RENORMALIZABILNA BAŽDARENJA

FIKSIRANJE BAŽDARENJA I POLJA DUHOVA

- ABELOVA QED
- NEABELOVA SU(2) & EW

Renormalizabilna R_ξ baždarenja

◊ Uvod – QED

(1) $\mathcal{L}_{tr} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \xrightarrow{\text{EOM}} \Box A_\nu + \partial_\nu \partial^\mu A_\mu = 0$

$\Rightarrow$ diferencijalni op.

$$K^{tr}_{\mu\nu} = \partial_\mu \partial_\nu - g_{\mu\nu} \partial^2$$

nema inverza

a foton se mora propagirati !

Modifikacija

$$\mathcal{L}_{tr} \to \mathcal{L}_{tr} - \frac{\lambda}{2}(\partial A)^2$$

EOM

$$\Rightarrow \quad \partial^\nu F_{\mu\nu} - \lambda \partial_\mu (\partial A) = \Box A_\mu + (1-\lambda)\partial_\mu (\partial A) = 0$$

$$\underbrace{\left(g_{\mu\nu} \Box + (1-\lambda)\partial_\mu \partial_\nu \right)}_{K_{\mu\nu}} A^\nu = 0$$

$$K_{\mu\nu} \to K_{\mu\nu}(k) = g_{\mu\nu} k^2 - (1-\lambda) k_\mu k_\nu$$

dijagonalno baždarenje.

$\lambda = 1$

$$K_{\mu\nu}(k)^{-1} = \frac{g_{\mu\nu}}{k^2}$$

DODAVANJE ČLANA FIKSIRANJA BAŽDARENJA I ČLANA DUHA

- Uz član fiksiranja baždarenja (koji nije baždarno invarijantan), dobije se dobar propagator;
- Cijena za to je propagiranje viška stupnjeva slobode (baždarnih modova);
- Kompenzacija se izvodi propagiranjem duhova (nefizikalnih, antikomutirajućih skalara);
- Umjesto baždarne simetrije pojavljuje se tzv. BRST-simetrija, koja osigurava fizikalne rezultate

◇ Primjer SU(2) YM

"(2)" $$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \frac{1}{2} (D^\mu \vec{\Phi})^\dagger (D_\mu \vec{\Phi}) - V(\vec{\Phi})$$

dovoljno blizu elektroslabe $SU(2) \times U(1)$

$\Longrightarrow$ član*) fiksiranja baždarenja " $-\frac{\xi}{2}(\underbrace{\partial^\mu A^a_\mu}_{\equiv \eta^a})$ "

"(7)" $$\mathcal{L}_{\xi,a} = -\frac{\xi}{2}\left(\partial^\mu W^+_\mu - \frac{M}{\xi}\phi^+\right)^2 - \frac{\xi}{2}\left(\partial^\mu W^-_\mu - \frac{M}{\xi}\phi^-\right)^2 - \frac{1}{2a}\left(\partial^\mu A_\mu\right)^2$$

Elektroslaba SU(2) × U(1) teorija

- uz triplet $(W_\mu^1, W_\mu^2, W_\mu^3)$
  dolazi i hipernabojni bozon B_μ
- 4-dimenzionalni multiplet realnih polja
  $$\vec{\Phi} = (\phi_1, \phi_2, \phi^0, \varphi^0)$$
  ekvivalentan je dubletu kompleksnih
  $$\begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \quad \text{– s konjugiranim} \quad \begin{pmatrix} \Phi^{0*} \\ -\Phi^- \end{pmatrix}$$

vev poprima neutralni φ^0

$$\vec{v} = \langle \vec{\phi} \rangle = (0, 0, 0, v)$$

naš ijeni $\phi^{\pm} = \frac{\phi_1 \pm i\phi_2}{\sqrt{2}}$

- pri tome neslomljeni generator odgovara mješavini T_3 i Y, el. naboji

$$Q = T_3 + Y/2$$

- 't Hooftovo R_ξ baždarenje

$$\partial^\mu W_\mu^+ - \frac{M}{\xi}\phi^+ = 0 \quad , \quad \partial^\mu W_\mu^- - \frac{M}{\xi}\phi^- = 0$$

$$\partial^\mu Z_\mu - \frac{M}{\cos\theta_W}\frac{1}{\xi_0}\phi^0 = 0 \quad , \quad \partial^\mu A_\mu^0 = 0$$

kao realizacija općeg R_ξ : $\partial^\mu W_\mu^a - \frac{g^a}{\xi}\langle v | T^a \vec{\phi} \rangle = 0$

odgovara članu fiksiranja baždarenja

$$\mathcal{L}_g = -\frac{\xi}{2}\sum_a \left(\partial^\mu W_\mu^a - \frac{g}{\xi}\langle \vec{v} | T^a \vec{\phi} \rangle\right)^2$$

→ Feynmanova pravila (Cheng & Li)

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A resource for signs and Feynman diagrams of the Standard Model

When performing a full calculation within the Standard Model or its extensions, it is crucial that one utilizes a consistent set of signs for the gauge couplings and gauge fields. Unfortunately, the literature is plagued with differing signs and notations. We present all Standard Model Feynman rules, including ghosts, in a convention-independent notation, and we table the conventions in close to 40 books and reviews.

2.2. Gauge Group $SU(2)_L \times U(1)_Y$

For the $SU(2)_L$ group, we have

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - \eta\, g \epsilon^{abc} W^b_\mu W^c_\nu \quad (a = 1, \ldots, 3),$$

$$D_\mu \psi_L = \left(\partial_\mu + i\, \eta\, g\, W^a_\mu T^a\right) \psi_L .$$

As for the Abelian $U(1)_Y$ group, we have

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu ,$$

with the covariant derivative given by

$$D_\mu \psi = (\partial_\mu + i\, \eta'\, g'\, \eta_Y Y\, B_\mu)\, \psi,$$

where Y is the hypercharge of the field, connected to the electric charge

$$Q = T_3 + \eta_Y Y \ .$$

As before $\eta', \eta_Y = \pm 1$. Some authors use

$$Q = T_3 + \eta_Y \frac{Y_{\text{theirs}}}{2} = \frac{\tau_3 + \eta_Y Y_{\text{theirs}}}{2},$$

2.3. *The gauge and fermion fields Lagrangian*

The gauge field Lagrangian is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

where the field strengths are given in Eqs. (1), (7) and (9).

The kinetic terms for the fermions, including the interaction with fields due to the covariant derivative, is written as

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\overline{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R$$

2.4. *The Higgs Lagrangian*

The SM includes a Higgs doublet with the following assignments,

$$\Phi = \begin{bmatrix} \varphi^+ \\ \dfrac{v + H + i\varphi_Z}{\sqrt{2}} \end{bmatrix}.$$

Since $\eta_Y Y_\Phi = +1/2$, the covariant derivative reads

$$D_\mu \Phi = \left[\partial_\mu + i\,\eta \frac{g}{\sqrt{2}} \left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) + i\,\eta \frac{g}{2} \tau_3 W_\mu^3 + i\,\eta' \frac{g'}{2} B_\mu\right]$$

$$= \left[\partial_\mu + i\,\eta \frac{g}{\sqrt{2}} \left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right) + i\,\eta_e e\, Q\, A_\mu\right.$$

$$\left. + i\,\eta \frac{g}{\cos\theta_W} \left(\frac{\tau_3}{2} - Q\, \sin^2\theta_W\right) \eta_Z Z_\mu\right] \Phi,$$

The Higgs Lagrangian is

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger D_\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 ,$$

leading to the relations,

$$v^2 = \frac{\mu^2}{\lambda}, \quad m_h^2 = 2\mu^2, \quad \lambda = \frac{g^2}{8} \frac{m_h^2}{m_W^2}.$$

Table 1. Values of T_3^f, Q and Y for the SM particles.

Field	ℓ_L	ℓ_R	ν_L	u_L	d_L	u_R	d_R	ϕ^+	ϕ^0
T_3	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
$\eta_Y Y$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
Q	-1	-1	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	1	0

Expanding this Lagrangian, we find the following terms quadratic in the fields:

$$\mathcal{L}_{\text{Higgs}} = \cdots + \frac{1}{8}g^2v^2W^3_\mu W^{\mu 3} + \frac{1}{8}g'^2v^2B_\mu B^\mu - \frac{1}{4}\eta\eta' gg' v^2 W^3_\mu B^\mu + \frac{1}{4}g^2v^2W^+_\mu W^{-\mu}$$

$$+\frac{1}{2}v\,\partial^\mu \varphi_Z\left(\eta' g' B_\mu - \eta\, g W^3_\mu\right) - \frac{i}{2}\eta\, gvW^-_\mu \partial^\mu \varphi^+ + \frac{i}{2}\eta\, gvW^+_\mu \partial^\mu \varphi^- \quad (28)$$

$$\mathcal{L}_{\text{Higgs}} = \cdots + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + m_W^2 W^+_\mu W^{-\mu}$$

$$-\eta\,\eta_Z\, m_Z Z_\mu \partial^\mu \varphi_Z - i\,\eta\, m_W\left(W^-_\mu \partial^\mu \varphi^+ - W^+_\mu \partial^\mu \varphi^-\right), \quad (29)$$

$$m_W = \frac{1}{2}gv, \quad m_Z = \frac{1}{\cos\theta_W}\frac{1}{2}gv = \frac{1}{\cos\theta_W}m_W. \quad (30)$$

2.6. *The gauge fixing*

In the R_ξ gauges, the gauge fixing Lagrangian reads:

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi_G} F_G^2 - \frac{1}{2\xi_A} F_A^2 - \frac{1}{2\xi_Z} F_Z^2 - \frac{1}{\xi_W} F_- F_+ ,$$

$$\begin{aligned} F_G^a &= \partial^\mu G_\mu^a , \\ F_A &= \partial^\mu A_\mu , \\ F_Z &= \partial^\mu Z_\mu + \eta\, \eta_Z\, \xi_Z m_Z \varphi_Z , \\ F_+ &= \partial^\mu W_\mu^+ + i\, \eta\, \xi_W m_W \varphi^+ , \\ F_- &= \partial^\mu W_\mu^- - i\, \eta\, \xi_W m_W \varphi^- . \end{aligned}$$

y verify that, with these definitions, $\mathcal{L}_{\rm GF}$ cancels the mixed second line of Eq. (29).

2.7. *The ghost Lagrangian*

The last piece needed for the SM Lagrangian is the ghost Lagrangian. For a linear gauge fixing condition, as in Eq. (39), this is given by the Fadeev-Popov prescription:

$$\mathcal{L}_{\rm Ghost} = \eta_G \sum_{i=1}^{4} \left[\overline{c}_+ \frac{\partial(\delta F_+)}{\partial \alpha^i} + \overline{c}_- \frac{\partial(\delta F_+)}{\partial \alpha^i} + \overline{c}_Z \frac{\partial(\delta F_Z)}{\partial \alpha^i} + \overline{c}_A \frac{\partial(\delta F_A)}{\partial \alpha^i} \right] c_i$$

by $c_\pm, c_A, c_Z$ the electroweak ghosts associated with

$$U = e^{i\,\eta\, g T^a \alpha^a} \quad (a = 1, \ldots, 3),$$

$$U = e^{i\,\eta'\,\eta_Y\, g' Y \alpha^4}.$$

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm Fermion} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm Ghost}$$

3. Notations found in the literature

$$D_\mu = \partial_\mu + i\eta\, g \frac{\tau_a}{2} W_\mu^a + i\eta'\, \eta_Y\, g' Y\, B_\mu, \qquad Q = T_3 + \eta_Y Y$$

$$\begin{cases} W_\mu^3 = \eta_Z\, Z_\mu \cos\theta_W + A_\mu \eta_\theta \sin\theta_W \\ B_\mu = -\eta_Z Z_\mu \eta_\theta\, \sin\theta_W + A_\mu \cos\theta_W \end{cases}, \quad \begin{cases} \eta_Z Z_\mu = W_\mu^3 \cos\theta_W - B_\mu \eta_\theta\, \sin\theta_W \\ A_\mu = W_\mu^3\, \eta_\theta \sin\theta_W + B_\mu \cos\theta_W \end{cases}. \tag{13}$$

sets the sign convention for η, η', η_Z, η_θ, and η_Y

in the literature are shown in Table 2.

Table 2. Sign conventions found in the literature. An asterisk, $*$, on the last column means that such authors have $Q = (\tau_3 + Y_{\text{theirs}})/2$ instead of our Eq. (11).

Ref.	η	η'	η_Z	η_θ	η_Y	η_e	Y
2–6, 46	+	+	+	+	+	+	
7–17	+	+	+	+	+	+	$*$
18, 19	−	−	+	+	+	−	
20–30	−	−	+	+	+	−	$*$
31, 32	−	−	+	−	+	+	
33	−	−	−	+	+	−	$*$
34	−	+	+	−		+	
35, 36	−	+	+	−	−	+	
37	−	+	+	−	+	+	
38	+	−	+	−	+	−	$*$

1. J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, (Mcgraw-Hill, New York, 1965).
2. D. Bailin and A. Love, *Introduction to Gauge Field Theory*, (Hilger, Bristol, UK, 1986); Errata available on-line `http://www.phys.susx.ac.uk/~mpfg9/fterta.htm`.
3. S. Pokorski, *Gauge Field Theories*, 2nd edition (Cambridge, Univ. Press, Cambridge, 2000); Errata available on-line `http://www.fuw.edu.pl/~pokorski/errata08.pdf`
4. J. P. Silva, invited lectures presented at the Central European School in Particle Physics, Faculty of Mathematics and Physics, Charles University, Prague, September 14-24, 2004; hep-ph/0410351.
5. A. Lahiri and P. B. Pal, *A First Book of Quantum Field Theory*, 2nd edition, (Alpha Sci. Int., Harrow, UK, 2007).
6. F. Mandl and G. Shaw, *Quantum Field Theory*, 2nd edition (Wiley, Chichester, UK, 2010).
7. C. Quigg, *Gauge Theories Of The Strong, Weak And Electromagnetic Interactions*, (The Benjamin/Cummings Publishing Company, NY, 1983).
8. F. Halzen and A. D. Martin, *Quarks And Leptons: An Introductory Course In Modern Particle Physics*, (Wiley, New York, USA, 1984).
9. R. J. Rivers, *Path Integral Methods In Quantum Field Theory*, (Cambridge Univ. Press, Cambridge, 1987).
10. I. J. R. Aitchison and A. J. G. Hey, *Gauge Theories In Particle Physics: A Practical Introduction*, 2nd edition (Hilger, Bristol, UK, 1989).
11. J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunter's Guide*, (Perseus Books, NY, 1990).
12. V. D. Barger and R. J. N. Phillips, *Collider Physics*, updated edition, (Westview, NY, 1991).
13. K. Huang, *Quarks, Leptons And Gauge Fields*, 2nd edition, (World Scientific, Singapore, 1992).

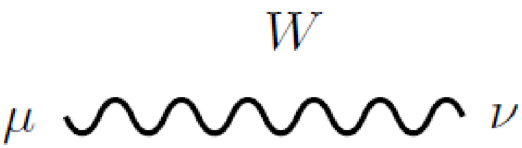
14. J. F. Donoghue, E. Golowich and B. R. Holstein, *Dynamics of the standard model*, (Cambridge Univ. Press, Cambridge, 1992).
15. G. F. Sterman, *An Introduction to Quantum Field Theory*, (Cambridge Univ. Press, Cambridge, 1993).
16. F. Gross, *Relativistic Quantum Mechanics and Field Theory* (Wiley, New York, 1993).
17. D. Griffiths, *Introduction to Elementary Particles*, revised 2nd edition (Wiley, Weinheim, Germany, 2008).
18. M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, (Addison-Wesley, Reading, USA, 1995).
19. L. Alvarez-Gaume and M. A. Vazquez-Mozo, *An Invitation to Quantum Field Theory*, (Springer, Berlin, 2012).
20. K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe and T. Muta, *Prog. Theor. Phys. Suppl.* **73**, 1 (1982).
21. L. B. Okun, *Leptons And Quarks*, (North-holland, Amsterdam, 1982).
22. L. H. Ryder, *Quantum Field Theory*, (Cambridge Univ. Press, Cambridge, 1985).
23. T. P. Cheng and L. F. Li, *Gauge Theory Of Elementary Particle Physics*, (Oxford Univ. Press, Oxford, 1988).
24. R. N. Mohapatra, *Unification And Supersymmetry. The Frontiers Of Quark - Lepton Physics*, 2nd edition (Springer, NY, 1992).
25. Fayyazuddin and Riazuddin, *A Modern Introduction to Particle Physics*, (World Scientific, Singapore, 1994).
26. W. B. Rolnick, *The Fundamental Particles and Their Interactions*, (Addison-Wesley,

Reading, USA, 1994).
27. G. L. Kane, *Modern Elementary Particle Physics*, updated edition (Addison-Wesley, Redwood City, USA, 1994).
28. V. P. Nair, *Quantum Field Theory: A Modern Perspective*, (Springer, NY, 2005).
29. A. Djouadi, *Phys. Rept.* **459**, 1 (2008) [hep-ph/0503173]; *ibid*, *Phys. Rept.* **457**, 1 (2008) [hep-ph/0503172].
30. A. Zee, *Quantum Field Theory in a Nutshell*, (Princeton Univ. Press, Princeton, 2010).
31. G. C. Branco, L. Lavoura and J. P. Silva, *CP Violation*, (Oxford University Press, Oxford, 1999).
32. W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *J. Phys. G* **35**, 075001 (2008) [arXiv:0711.4022 [hep-ph]].
33. C. Itzykson and J. B. Zuber, *Quantum Field Theory*, (Mcgraw-hill, NY, 1986).
34. S. Sakakibara, *Phys. Rev. D* **24**, 1149 (1981).
35. A. Barroso, J. Pulido and J. C. Romao, *Nucl. Phys. B* **267**, 509 (1986).
36. P. Nogueira, *"Ward Identities for $Z\gamma\gamma$ and $ZZ\gamma$ Green's Functions"*, Master thesis, IST, Lisbon (1987).
37. J. C. Romao, *Advanced Quantum Field Theory*, available on-line from http://porthos.ist.utl.pt/ftp/textos/tca.pdf (2012).
38. A. Das, *Lectures on Quantum Field Theory*, (World Scientific, Hackensack, USA, 2008).
39. N. D. Christensen and C. Duhr, *Comput. Phys. Commun.* **180**, 1614 (2009), [arXiv:0806.4194].
40. M. Srednicki, *Quantum field theory*, (Cambridge University Press, UK, 2007).
41. S. Weinberg, *The quantum theory of fields. Vol. 2: Modern applications*, (Cambridge University Press, UK, 1996).
42. D. Y. .Bardin and G. Passarino, *The standard model in the making: Precision study of the electroweak interactions* , (Oxford University Press, UK, 1999).
43. C. Becchi, A. Rouet and R. Stora, *Annals Phys.* **98**, 287 (1976).
44. I. Tyutin, arXiv:0812.0580.
45. J. C. Romao and A. Barroso, *Phys. Rev.* **D35**, 2836 (1987).
46. P. Langacker, *The Standard Model and Beyond*, (CRC Press, New York, 2009).

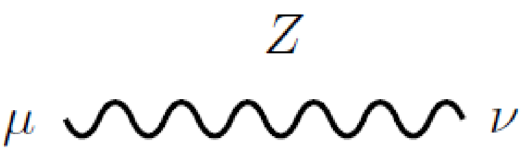
EW THEORY

5.1. *Propagators*

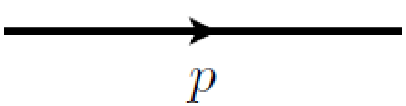
$$-i\left[\frac{g_{\mu\nu}}{k^2+i\epsilon}-(1-\xi_A)\frac{k_\mu k_\nu}{(k^2)^2}\right]$$



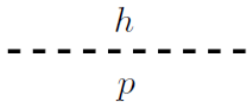
$$-i\,\frac{1}{k^2-m_W^2+i\epsilon}\left[g_{\mu\nu}-(1-\xi_W)\frac{k_\mu k_\nu}{k^2-\xi_W m_W^2}\right]$$



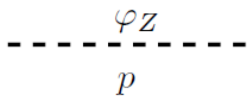
$$-i\,\frac{1}{k^2-m_Z^2+i\epsilon}\left[g_{\mu\nu}-(1-\xi_Z)\frac{k_\mu k_\nu}{k^2-\xi_Z m_Z^2}\right]$$



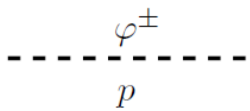
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2+i\epsilon}$$



$$\frac{i}{p^2 - m_h^2 + i\epsilon}$$

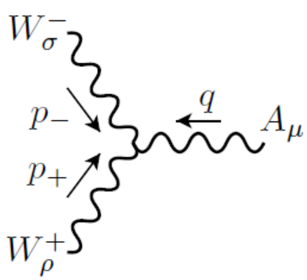


$$\frac{i}{p^2 - \xi_Z m_Z^2 + i\epsilon}$$

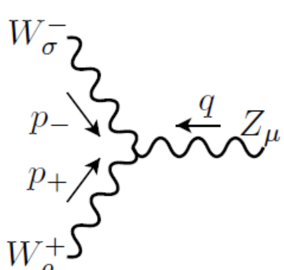


$$\frac{i}{p^2 - \xi_W m_W^2 + i\epsilon}$$

5.2. *Triple Gauge Interactions*

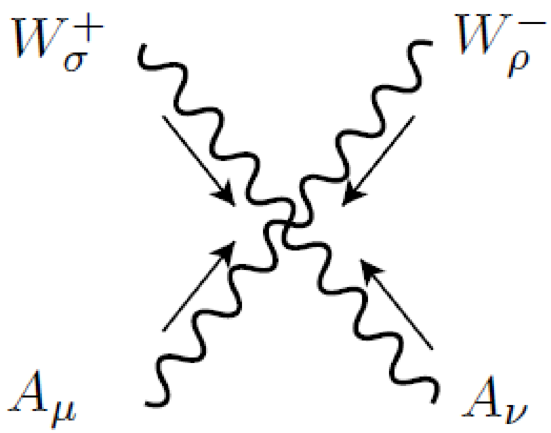


$$-i\,\eta_e\,e\,[g_{\sigma\rho}(p_- - p_+)_\mu + g_{\rho\mu}(p_+ - q)_\sigma + g_{\mu\sigma}(q - p_-)_\rho]$$

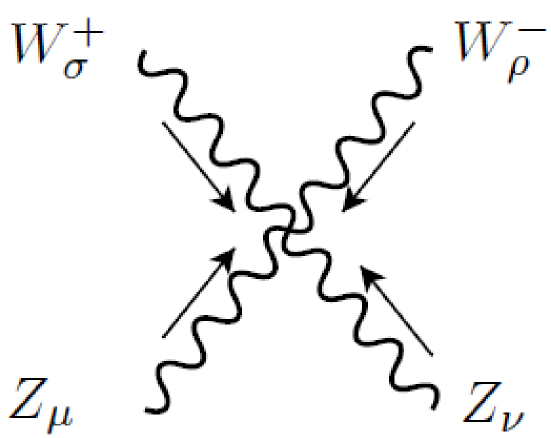


$$-i\eta\eta_Z g\cos\theta_W[g_{\sigma\rho}(p_- - p_+)_\mu + g_{\rho\mu}(p_+ - q)_\sigma + g_{\mu\sigma}(q - p_-)_\rho]$$

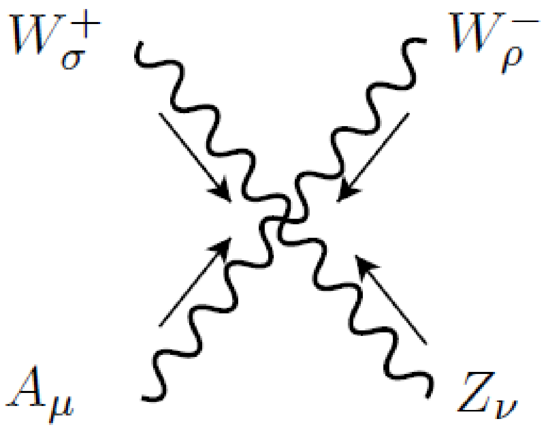
5.3. *Quartic Gauge Interactions*



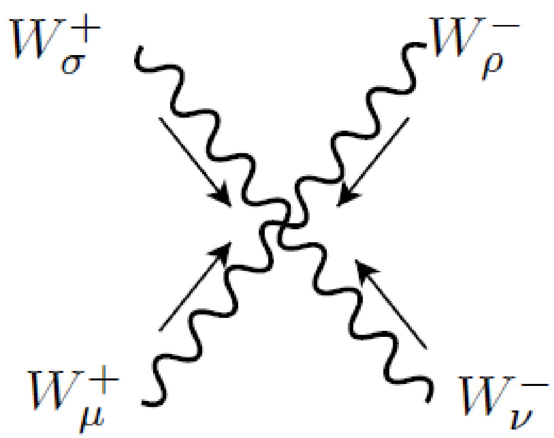
$$-ie^2\left[2g_{\sigma\rho}g_{\mu\nu} - g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\nu}g_{\rho\mu}\right]$$



$$-ig^2\cos^2\theta_W\left[2g_{\sigma\rho}g_{\mu\nu} - g_{\sigma\mu}g_{\rho\nu} - g_{\sigma\nu}g_{\rho\mu}\right]$$



$$- i\, \eta_e \eta\, \eta_Z\, e g \cos\theta_W \left[2 g_{\sigma\rho} g_{\mu\nu} - g_{\sigma\mu} g_{\rho\nu} - g_{\sigma\nu} g_{\rho\mu} \right]$$



$$i g^2 \left[2 g_{\sigma\mu} g_{\rho\nu} - g_{\sigma\rho} g_{\mu\nu} - g_{\sigma\nu} g_{\rho\mu} \right]$$