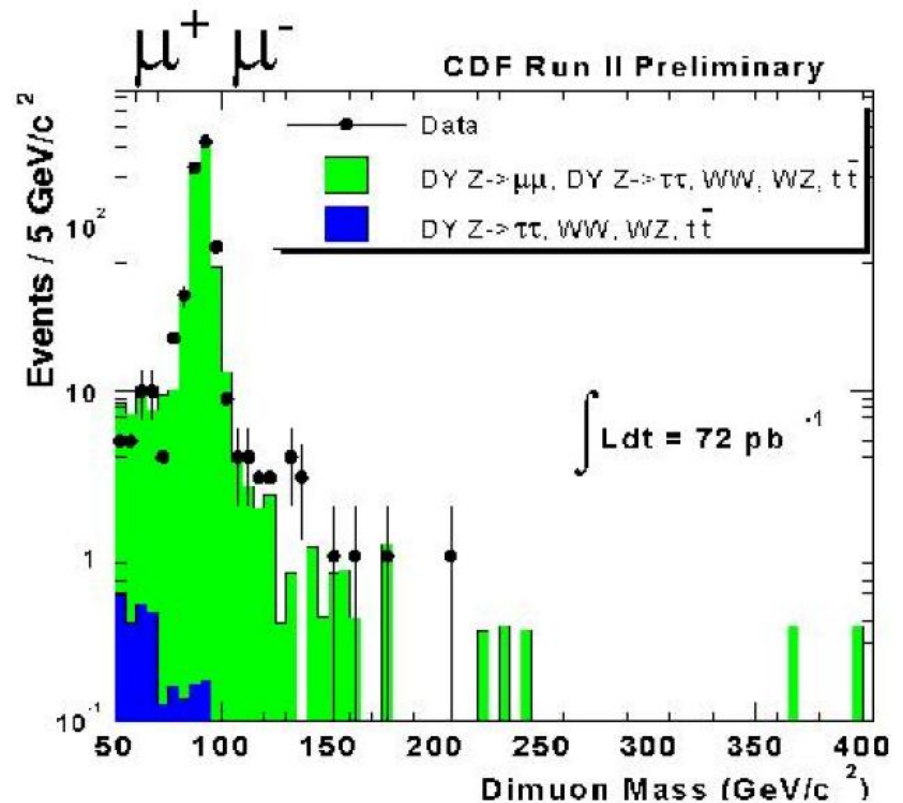
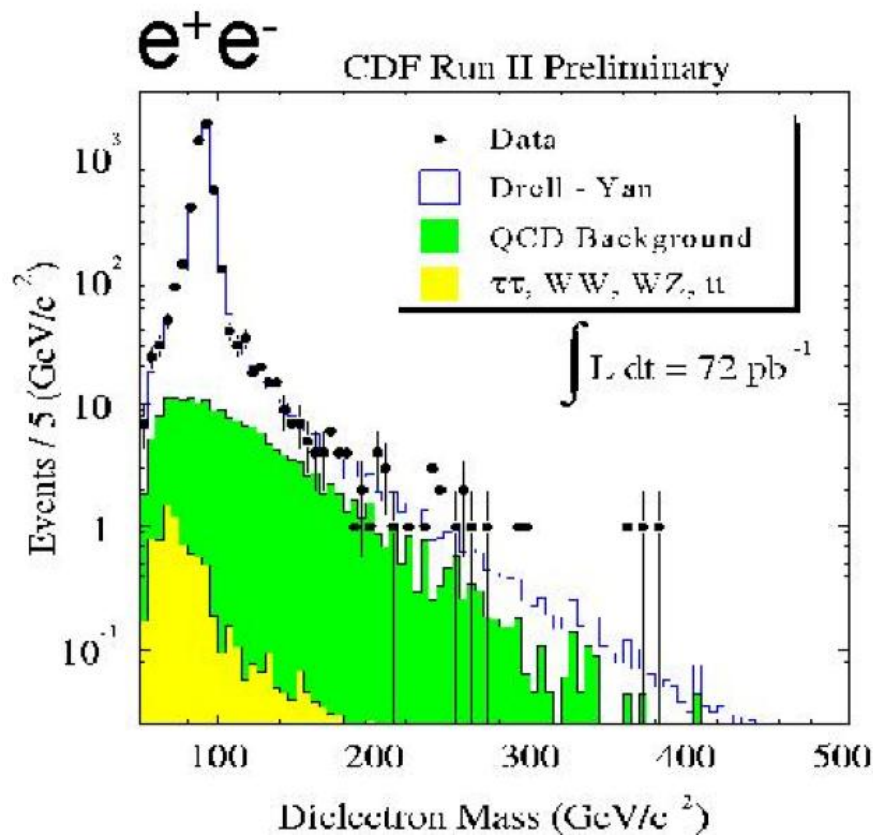


- 27370 W $\rightarrow ev$ candidates
 - $|\eta^e| < 1.1$, E_T & $\cancel{E}_T > 25 \text{ GeV}$
- $\epsilon(W) \approx 16\%$
- bkgd $\sim 3\%$ QCD, $\sim 1.5\%$ τ

$$\sigma(W)\text{Br}(W \rightarrow e\nu) = 3054 \pm 100(N_W) \pm 86(\text{sys}) \pm 305(\text{lumi}) \text{ pb}$$

$$\sigma(Z)\text{Br}(Z \rightarrow ee) = 294 \pm 11(N_Z) \pm 8(\text{sys}) \pm 29(\text{lumi}) \text{ pb}$$

DRELL-YAN-ove PRODUKCIJE na TEVATRON-u



Quark-Quark Scattering

Consider the process $u + d \rightarrow u + d$ which can occur in the high energy proton-proton scattering

There are nine possible colour configurations of the colliding quarks which are all equally likely.

Need to determine the average matrix element which is the sum over all possible colours divided by the number of possible initial colour states

$$\langle |M_{fi}|^2 \rangle = \frac{1}{3} \cdot \frac{1}{3} \sum_{i,j,k,l=1}^3 |M_{fi}(ij \rightarrow kl)|^2$$

