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Observing New Particles at High-Energy Colliders
The Standard Model at Lepton and Hadron Colliders (Lecture 1)

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Observing New Particles at High-Energy Colliders

1. The Standard Model at Lepton and Hadron Colliders

M. E. Peskin
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In this series of lectures, I will discuss the physics of particle production at high energy colliders.

The three lectures will cover:

1. The Standard Model at Lepton and Hadron Colliders

2. Discovery of New Physics at the LHC

3. Measurement of the Supersymmetry Spectrum

My main concern will be with the observation and characterization of new particles, beyond those of the Standard Model. However, in order to understand both the properties of new particles and the issues in discovering them, it is necessary to have a clear idea of the predictions of the Standard Model. Most of the lectures will be devoted to this topic.

We are entering an era in which we realistically expect to see the discovery of new particles from the next level in physics beyond the current Standard Model. It should begin next year with the startup of the LHC.

We are used to thinking of exotic particles as being entitites that appear in loops and give rise to virtual corrections. We use them to explain anomalies in flavor physics and constrain them in precision tests.

Those days are over. We are about to meet these particles face to face.

To work in this new era, it will be important to have a concrete understanding of how the standard quarks, lepton, and gauge bosons appear at colliders. We will use that understanding to demonstrate that the new particles exist, and to work out their properties.

In this lecture, I will review some aspects of Standard Model physics and show how they are reflected in high-energy experiments. I am sure that the physics of the Standard Model is very familiar to you.   But you need to be equally familiar with the direct manifestations of Standard Model physics in the data.

Also, it is amazing to see that the Standard Model formulae really work!

At the colliders of the next generation, every experiment that is currently a precision test of the Standard Model will become a window to new physics.

This is especially clear at the next-generation e+e- collider, the ILC. The predictions for cross sections and asymmetries that I will describe also apply to new particles with given SU(2)xU(1) quantum numbers. The predictions for standard Model particle pair production can also be corrected by the effects of new s-channel resonances -- due to Z's, quark and lepton compositeness, Kaluza-Klein or Randall-Sundrum recurrences, etc.

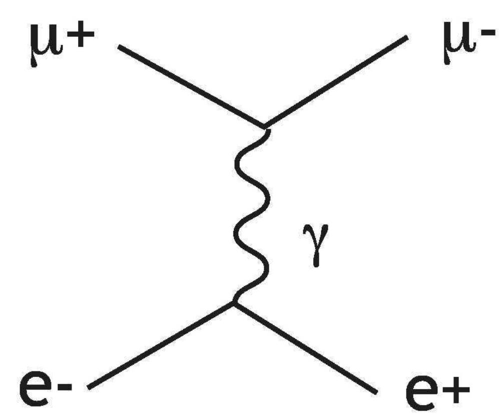
At both the ILC and the LHC, we will study new particles through their decays to Standard Model particles. The special properties of τ, b, W, Z, t that I will discuss provide diagnostics of the properties of the exotic parent states.

At high energy, the Standard Model gives a very simple picture of quarks and leptons. The masses of these particles can be ignored. The particles are best thought of as states of definite conserved helicity

$$h = \vec{S} \cdot \hat{P}$$

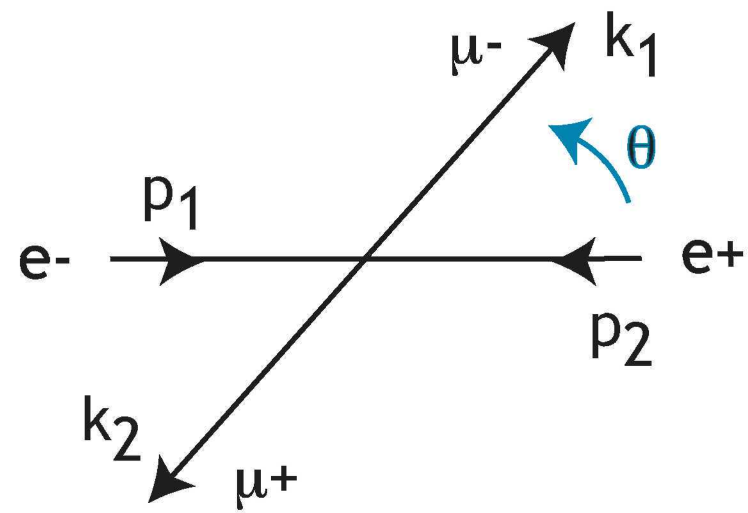
A subtlety is that SU(2) couples only to left-handed quarks and leptons and their right-handed antiparticles. More generally, the left- and right-handed states have different SU(2)xU(1) quantum numbers. This is directly manifested in the experiments, as we will see.

Much of the dynamics of the Standard Model particle production in e^+e^- annihilation is exhibited by studying the very simplest reaction: $e^+e^- \rightarrow \mu^+\mu^-$.



The amplitudes for $e^+e^- \to \mu^+\mu^-$ between states of definite helicity are very simple:

$$
\begin{aligned}
&i\mathcal{M}(e^-_L e^+_R \to \mu^+\mu^-) \\
&\qquad = 2ie^2 \frac{u}{s} \\
&\qquad = ie^2(1+\cos\theta)
\end{aligned}
$$



that is,

$$i\mathcal{M}(e^-_L e^+_R \to \mu^-_L \mu^+_R) = i\mathcal{M}(e^-_R e^+_L \to \mu^-_R \mu^+_L) = ie^2(1+\cos\theta)$$
$$i\mathcal{M}(e^-_L e^+_R \to \mu^-_R \mu^+_L) = i\mathcal{M}(e^-_R e^+_L \to \mu^-_L \mu^+_R) = ie^2(1-\cos\theta)$$

These formulae lead to the regularities:

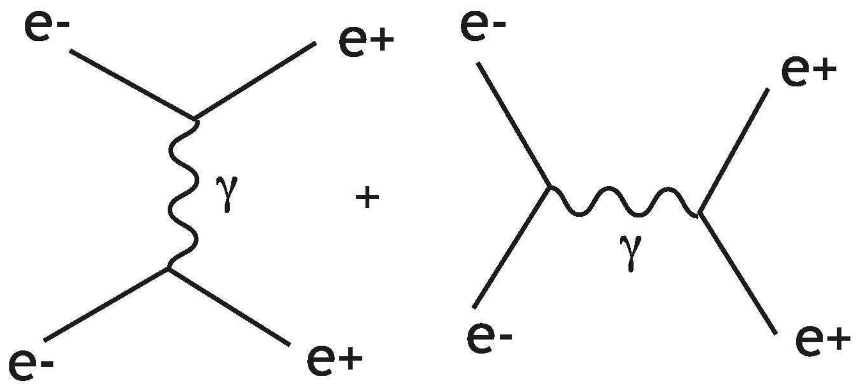
$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) \qquad\qquad \sigma = \frac{4\pi\alpha^2}{3s} = \frac{87.\ \mathrm{fb}}{(E_{CM}\ \mathrm{TeV})^2}$$

with $s = q^2 = (E_{CM})^2$. The second formula sets the size of cross sections for all QED and electroweak processes.

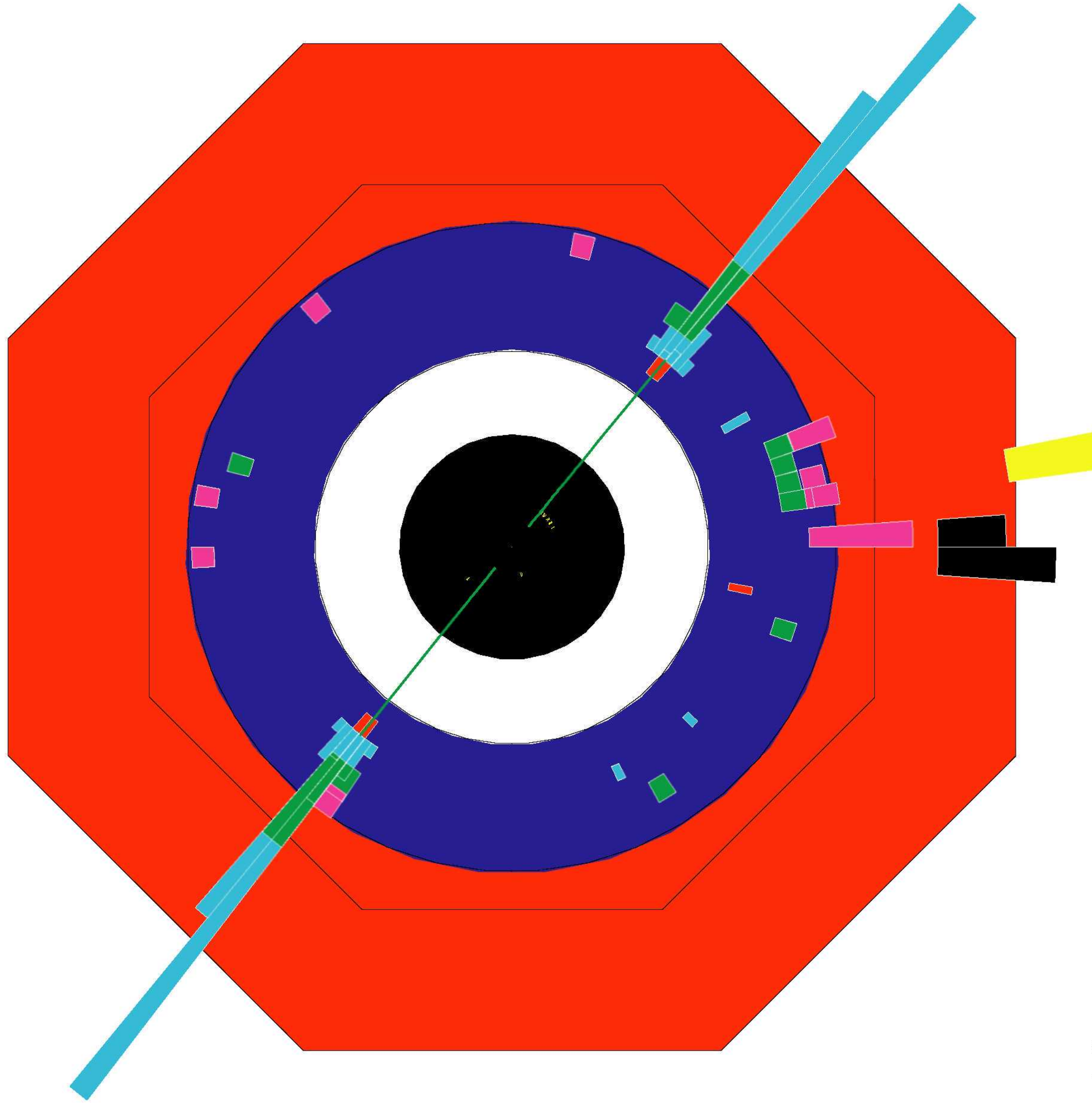
Here are some examples of e+e- annihilation to leptons

$$e^+e^- \to \mu^+\mu^- \qquad\qquad e^+e^- \to \tau^+\tau^-$$

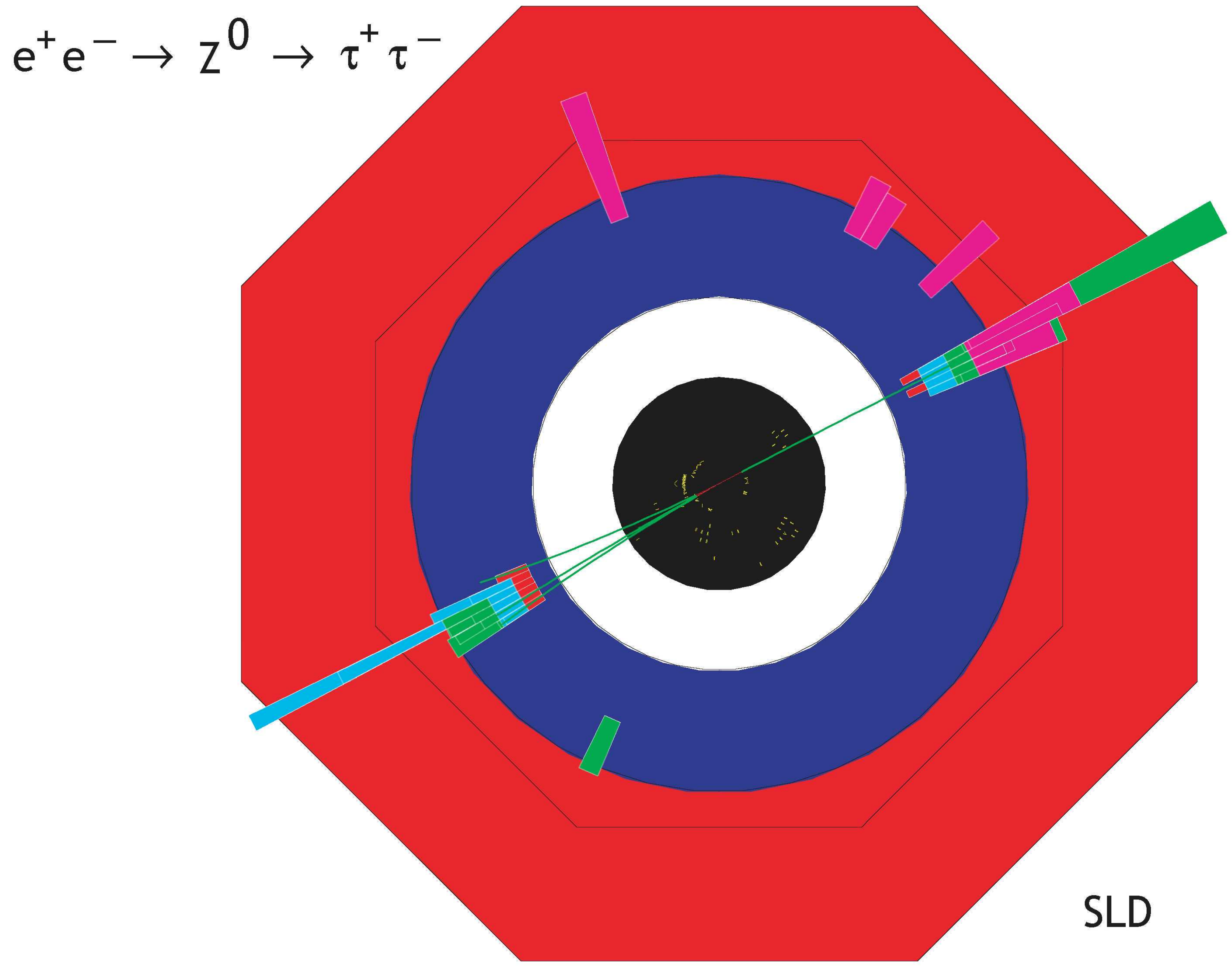
and the related process of Bhabha scattering: $e^+e^- \to e^+e^-$

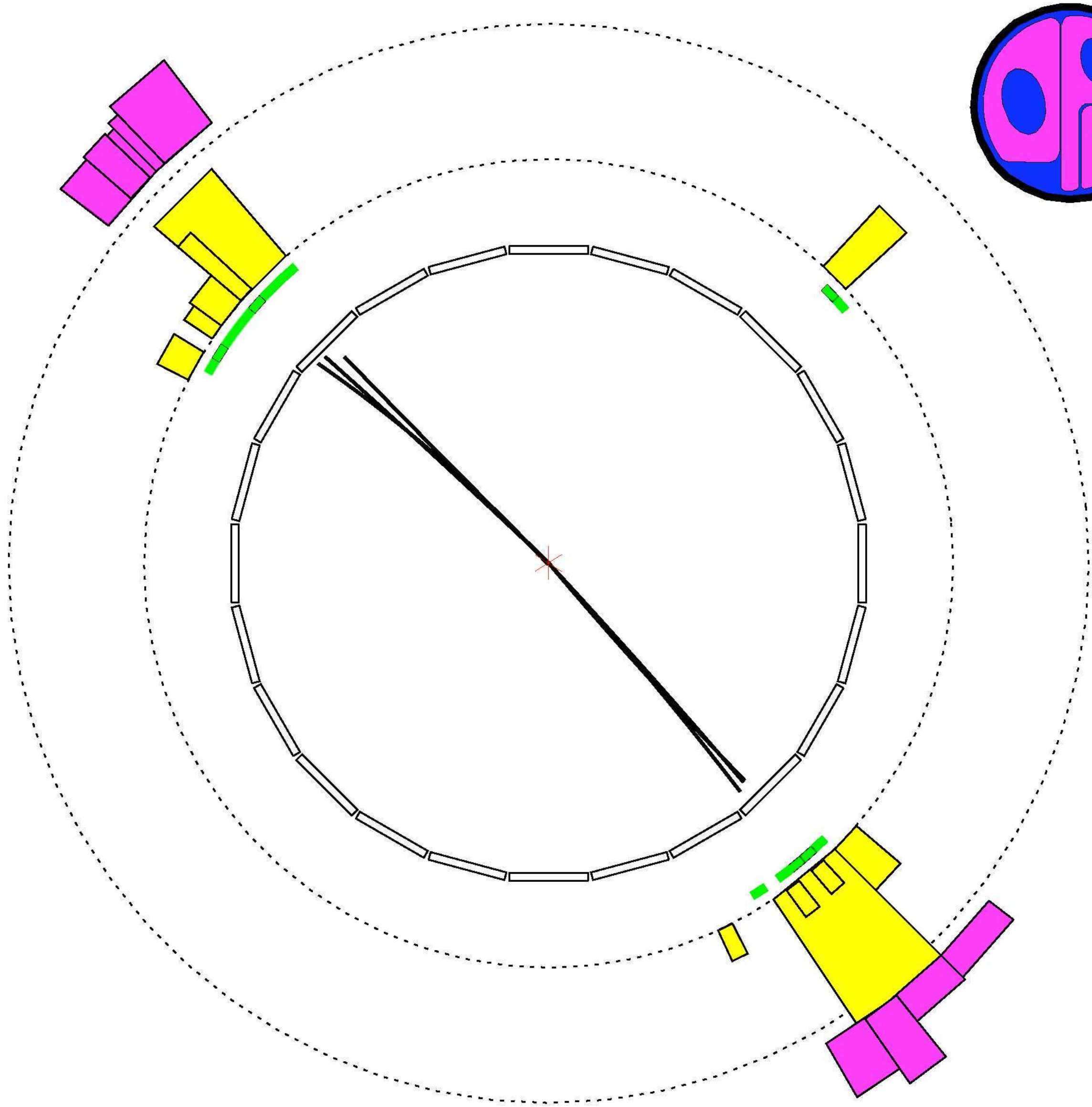


$e^+e^- \rightarrow Z^0 \rightarrow e^+e^-$



SLD


$e^+e^- \to Z^0 \to \tau^+\tau^-$
SLD



The leading-order prediction of the Standard Model is that this process $e^+e^- \to q\bar{q}$ is described by the same formulae, taking account of the color and charges of the quarks.

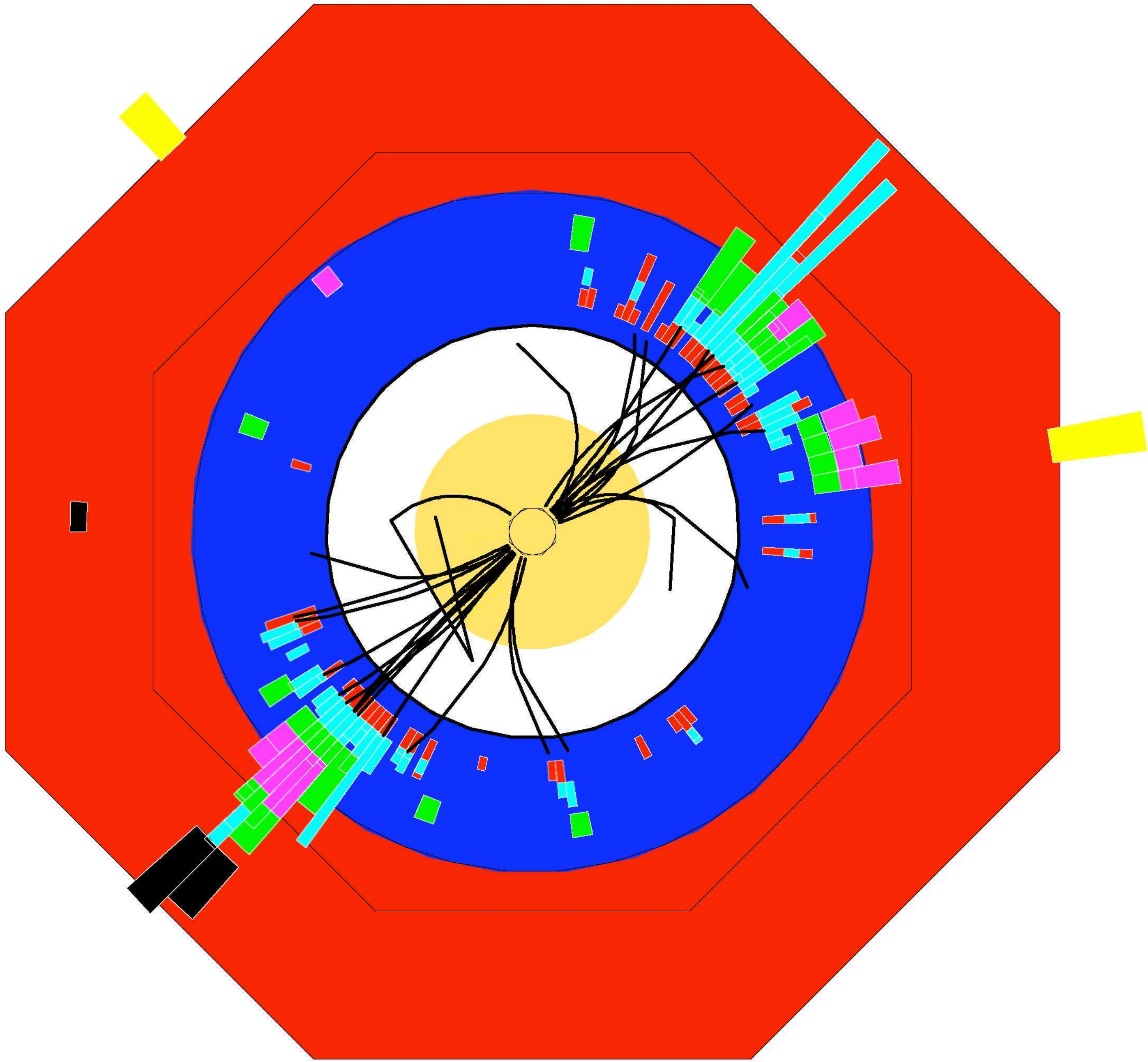
The final states are jets of mesons and baryons moving in the direction of the quark that initiated the jet. I will have much more to say about jets in the second lecture.

The predictions of the QED formulae are:

angular distribution: $$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta)$$

total cross section: $$\sigma = \frac{4\pi\alpha^2}{3s} \cdot 3 \cdot \sum_f Q_f^2$$

This receives a relevant QCD correction: $$\cdot(1 + \frac{\alpha_s(s)}{\pi})$$



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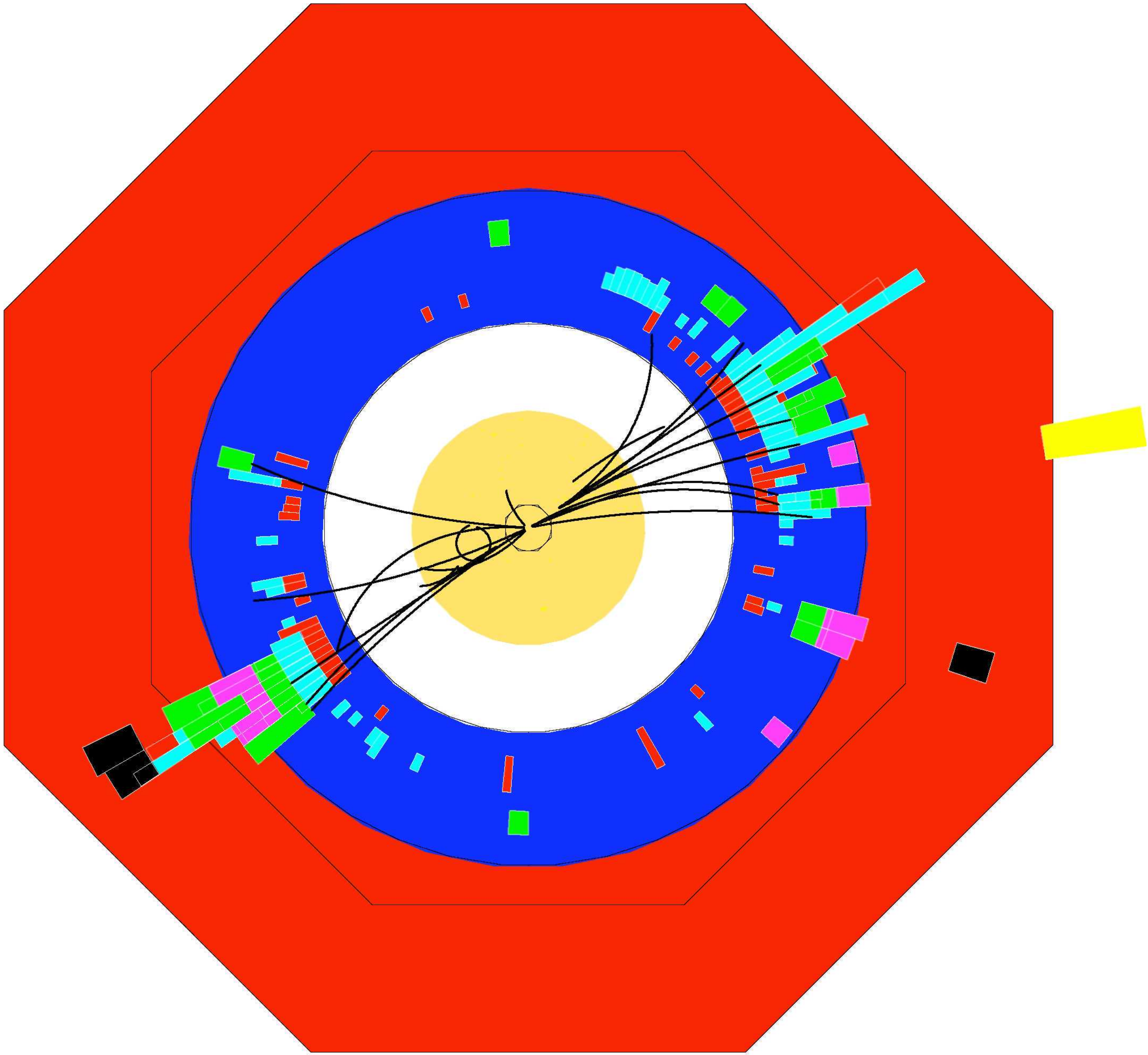
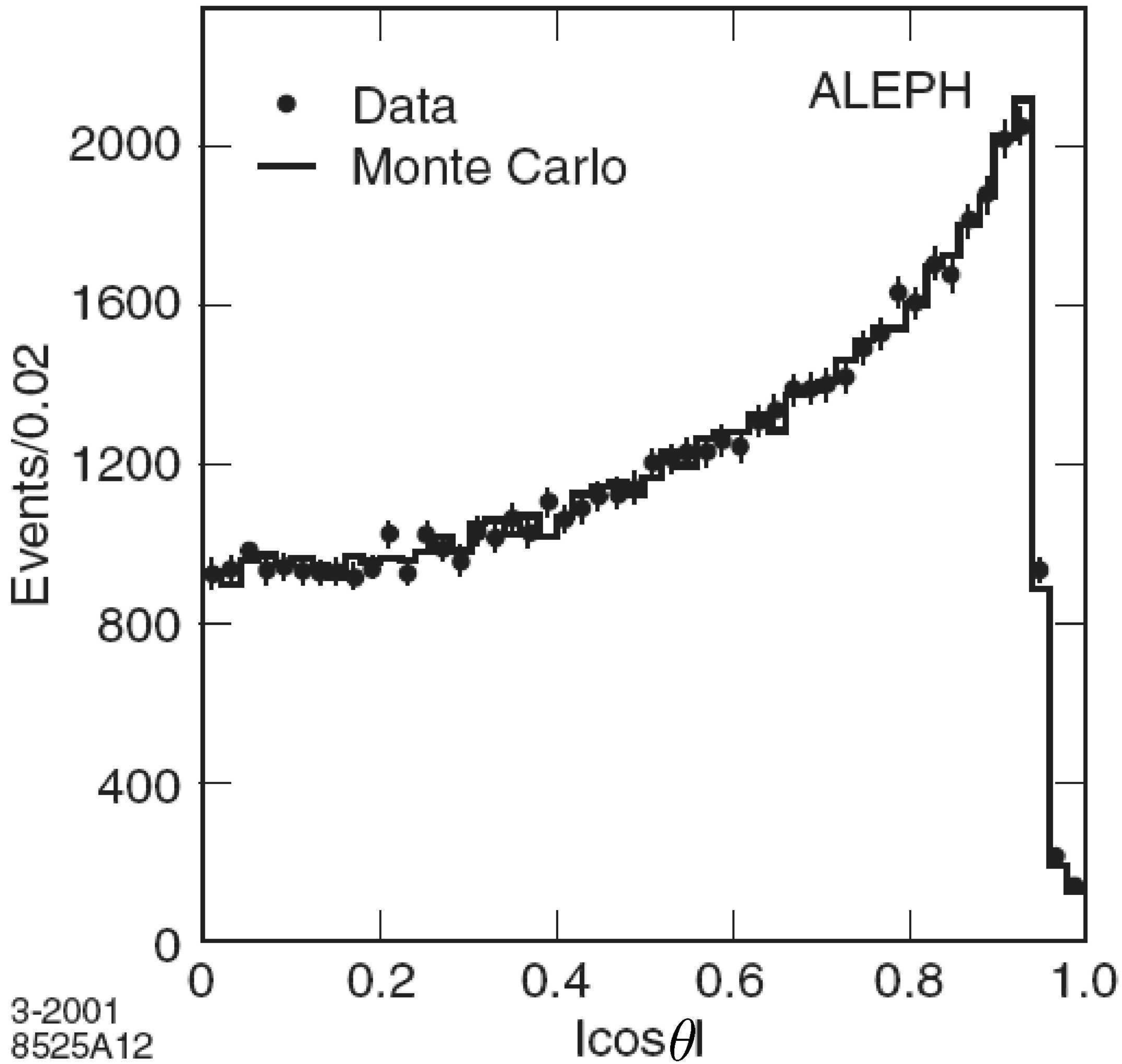
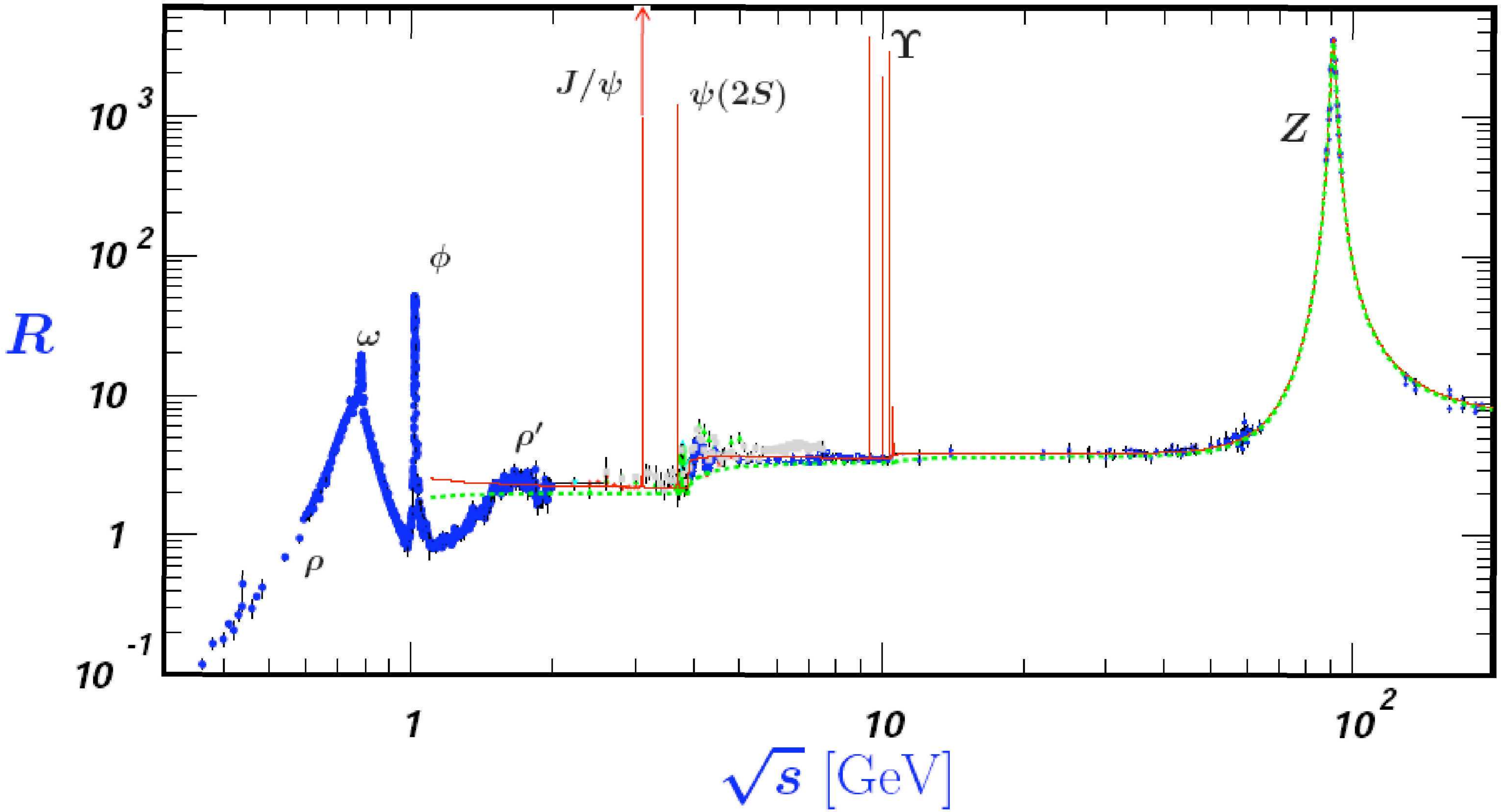


SLD



Data
Monte Carlo
ALEPH
Events/0.02
2000
1600
1200
800
400
0
0
0.2
0.4
0.6
0.8
1.0
|cosθ|
3-2001
8525A12



PDG compilation

It is easy enough to add the full SU(2)xU(1) gauge theory to these predictions. First, review the basic structural formulae.

There are three parameters: $g\ ,\ g'\ ,\ v = \langle\phi\rangle = 246\ \mathrm{GeV}$

The weak-interaction boson masses are given by:

$$m_W = gv/2 \qquad m_Z = (g^2 + g'^2)^{1/2} v/2 \qquad m_\gamma = 0$$

$\gamma\ ,\ Z$ are linear combinations of $A^0\ ,\ B^0$

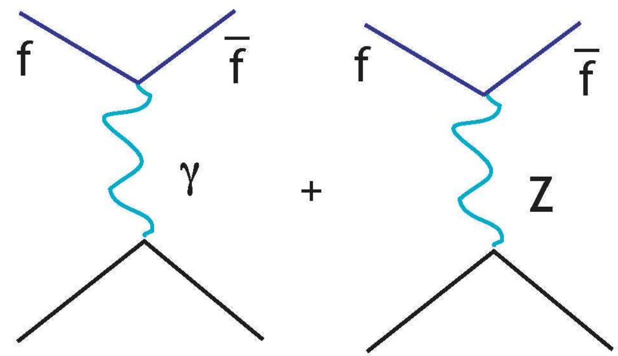
$$\gamma = \sin\theta_w A^0 + \cos\theta_w B^0$$

γ couples to Q ; Z couples to

$$Q_Z = I^3 - \sin^2\theta_w Q$$

The values of I^3 and Q are very different for different fermion states, so this single formula induces a variety of cross sections and asymmetries.

These couplings lead to modifications of the cross sections for $e^- e^+ \to f \overline{f}$



$$i\mathcal{M}(e^-_L e^+_R \to f_L \overline{f}_R) = ie^2(1+\cos\theta)\cdot s$$
$$\cdot\left[-\frac{Q}{s} + \frac{(-\frac{1}{2}+s_w^2)(I^3 - Qs_w^2)}{s_w^2 c_w^2}\frac{1}{s-m_Z^2}\right]$$
$$i\mathcal{M}(e^-_L e^+_R \to f_R \overline{f}_L) = ie^2(1-\cos\theta)\cdot s$$
$$\cdot\left[-\frac{Q}{s} + \frac{(-\frac{1}{2}+s_w^2)(-Qs_w^2)}{s_w^2 c_w^2}\frac{1}{s-m_Z^2}\right]$$

Notice that these formulae lead to all possible P, C violating effects - dependence on e- polarization, dependence of f polarization, forward-backward asymmetry in $e^- e^+ \to f \overline{f}$

It is worth pausing to give a precise definition of asymmetries. Take the forward-backward asymmetry as an example

$$A_{FB} = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)}$$

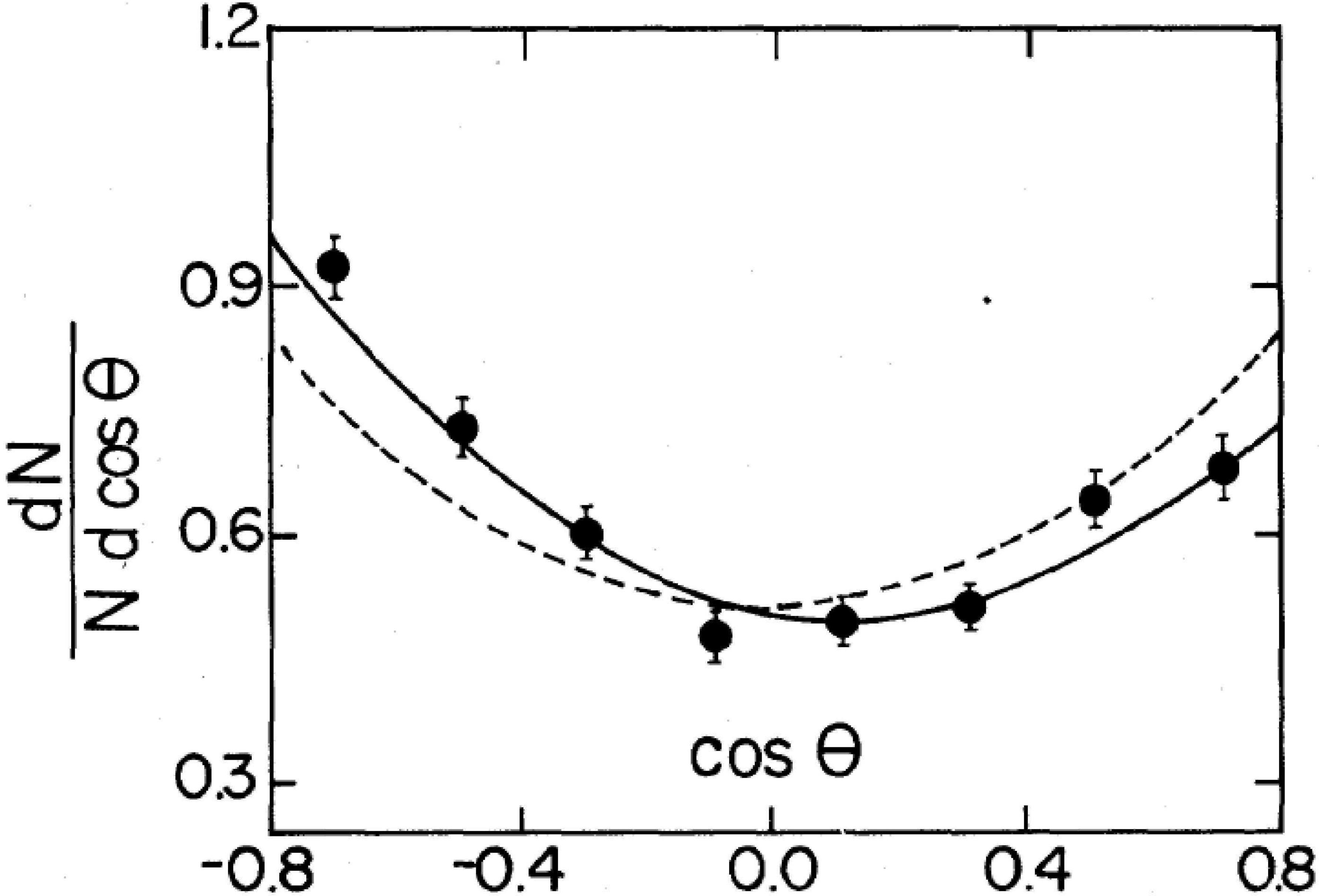
for pure $e^-_L e^+_R \to f_L \overline{f}_R$ $\qquad \frac{d\sigma}{d\cos\theta} \sim (1+\cos\theta)^2$

$$N(\cos\theta > 0) \sim \int_0^1 d\cos\theta\ (1+\cos\theta)^2 = \frac{7}{3}$$

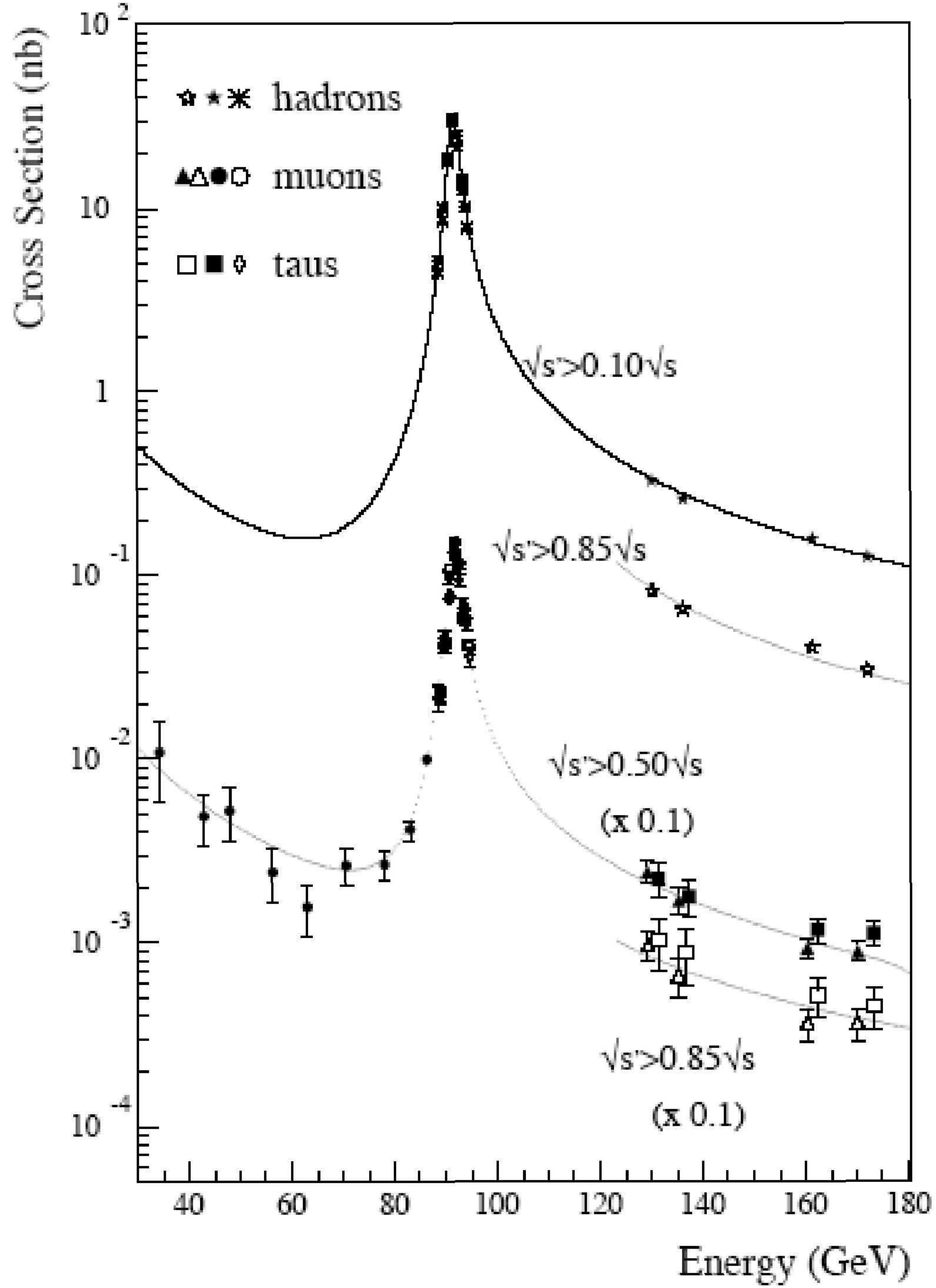
$$N(\cos\theta < 0) \sim \int_{-1}^0 d\cos\theta\ (1+\cos\theta)^2 = \frac{1}{3}$$

$$A_{FB} = \frac{7-1}{7+1} = \frac{3}{4}$$

This corresponds to 88% forward, 12% backward.

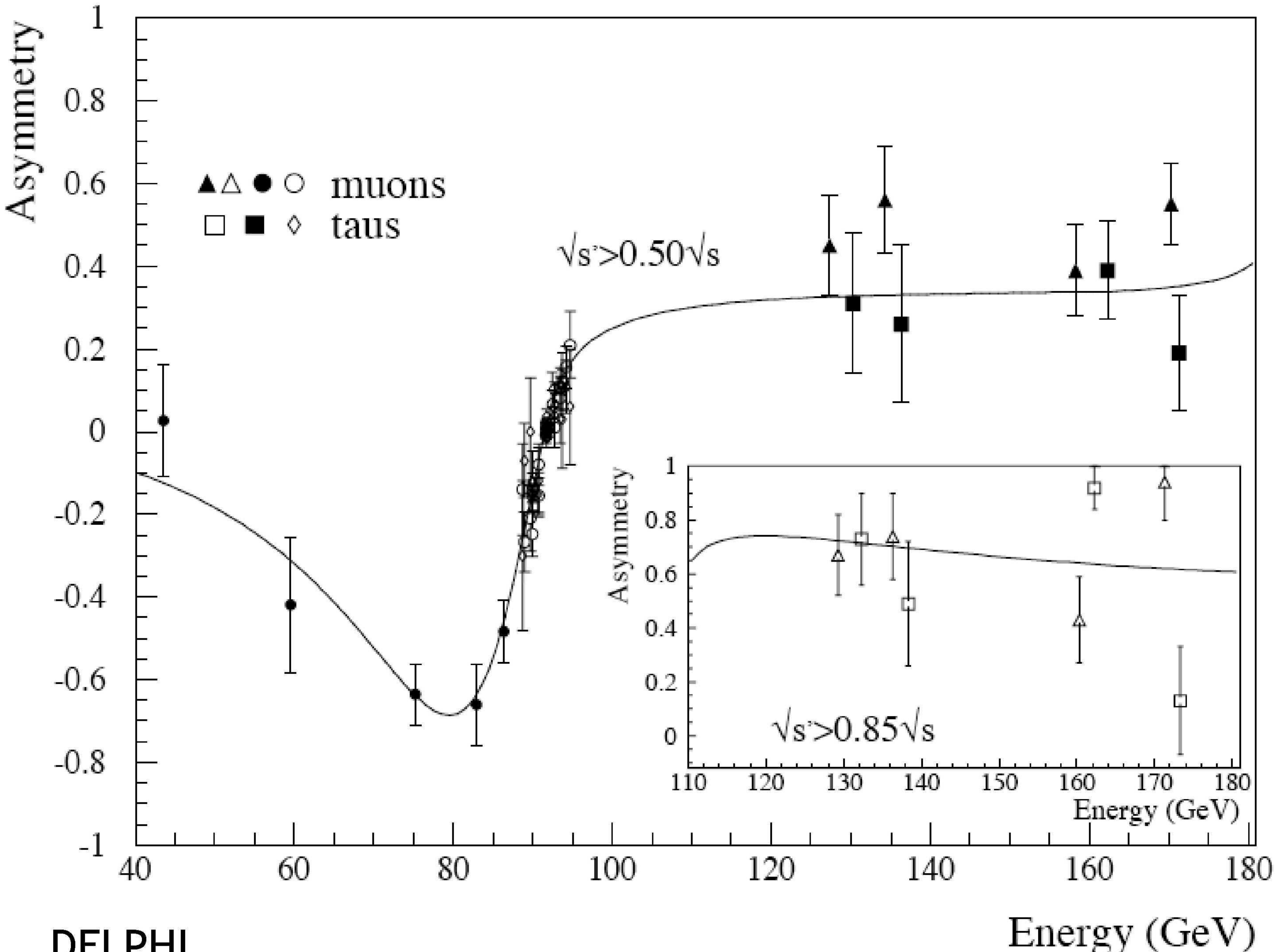


Mark-J at 34.6 GeV



Cross Section (nb)
10 2
10
1
10 -1
10 -2
10 -3
10 -4
hadrons
muons
taus
√s'>0.10√s
√s'>0.85√s
√s'>0.50√s
(x 0.1)
√s'>0.85√s
(x 0.1)
40
60
80
100
120
140
160
180
Energy (GeV)

DELPHI


Asymmetry
muons
taus
√s'>0.50√s
Asymmetry
√s'>0.85√s
Energy (GeV)
Energy (GeV)

DELPHI

In the limit of very high energies $\quad Q_e = -1 \quad I_e^3 = -\frac{1}{2},\ 0$

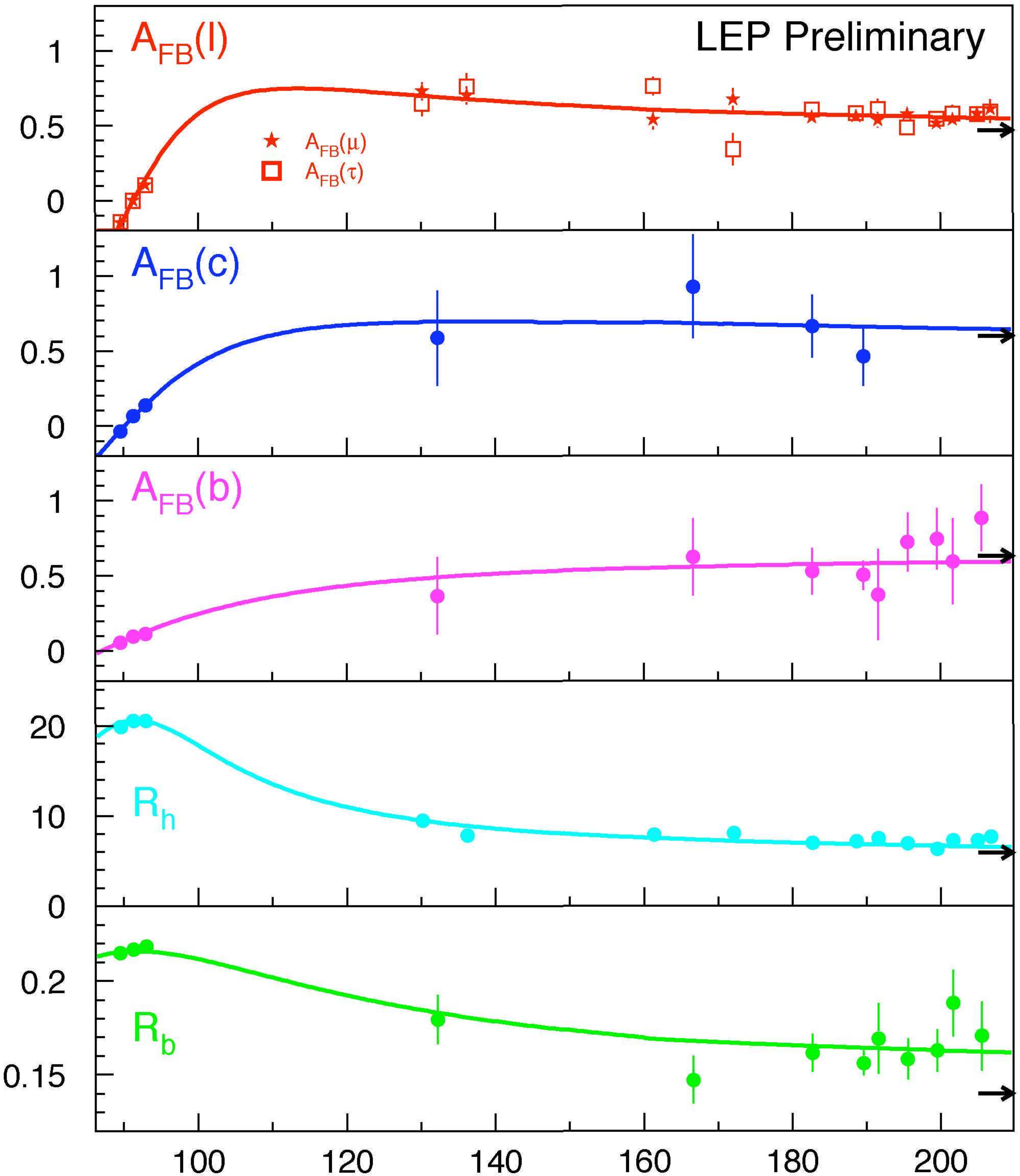
$$e^2\left[\frac{Q_eQ_f}{s} + \frac{(I_e^3 - Q_e s_w^2)(I_f^3 - Q_f s_w^2)}{s_w^2 c_w^2}\frac{1}{s - m_Z^2}\right]$$

$$\to \frac{e^2}{s}\left[\frac{I_e^3 I_f^3}{s_w^2} + \frac{I_e^3 I_f^3}{c_w^2} - \frac{Q_e I_f^3 + I_e^3 Q_f}{c_w^2} + Q_e Q_f (1 + \frac{s_w^2}{c_w^2})\right]$$

$$= \frac{e^2}{s}\left[\frac{I_e^3 I_f^3}{s_w^2} + \frac{(I_e^3 - Q_3)(I_f^3 - Q_f)}{c_w^2}\right]$$

$$= \frac{1}{s}[g^2 I_e^3 I_f^3 + g'^2 Y_e Y_f]$$

that is, SU(2) x U(1) is restored at high energies.



data compilation
by Hildreth

The coupling of each chiral fermion flavor to the Z boson is proportional to the charge

$$Q_Z = I^3 - Q s_w^2$$

It is a powerful test the Standard Model to measure these charges individually and see whether they are accounted for by a single universal parameter s_w^2 .

The accuracy of the experiments was such that order- α radiative correction must be included to interpret the results.

To begin, let's make a table of the Q_Z for an illustrative value $s_w^2 = 0.231$

species	Q_{ZL}	Q_{ZR}	S_f	A_f
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	—	0.25	1
e, μ, τ	$-\frac{1}{2} + \sin^2\theta_w$	$\sin^2\theta_w$	0.126	0.15
u, c, t	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_w$	$-\frac{2}{3}\sin^2\theta_w$	0.144	0.67
d, s, b	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_w$	$\frac{1}{3}\sin^2\theta_w$	0.185	0.94

where

$$S_f = Q_{ZL}^2 + Q_{ZR}^2 \qquad A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{Q_{ZL}^2 + Q_{ZR}^2}$$

The S_f give the total rate of Z decay to that species.

The A_f give the parity asymmetries in Z decays.

The partial width of the Z into a fermion species f is given by:

$$\Gamma(Z \to f\overline{f}) = \frac{\alpha m_Z}{6 s_w^2 c_w^2} \cdot S_f$$

times the factor $3(1 + \alpha_s/\pi)$ = 3.11 for quarks.

This gives the following table of partial widths and branching ratios:

species	$\Gamma(Z \to f\overline{f})$	BR
ν_e, ν_μ, ν_τ	167 MeV	6.7%
e, μ, τ	84 MeV	3.4%
u, c	300 MeV	12.0%
d, s, b	383 MeV	15.3%

Including a small correction for the case of $\Gamma(Z \to b\bar{b})$, we find a total width

$$\Gamma_Z = 2.50 \text{ GeV}$$

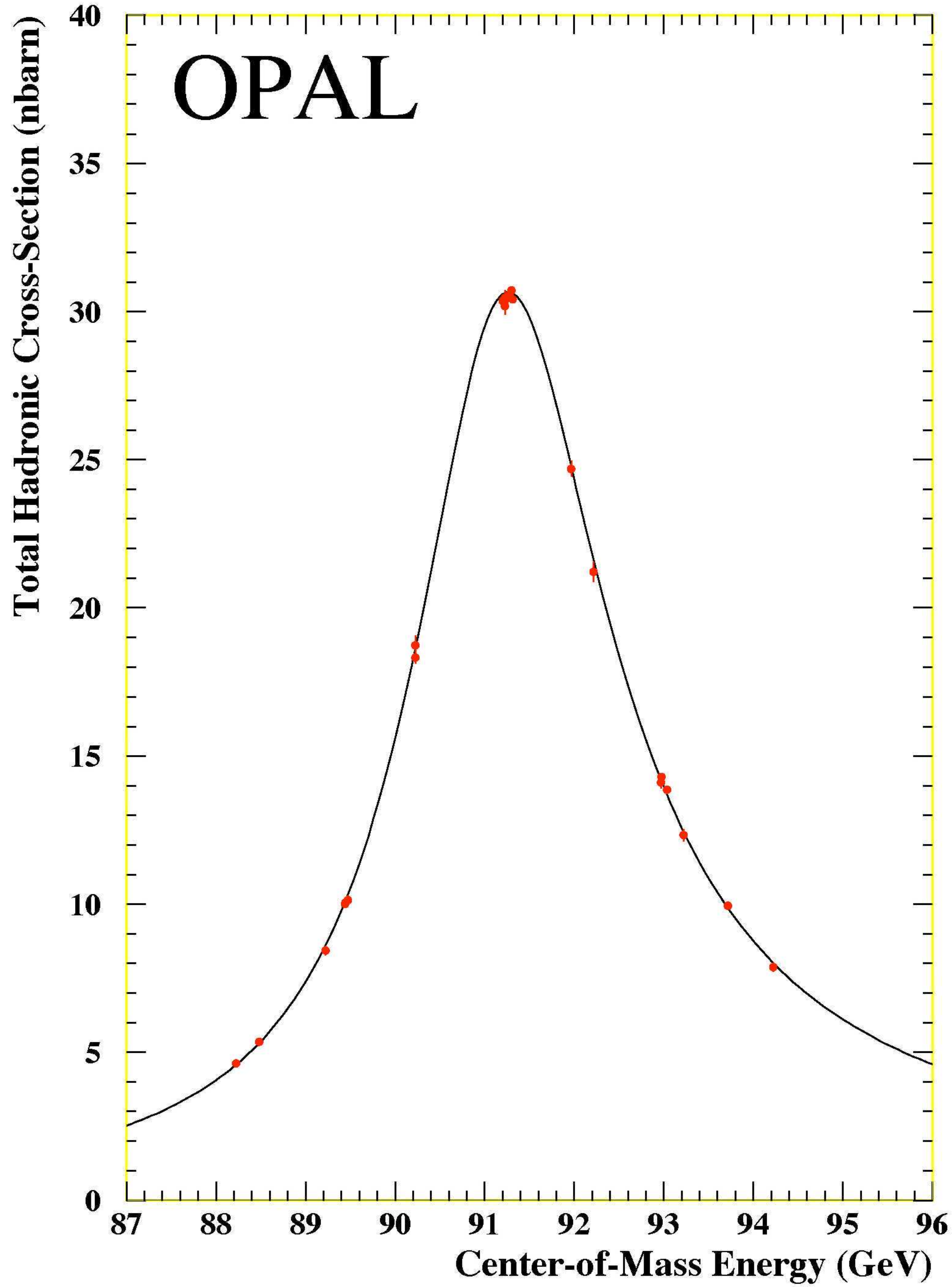
To test these predictions, we first measure e+e- annihilation at the Z resonance and measure the relative branching ratios to hadrons and to visible leptons.

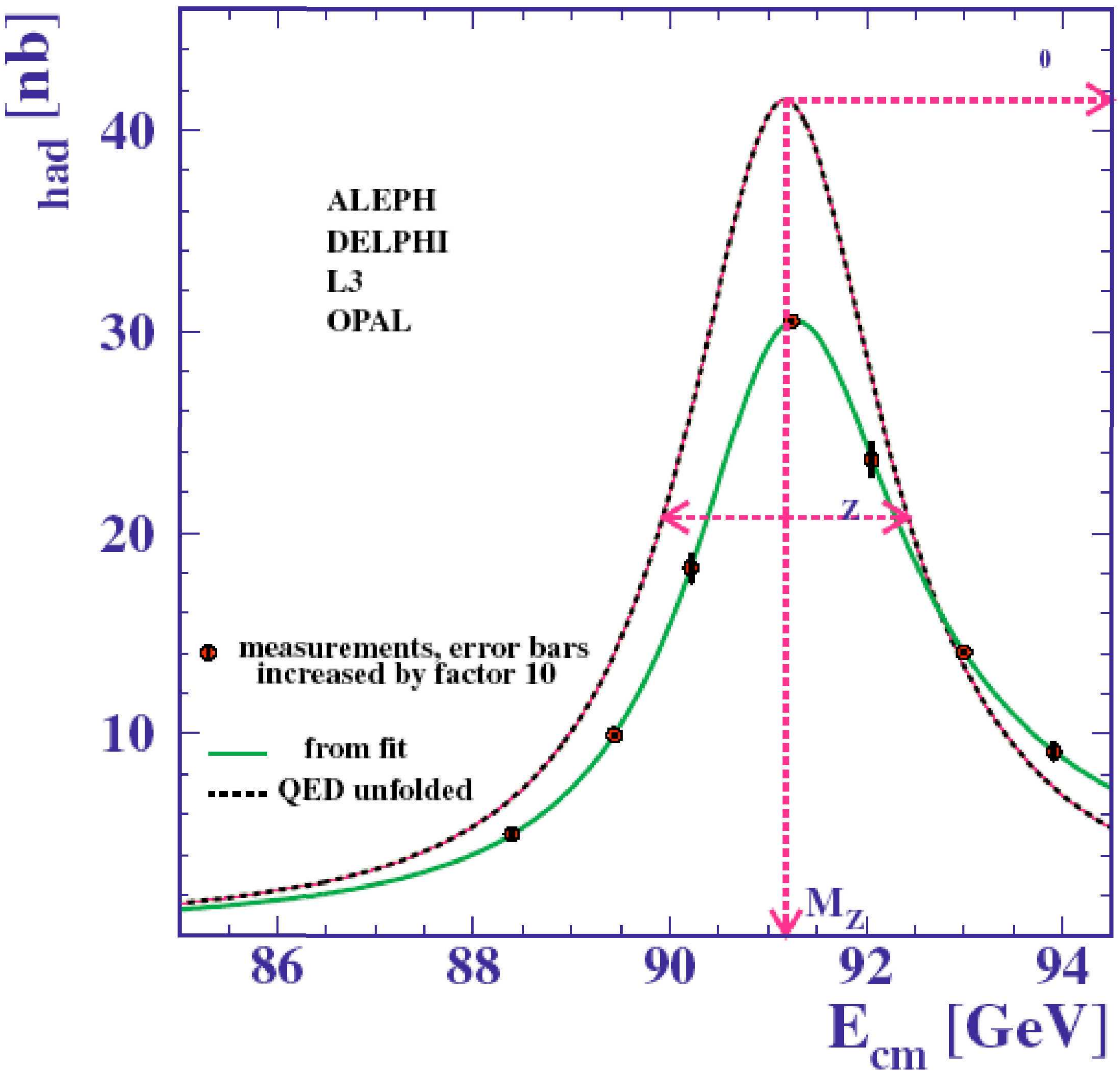
Then we must determine the total width.

The shape of the resonance is distorted by initial-state photon radiation. Thus, it is necessary to measure the detailed shape of the resonance to extract Γ_Z.

It is amusing to note that all three of the Standard Model interactions - QED, QCD, and of course SU(2) x U(1) contribute to the Z line-shape.

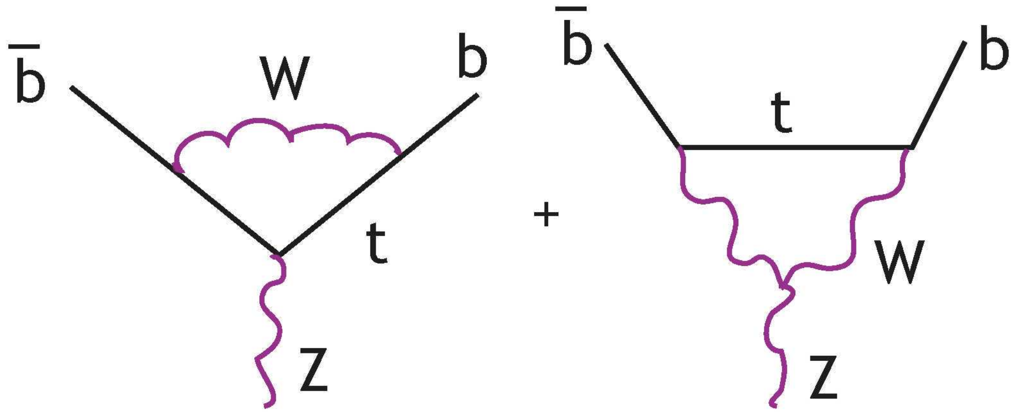
The result is: $\Gamma_Z = 2.4952 \pm .0023 \text{ MeV}$


OPAL
Total Hadronic Cross-Section (nbarn)
0
5
10
15
20
25
30
35
40
87
88
89
90
91
92
93
94
95
96
Center-of-Mass Energy (GeV)



composite of the four LEP experiments, showing the effect of ISR

There is a special consideration for the b quark. The diagrams



contribute a correction to the b_L Z charge,

$$Q_{ZbL} = -\left(\frac{1}{2} - \frac{1}{3}s_w^2 - \frac{\alpha}{16\pi s_w^2}\frac{m_t^2}{m_W^2}\right)$$

This is a -2% correction to the partial width. It is easier to measure the quantity

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \ \text{hadrons})}$$

which is almost independent of s_w^2 and so directly tests the above correction.

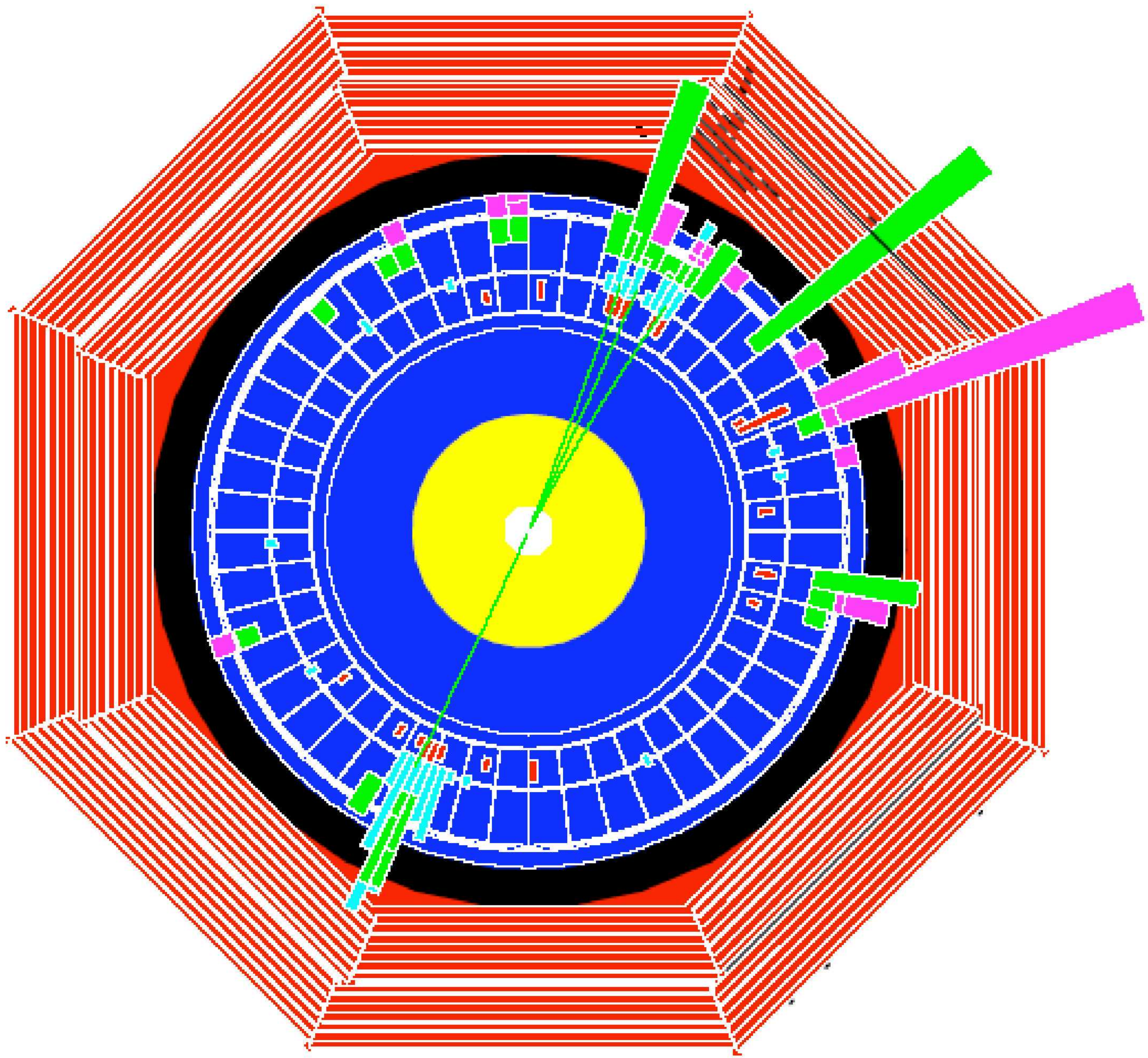
But how do we know which hadronic events contain b quarks ?

The three heavy fermions τ , c , b have weak-interaction decay times that are small but measurable. With a special-purpose device based on silicon strips or pixels, one can locate the decay vertices and identify the short-lived particles that they indicate.

	τ (ps)	$c\tau$ (mm)
τ	0.29	0.09
c (D^0)	0.41	0.12
b (B^0)	1.55	0.46

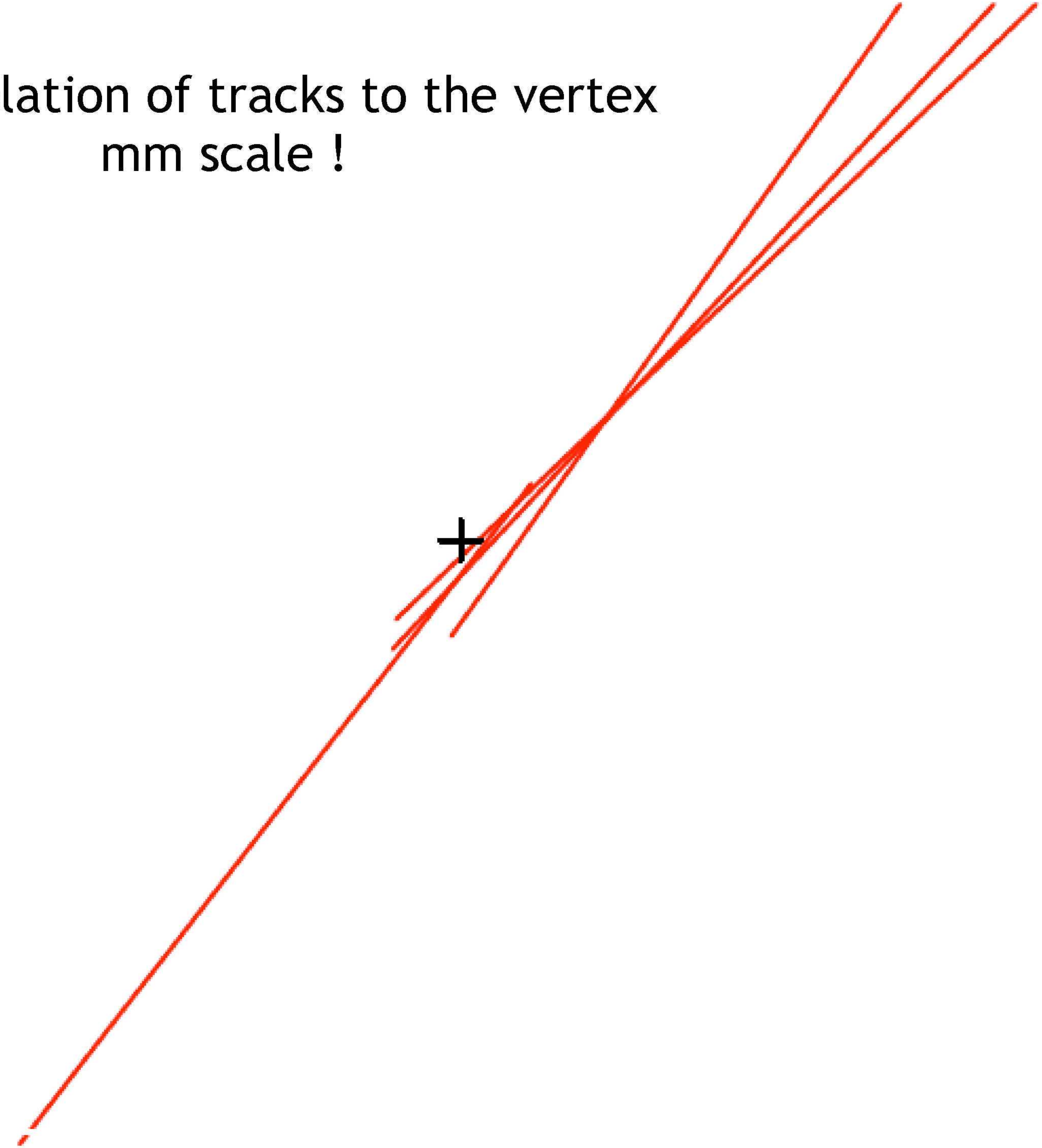
In analyses of this type, it is a challenge to the experimenters to choose a signal criterion that maximizes the efficiency of observing heavy quarks vertices while minimizing fakes.

$$e^+e^- \to \tau^+\tau^-$$

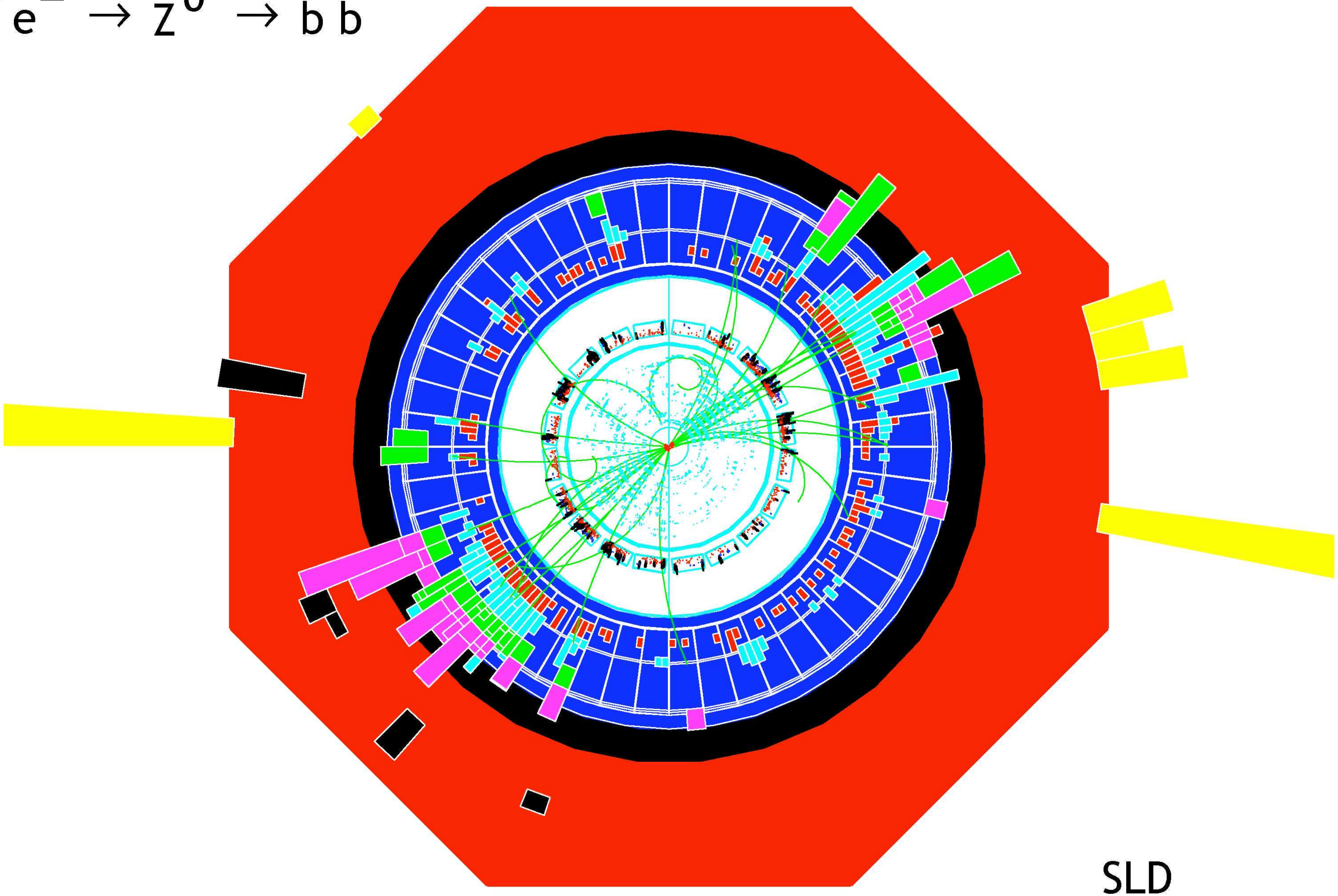


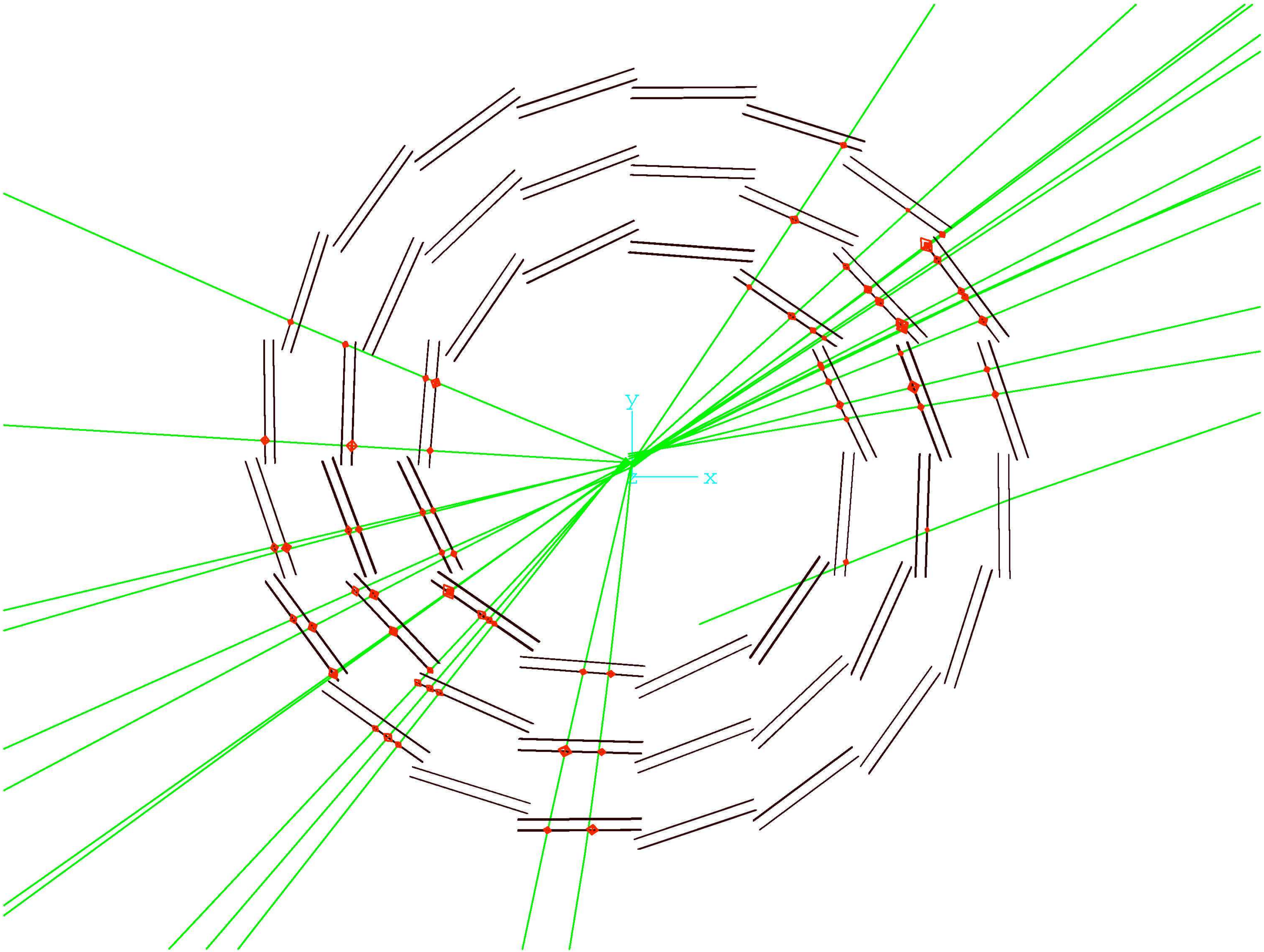
SLD

extrapolation of tracks to the vertex
mm scale !

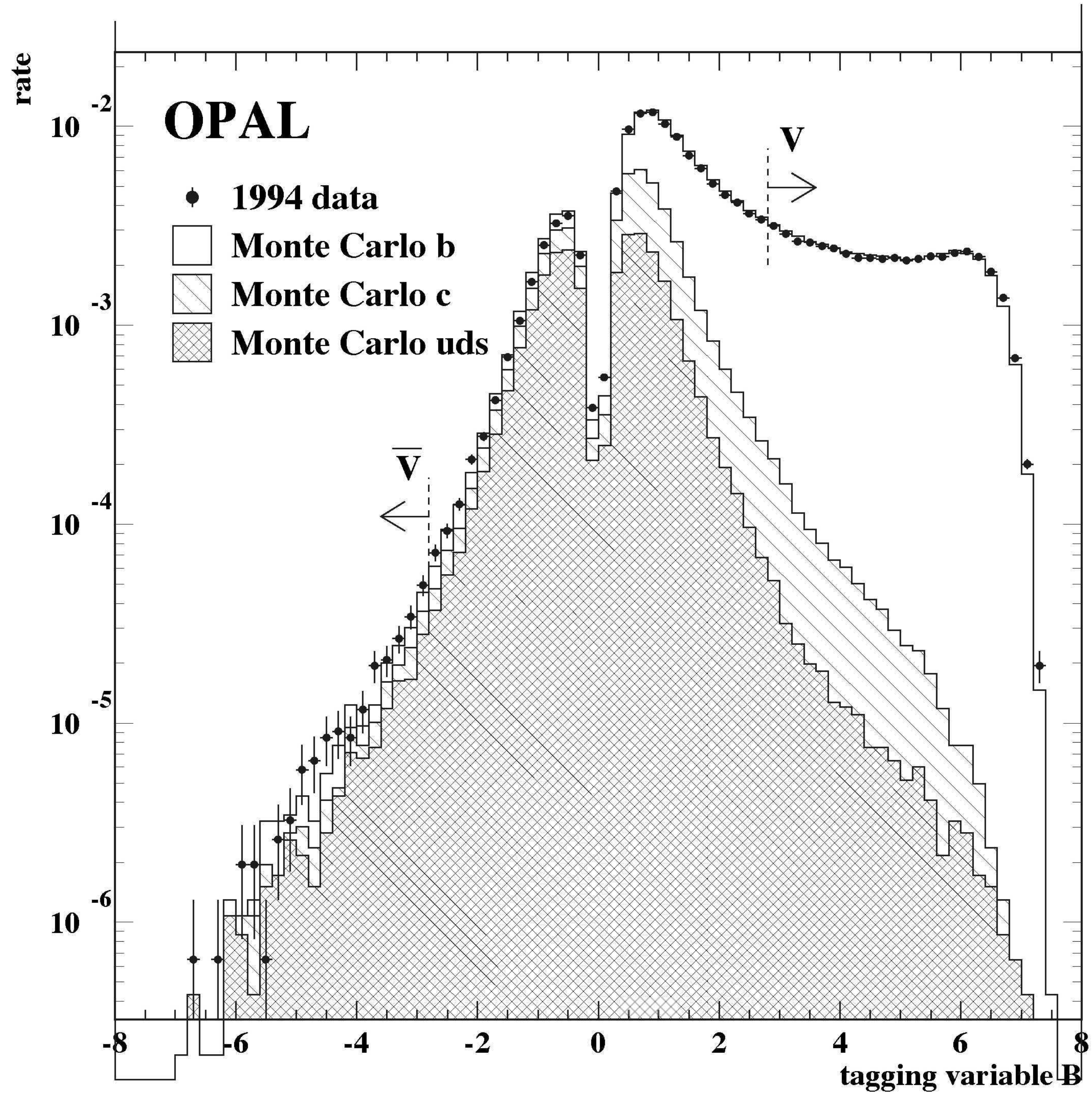


$e^+ e^- \rightarrow Z^0 \rightarrow b\,\bar{b}$

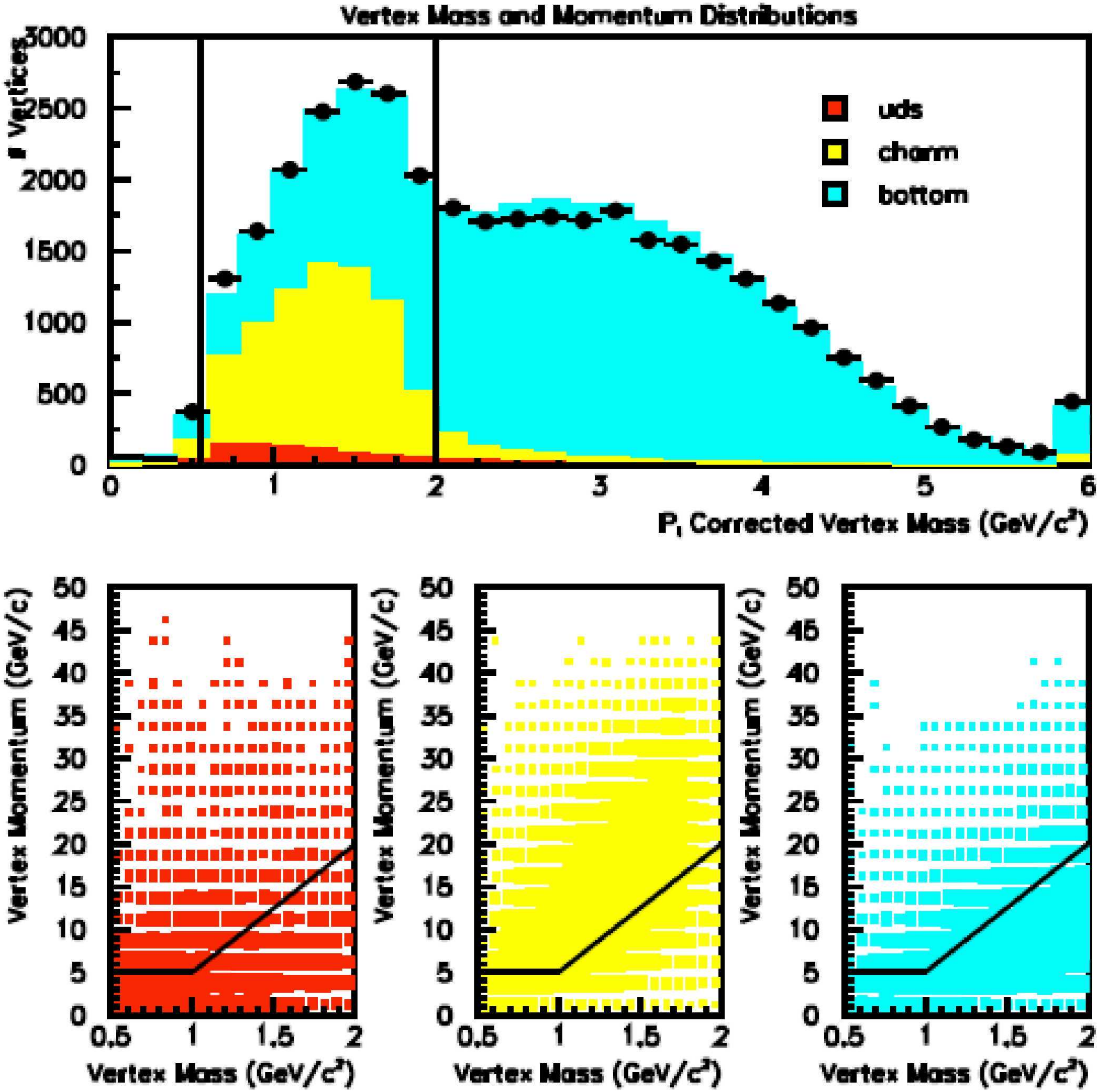




y
x
z



OPAL
rate
1994 data
Monte Carlo b
Monte Carlo c
Monte Carlo uds
V
$\overline{V}$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
-8
-6
-4
-2
0
2
4
6
8
tagging variable B



Vertex Mass and Momentum Distributions
Vertices
uds
charm
bottom
P_t Corrected Vertex Mass (GeV/c²)
Vertex Momentum (GeV/c)
Vertex Mass (GeV/c²)

SLD

The final result is:

$$R_b = 0.21643 \pm 0.00073$$

in excellent agreement with the Standard Model and confirming the -2% shift due to the t-W diagrams.

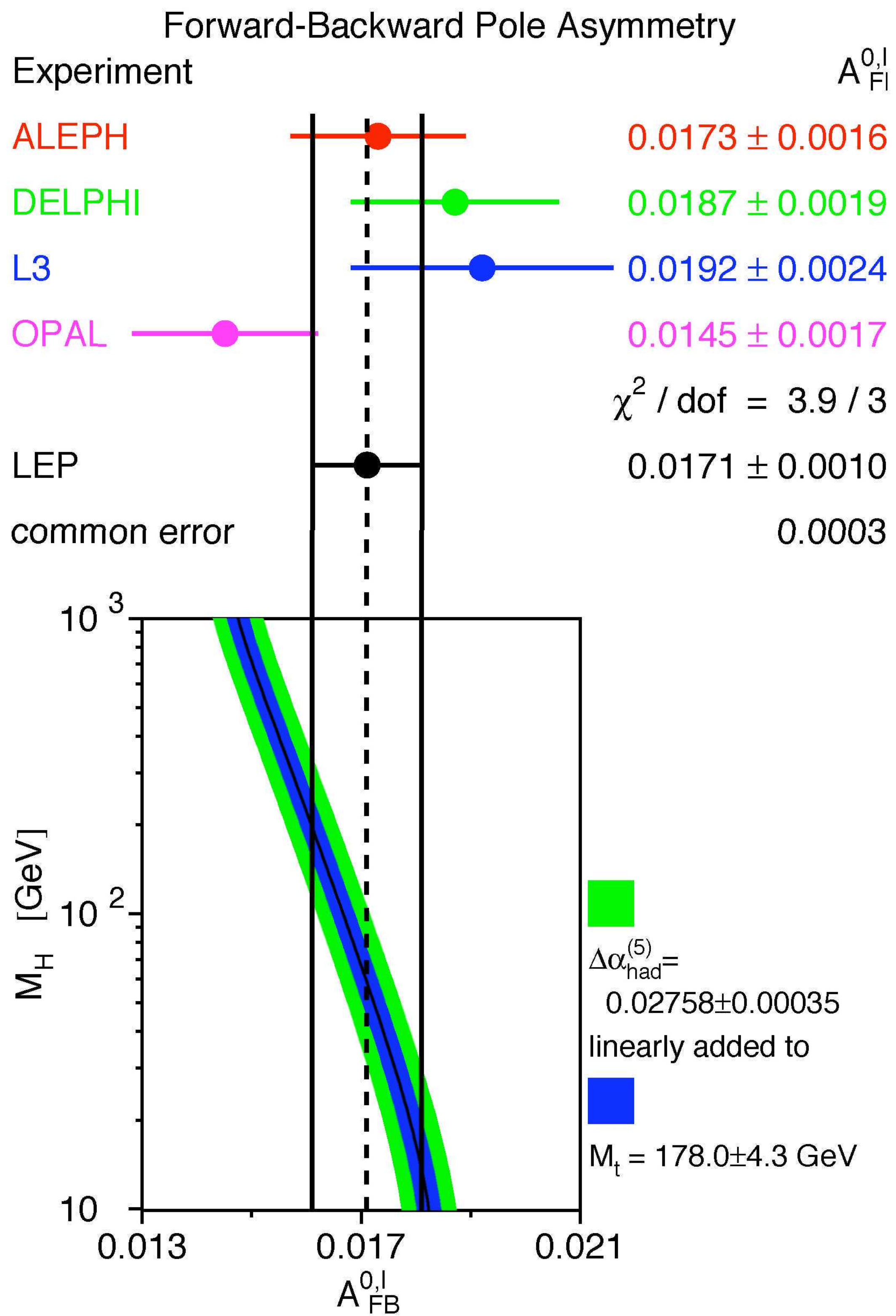
Next, consider the measurement of the A_f. There are three different techniques.

The first is to measure the unpolarized forward-backward asymmetry. For $e^+e^- \to f\overline{f}$ just at the Z resonance,

$$A_{FB} = \frac{3}{4} A_e A_f$$

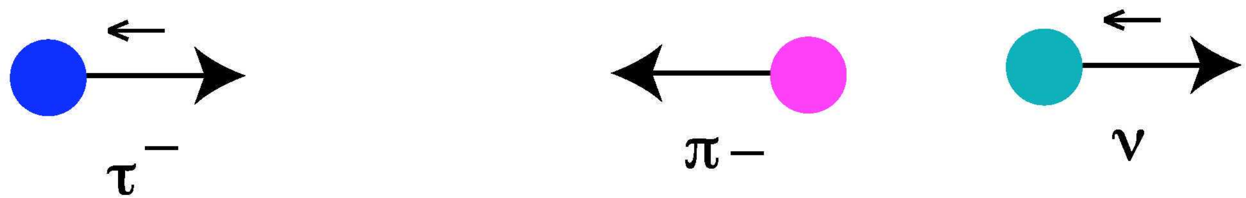
where the factor 3/4 comes from the slide on asymmetries.

Unfortunately, for leptonic final states, this is a 2% effect, reduced to 1% by radiative corrections. Nevertheless, the effect can be observed and measured.


Forward-Backward Pole Asymmetry
Experiment
$A_{FB}^{0,l}$
ALEPH 0.0173 ± 0.0016
DELPHI 0.0187 ± 0.0019
L3 0.0192 ± 0.0024
OPAL 0.0145 ± 0.0017
χ^2 / dof = 3.9 / 3
LEP 0.0171 ± 0.0010
common error 0.0003
10^3
10^2
10
M_H [GeV]
0.013
0.017
0.021
$A_{FB}^{0,l}$
$\Delta\alpha_{had}^{(5)}$ =
0.02758±0.00035
linearly added to
M_t = 178.0±4.3 GeV

Second, one can directly measure the polarization of τ leptons.

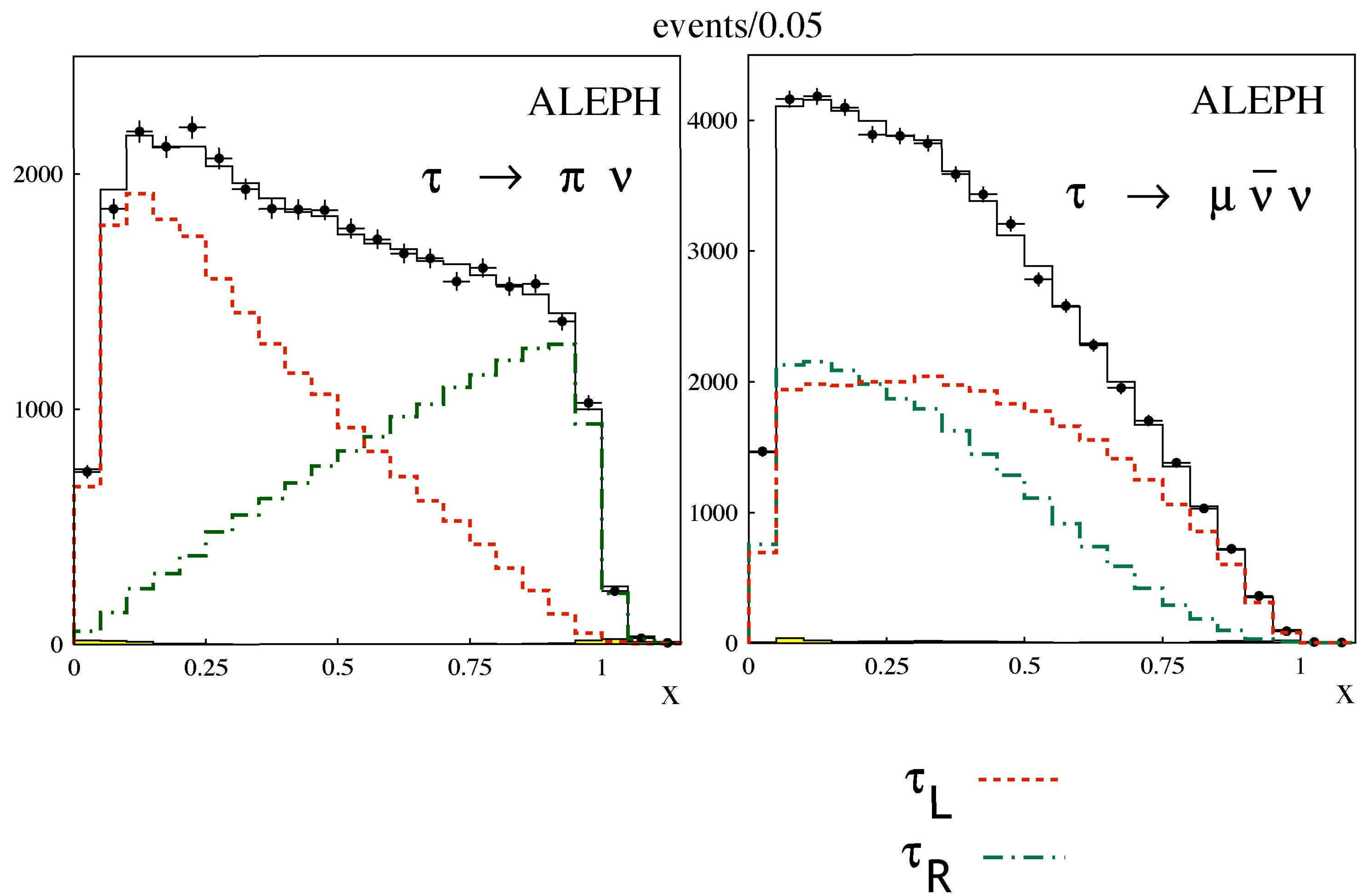
When τ decays to $\nu_\tau \pi$, the pion goes dominantly in the spin direction:



$$\frac{d\Gamma}{d\cos\theta} \sim (1 + \cos\theta)$$

Boosting to a relativistic τ, $$\frac{d\Gamma}{dE_\pi} \sim E_\pi$$

Similar effects are seen in other τ decay channels; for example, in $\tau \longrightarrow \nu_\tau \overline{\nu}_\mu \mu$, the muon goes in the direction opposite to the τ spin.


events/0.05
ALEPH
$\tau \rightarrow \pi\ \nu$
2000
1000
0
0
0.25
0.5
0.75
1
X
ALEPH
$\tau \rightarrow \mu\ \bar{\nu}\ \nu$
4000
3000
2000
1000
0
0
0.25
0.5
0.75
1
X
τ_L
τ_R

Combining these effects, one obtains:

$$A_\ell = 0.1465 \pm 0.0033$$

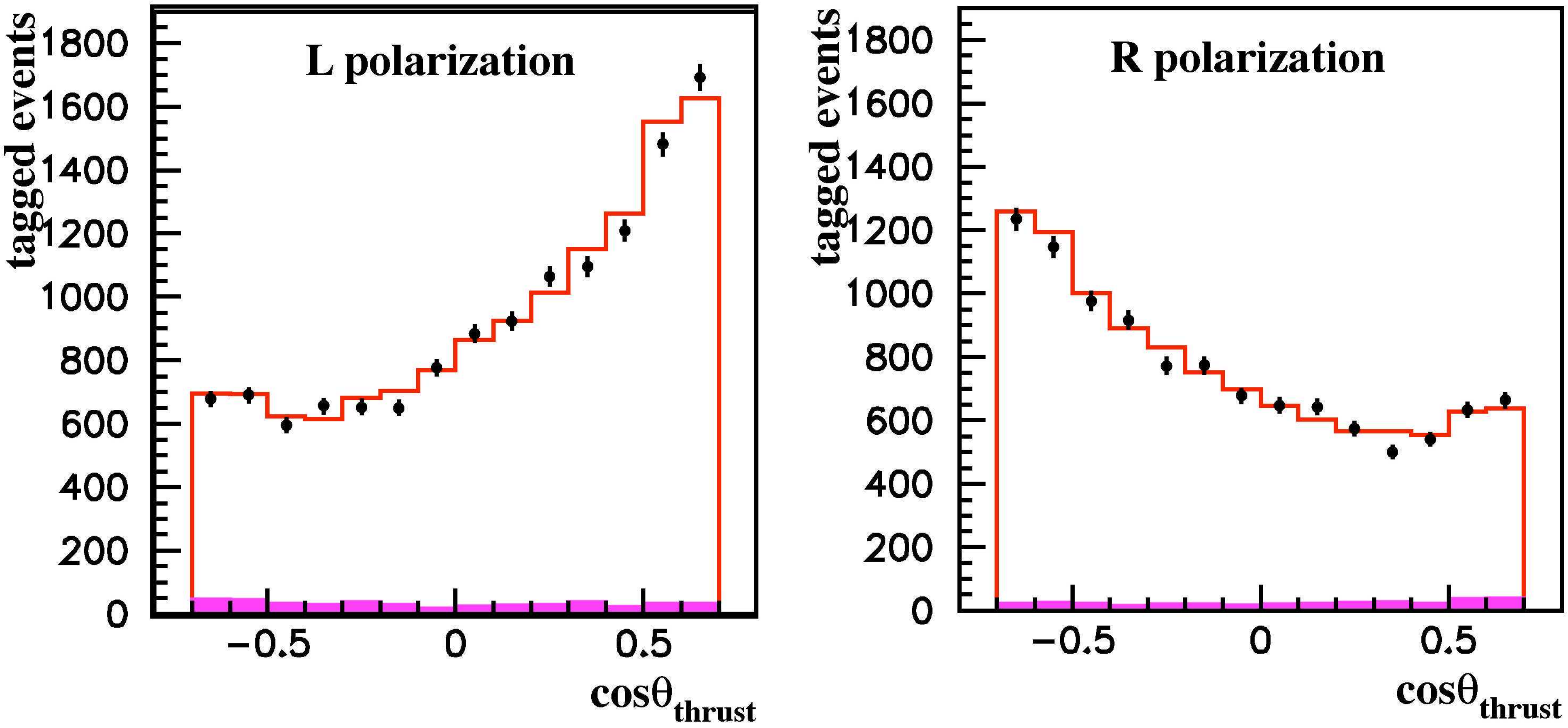
It was also possible at SLAC to polarize the electrons and measure A_e directly as an asymmetry in the total cross section on the Z resonances. This gives:

$$A_e = 0.1513 \pm 0.0021$$

Using the sample of heavy quark events, we can measure A_b and A_c from the forward-backward asymmetries.

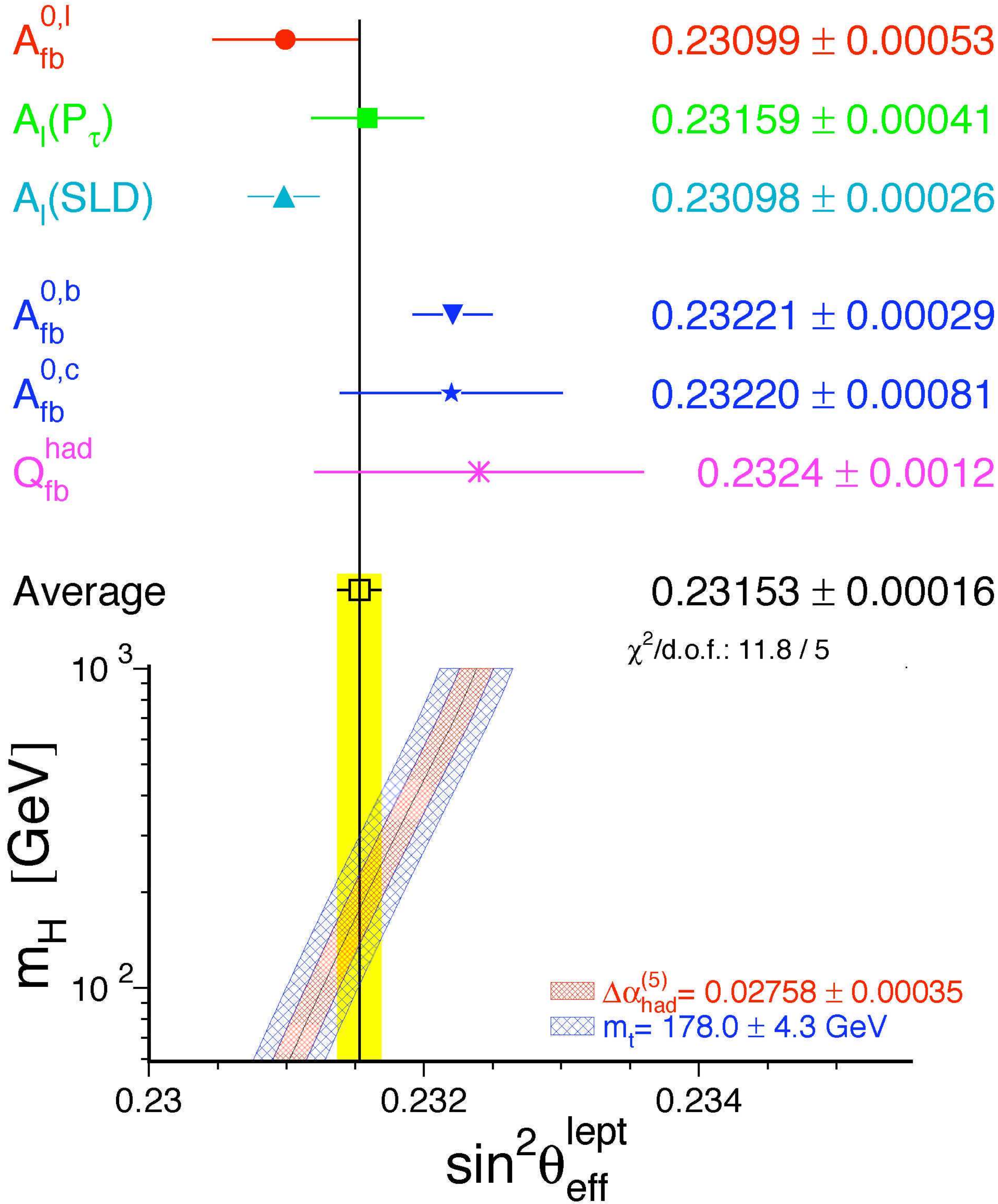
This depends on the ability to tell b from $\overline{b}$, c from $\overline{c}$, by the charges of leptons or K's from the weak decays.

With polarized electrons, there is a remarkable effect showing that A_b is almost maximal, as the Standard Model predicts.



$A_b = 0.94$ at the Z^0 SLD

The various measurements are compatible, and they give a very precise value for s_w^2.



Next, look at the W boson. What do W's look like ?

W^+ decays to $d_L + \overline{u}_R$ in each (light) SU(2) doublet of the SM

$$i\mathcal{M}(W^+ \to \nu_e e^+) = i\frac{g}{\sqrt{2}}\overline{u}_L(\nu_e)\gamma_\mu v_L(e^+)\epsilon^\mu(W^+)$$

$$\Gamma(W^+ \to \nu_e e^+) = \frac{g^2}{48\pi}m_W = 220 \text{ MeV}$$

the partial widths to $\nu_\mu\mu^+$, $\nu_\tau\tau^+$ are the same.
the partial widths to quarks are multiplied by $3 \cdot (1 + \alpha_s/\pi)$

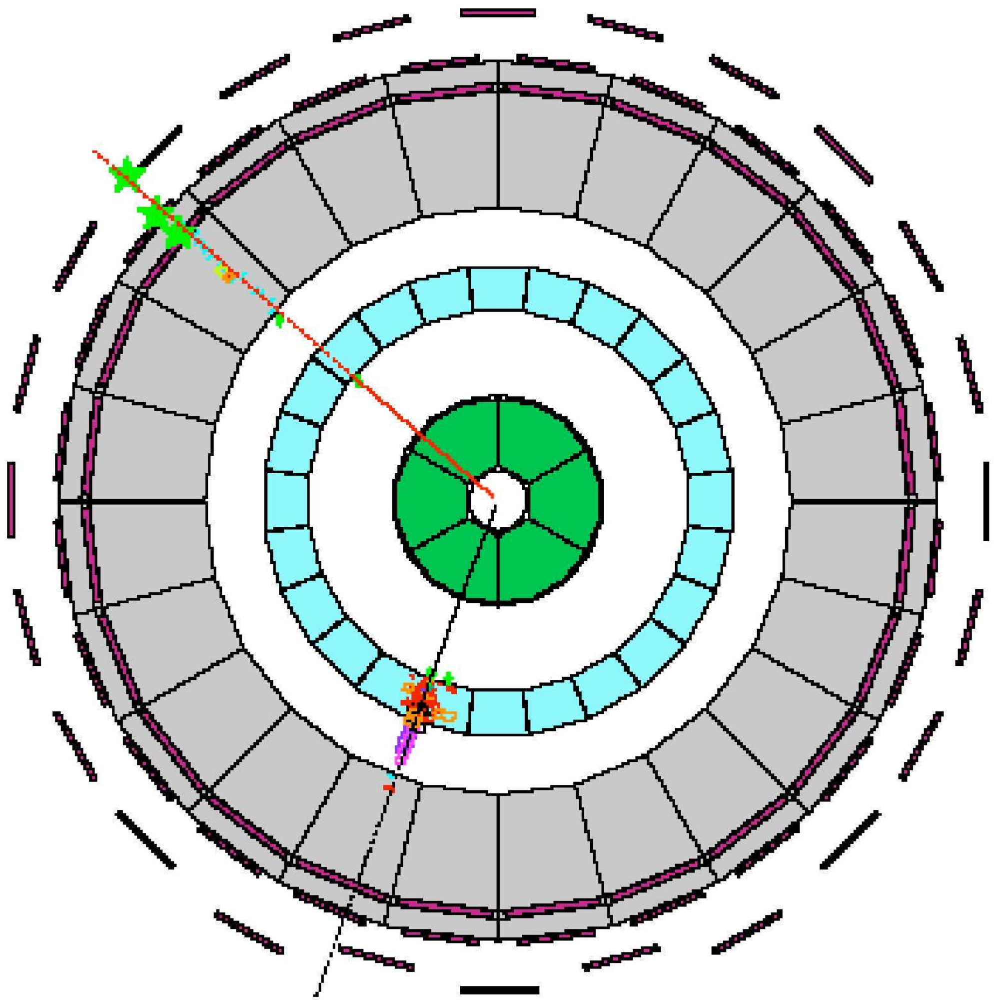
Then
$$BR(W^+ \to \nu_e e^+) = 11\ \%$$
$$BR(W^+ \to u\overline{d}) = 34\ \%$$

This leads to an interesting variety of $e^+e^- \to W^+W^-$ events

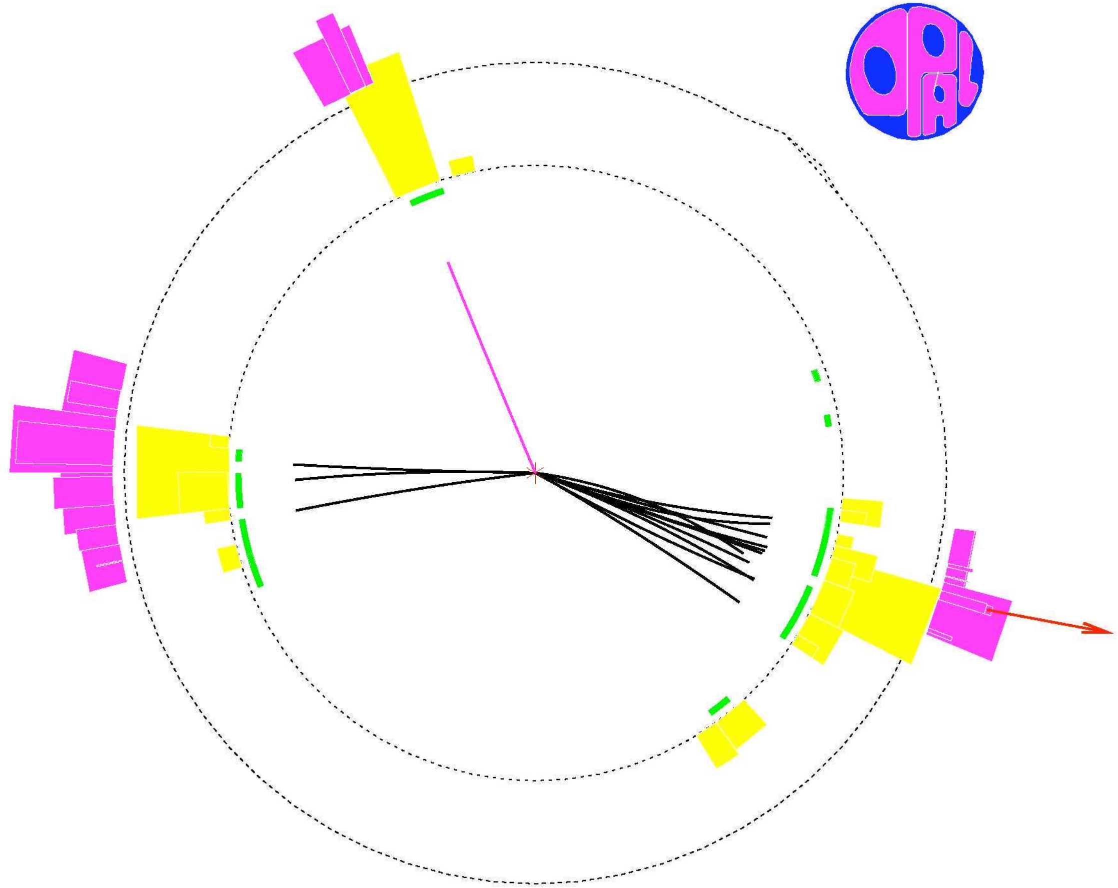
$$e^+e^- \to W^+W^-$$

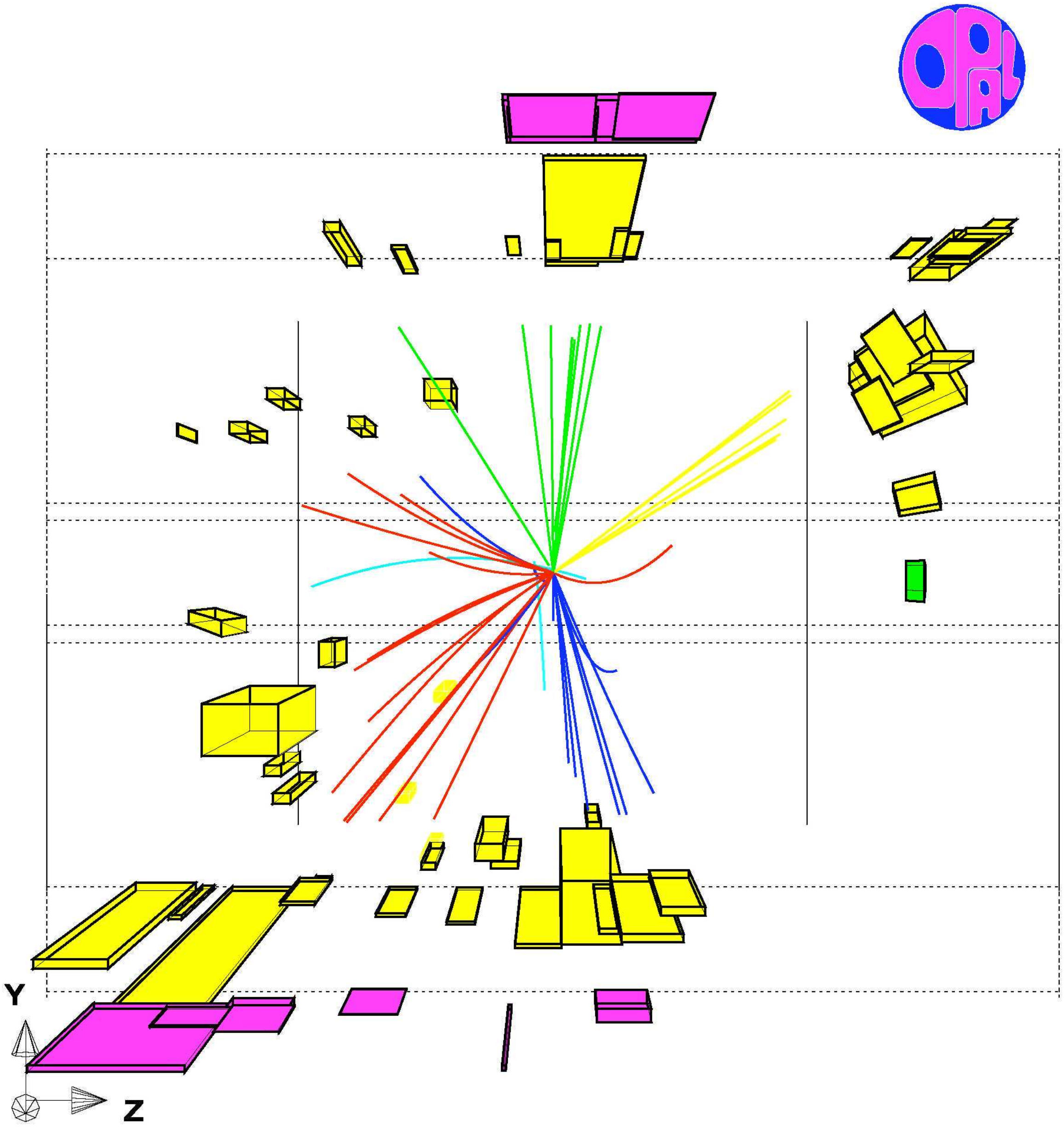
$$\to e\ \bar{\nu}\ \mu\ \nu$$

DELPHI



$$e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}\tau\nu$$



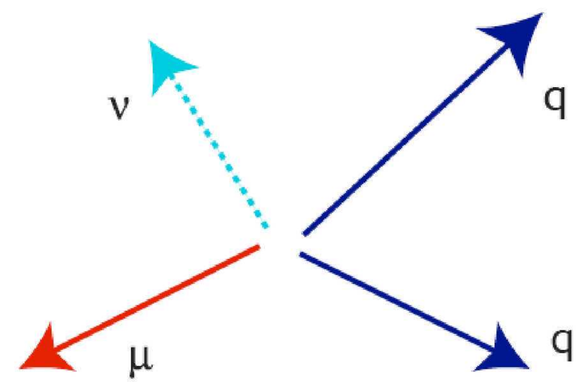


Y
Z

These events can be used to make a precision measurement of the W boson mass.

For example, look at $e^+e^- \to W^+W^- \to (q\overline{q})(\ell\nu)$

This leads to final states with 2 jets,
an isolated lepton, and
missing (unseen) momentum.

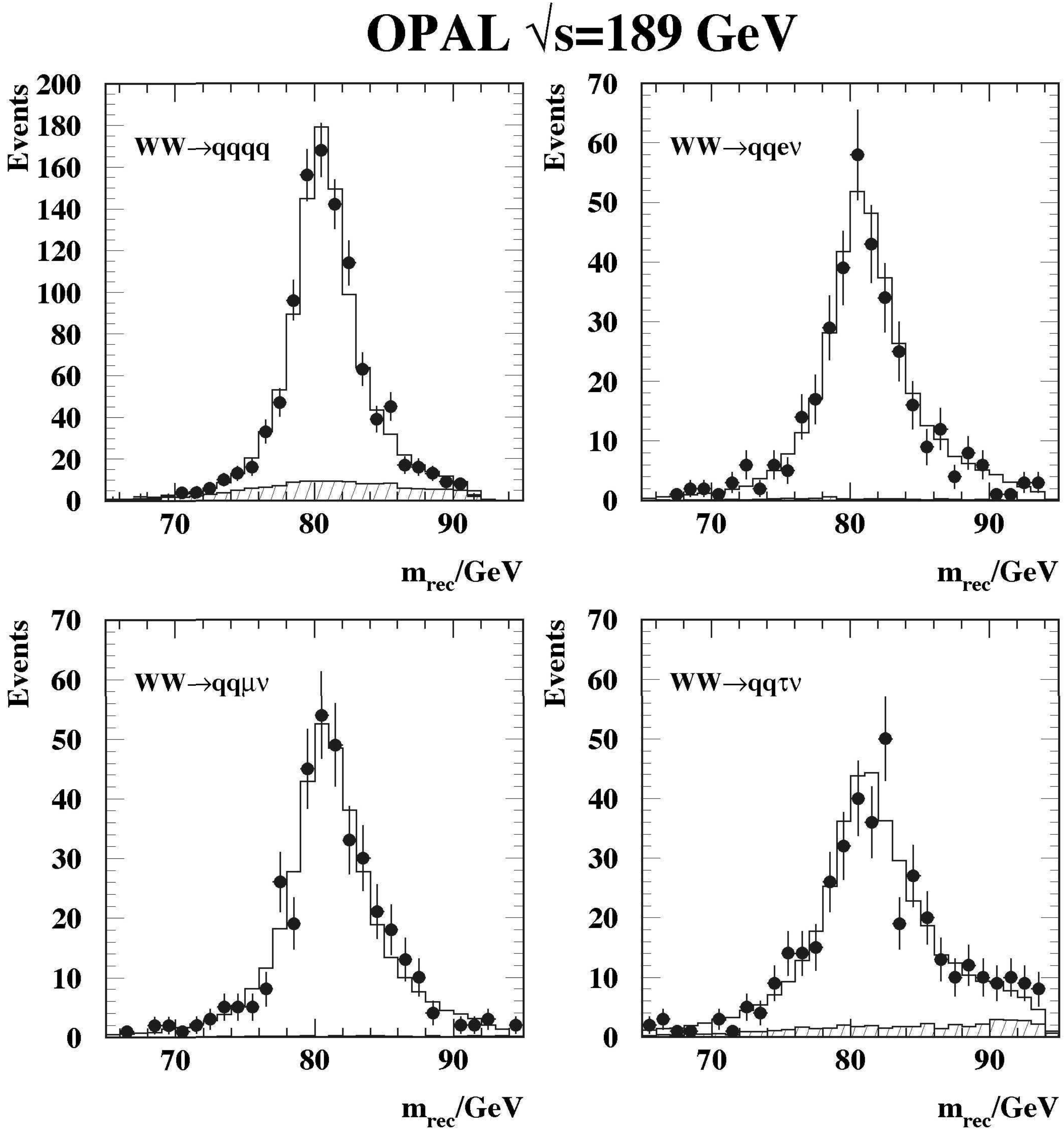


Determine the axes of the jets, leaving the energies unkown.

$$\vec{p}_1 = E_1\hat{n}_1 \quad \vec{p}_2 = E_2\hat{n}_2 \qquad \vec{p}_\ell = E_\ell\hat{n}_\ell \quad \vec{p}_\nu = E_\nu\hat{n}_\nu$$

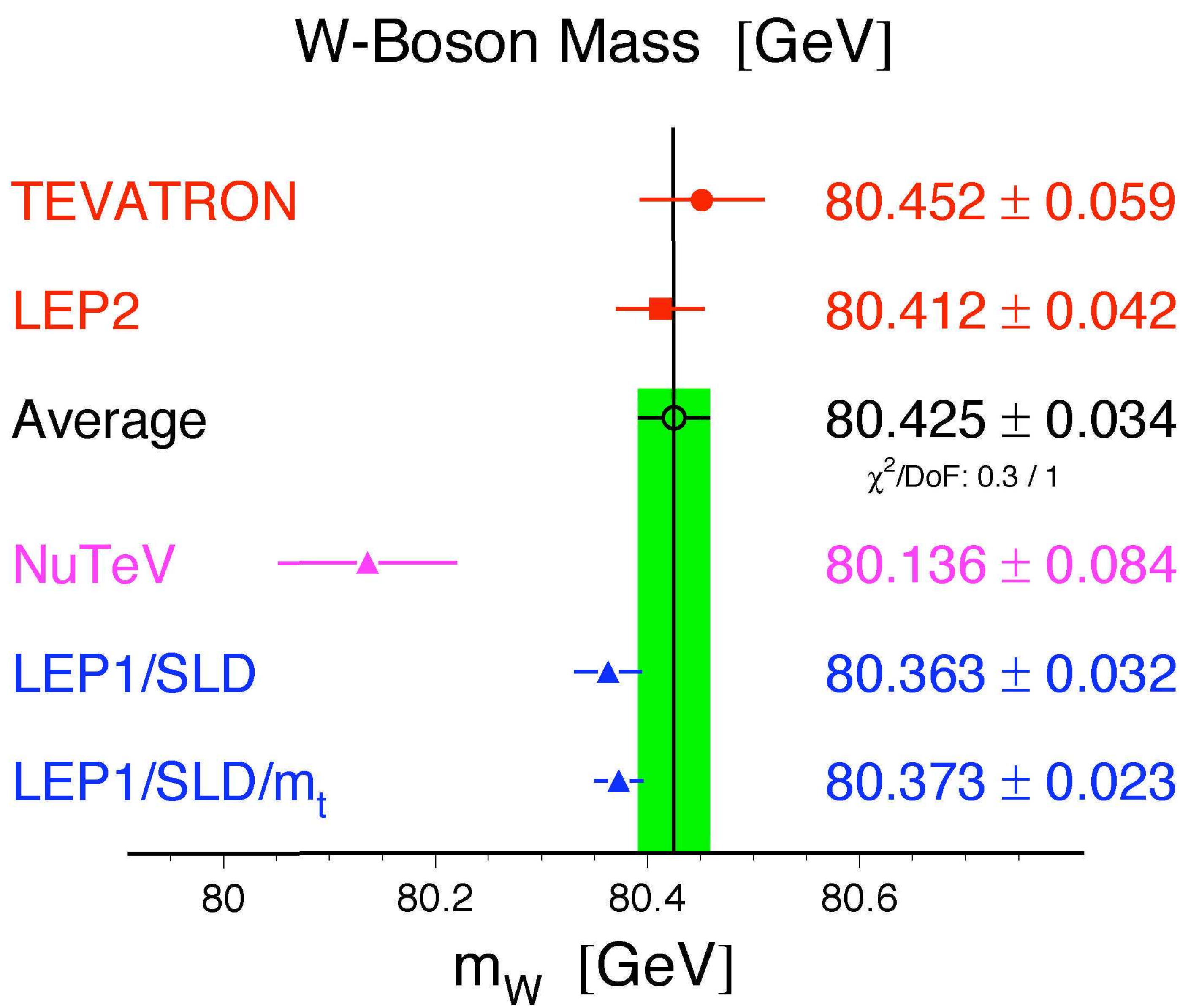
Fix the jet energies and the neutrino parameters from EM conservation and $(p_1 + p_2)^2 = (p_\ell + p_\nu)^2 = m_W^2$

There are 5 parameters, 5 unknowns. ISR, quark fragmentation, give small, estimable, corrections.


OPAL √s=189 GeV
WW→qqqq
WW→qqeν
WW→qqμν
WW→qqτν
Events
m_rec/GeV
0
20
40
60
80
100
120
140
160
180
200
10
30
50
70
90

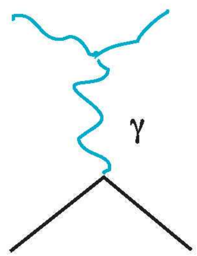
This very accurate value of m_W tests the fundamental SU(2)xU(1) prediction

$$m_W = m_Z \cos\theta_w$$



Finally, study the cross section $\sigma(e^+e^- \to W^+W^-)$

Immediately, there is a problem



$$\sim \overline{v}\gamma^\mu u \; \frac{1}{s} \; (k_+ - k_-)_\mu \epsilon_+^* \cdot \epsilon_-^*$$

W^+ has 3 polarization states. In the rest frame $\epsilon^\mu = (0, \hat{n})^\mu$

but for a W in motion

$$p^\mu = (E_W, 0, 0, k_W)$$

$$\epsilon_R = \frac{1}{\sqrt{2}}(0, 1, i, 0)$$

$$\epsilon_L = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$$

$$\epsilon_0 = \frac{1}{m_W}(k_W, 0, 0, E_W) \approx p^\mu / m_W$$

Notice that $\epsilon_{+0}^* \cdot \epsilon_{-0}^* = \dfrac{E_W^2 + k_W^2}{m_W^2} \approx \dfrac{s}{2m_W^2}$

This is trouble; unitarity requires $|i\mathcal{M}(e^+e^- \to W^+W^-)| \; < \; \mathrm{const}$
in each partial wave.

Similar issues affect the study of another SM object that I should introduce you to, the top quark.

$$m_t \approx 175 \text{ GeV} \qquad m_b \approx 4 \text{ GeV}$$

so t decays, not by a Fermi weak interaction, but rather by the direct 2-body decay process $t \to W^+ b$

One would expect $\Gamma_t \sim \frac{\alpha}{6 s_w^2} m_t \approx 1 \text{ GeV}$

In fact, Γ_t is slightly larger. Consider $t \to W_0^+ b$

$$\begin{aligned} i\mathcal{M} &= i\frac{g}{\sqrt{2}} \overline{u}_L(b) \gamma^\mu \epsilon_\mu^* u_R(t) \\ &\approx i\frac{g}{\sqrt{2}} \overline{u}_L(b) \frac{\not{k}_+}{m_W} u_R(t) = i\frac{g}{\sqrt{2}} \overline{u}_L(b) \frac{\not{p}_t - \not{p}_b}{m_W} u_R(t) \\ &\approx i\frac{g}{\sqrt{2}} \frac{m_t}{m_W} \overline{u}_L(b) u_R(t) \end{aligned}$$

This leads to

$$\Gamma_t = \frac{g^2}{64\pi}\frac{m_t^3}{m_W^2}(1 - \frac{m_W^2}{m_t^2})^2(1 + 2\frac{m_W^2}{m_t^2})$$

which gives

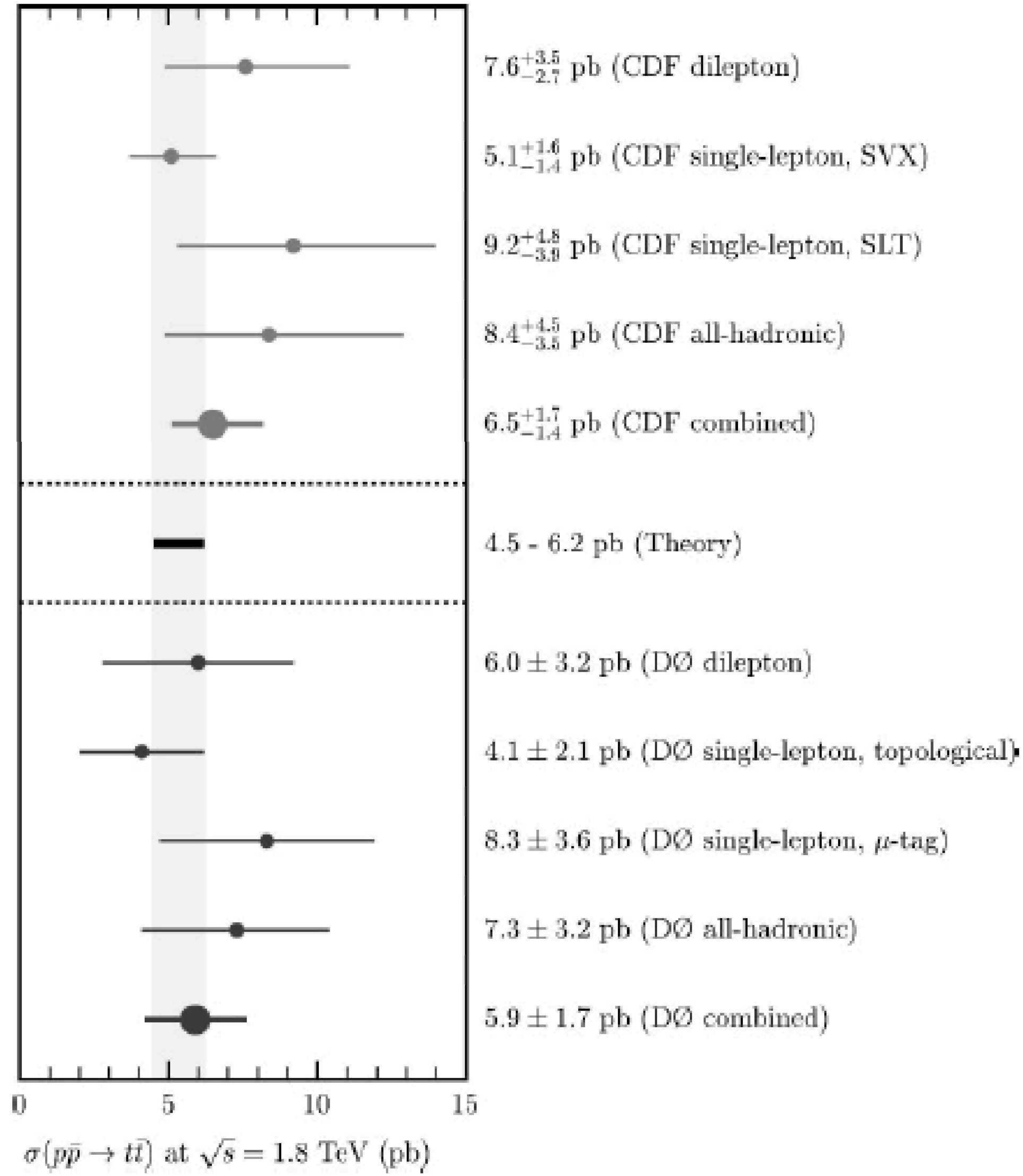
$$\Gamma_t \approx 1.7 \text{ GeV}$$

and

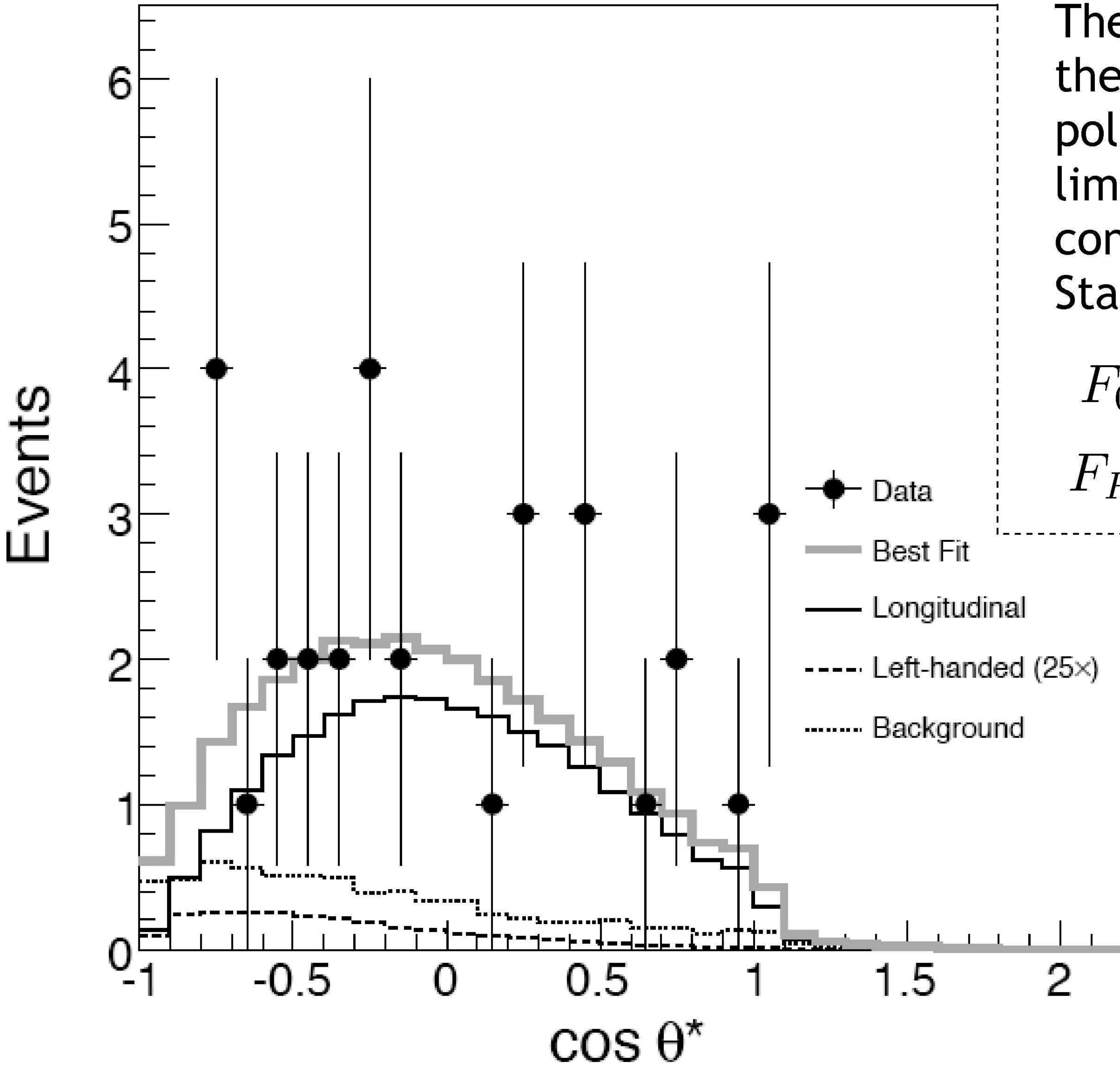
$$P_0 = \frac{\Gamma_t(\to W_0^+)}{\Gamma_t} = 70\ \%$$

The final t decay products are $t \longrightarrow bq\overline{q}$, $t \longrightarrow b\ell\nu$ with the same fractions as for W decay.

The equality of the top and W branching ratios is tested by the consistency of ttbar total cross section mesurements at the Tevatron for different final state channels.



Chakraborty, Konigsberg, Rainwater review



The evidence on the top decay polarization is limited but is consistent with the Standard Model.

$$F_0 = 0.74^{+0.22}_{-0.34}$$

$$F_R = 0.00^{+0.20}_{-0.19}$$

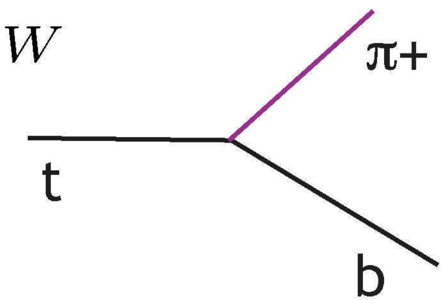
CDF

Does the dependence m_t^3/m_W^2 make sense ?

Think about the unbroken gauge theory of SU(2) x U(1). In this theory, the Higgs doublet ϕ couples to t. Its Yukawa coupling is large,

$$\lambda_t = \frac{\sqrt{2}m_t}{v} = \frac{g}{\sqrt{2}}\frac{m_t}{m_W}$$

This unbroken theory contains the process



where π^+ is the Goldstone boson eaten by the W^+ in the Higgs mechanism. The matrix element is

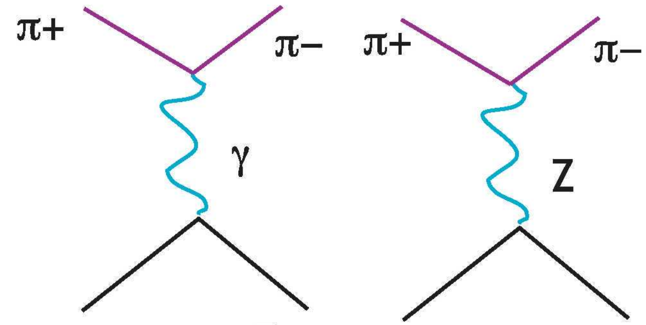
$$i\mathcal{M} = i\lambda_t \overline{u}_L(b)u_R(t) = i\frac{g}{\sqrt{2}}\frac{m_t}{m_W}\overline{u}_L(b)u_R(t)$$

which is exactly what we found for $t \to W_0^+ b$

This illustrates Goldstone boson equivalence: a W_0^+ at high energy has the couplings of the Goldstone boson that it ate to obtain mass.

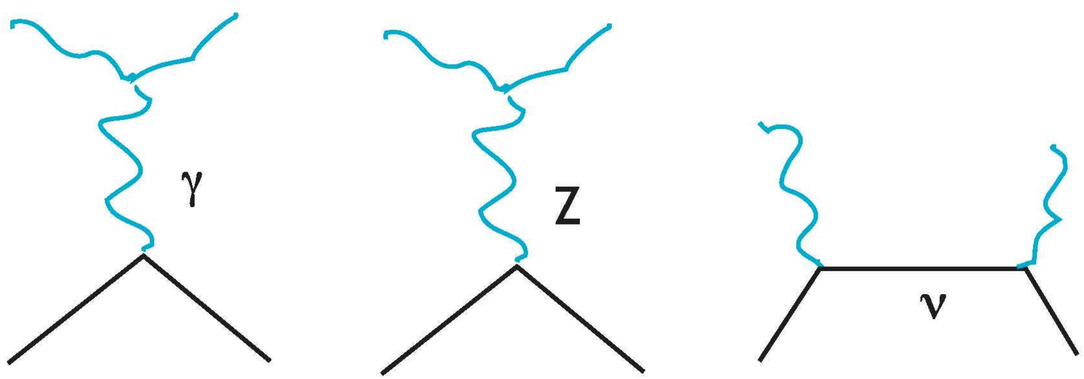
In $e^+e^- \to W^+W^-$, Goldstone boson equivalence implies

$$i\mathcal{M}(e_L^- e_R^+ \to W_0^+ W_0^-) =$$



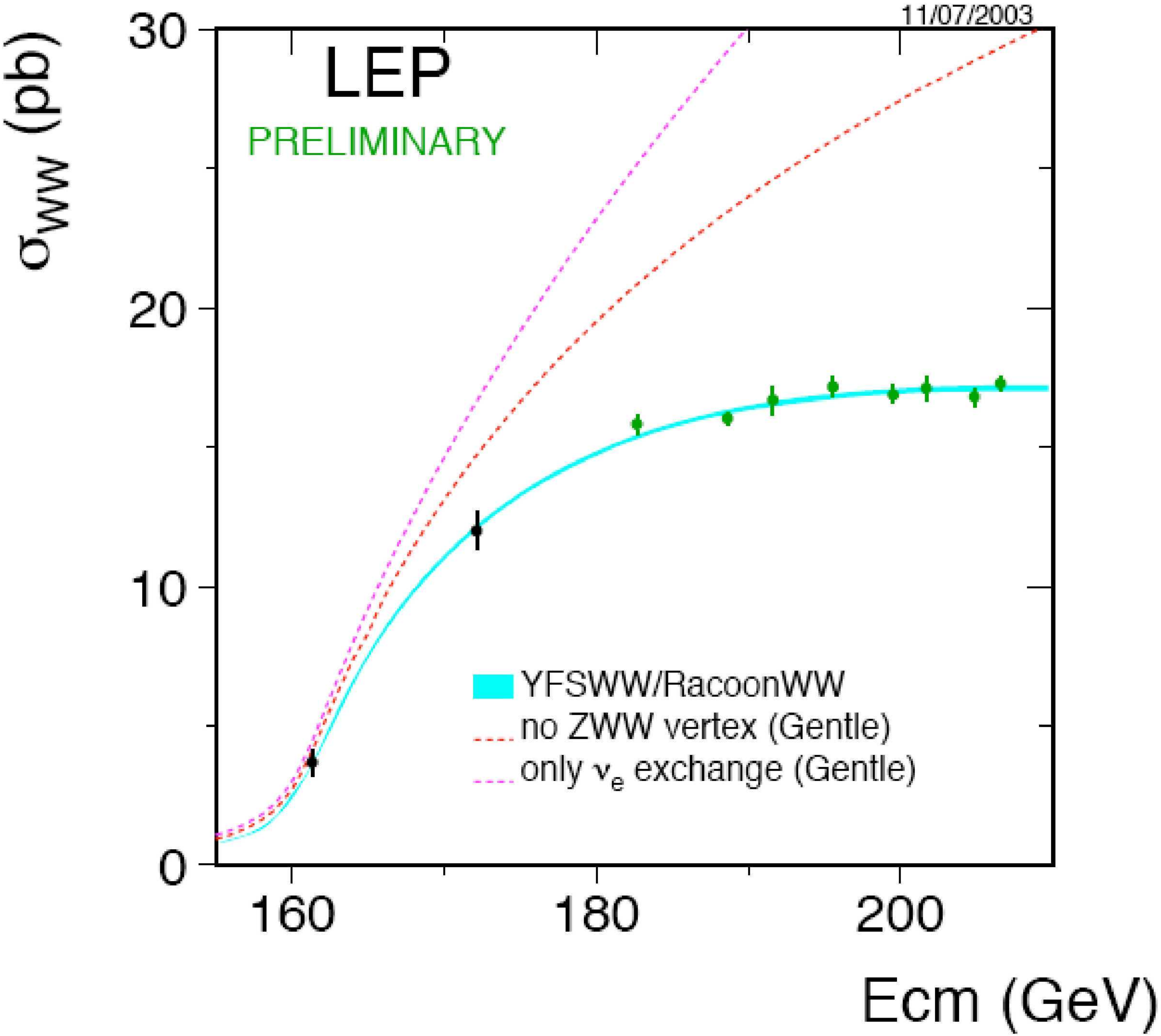
$$= ie^2\left[\frac{1}{4c_w^2} + \frac{1}{4s_w^2}\right]\overline{v}\gamma^\mu u \frac{1}{s}(k_+ - k_-)_\mu$$

This requires a delicate cancellation among the diagrams



You can check that this cancellation occurs in the SU(2)xU(1) gauge theory. It takes place only if the form of the 3-boson vertex is exactly that given by Yang-Mills theory.

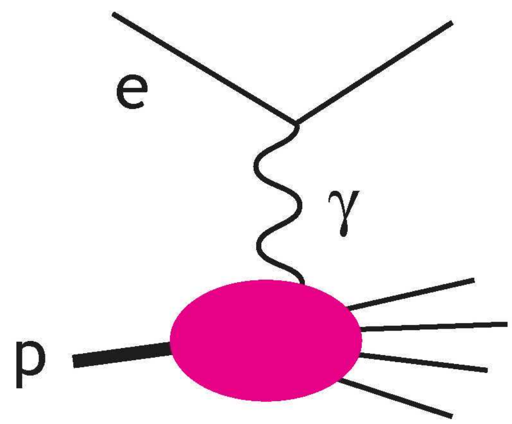
What does experiment have to say about this ?

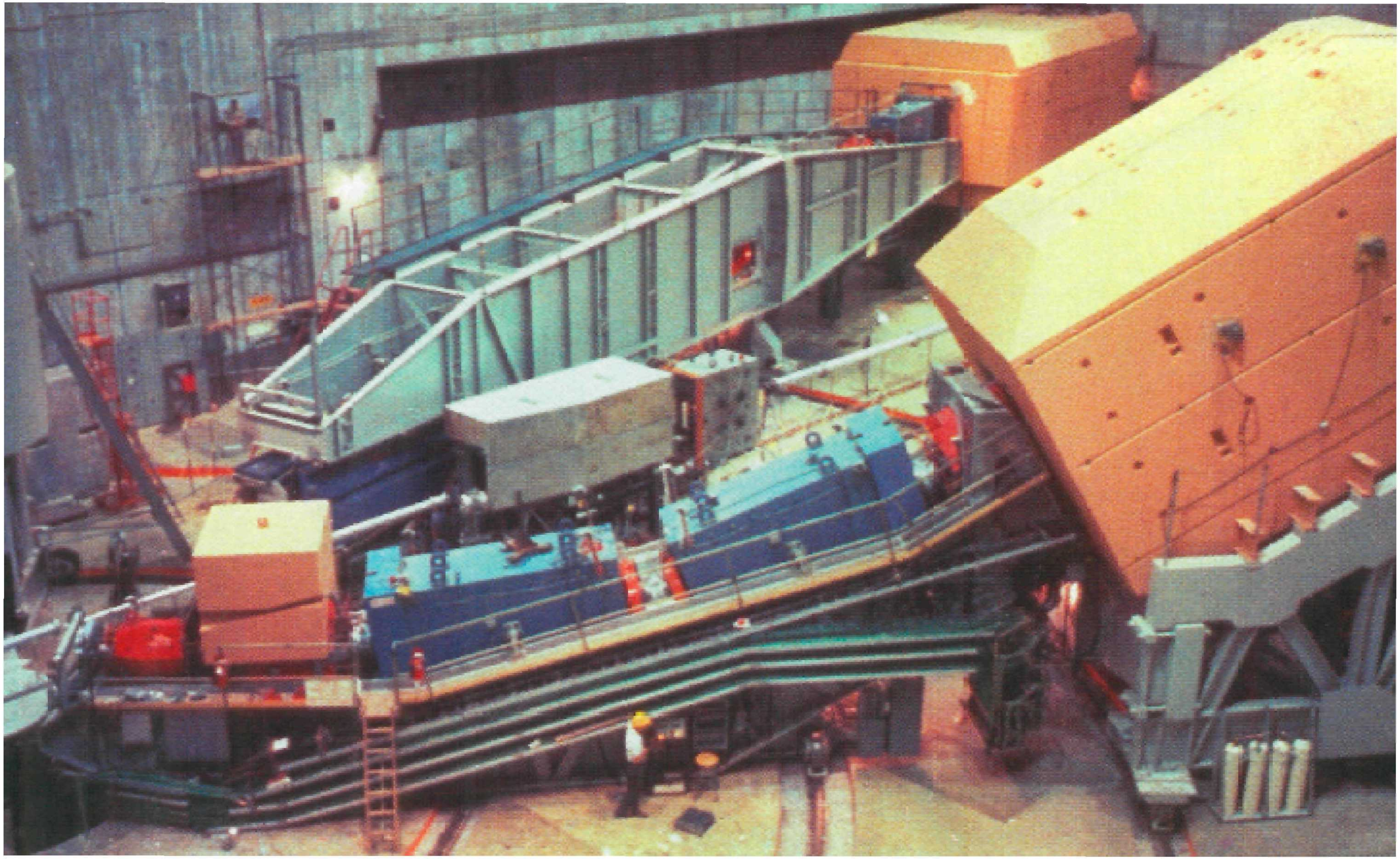

11/07/2003
30
20
10
0
σ_{WW} (pb)
LEP
PRELIMINARY
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)
160
180
200
Ecm (GeV)

It is more complex to describe processes with proton initial states. Here we must treat the proton wavefunction non-perturbatively.

It is too hard a problem to solve for the structure of the proton. We need an experimental setting in which we can measure it.

This is provided by the process deep inelastic electron scattering from protons, measured in the famous SLAC-MIT experiment.





the SLAC-MIT deep inelastic scattering experiment
1967

There is some wonderful kinematics, due to Feynman, that makes this process very effective for measuring the proton structure.

Work in the ep CM frame. The initial proton is coming in at high momentum. Because of asymptotic freedom, a quark in the proton cannot be at high pT with respect to the proton, except through perturbative QCD corrections. So write (ignoring all masses)

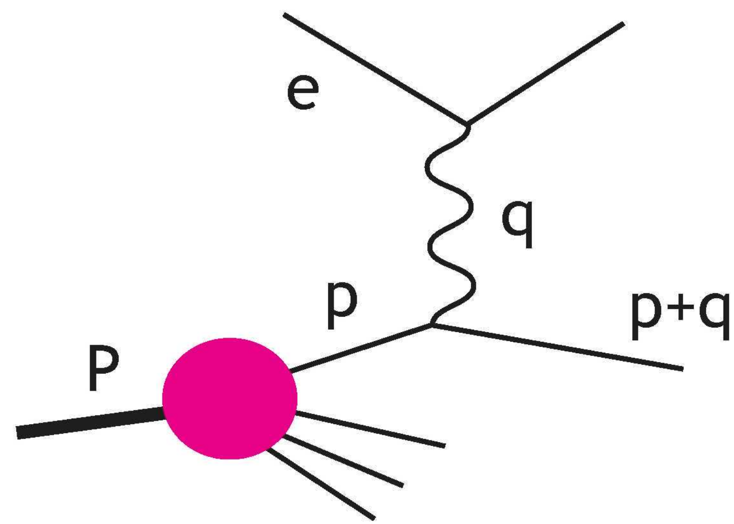
$$p^\mu = \xi P^\mu \quad 0 < \xi < 1$$

The mass of the final quark is

$$(p+q)^2 = 2p\cdot q + q^2 = 2\xi P\cdot q - Q^2$$

But this is small! so we can solve for

$$\xi = \frac{Q^2}{2P\cdot Q} \equiv x$$



Then ξ is precisely determined by the final electron momentum vector.

We can now represent the proton structure by giving the probability that we find a quark at a given value of ξ.

$$d\xi \, f_q(\xi)$$

$f_q(\xi)$ is called the parton distribution function.

We can fold this distribution together with the electron-quark scattering process. This requires a QED matrix element, but it is just the cross of the simple one discussed previously.

$$\begin{aligned} i\mathcal{M}(e_L q_L \to e_L q_L) &= 2ie^2 \, s/t \\ i\mathcal{M}(e_L q_R \to e_L q_R) &= 2ie^2 \, u/t \end{aligned}$$

For electron momentum k, define

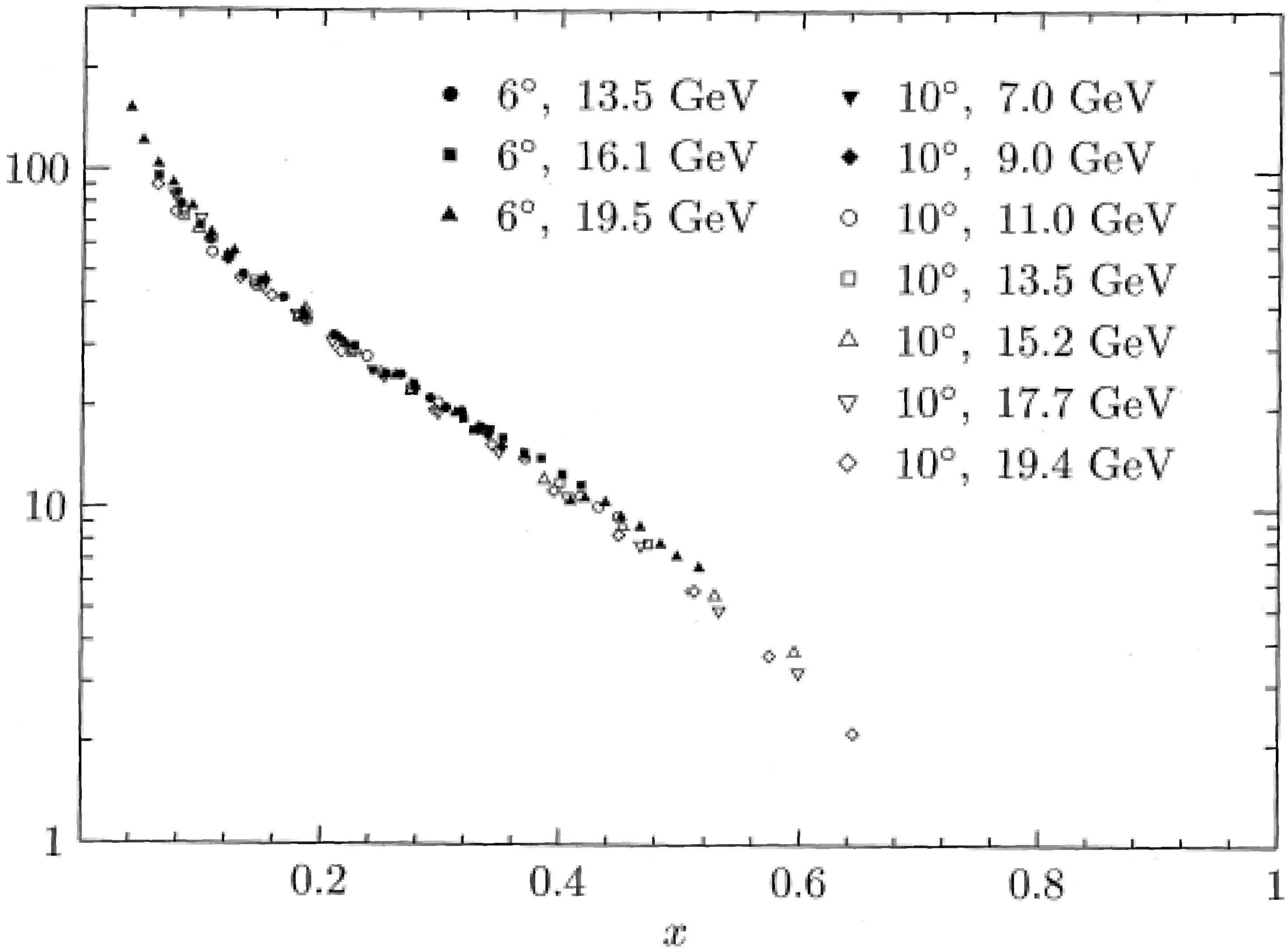
$$y = \frac{2P \cdot q}{2P \cdot k} = \frac{t}{s} \qquad (1-y) = -\frac{u}{s}$$

Then it is straightforward to derive the formula

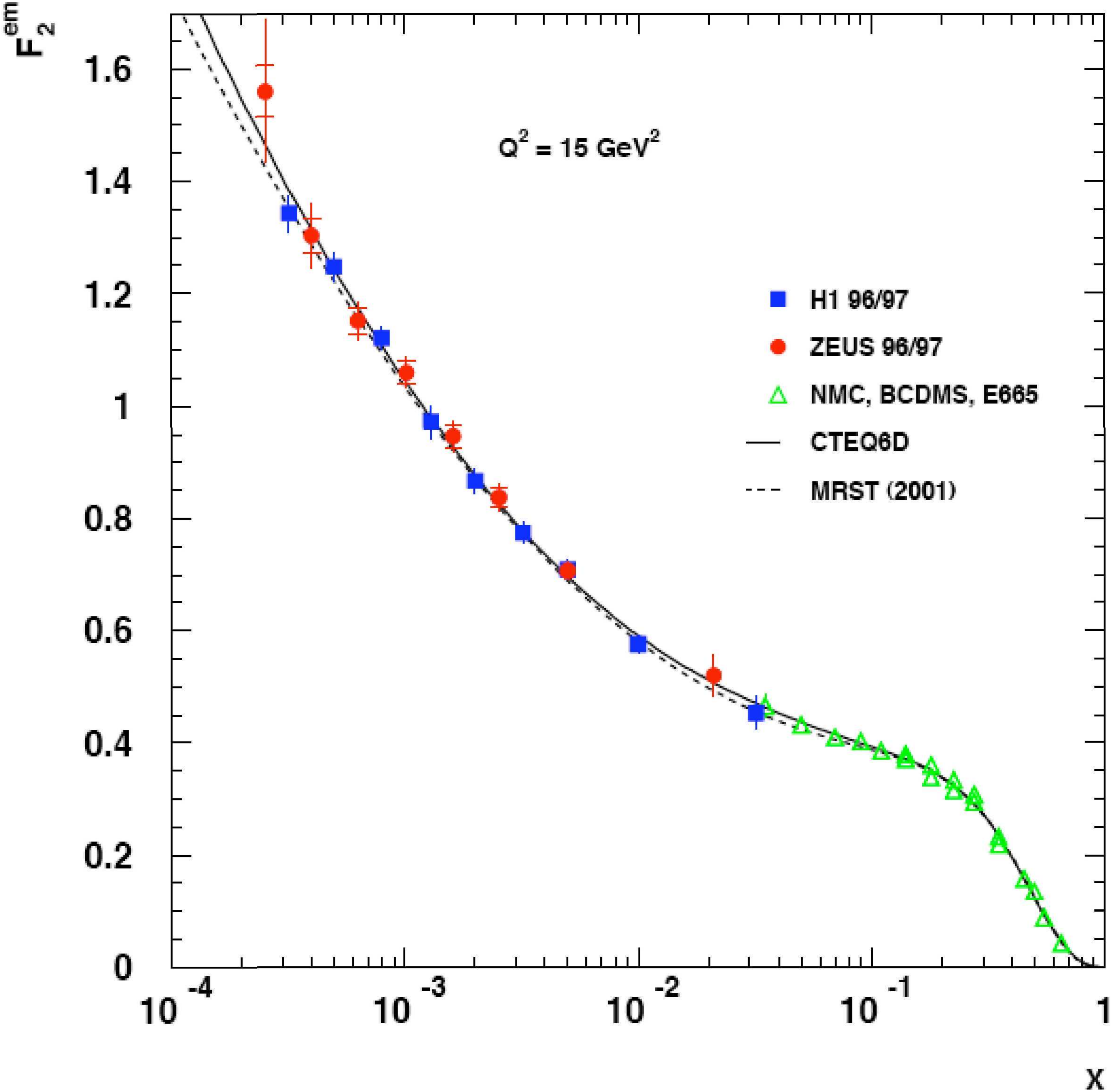
$$\frac{d\sigma}{dxdy} = F_2(x)\frac{2\pi\alpha^2 s}{Q^4}[1 + (1-y)^2] \qquad F_2(x) = \sum_f Q_f^2 f_f(x)$$

This formula exhibits Bjorken scaling: $F_2(x)$ is only a function of x and is independent of Q^2.

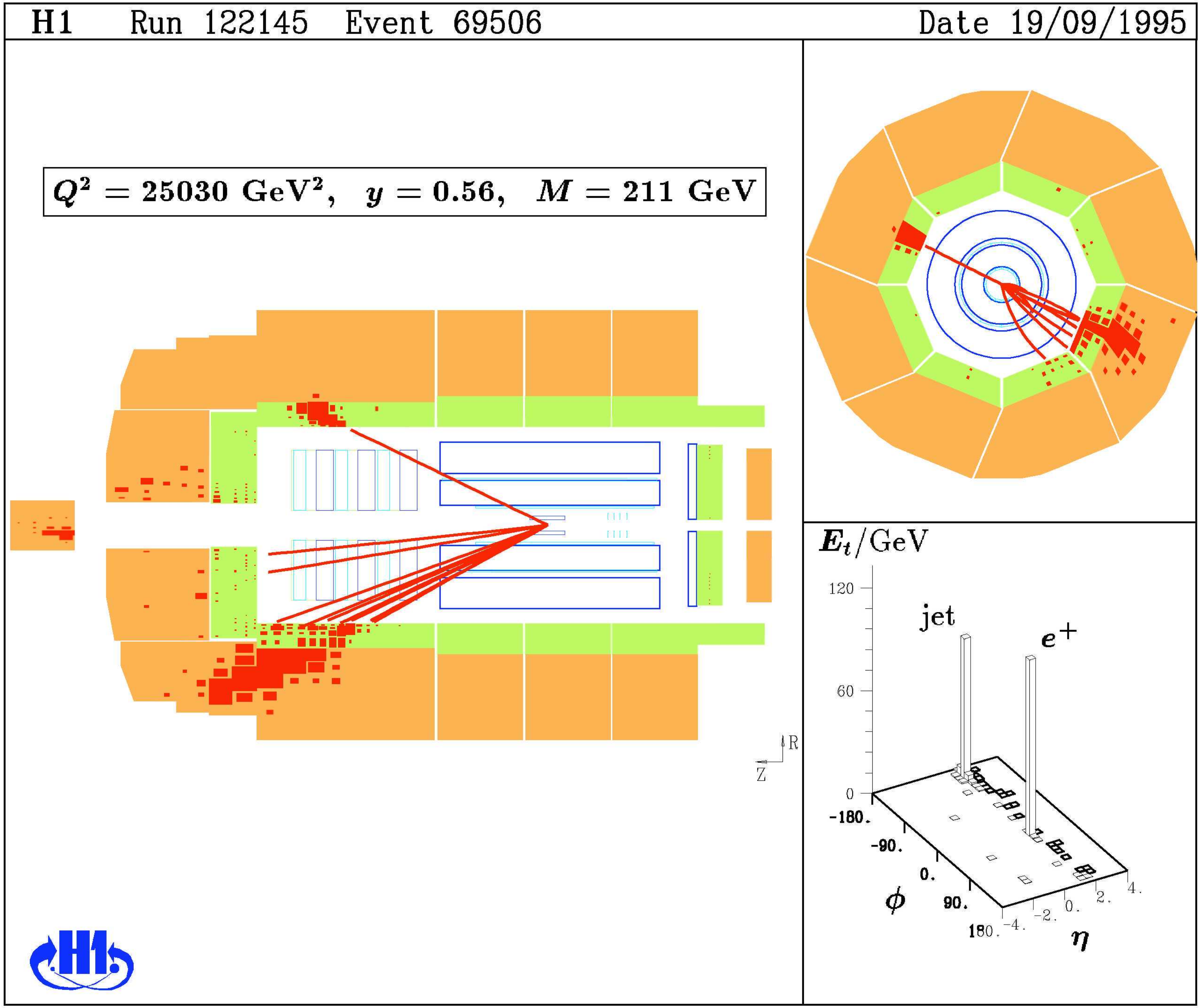
$F_2(x)$ is a combination of contributions from the various quark flavors. We can disentangle this by looking also at cross sections from neutrino scattering, and from other probes that I will discuss tomorrow.


6°, 13.5 GeV
6°, 16.1 GeV
6°, 19.5 GeV
10°, 7.0 GeV
10°, 9.0 GeV
10°, 11.0 GeV
10°, 13.5 GeV
10°, 15.2 GeV
10°, 17.7 GeV
10°, 19.4 GeV
100
10
1
0.2
0.4
0.6
0.8
1
x

SLAC-MIT



F_2^{em}
$Q^2 = 15\ GeV^2$
H1 96/97
ZEUS 96/97
NMC, BCDMS, E665
CTEQ6D
MRST (2001)
x



H1 - 920 GeV p x 27.6 GeV e-

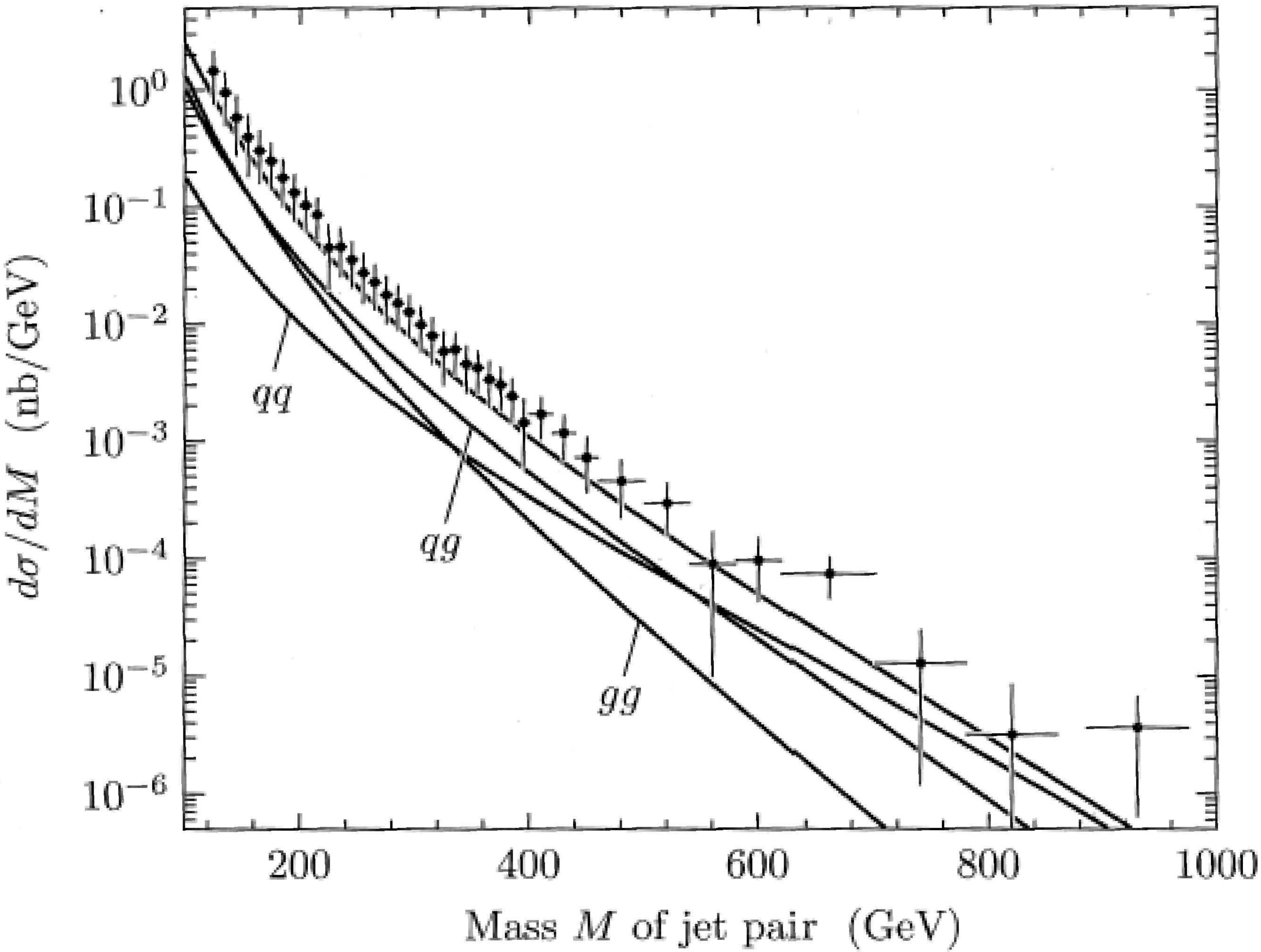
The parton distribution functions give us a very simple model for the cross section for hard-scattering events in proton-proton collisions. Take a parton from each proton, and fold the distributions with the cross sections computed in perturbative QCD:

$$\sigma(pp \to ab + X) = \int d\xi_1 d\xi_2 \sum_{f1,f2} f_{f1}(\xi_1) f_{f2}(\xi_2) \int d\cos\theta_* \frac{d\sigma}{d\cos\theta_*}(f_1 f_2 \to ab)$$

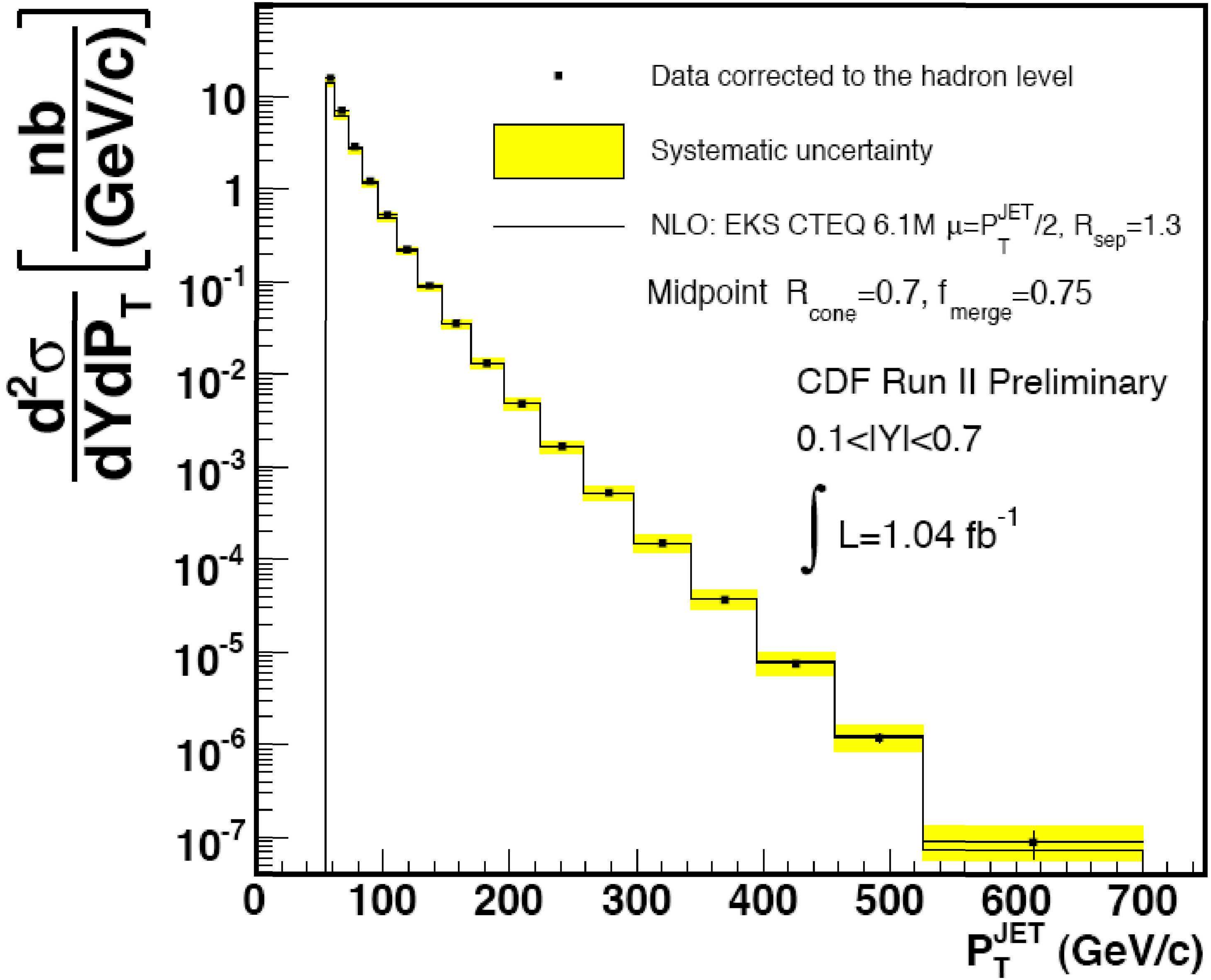
For example,

$$\frac{d\sigma}{d\cos\theta_*}(gg \to u\overline{u}) = \frac{\pi\alpha_s^2}{12s}\left[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{u^2+t^2}{s^2}\right]$$

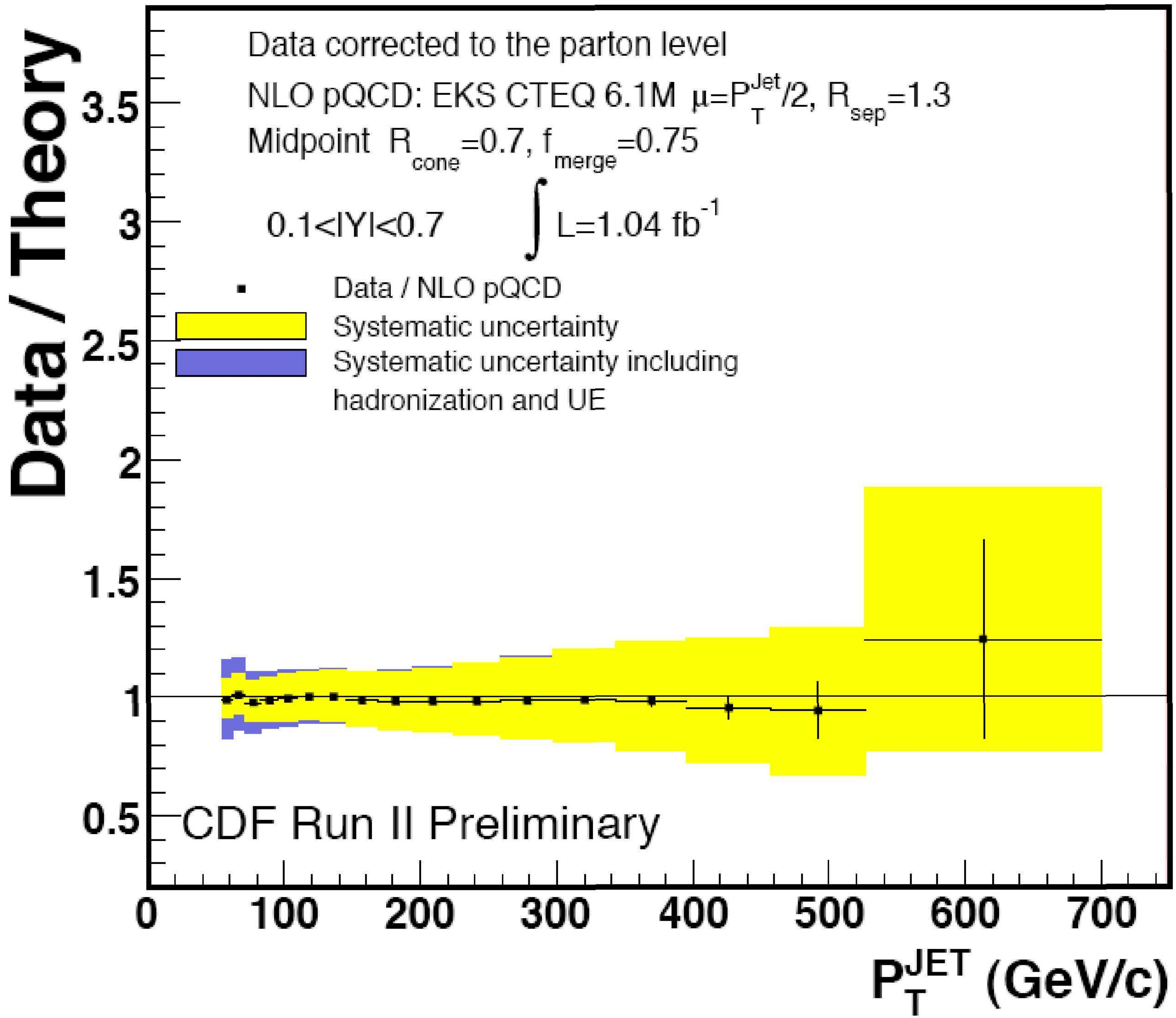
simple leading-order analysis:



CDF - 1.8 TeV



Data corrected to the hadron level
Systematic uncertainty
NLO: EKS CTEQ 6.1M $\mu=P_T^{JET}/2$, $R_{sep}=1.3$
Midpoint $R_{cone}=0.7$, $f_{merge}=0.75$
CDF Run II Preliminary
$0.1<|Y|<0.7$
$\int L=1.04\ fb^{-1}$
$\frac{d^2\sigma}{dYdP_T}\left[\frac{nb}{(GeV/c)}\right]$
P_T^{JET} (GeV/c)
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
400
500
600
700



Data corrected to the parton level
NLO pQCD: EKS CTEQ 6.1M $\mu=P_T^{Jet}/2$, $R_{sep}=1.3$
Midpoint $R_{cone}=0.7$, $f_{merge}=0.75$
0.1<|Y|<0.7
$\int L=1.04\ fb^{-1}$
Data / NLO pQCD
Systematic uncertainty
Systematic uncertainty including hadronization and UE
CDF Run II Preliminary
Data / Theory
P_T^{JET} (GeV/c)
0.5
1
1.5
2
2.5
3
3.5
0
100
200
300
400
500
600
700

In principle, then, we can measure the parton distributions, integrate these with QCD cross sections, and predict the rates of Standard Model and exotic processes at hadron colliders.

This not only gives a simple method of calculating, but it predicts that new particle production will be very distinctive. If the dominant Standard Model processes are 2-body quark and gluon scattering, these would give 2-jet final states that are easily distinguished from the processes we wish to find.

However, there is another level of complication that I have not yet described. We will discuss it in the next lecture.