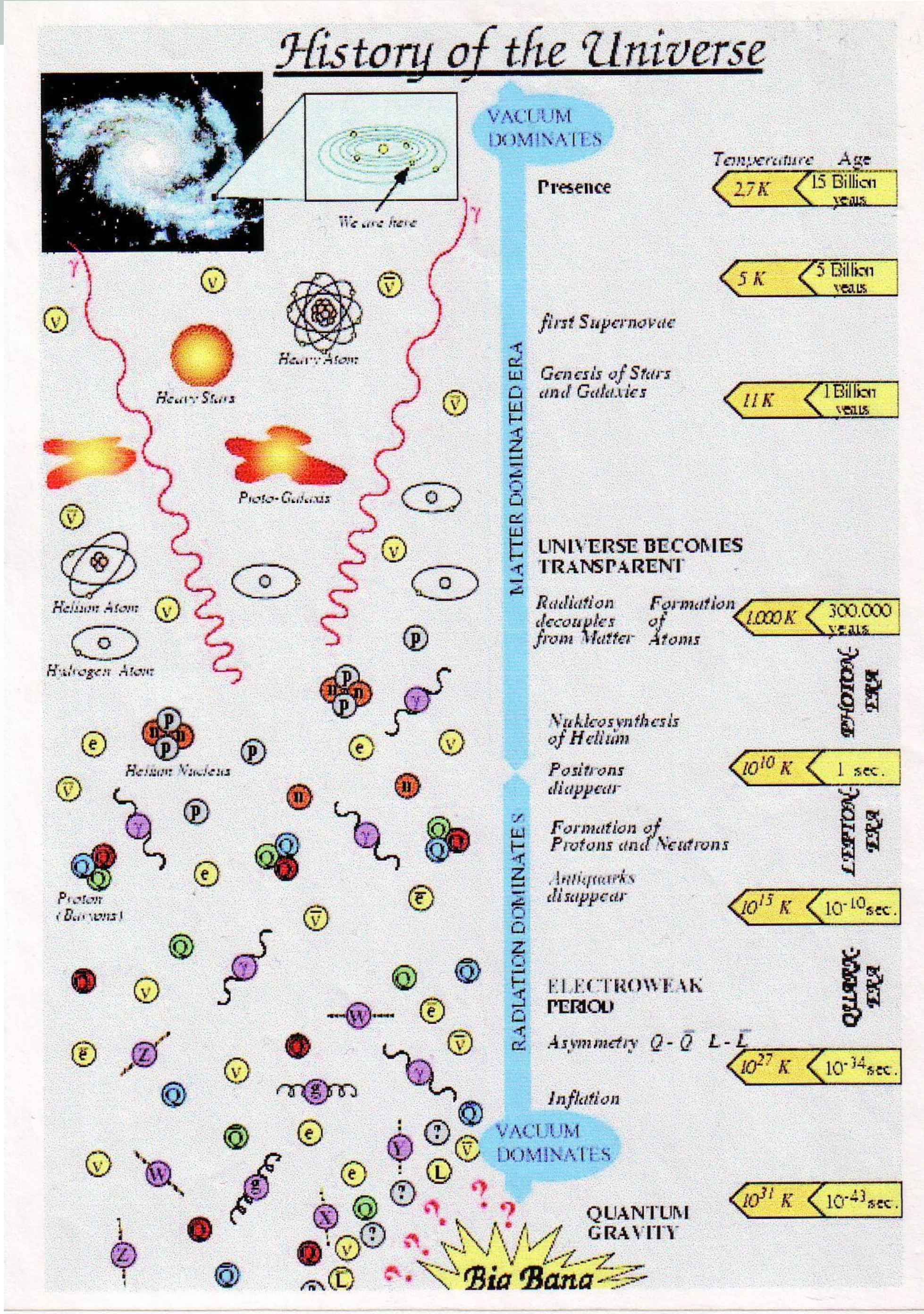
FIZIKALNA

KOZMOLOGIJA

IV. FRIEDMANNOVE JEDNADŽBE ZA 1-KOMPONENTNE SVEMIRE



Starting from the Friedmann-Lemaître equation:

$$H^2(t) = \frac{8\pi G}{3}(\rho_m(t) + \rho_r(t)) - \frac{k}{a^2(t)} + \frac{\Lambda}{3}$$

$$\Rightarrow \quad 1 = \frac{8\pi G \rho_m(t)}{3H^2(t)} + \frac{8\pi G \rho_r(t)}{3H^2(t)} - \frac{k}{a^2(t)H^2(t)} + \frac{\Lambda}{3H^2(t)}$$

$$\Rightarrow \quad 1 = \Omega_m(t) + \Omega_r(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

Ω_i = H^2-normalised energy density, varies with t

$$\rho_c(t) \equiv \frac{3H^2(t)}{8\pi G}$$ critical density at time t

$$\rho_c^0 \equiv \rho_c(t_0) = \frac{3H_0^2}{8\pi G}$$ critical density today, ~1gal/Mpc^3~5p/m^3

Starting again from the Friedmann-Lemaître equation:

$$H^2(t)=\frac{8\pi G}{3}\left(\rho_m(t)+\rho_r(t)\right)-\frac{k}{a^2(t)}+\frac{\Lambda}{3}$$

$$\Rightarrow \frac{H^2(t)}{H_0^2}=\frac{8\pi G\rho_m(t)}{3H_0^2}+\frac{8\pi G\rho_r(t)}{3H_0^2}-\frac{k}{a^2(t)H_0^2}+\frac{\Lambda}{3H_0^2}$$

$$\Rightarrow \boxed{\frac{H^2(t)}{H_0^2}=\Omega_m^0\left(\frac{a_0}{a(t)}\right)^3+\Omega_r^0\left(\frac{a_0}{a(t)}\right)^4+\Omega_k^0\left(\frac{a_0}{a(t)}\right)^2+\Omega_\Lambda^0}$$

each contribution to $H^2(t)$ varies differently with time

reminder: $a_0/a(t)=1+z \Rightarrow$ easy to derive $H(z)$

$$H^2(z)=H_0^2\left(\Omega_m^0(1+z)^3+\Omega_r^0(1+z)^4+\Omega_k^0(1+z)^2+\Omega_\Lambda^0\right)$$

DANAŠNJE VRIJEDNOSTI PARAMETARA GUSTOĆE

CMB temperature:

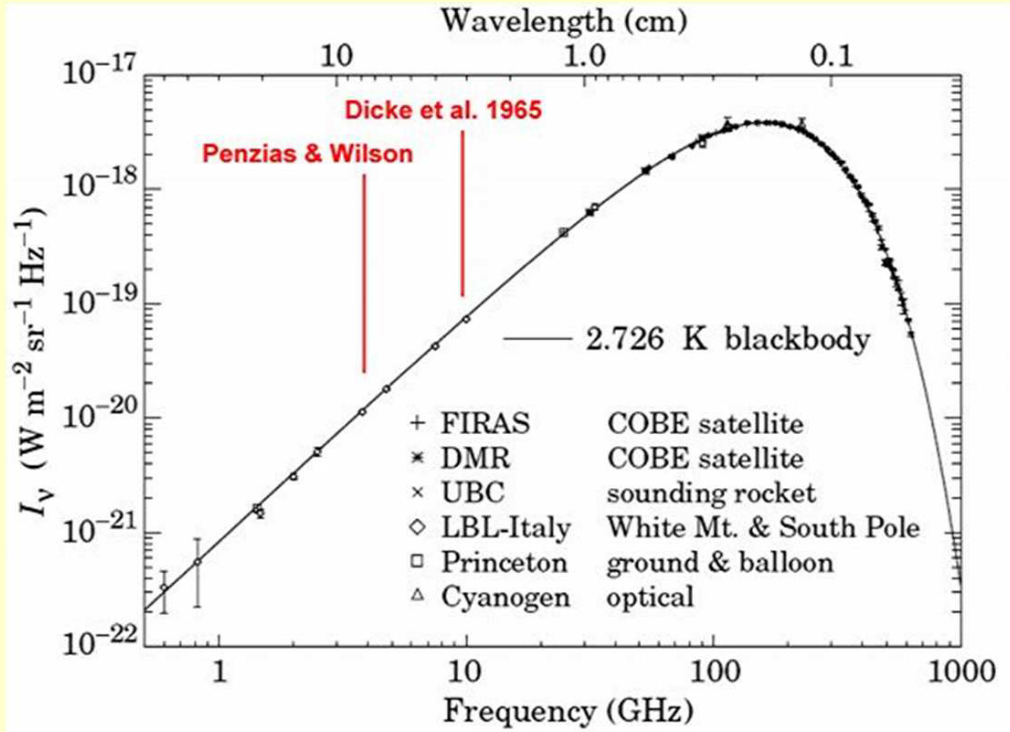
$$T_{CMB} = 2.7255 \pm 0.0006 K$$

Fixsen D.J., 2009, ApJ, 707, 916

$$\Rightarrow n_\gamma \approx 411 cm^{-3}$$

$$\rho_\gamma^0 = (\pi^2/15) T_\gamma^4 \approx 0.26 eV cm^{-3}$$

$$\Rightarrow \Omega_r^0 \approx 10^{-5}$$

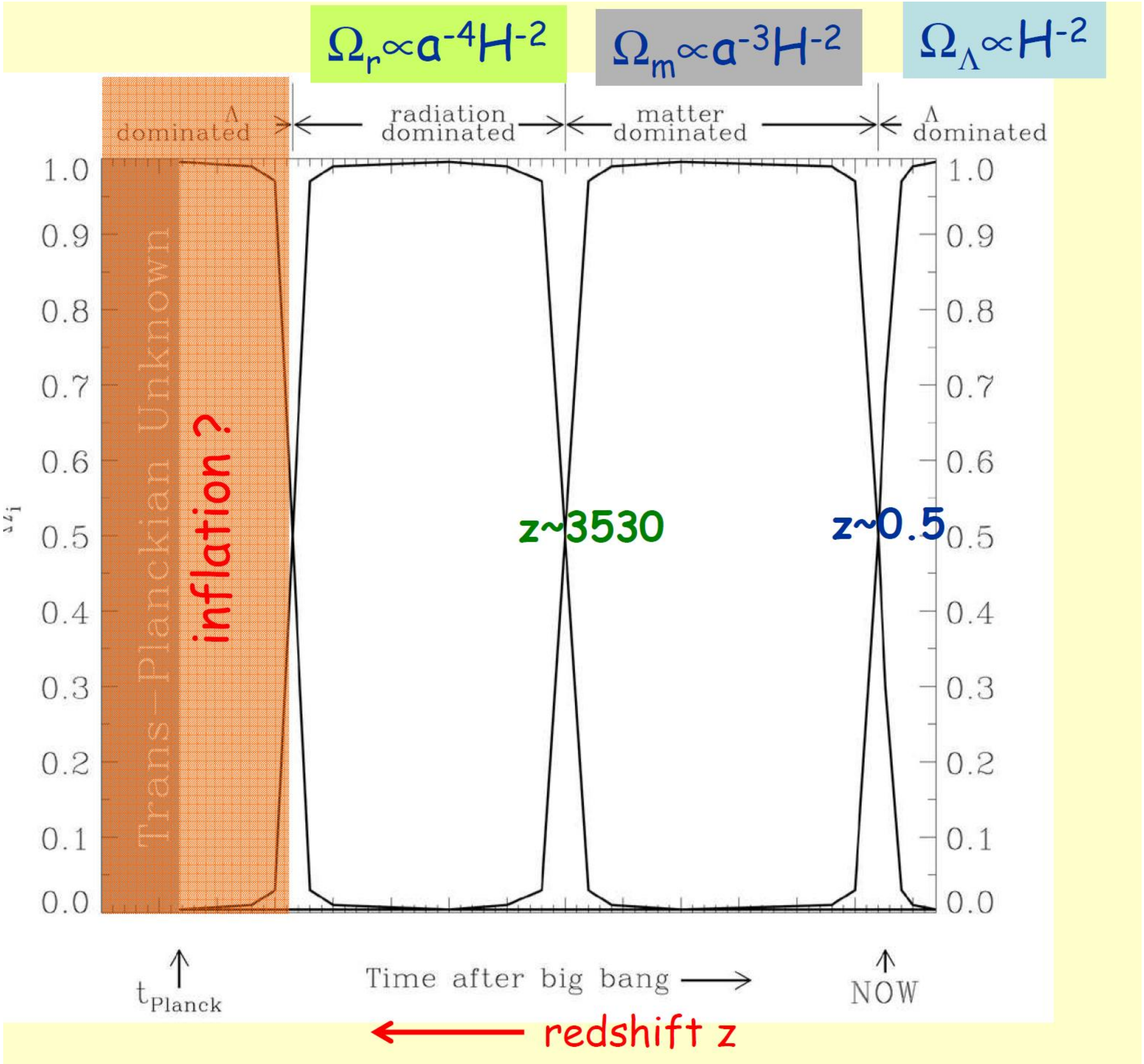


radiation is negligible today

CMB + BAO + SNe Ia : curvature is negligible today

$$\Omega_k^0 = -0.0057 \pm 0.0067$$

Komatsu et al, 2011, ApJS, 192, 18K



$\Omega_r \propto a^{-4}H^{-2}$
$\Omega_m \propto a^{-3}H^{-2}$
$\Omega_\Lambda \propto H^{-2}$
Λ dominated
radiation dominated
matter dominated
Λ dominated
Trans-Planckian Unknown
inflation ?
z~3530
z~0.5
Ω_i
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
t_{Planck}
Time after big bang
NOW
redshift z

Evolucija parametra gustoće - za primjer svemirske prašine

Iz ponašanja $\frac{\Omega}{\Omega_0} = \frac{\rho}{\rho_0} \frac{H_0^2}{H^2}$

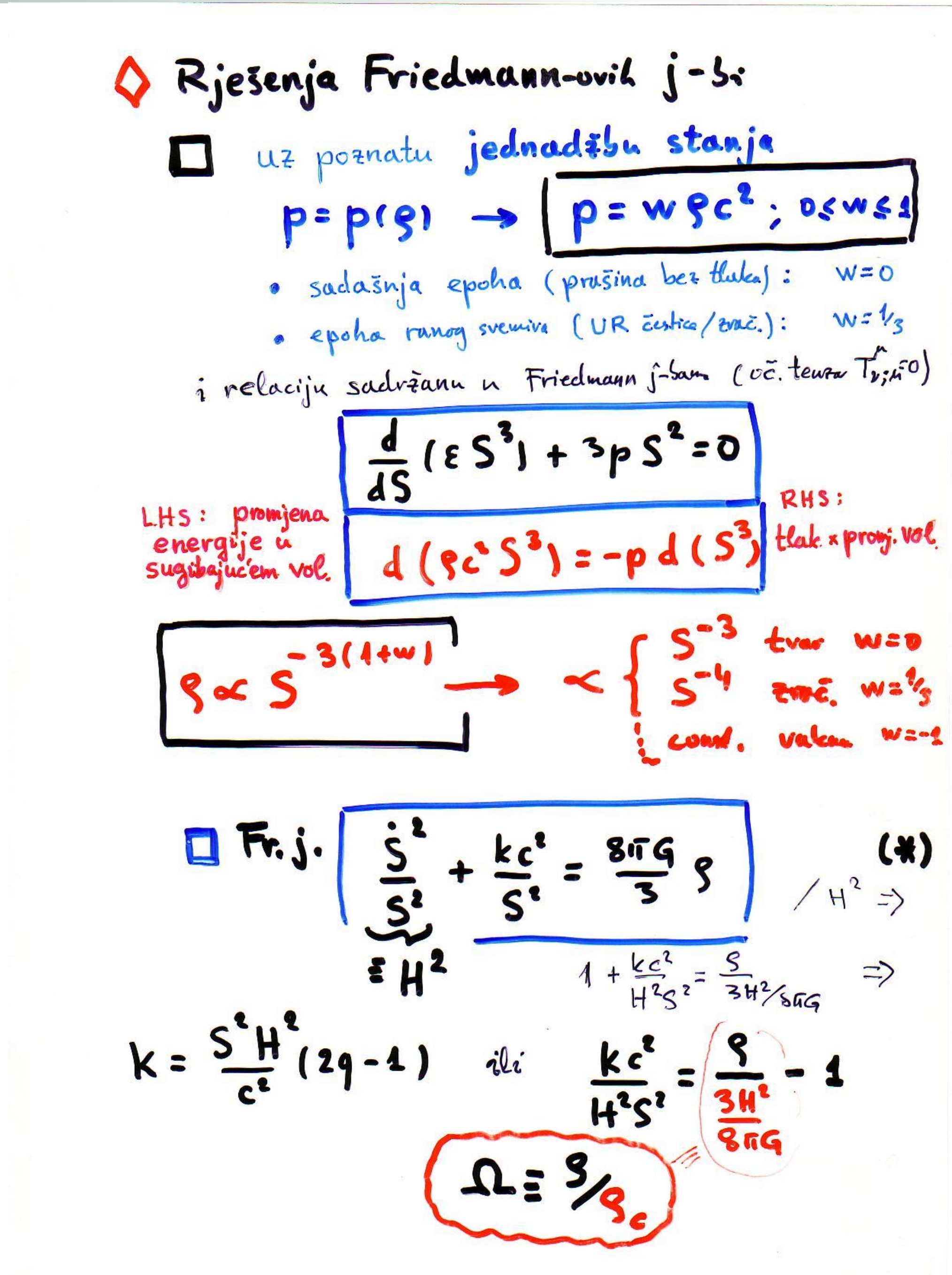
$\rho_0 / \rho \leftarrow (1+z)^3$ — $\rho S^3 = \text{const.}$ za $P = 0$

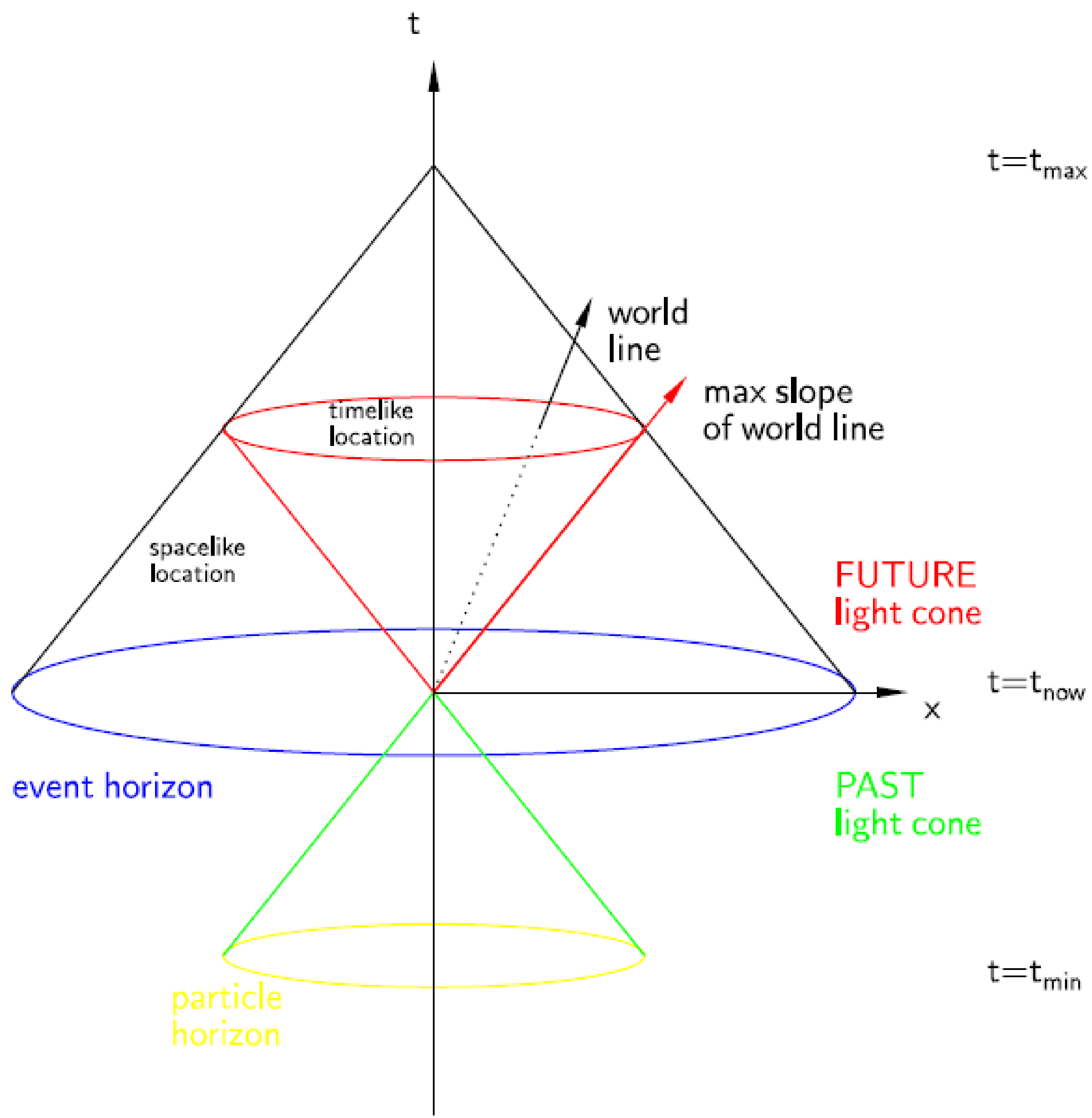
$$\Rightarrow \Omega H^2 = (1+z)^3 \Omega_0 H_0^2$$

$\uparrow$ iz Friedmannove j. $\Rightarrow$

$$\Omega = \frac{\Omega_0 (1+z)}{1+\Omega_0 z}$$

Tri slučaja: w=1/3, 0, -1




t
$t=t_{max}$
world
line
max slope
of world line
timelike
location
spacelike
location
FUTURE
light cone
$t=t_{now}$
x
event horizon
PAST
light cone
$t=t_{min}$
particle
horizon

ČESTIČNI HORIZONT (PROŠLOSTI) & HORIZONT DOGAĐAJA (BUDUĆNOSTI)

Čestični horizont

$$d_H(t_0) = S(t_0)\int_0^{r_e} \frac{dr}{\sqrt{1-kr^2}} = S(t_0)\int_0^{t_0} \frac{c\,dt}{S(t)}$$

Horizont događaja

$$d_E = S(t)\int_t^{\infty} \frac{c\,dt'}{S(t')}$$

DANAŠNJA EPOHA – EPOHA MATERIJE

U današnjoj epohi (s faktorom širenja S_0)

gustoća zračenja

$\varepsilon_0 \simeq 10^{-13}\,\mathrm{erg\,cm^{-3}}$ $\ll$

gustoća tvari $\gtrsim$

$\rho_0 c^2 \gtrsim 10^{-10}\,\mathrm{erg\,cm^{-3}}$

indicira prijelaz na $S \simeq 10^{-3} S_0$

svemira dominiranog zračenjem $\rightarrow$ u svemir dominiran materijom

Sugibajući volumen

$V = V_0 \left(\frac{S}{S_0}\right)^3$

uz konstantan broj čestica

$\Rightarrow \quad \rho = \rho_0 \left(\frac{S_0}{S}\right)^3$

Izbor $\begin{cases} T_0^0 = \rho_0 c^2 \dfrac{S_0^3}{S^3} \\ T_i^i = 0 \end{cases}$

vodi na Friedmannove j-be:

$$2\frac{\ddot{S}}{S} + \frac{\dot{S}^2 + kc^2}{S^2} = 0$$

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G \rho_0}{3} \cdot \frac{S_0^3}{S^3}$$

– nisu nezavisne $\rightarrow$ iz jedne od njih određuje se $S(t)$ za 3 slučaja $(k = 0, 1, -1)$, zasebno.

Ravni svemir u eri tvari: w= 0

- **Rješenje za k=0**
  Einstein-deSitterov model (1932)

(*)$|_{t=t_0}$ uz Hubbleovu konstantu sadašnje epohe $H_0 \equiv \frac{\dot S}{S}\Big|_{t_0}$ $\Rightarrow$

$$\rho_0 = \frac{3H_0^2}{8\pi G} \equiv \rho_c$$

$$= 2\cdot 10^{-29}\, h_0^2\, g\, cm^{-3} \simeq 10^2 \rho_B$$

(ρ_B: gustoća opažene barionske tvari $\simeq 10^{-31}\, g\, cm^{-3}$)

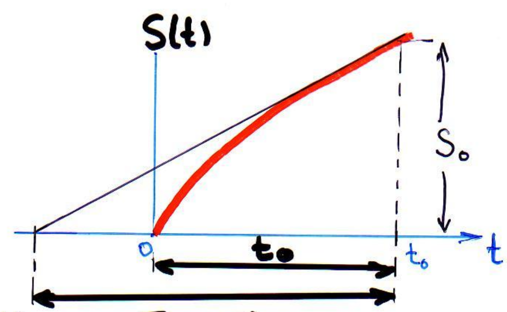
$$\frac{\dot S}{S} \equiv H(t) = \left(\frac{8\pi G}{3}\rho\right)^{1/2}$$

$\leftarrow$ $\rho \propto S^{-3}$ u sadašnjoj (w=0) epohi

$$\Rightarrow \int dS\, S^{-1+3/2} \propto \int dt$$

$$\Rightarrow \frac{2}{3} S^{3/2} \propto t - t_{poč}$$

$$\frac{S(t)}{S_0} = \left(\frac{t}{t_0}\right)^{2/3} \quad ; \quad S(t_{poč}) = 0$$



- Hubble-ovo vrijeme $T_H = H_0^{-1}$
- $\frac{1}{S_0}\dot S\Big|_{t=t_0} = \frac{2}{3}\frac{1}{t_0}$ $\Rightarrow$ starost svemira $t_0 = \frac{2}{3} H_0^{-1}$

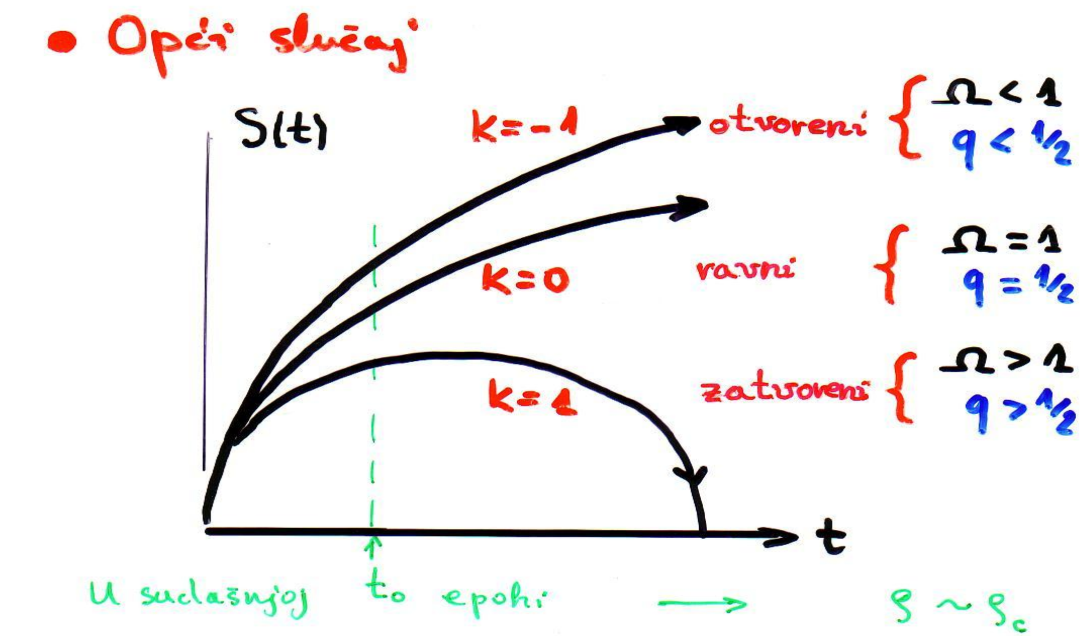
Vježba:

4.1 svemirska lukovica

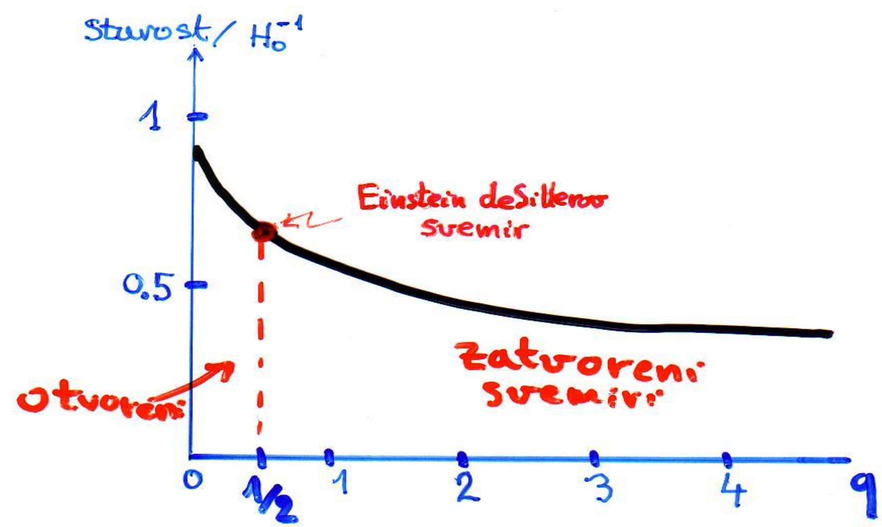
4.2 primjeri neobične

...

Dodajmo k=1, -1



• Opći slučaj
S(t)
k=-1
otvoreni
Ω < 1
q < 1/2
k=0
ravni
Ω = 1
q = 1/2
k=1
zatvoreni
Ω > 1
q > 1/2
t
U sadašnjoj t0 epohi → ρ ~ ρc



Starost / H0^-1
1
0.5
Einstein deSitterov svemir
otvoreni
zatvoreni svemiri
0
1/2
1
2
3
4
q