FIZIKALNA

KOZMOLOGIJA

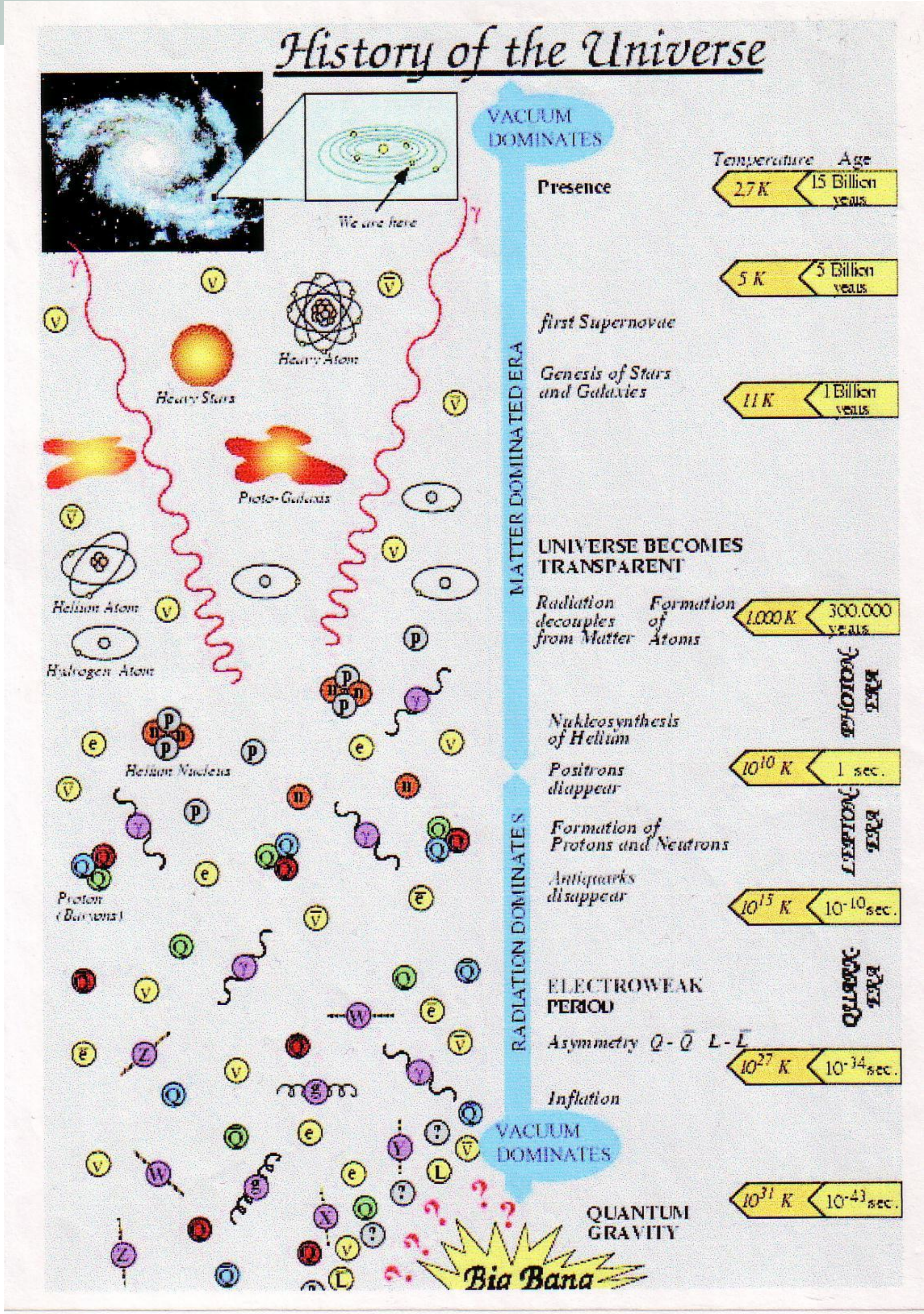
V. RANI SVEMIR

Počinjemo na 10^{13} K

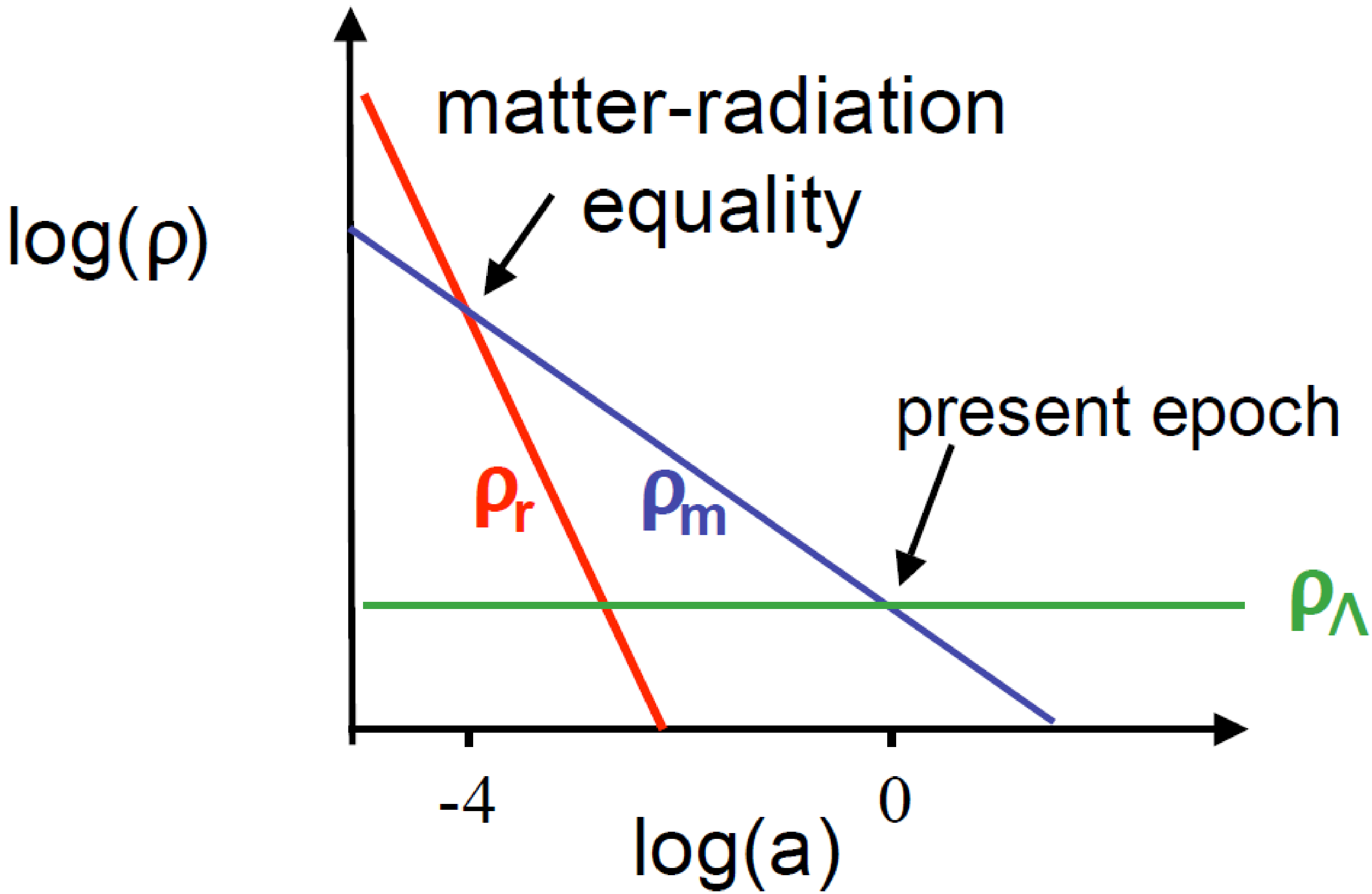
~ 900 MeV

ispod praga produkcije

$$e^+ + e^- \to \gamma^* \to p + \bar{p}$$



EVOLUCIJA RAZLIČITIH KOMPONENTI ENERGIJE



SVEMIR U ERI ZRAČENJA

UKAZUJE NA VRUĆI VELIKI PRASAK

RANI SVEMIR

Narlikar '93, Ch 5

$$\frac{\dot{S}^2 + \cancel{kc^2}}{S^2} = \frac{8\pi G}{3c^2} \left(T_0^0\right)$$

← zrač. crnog tijela tem T

$u = aT^4$

– povezano s gustoćom zračenja u_0 u sadašnjem trenutku

$$u = u_0 \frac{S_0^4}{S^4}$$

⇓ $S|_{t=0} = 0$

$$S = \text{const.}\left(\frac{32\pi G}{3c^2}\right)^{1/4} t^{1/2}$$

$$T = \frac{\text{const.}}{S}$$

$$T(K) = 1.52 \cdot 10^{10}\, t_{(s)}^{-1/2}$$

◊ Termodinamika ranog svemira

STATISTIČKI ansambli
&
RAVNOTEŽNA T-dn.

$$dU = TdS - pdV + \mu dN$$

- termodinamička ravnoteža
- čestice idealnog plina

RAVNOTEŽNA TERMODINAMIKA RANOG SVEMIRA

sudari ravnoteža

elastični – kinetička

neelast. – kemijska

$\Gamma \gg \frac{\dot{S}}{S}$

$\ll$

bez sudara: odvezivanje CMB-a

zamrzavanje nukleosint.

QM čestica u kutiji vol. $V = L^3$

→ dopušteni valni brojevi

$k_x = n \frac{2\pi}{L}$ (harmonički R.u.)

$$\sum_i \rightarrow \frac{V}{h^3} \int d^3p \quad \& \quad \hbar = \frac{h}{2\pi}$$

$$\rightarrow \frac{V}{(2\pi)^3 \hbar^3} 4\pi \int p^2 dp$$

Broj č. u jedin. vol, impulsa $\in (p, p+dp)$

$$dN = n(p)dp = \frac{g_A}{2\pi^2 \hbar^3} \frac{p^2\, dp}{e^{\beta(E_A(p) - \mu_A)} \pm 1}$$

g_A = faktor degeneracije ; $E_A = \sqrt{c^2p^2 + m_A^2 c^4}$;

Helmholtzova slobodna energija

$$F = U - TS$$

– minimizirana u ravnoteži za konstantne T & V :

$$dF = -S\,dT - P\,dV + \mu\,dN$$

$\& \quad \frac{dF}{dN} = 0 \Rightarrow \mu = 0$

PROMJENU ČESTICA (KEMIJSKE POTENCIJALE) KONTROLIRAJU ZAKONI OČUVANJA (Q,B,L –FEČ)

- **ČESTICE U FLUIDU MOŽEMO PODIJELITI U GRUPE ONIH, KOJE MEĐUSOBNO INTERAGIRAJU**
- **GUSTOĆU BROJA ČESTICA ODREĐUJU 3 UČINKA KOJA ULAZE U BOLTZMANNOVU JEDNADŽBU**

$$\frac{dN_A}{dt} = -3\underbrace{\frac{\dot{S}}{S}}_{\text{ekspanzija}} N_A + N_C \Gamma_{CD\to AB} - N_A \Gamma_{AB\to CD}$$

stvaranje č — poništenje č. — određeno brzinom reakcije

$$\Gamma_{AB\to CD} = N_A \langle \sigma_{AB\to CD} v \rangle$$

RELEVANTNE OČUVANE VELIČINE Q, L, B

	e	ν_e	ν_μ	p	n	& anti-č.
Q	-1	0	0	1	0	
L_e	1	1	0	0	0	
L_μ	0	0	1	0	0	
B	0	0	0	1	1	

s pridruženim kemijskim potencijalima

$$\mu_Q \ , \ \mu_{L_e} \ , \ \mu_{L_\mu} \ , \ \mu_B$$

IZRAZIMO POMOĆU BROJA ČESTICA

$$\mu_Q \hat{Q} + \mu_{L_e} \hat{L}_e + \mu_{L_\mu} \hat{L}_\mu + \mu_B \hat{B} =$$

$$= \mu_Q \left[\hat{N}_{e^+} - \hat{N}_{e^-} + \hat{N}_p \right]$$

$$+ \mu_{L_e} \left[\hat{N}_{e^-} + \hat{N}_{\nu_e} - \hat{N}_{e^+} - \hat{N}_{\bar{\nu}_e} \right]$$

$$+ \mu_{L_\mu} \left[\hat{N}_{\nu_\mu} - \hat{N}_{\bar{\nu}_\mu} \right]$$

$$+ \mu_B \left[\hat{N}_p + \hat{N}_n \right]$$

Očuvanja Q, B, L selektiraju dopuštene reakcije

$n \to p + e^- + \bar{\nu}_e$

& inverzne $\quad p + \bar{\nu}_e \to e^+ + n$

te mionske inačice $\quad p + \bar{\nu}_\mu \to \mu^+ + n$

$e^- + \mu^+ \to \nu_e + \bar{\nu}_\mu \quad \Rightarrow$

$\mu_{e^-} - \mu_{\nu_e} = \mu_{\mu^-} - \mu_{\nu_\mu} = \mu_n - \mu_p$ itd.

TERMODINAMIKA RANOG SVEMIRA

Mala gustoća bariona

$N_B / N_\gamma \sim 10^{-9}$

i jednako mala gustoća leptona

⇓

opravdanje

$\mu_A = 0$ za sve A

Gustoće (uz $\mu_A = 0$)

- broja čestica $$N_A = \frac{g_A}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2\,dp}{e^{E_A(p)/k_BT} \pm 1}$$
- energije $$\varepsilon_A = \frac{g_A}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2 E_A(p)\,dp}{e^{E_A(p)/k_BT} \pm 1}$$
- tlaka $$p_A = \frac{g_A}{2\pi^2\hbar^3}\int_0^\infty \frac{\left[\frac{c^2p^2}{E_A(p)}\right] p^2\,dp}{e^{E_A(p)/k_BT} \pm 1}$$
- entropije $$S_A = \frac{p_A + \varepsilon_A}{T}$$

Visoko-temperaturna granica

(relativist. č. $E_A \simeq pc$)

$$\rho_B c^2 = \frac{g_B}{2} a T^4$$

$$\rho_F c^2 = \frac{7}{16} g_F a T^4$$

ukup. $$\rho c^2 = \frac{1}{2} g_* a T^4$$

$$g_* = g_B + \frac{7}{8} g_F$$

TERMODINAMIKA ULTRA RELATIVISTIČKE PLAZME

Number density: $$n = \frac{\xi(3)}{\pi^2} g'(T) T^3$$

Energy density: $$\rho = 3p = \frac{\pi^2}{30} g(T) T^4$$

Entropy density: $$s \equiv \frac{p+\rho}{T} = \frac{2\pi^2}{45} g(T) T^3$$

Where, the number of relativistic degrees of freedom *sum* over all bosons and fermions with appropriate weight:

$$g'(T) = g_b(T) + \frac{3}{4} g_f(T)$$

$$g(T) = g_b(T) + \frac{7}{8} g_f(T)$$

In the absence of dissipative processes (e.g. phase transitions which generate entropy) the **comoving entropy** is *conserved*:

$$\frac{d}{dt}(sa^3) = 0 \quad \Rightarrow \quad s \propto 1/a^3 \qquad \text{i.e.} \quad T \propto 1/a$$

At early times the curvature term becomes negligible (compared to radiation) so the Friedmann equation simplifies to:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}$$

Integrating this yields the time-temperature relationship:

t (s) = 2.42 $g^{-1/2}$ $(T/\mathrm{MeV})^{-2}$

Jednoznačna veza temperature i vremena:

$$\sqrt{t(s)} = \frac{1.8 \cdot 10^{10}}{g_*^{1/4}\, T(K)}$$

Termička povijest svemira

— u eri zračenja —

Efektivni broj st. slobode $g_* = g_B + \frac{7}{8} g_F$

u ovisnosti o

temperaturi / česticama u ravnoteži

$k_B T < m_A c^2$

$m_s c^2$ — γ, $e^{\pm}$, ν_e, ν_μ, ν_τ, $\mu^{\pm}$, 37 $\{u\bar{u}, d\bar{d}, g\}$ — $\frac{205}{4}$

— $\Lambda_{QCD} \simeq 200$ MeV — ↓ 3 $\{\pi^+, \pi^-, \pi^0\}$ — $\frac{69}{4}$

— $m_\pi c^2 \simeq 140$ MeV — $\frac{57}{4}$

— $m_\mu c^2 \simeq 106$ MeV — $\frac{43}{4}$

— 3.5 MeV

2.3 MeV — $\frac{11}{2}$

— 1 MeV

0.2 MeV — jednosmjerno podgrijavanje γ-zraka: $e^+e^- \to \gamma + \gamma \Rightarrow T_\gamma = 1.4\, T_\nu$

— 1 eV

0.3 eV — 2 — ploha zadnjeg raspršenja

CMB

RANI SVEMIR

- ERA ZRAČENJA

za $T \lesssim 10^{12}$ K

$\Leftrightarrow$ $t \gtrsim 10^{-4}$ s

obilje γ, e^-, ν_e, ν_μ

i njihovih a-č

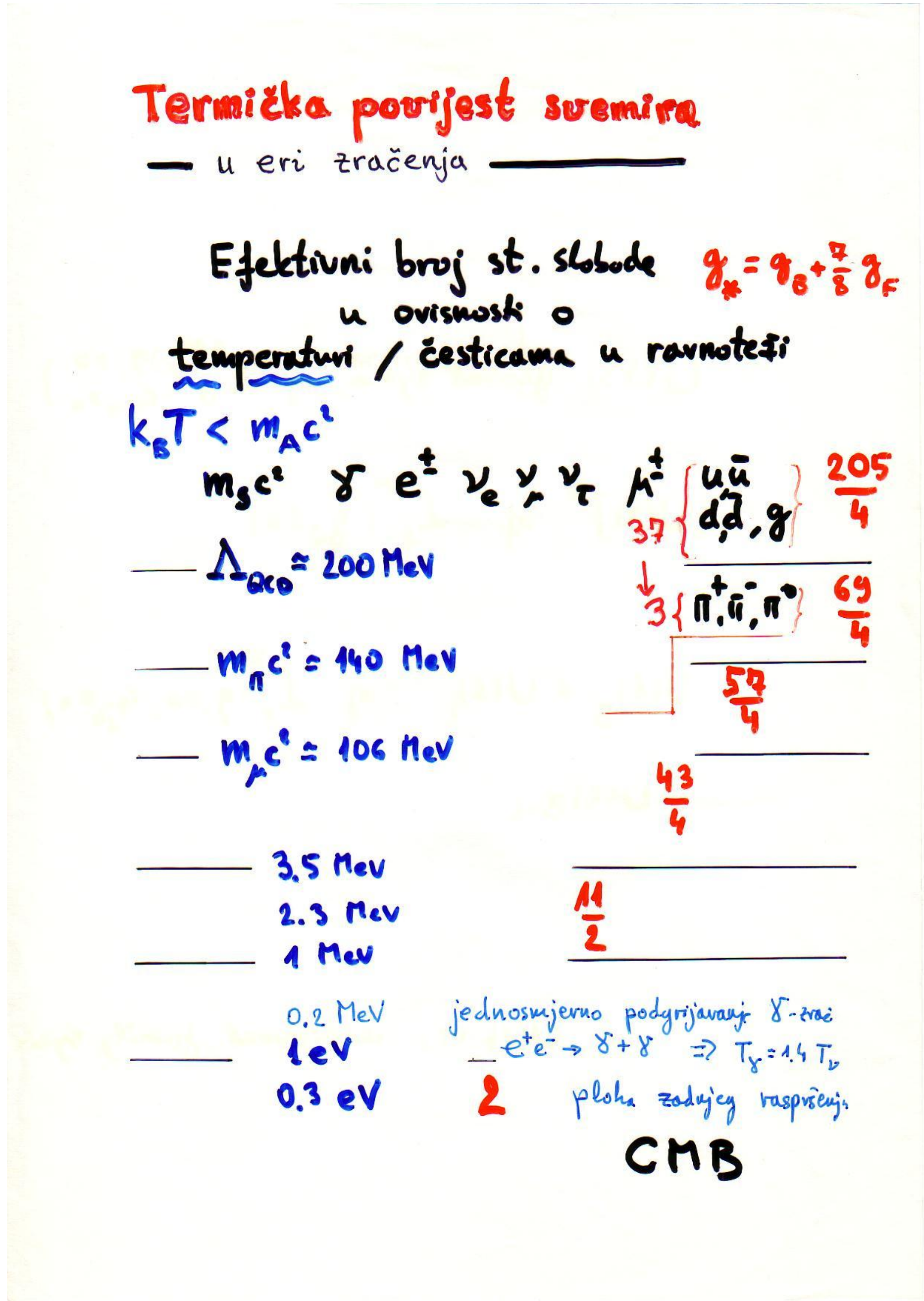


Table 5.1 *Thermodynamic quantities for various particle species at $T \gg T_A$*

Particle species A	Symbol	T_A (K)	g_A	N_A/N_γ	$\varepsilon_A/\varepsilon_\gamma$	S_A/S_γ
Electron	e^-	5.93×10^{6}	2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$
Positron	e^+		2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$
Muon	μ^-	1.22×10^{12}	2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$
Antimuon	μ^+		2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$
Muon, electron	ν_μ, ν_e	0	1	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{7}{16}$
neutrinos and their antineutrinos	$\bar{\nu}_\mu, \bar{\nu}_e$		1	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{7}{16}$
Pions	π^+		1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	π^-	1.6×10^{12}	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	π^0		1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Proton	p	10^{13}	2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$
Neutron	n	$T_n - T_p \sim 1.5 \times 10^{10}$	2	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$