MASE FERMIONA U SM

- MASE KVARKOVA i NABIJENIH LEPTONA
- MASE NEUTRINA
- ČAROLIJA i ENIGMA HIGGSOVOG SEKTORA

MASE FERMIONA ILI YUKAWINA VEZANJA

Mase fermiona generirane $SU(2) \times U(1)$ simetričnim Yukawinim međudjelovanjem

12 fundamentalnih fermiona u 3 obitelji

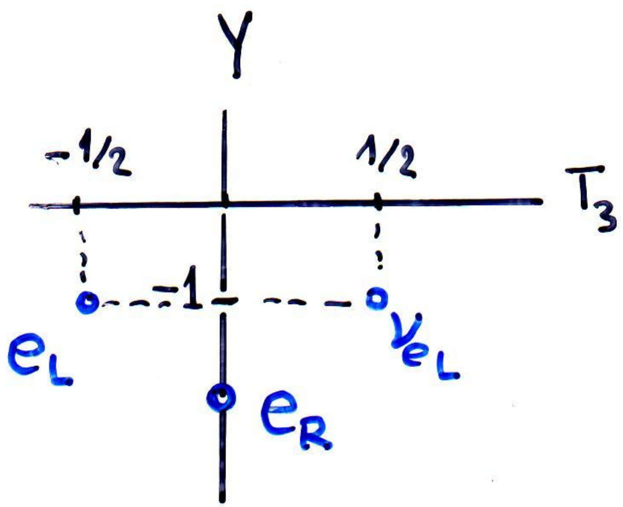
u, d, ν_e, e

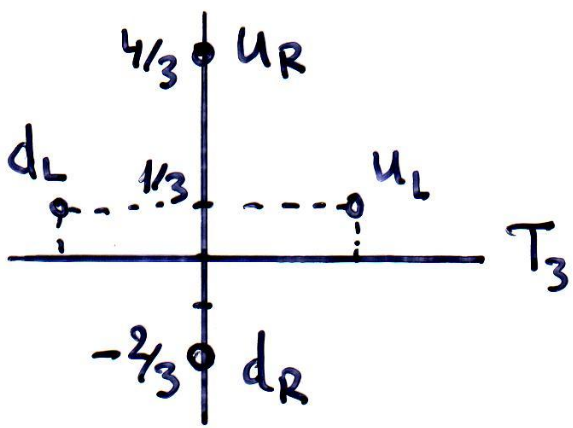
c, s, ν_μ, μ

t, b, ν_τ, τ

15 stanja heliciteta unutar jedne obitelji

Obitelj fermiona realizirana s pet reprezentacija SM-a





	$SU(3)_{boje} \times SU(2) \times U(1)$
$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$(1, 2, -1)$
e_R	$(1, 1, -2)$
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$(3, 2, 1/3)$
u_R	$(3, 1, 4/3)$
d_R	$(3, 1, -2/3)$

Izvor mase u Higgsovom kondenzatu

$$\mathcal{L}_H = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi)$$

- Čarolija dubleta kompleksnih skalara

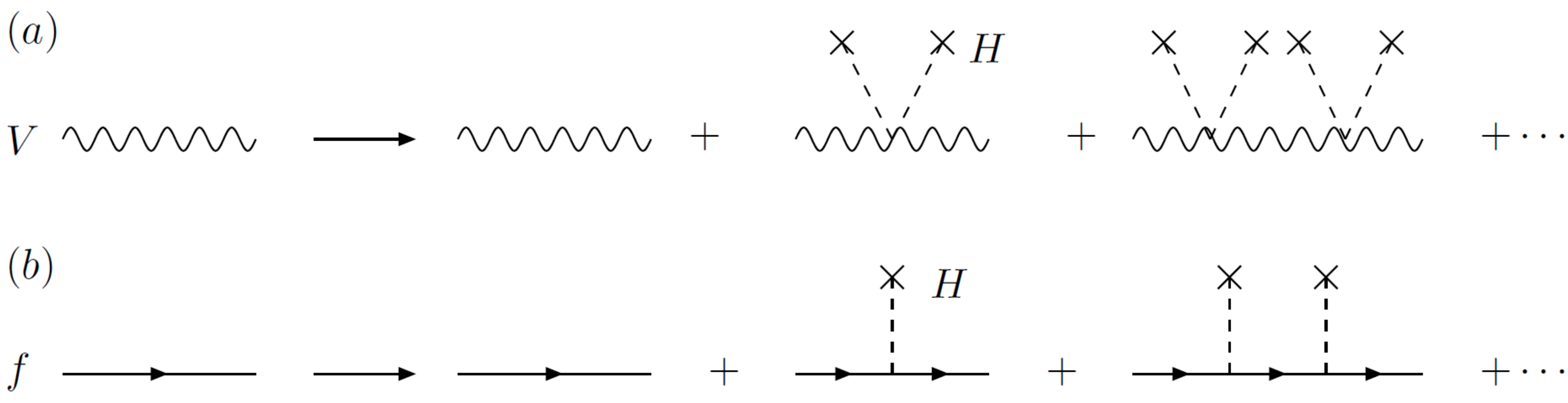
$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1^+(x) + i\phi_2^+(x) \\ \phi_1^0(x) + i\phi_2^0(x) \end{pmatrix}$$

- Yukawino vezanje Dim-4

$$\Phi' = e^{-i\zeta(x)\cdot\tau}\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\mathcal{L}_{H-L} = -g_f^e\left[\overline{L}\mathbf{\Phi}e_R + \overline{e}_R\mathbf{\Phi}^\dagger L\right] \qquad m_e = \frac{g_f^e v}{\sqrt{2}}$$

Masivna polja SM-a crpu masu iz kondenzata Higgsovog polja



$$(a) \qquad \frac{1}{q^2} \;\rightarrow\; \frac{1}{q^2} + \sum_j \frac{1}{q^2}\left[\left(\frac{gv}{2}\right)^2 \frac{1}{q^2}\right]^j \;=\; \frac{1}{q^2 - M^2} \quad : \quad M^2 = g^2 \frac{v^2}{4}$$

$$(b) \qquad \frac{1}{\not{q}} \;\rightarrow\; \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}}\left[\frac{g_f v}{\sqrt{2}} \frac{1}{\not{q}}\right]^j \;=\; \frac{1}{\not{q} - m_f} \quad : \quad m_f = g_f \frac{v}{\sqrt{2}}$$

MASE LEPTONA – u unitarnom bažd.

$$m_e = \frac{y_e v}{\sqrt{2}}$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_e \bar{e}_R \Phi^\dagger L_L + y_e^* \bar{L}_L \Phi e_R\right]$$

$$L_L = (\nu_L, e_L)^T$$

$$\Phi = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}$$

- Uz umnožak dva dubleta

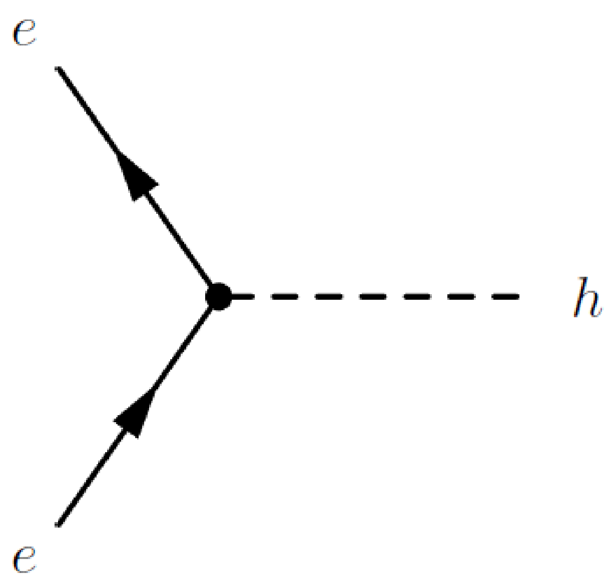
$$\Phi^\dagger L_L = \left(0, \frac{v+h}{\sqrt{2}}\right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L = \frac{v+h}{\sqrt{2}} e_L$$

$$\mathcal{L}_{\text{Yukawa}} \supset -y_e \frac{1}{\sqrt{2}} \left[(v+h)\bar{e}_R e_L + (v+h)\bar{e}_L e_R\right]$$

$$= -\frac{y_e}{\sqrt{2}}(v+h)\bar{e}e = -\left(\frac{y_e v}{\sqrt{2}}\right)\bar{e}e - \frac{y_e}{\sqrt{2}} h\bar{e}e$$

Feynmanovo pravilo Yukawinog vrha

$$h\bar{e}e: \quad \frac{-iy_e}{\sqrt{2}} = \frac{-im_e}{v}$$



$$= -i\frac{y_e}{\sqrt{2}} = -i\frac{m_e}{v}$$

- Odražava afinitet fermiona na higgs

$$\frac{y_e}{\sqrt{2}} = \frac{m_e}{v} = \frac{511\ \text{keV}}{246\ \text{GeV}} \simeq 2.1 \times 10^{-6} \qquad \frac{y_\tau}{\sqrt{2}} = \frac{m_\tau}{v} = \frac{1.78\ \text{GeV}}{246\ \text{GeV}} \simeq 7.2 \times 10^{-3}$$

MASE KVARKOVA – donjih i gornjih, za realno Yukawino vezanje

$$m_d = y_d v/\sqrt{2},$$

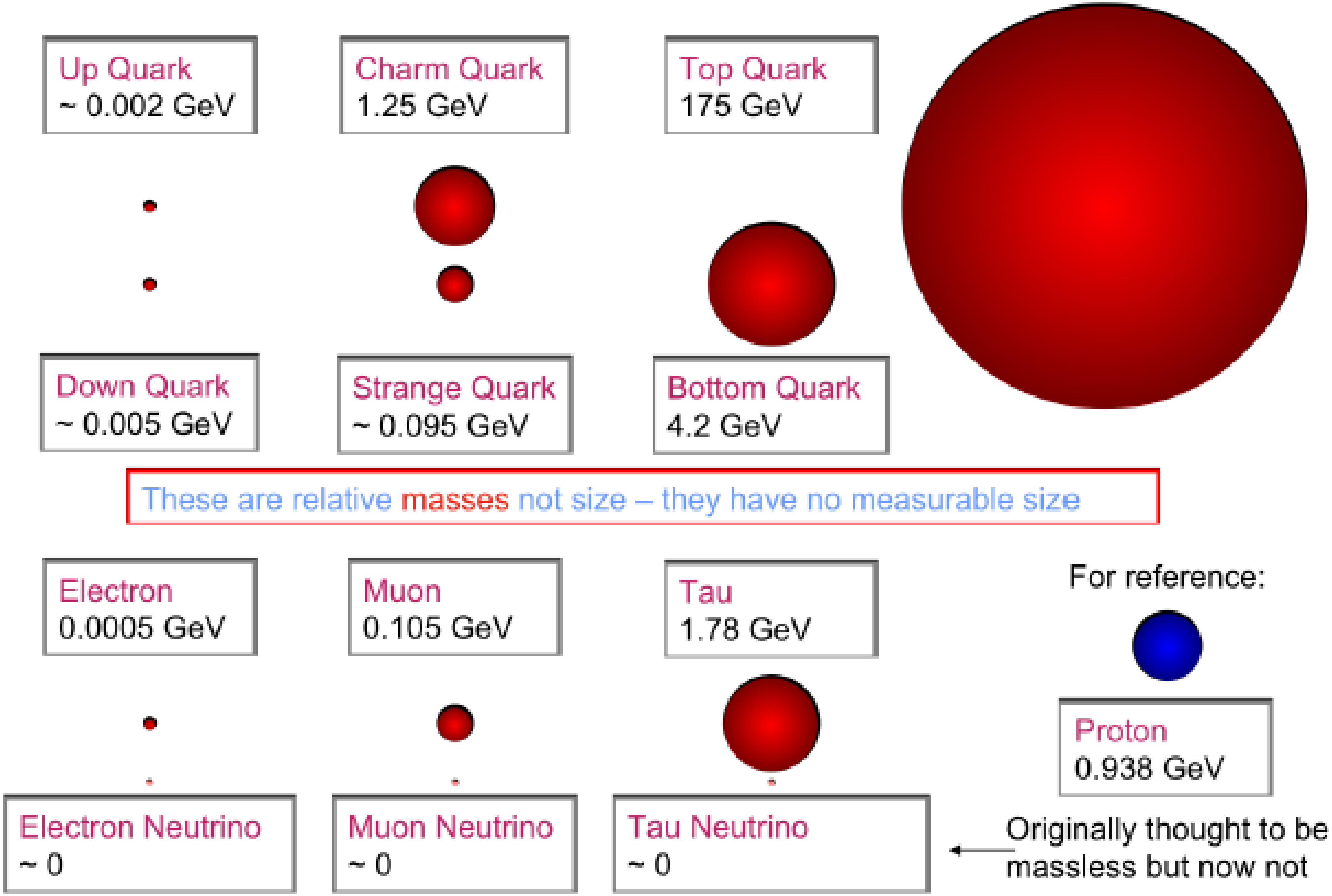
$$m_u = y_u v/\sqrt{2}$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left[y_d \bar{d}_R \Phi^\dagger Q_L + y_d^* \bar{Q}_L \Phi d_R\right] - \left[y_u \bar{u}_R \tilde{\Phi}^\dagger Q_L + y_u^* \bar{Q}_L \tilde{\Phi} u_R\right]$$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left(\frac{y_d v}{\sqrt{2}}\right)\bar{d}d - \frac{y_d}{\sqrt{2}} h\bar{d}d \qquad \Phi^\dagger Q_L = \left(0, \frac{v+h}{\sqrt{2}}\right)\begin{pmatrix} u_L \\ d_L \end{pmatrix} = \frac{v+h}{\sqrt{2}} d_L$$

- Uz konjugiran dublet $\tilde{\Phi} \equiv i\sigma^2 \Phi^* = i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$

$$\mathcal{L}_{\text{Yukawa}} \supset -\left(\frac{y_u v}{\sqrt{2}}\right)\bar{u}u - \frac{y_u}{\sqrt{2}} h\bar{u}u \qquad \tilde{\Phi}^\dagger Q_L = \left(\frac{v+h}{\sqrt{2}}, 0\right)\begin{pmatrix} u_L \\ d_L \end{pmatrix} = \frac{v+h}{\sqrt{2}} u_L$$


Up Quark
~ 0.002 GeV
Charm Quark
1.25 GeV
Top Quark
175 GeV
Down Quark
~ 0.005 GeV
Strange Quark
~ 0.095 GeV
Bottom Quark
4.2 GeV
These are relative masses not size – they have no measurable size
Electron
0.0005 GeV
Muon
0.105 GeV
Tau
1.78 GeV
For reference:
Proton
0.938 GeV
Electron Neutrino
~ 0
Muon Neutrino
~ 0
Tau Neutrino
~ 0
Originally thought to be massless but now not

Raspad Higgsovog bozona na fermionsko-antiferminski par

- Invarijantna amplituda

$$i\mathcal{M} = \bar{u}_f \left(\frac{-im_f}{v} \right) v_{\bar{f}}$$

čije kvadriranje, sumiranje po polarizacijama (i bojama) i integracija po 2-čestičnom faznom prostoru, daje parcijalnu širinu raspada:

$$\Gamma(h \to f\bar{f}) = \frac{N_c}{8\pi} \frac{m_f^2}{v^2} m_h \left[1 - \frac{4m_f^2}{m_h^2} \right]^{3/2}$$

Miješanje kvarkovskih generacija

$$Q_{Lj}, \quad u_{Rj}, \quad d_{Rj}, \qquad j = 1, 2, 3$$

$$\mathcal{L}^q_{\text{Yukawa}} = -\sum_{i=1}^{3}\sum_{j=1}^{3}\left[y^u_{ij}\bar{u}_{Ri}\tilde{\Phi}^\dagger Q_{Lj} + y^d_{ij}\bar{d}_{Ri}\Phi^\dagger Q_{Lj}\right] + \text{h.c.}$$

$$\mathcal{L}^q_{\text{Yukawa}} \supset -(\bar{u}_1, \bar{u}_2, \bar{u}_3)_R \mathcal{M}^u \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L - (\bar{d}_1, \bar{d}_2, \bar{d}_3)_R \mathcal{M}^d \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L + \text{h.c.}$$

- u unit. bažd. kompleksne Yukawine (3x3) matrice vode na matrice masa

$$\mathcal{M}^u_{ij} = \frac{v}{\sqrt{2}} y^u_{ij}, \qquad \mathcal{M}^d_{ij} = \frac{v}{\sqrt{2}} y^d_{ij}$$

BAZA KVARKOVSKIH MASA – dijagonalizacijom biunitarnim transform. $U^{-1} = U^{\dagger}$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R} = U_{L,R} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}, \qquad \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R} = D_{L,R} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}$$

- Dijagonalizacija masa/Yukawinih matrica

$$U_R^{-1} \mathcal{M}^u U_L = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \qquad D_R^{-1} \mathcal{M}^d D_L = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

- Dijagonalna i realna Higgsova vezanja u bazi masa

$$h\bar{q}q : \quad \frac{-iy_q}{\sqrt{2}} = \frac{-im_q}{v}$$

CKM miješanje u nabijenoj struji & okusno dijagonalna neutralna

$$J_L^{+\mu} = (\bar{u}_1, \bar{u}_2, \bar{u}_3)_L \gamma^\mu \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L U_L^\dagger \gamma^\mu D_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$

CKM matrica $\qquad U_L^\dagger D_L \equiv V$

je unitarna $\qquad V^\dagger V = \left(U_L^\dagger D_L\right)^\dagger \left(U_L^\dagger D_L\right) = D_L^\dagger U_L U_L^\dagger D_L = 1$

- Univerzalno vezanje fotona i Z bozona: $\quad Q$
  GIM i granasta odsutnost FCNC $\quad (T^3 - s_W^2 Q)$

$$(\bar{u}_1, \bar{u}_2, \bar{u}_3)_L \gamma^\mu \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L U_L^\dagger \gamma^\mu U_L \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L = (\bar{u}, \bar{c}, \bar{t})_L \gamma^\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

Za više obitelji (generacija) -općenito Yukawin lagrangian nije dijagonalan u okusu

$$-\mathcal{L}_Y = G^{(d)}_{ij}\,\bar{Q}'_{L_i}\Phi D'_{R_j} + G^{(u)}_{ij}\,\bar{Q}'_{L_i}\tilde{\Phi} U'_{R_j} + h.c.$$

Nakon SSB :

$$-\mathcal{L}_Y = \left(1 + \frac{H(x)}{v}\right)\sum_{i,j=1}^{3}\left\{(m_U)_{ij}\,\bar{U}'_{L_i}U'_{R_j} + (m_D)_{ij}\,\bar{D}'_{L_i}D'_{R_j} + h.c.\right\}$$

$$U' = \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} \qquad D' = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

matrice mase u "baždarnoj" bazi

nisu dijagonalne

$$(m_U)_{ij} = -\frac{v}{\sqrt{2}} G^{(u)}_{ij} \quad ; \quad (m_D)_{ij} = -\frac{v}{\sqrt{2}} G^{(d)}_{ij}$$

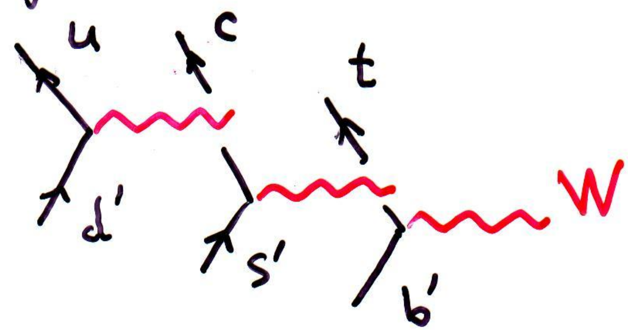
Prijelaz na "fizikalnu" bazu

$$U_{L(R)} = V^U_{L(R)} U'_{L(R)} = V^U_{L(R)} \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}_{L(R)} \quad ; \quad D_L = V^D_L D'_L = V^D_L \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L$$

daje dijagonalne matrice mase

$$\mathcal{L}_M = -[\bar{U}_L \, \mathrm{diag}(m_u, m_c, m_t) U_R + \bar{D}_L \, \mathrm{diag}(m_d, m_s, m_b) D_R + h.c.]$$

U fizikalnoj bazi (dobro def. masa)
"zakomplicira" se nabijena slaba struja



$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left(\bar{U}'_L \gamma^\mu W^\dagger_\mu D'_L + h.c. \right)$$

$$\to \frac{g}{\sqrt{2}} \left[\bar{U}_L \gamma^\mu \left(V_L^U V_L^{D\dagger} \right) D_L + h.c. \right]$$

volimo oznaku

$$U_L^\dagger D_L \equiv V$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$(n \times n)$

$$n^2 - (2n-1) = (n-1)^2 = \frac{n(n-1)}{2} + \frac{(n-1)(n-2)}{2}$$

kutova   komplex. faza

Pogodan je odabir slabe baze u kojoj su gornji kvarkovi masena stanja. Izospinski dubleti tada imaju zapis

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L,$$

- U prostoru generacija je

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L .$$

Miješanje leptona (Pontecorvo-Maki-Nakagawa-Sakata)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

utvrđeno na temelju pojave
OSCILACIJA NEUTRINA

prva opipljiva naznaka fizike izvan standardnog modela

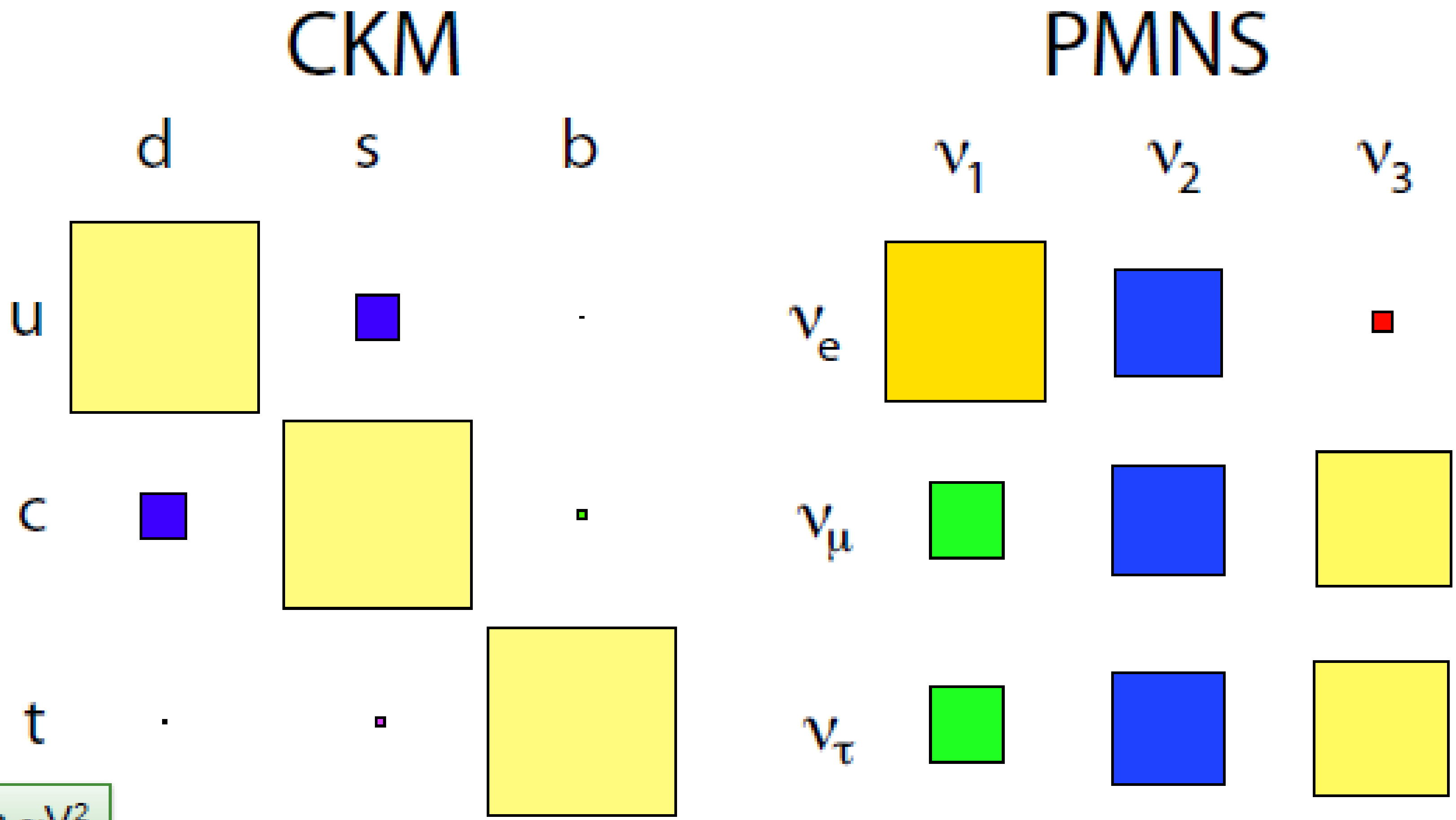
Kao za kvarkove, moguć je odabir slabe baze u kojoj su nabijeni leptoni masena stanja. Izospinski dubleti tada imaju zapis

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L,$$

- tako da PMNS matrica povezuje okusna stanja neutrina s masenim (1,2,3)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

CKM vs. PMNS



Area ~V^2

Why these values? Are the two related? Are they related to masses?

$$|U_{\mathrm{LEP}}| = \begin{pmatrix} 0.73-0.89 & 0.44-0.66 & <0.24 \\ 0.23-0.66 & 0.24-0.75 & 0.51-0.87 \\ 0.06-0.57 & 0.40-0.82 & 0.48-0.85 \end{pmatrix}.$$

$$|U_{\mathrm{LEP}}| \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1+\mathcal{O}(\lambda)) & \frac{1}{\sqrt{2}}(1-\mathcal{O}(\lambda)) & \epsilon \\ -\frac{1}{2}(1-\mathcal{O}(\lambda)+\epsilon) & \frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\mathcal{O}(\lambda)-\epsilon) & -\frac{1}{2}(1+\mathcal{O}(\lambda)-\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \begin{matrix} \lambda \sim 0.2 \\ \epsilon < 0.25 \end{matrix}$$

from quark's

$$|U_{\mathrm{CKM}}| \simeq \begin{pmatrix} 1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix} \quad \lambda \sim 0.2$$

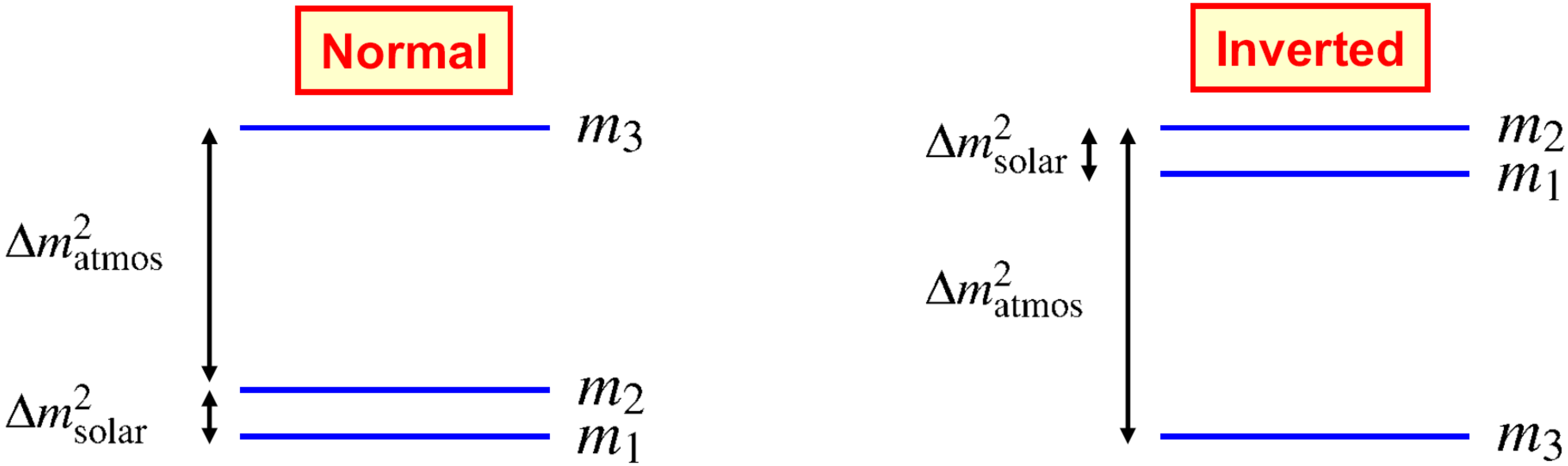
★ **To date, results on neutrino oscillations only determine**

$$|\Delta m^2_{ji}| = |m^2_j - m^2_i|$$

★ **Two distinct and very different mass scales:**

- **Atmospheric neutrino oscillations :** $|\Delta m^2|_{\rm atmos} \sim 2.5 \times 10^{-3}\,{\rm eV}^2$
- **Solar neutrino oscillations:** $|\Delta m^2|_{\rm solar} \sim 8 \times 10^{-5}\,{\rm eV}^2$

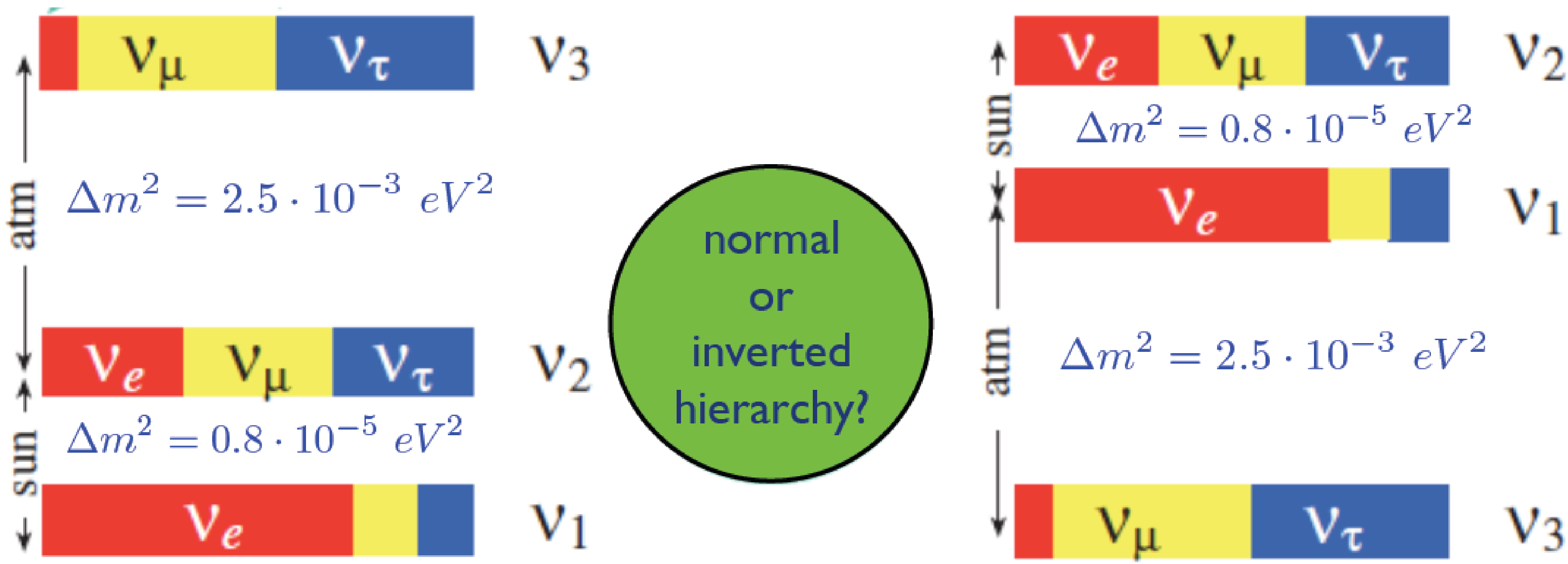
•**Two possible assignments of mass hierarchy:**



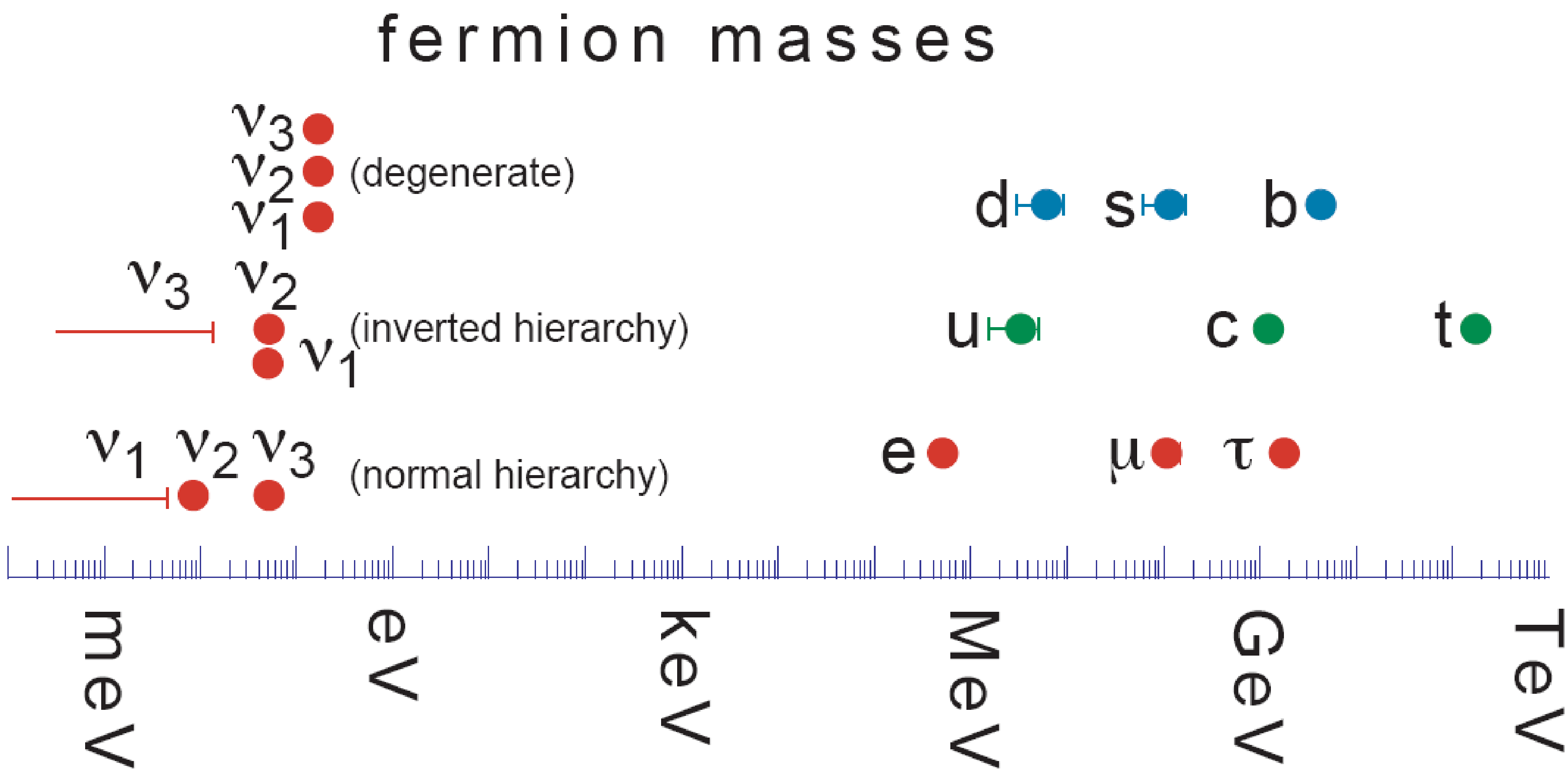
•**In both cases:** $\Delta m^2_{21} \sim 8 \times 10^{-5}\,{\rm eV}^2$ **(solar)**

$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,{\rm eV}^2$ **(atmospheric)**

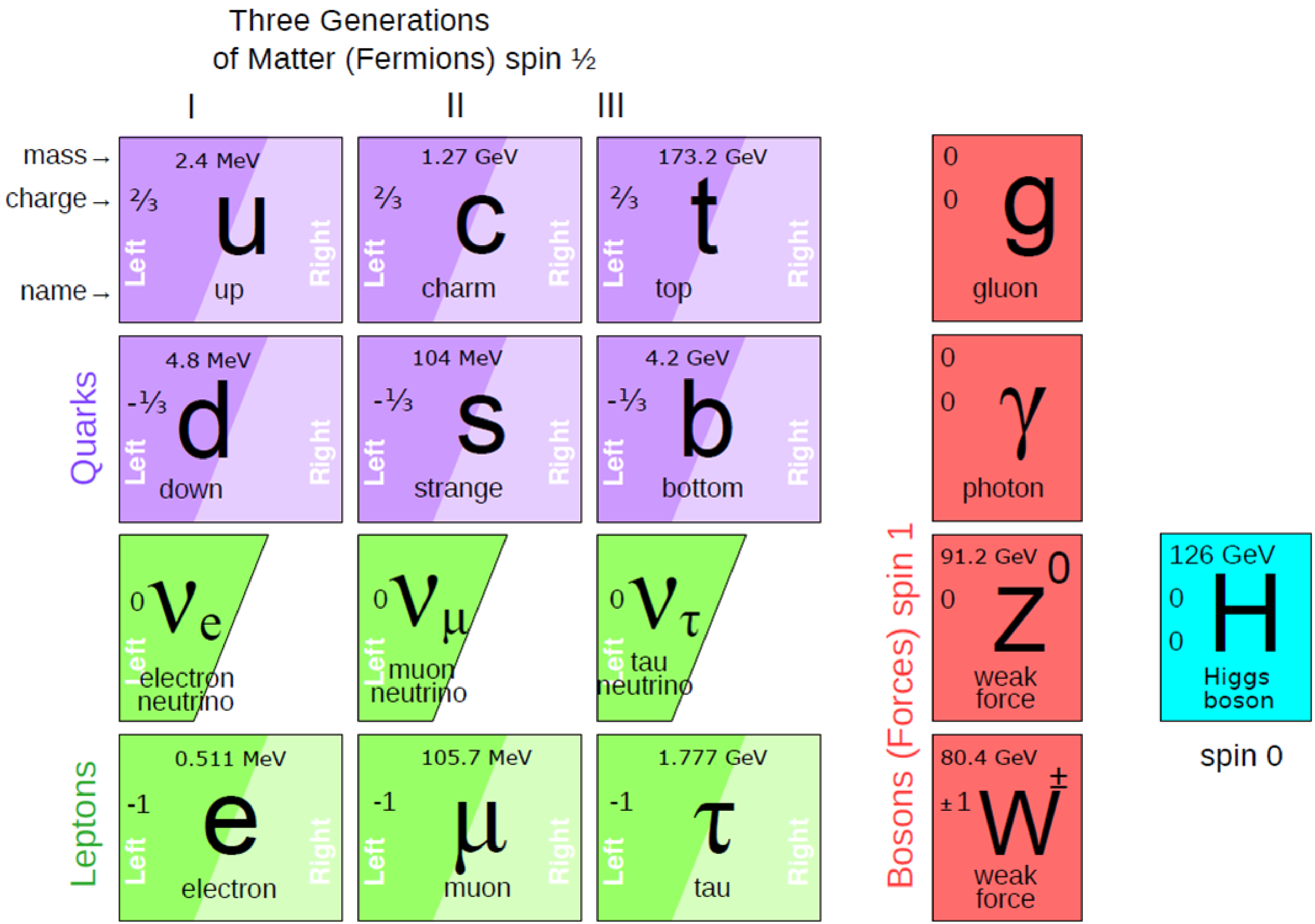
NORMALNA I INVERZNA HIJERARHIJA MASA NEUTRINA



Neutrinske mase kao opipljivo odstupanje od SM-a Fig.Murayama'08



PERIODIČKA TABLICA SM-a



Diracove mase neutrina ukoliko postoje i desni neutrini ν_{Ri} $(i = 1, 2, 3)$

$$\mathcal{L}_{\text{Yukawa}} \supset -y_\nu \bar{\nu}_R \tilde{\Phi}^\dagger L_L + \text{h.c.}$$

$$\mathcal{L}^{\ell}_{\text{Yukawa}} = -\sum_{i=1}^{3}\sum_{j=1}^{3}\left[y^{\nu}_{ij}\bar{\nu}_{R_i}\tilde{\Phi}^\dagger L_{Lj} + y^{\ell}_{ij}\bar{e}_{Ri}\Phi^\dagger L_{Lj}\right] + \text{h.c.}$$

- PMNS miješanje okusnih i masenih (1,2,3) stanja, pri čemu male mase neutrina

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

expt.

$$m_\nu \sim 0.1 \text{ eV} \implies \frac{y_\nu}{\sqrt{2}} = \frac{m_\nu}{v} \simeq 4 \times 10^{-13}$$

!!

Dim-5 Seesaw Operator

Only 3 realizations at tree-level

- single BSM particle with SM charges

Type I — three heavy right-handed neutrinos

$N_R \sim (1,1,0)$

Type II — one heavy Higgs triplet $\Delta \equiv \begin{pmatrix} \Delta^- & -\sqrt{2}\,\Delta^0 \\ \sqrt{2}\,\Delta^{--} & -\Delta^- \end{pmatrix}$

$\Delta \sim (1,3,-2)$

Type III — three heavy triplet fermions $\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}$

$\Sigma \sim (1,3,0)$

E. Ma, Phys. Rev. Lett. **81**, 1171 (1998)

Weinbergov operator dim 5, generiran česticama nove fizike skale Λ, daje Majoraninu masu neutrina

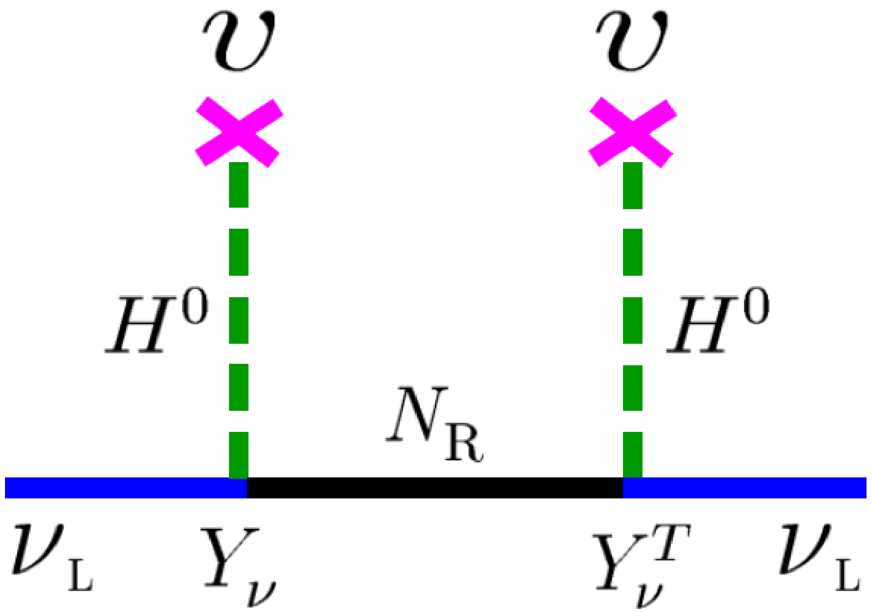
$$\mathcal{L}_{\text{Majorana}} = -\frac{(\tilde{\Phi}^\dagger L_L)^2}{\Lambda} \qquad m\nu_L\nu_L$$

- nakon SSB, iz korektno napisanog operatora izraženog konjugiranim "L" spinorom koji se Lor-transf. kao "R"

$$\bar{L}_L^c \equiv -L_L^T C, \quad C = -i\gamma^2\gamma^0$$

$$\mathcal{L}_{\text{Majorana}} = -\frac{y_{ij}^{\text{Maj}}}{\Lambda} \bar{L}_{Li}^c \tilde{\Phi}^* \tilde{\Phi}^\dagger L_{Lj} \qquad \Delta L = 2$$

Fermioni s "GUT-skale" u njigalici Tipa I & III



$$M_\nu \approx -v^2 Y_\nu \frac{1}{M_R} Y_\nu^T$$

$$M_\nu \approx M_D M_R^{-1} M_D^T$$

0.01 eV

10^{15} GeV

100 GeV

Tip I

$$f_l(\bar{\nu}_L\phi^+ + \bar{l}_L\phi^0)l_R \Rightarrow m_l = f_l\langle\phi^0\rangle$$

$$f_\nu(\bar{\nu}_L\bar{\phi}^0 - \bar{l}_L\phi^-)\nu_R \Rightarrow m_D = f_\nu\langle\bar{\phi}^0\rangle$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

$$m_{1,2} = M/2 \mp \sqrt{(M/2)^2 + m_D^2}$$

(a) If $M = 0$, then $m_{1,2} = \mp m_D$ and ν_L pairs with ν_R to form a Dirac fermion

Dirac fermion may be regarded as two mass-degenerate Majorana fermions of opposite CP.

(b) Since M is an invariant mass term, it is presumably very large

In that case, $m_D << M$, and $m_1 \simeq -m_D^2/M$, $m_2 \simeq M$.

Tip II

triplet $(\xi^{++}, \xi^{+}, \xi^{0})$ which couples directly to the symmetric triplet combination of two $(\nu, l)_L$

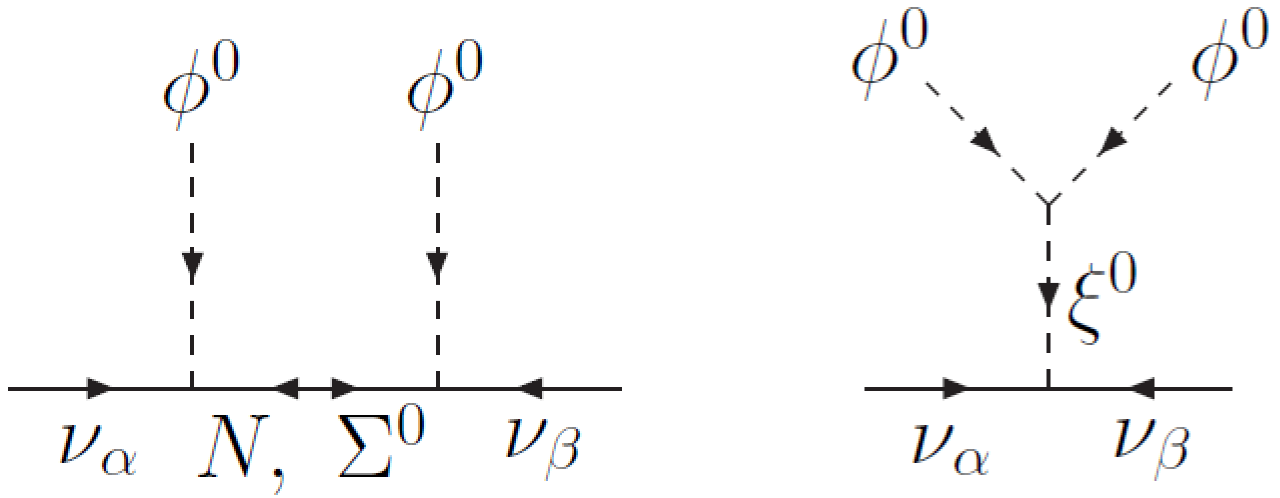
$$\frac{h_\nu}{2}\left[\nu\nu\xi^0 - \frac{(\nu l + l\nu)}{\sqrt{2}}\xi^+ + ll\xi^{++}\right] \Rightarrow m_\nu = h_\nu\langle\xi^0\rangle$$

with $\langle\xi^0\rangle << \langle\phi^0\rangle$

$$(\nu_\alpha\phi^0 - l_\alpha\phi^+)(\nu_\beta\phi^0 - l_\beta\phi^+) =$$

$$\nu_\alpha\nu_\beta\phi^0\phi^0 - (\nu_\alpha l_\beta + l_\alpha\nu_\beta)\phi^+\phi^0 + l_\alpha l_\beta\phi^+\phi^+$$

$$\mathcal{L}_5 = \frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha\phi^0 - l_\alpha\phi^+)(\nu_\beta\phi^0 - l_\beta\phi^+)$$

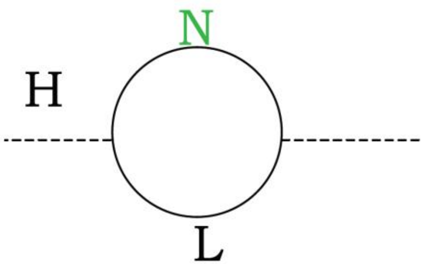


Three tree-level realizations of seesaw Majorana neutrino mass

Tip III $(\nu_\alpha\phi^0 - l_\alpha\phi^+)(\nu_\beta\phi^0 - l_\beta\phi^+) =$

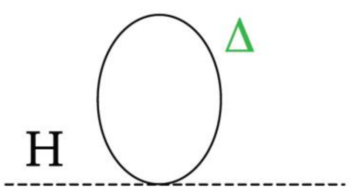
$$-2\nu_\alpha\phi^+ l_\beta\phi^0 + (\nu_\alpha\phi^0 + l_\alpha\phi^+)(\nu_\beta\phi^0 + l_\beta\phi^+) - 2l_\alpha\phi^0\nu_\beta\phi^+$$

TeV-seesaw scale is suggested by EW hierarchy problem



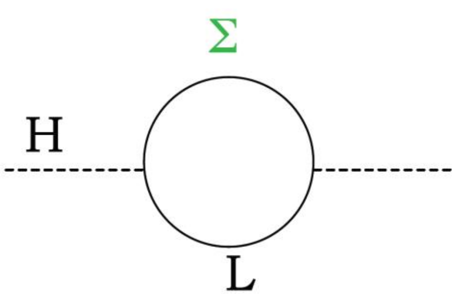
$$\delta m_H^2 = -\frac{Y_N^\dagger Y_N}{16\pi^2}\left[2\Lambda^2 + 2M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

(Vissani)



$$\delta m_H^2 = -3\frac{\lambda_3}{16\pi^2}\left[\Lambda^2 + M_\Delta^2\left(\log\frac{M_\Delta^2}{\Lambda^2} - 1\right)\right] - \frac{\mu_\Delta^2}{2\pi^2}\log\left(\left|\frac{M_\Delta^2 - \Lambda^2}{M_\Delta^2}\right|\right)$$

(Abada, Biggio, Bonnet, Hambye, M.B.G.)



$$\delta m_H{}^2 = -3\frac{Y_\Sigma^\dagger Y_\Sigma}{16\pi^2}\left[2\Lambda^2 + 2M_\Sigma^2 \log\frac{M_\Sigma^2}{\Lambda^2}\right]$$

Lowering the seesaw scale
by going to Dim>5 operators

$$\mathcal{O}^{D=5} = \mathcal{O}_{\text{Weinberg}} = LLHH$$
$$\mathcal{O}^{D=7} = (LLHH)(H^\dagger H)$$
$$\mathcal{O}^{D=9} = LLHH(H^\dagger H)(H^\dagger H)$$
$$\ldots$$

$$m_\nu \sim v \left(\frac{v}{M}\right)^{D-4}$$

D=9 and $M \sim \mathrm{TeV}$ is enough to get sub-eV neutrino mass

F. Bonnet, D. Hernandez, T. Ota and W. Winter, JHEP **0910**:076,2009 [0907.3143 [hep-ph]];

Scale of New Physics for Type I, II and III Seesaw

- Type I [Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanović 79]
- Type II [Magg, Wetterich 80; Schechter, Valle 80; Lazarides et al. 80; Mohapatra, Senjanović 80; Gelmini, Roncadelli 80]
- Type III [Foot, Lew, He, Joshi 89]

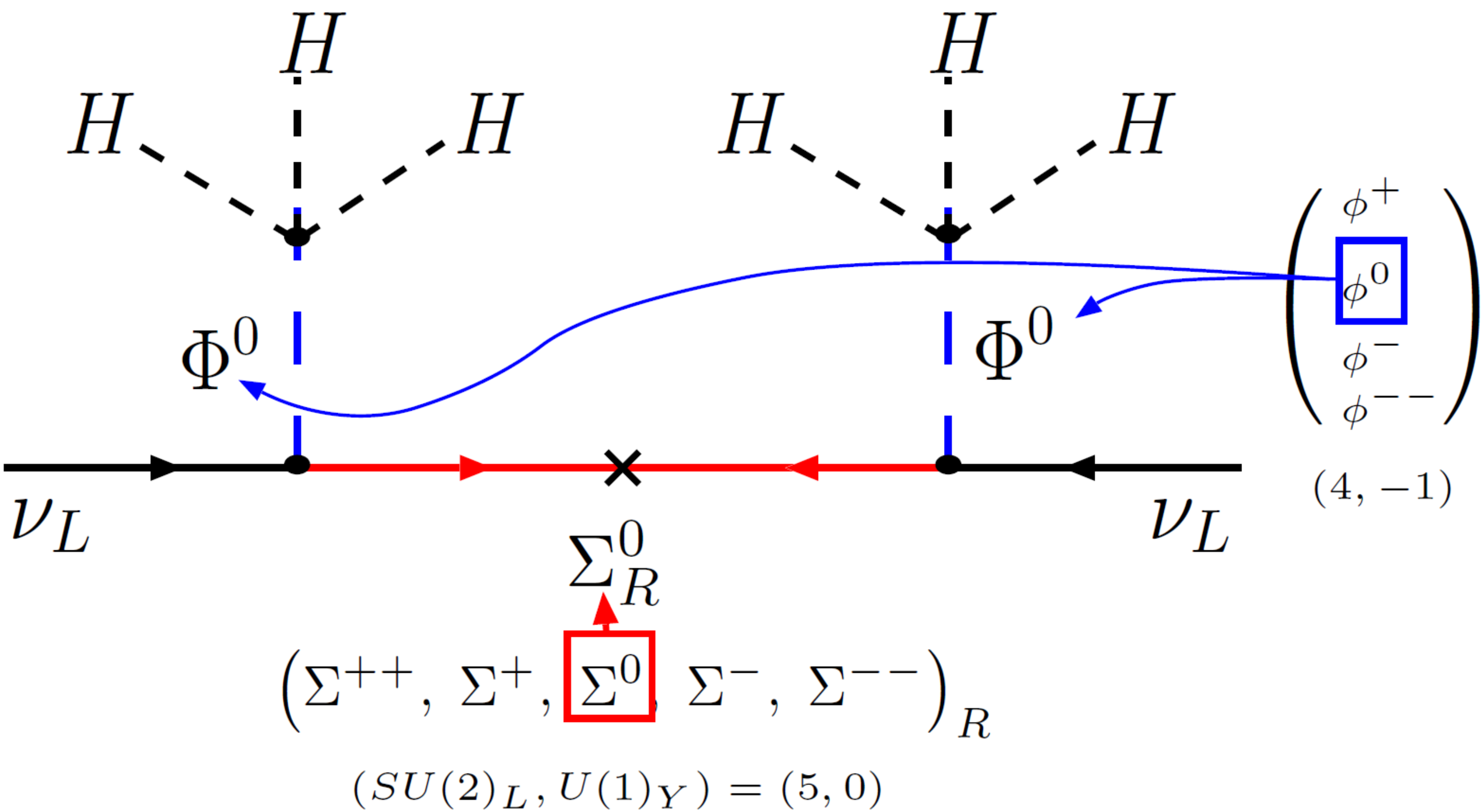
Taking empirical values $m_\nu \sim 0.1\,\mathrm{eV}$, $v = 246\,\mathrm{GeV}$ and natural value $\lambda = O(1)$ one gets

$$M \sim 10^{14}\,\mathrm{GeV}$$

But for such large M there is no hope of direct observation of either M-particles or $O(1/M^2)$ effects $\Rightarrow$ No new discoveries at LHC!

"Type V" Seesaw

(5, 0) instead of (3, 0) in Type III



EWSB in usual way from the Higgs doublet

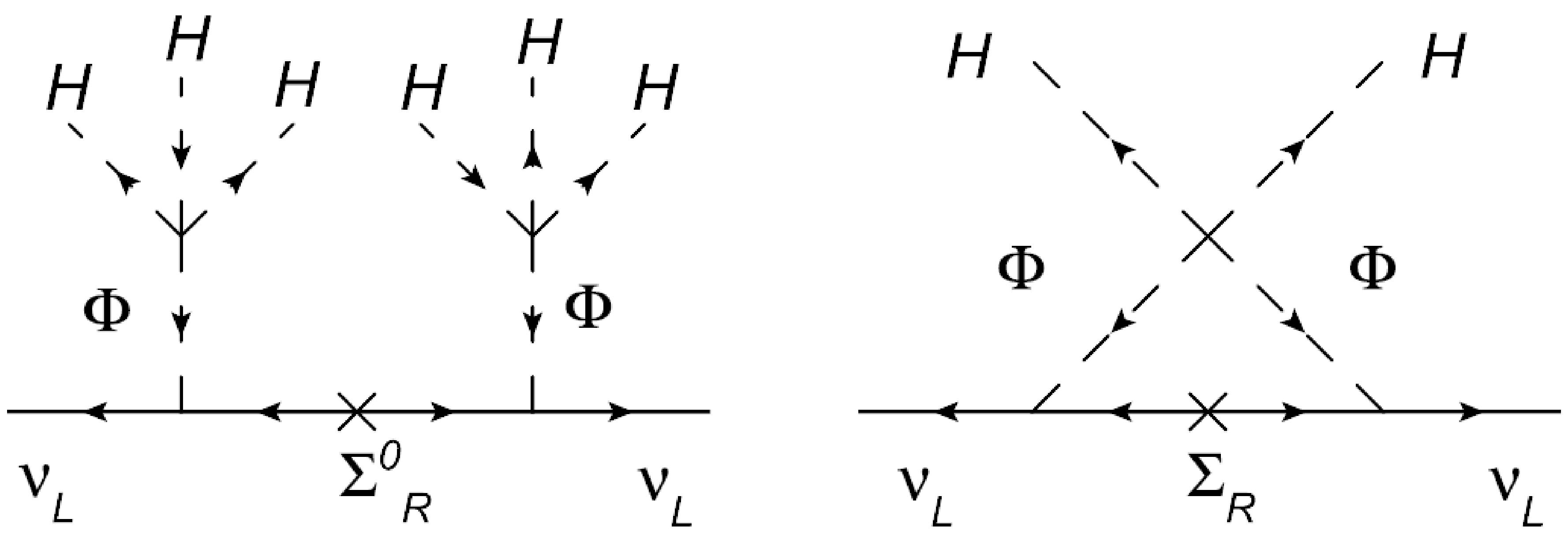
$$V(H,\Phi) = -\mu_H^2 H^\dagger H + \mu_\Phi^2 \Phi^\dagger \Phi + \lambda_1 \left(H^\dagger H\right)^2 + \lambda_2 H^\dagger H \Phi^\dagger \Phi$$
$$+ \lambda_3 H^* H \Phi^* \Phi + \left(\lambda_4 H^* H H \Phi + \text{H.c.}\right)$$
$$+ (\lambda_5 H H \Phi \Phi + \text{H.c.}) + \left(\lambda_6 H \Phi^* \Phi \Phi + \text{H.c.}\right)$$
$$+ \lambda_7 \left(\Phi^\dagger \Phi\right)^2 + \lambda_8 \Phi^* \Phi \Phi^* \Phi.$$

- Induced vev for quadruplet scalar Φ

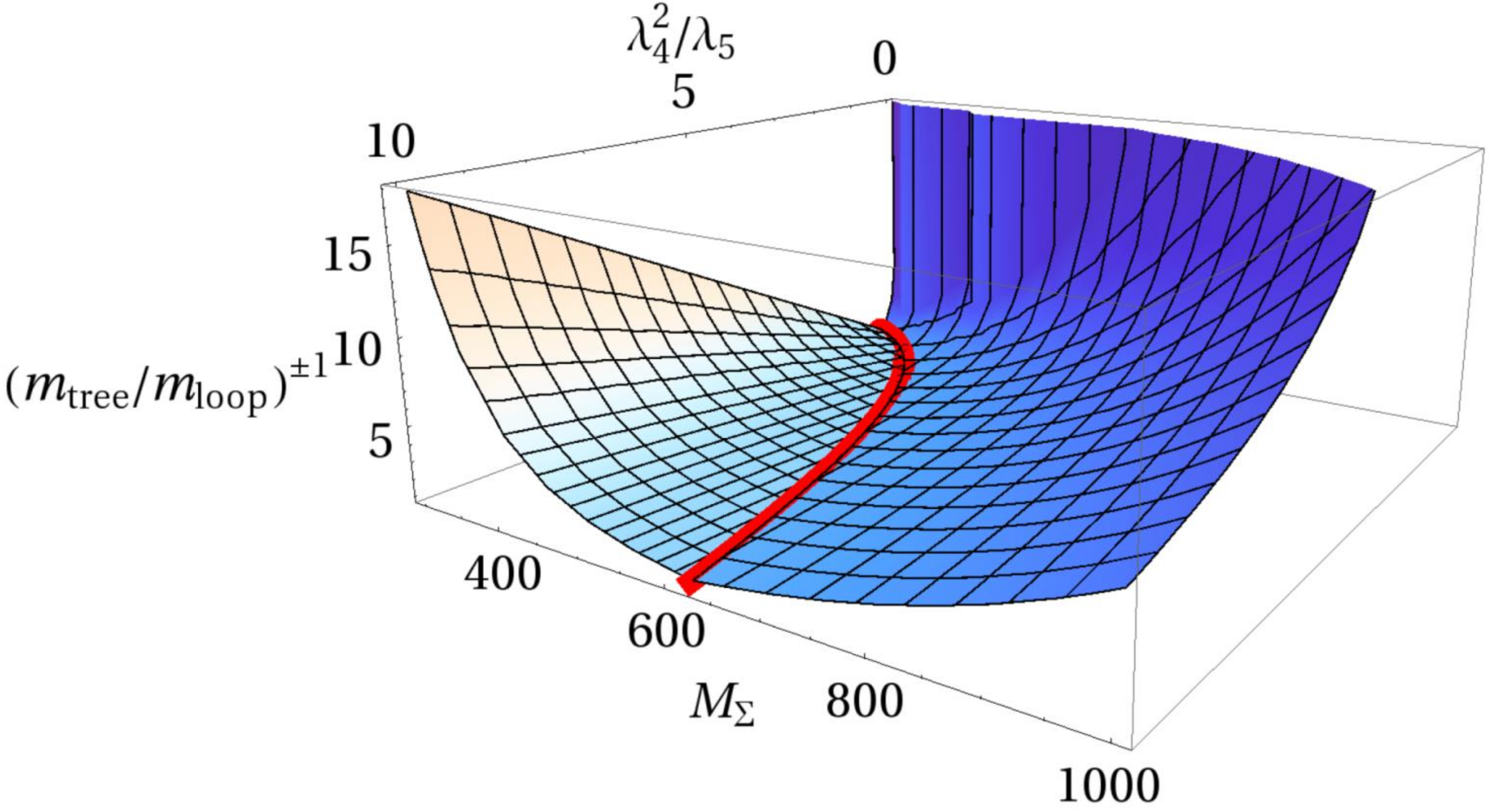
$$v_\Phi \simeq -\frac{1}{\sqrt{3}} \lambda_4^* \frac{v^3}{\mu_\Phi^2}$$

Seesaw w.r.t. Radiative

Mass ~ $\frac{-1}{6}(\lambda_4^*)^2\frac{v^6}{\mu_\Phi^4}\sum_k\frac{Y_{ik}Y_{jk}}{M_k}+\frac{-5\lambda_5^*v^2}{24\pi^2}\sum_k\frac{Y_{ik}Y_{jk}M_k}{m_\Phi^2-M_k^2}$

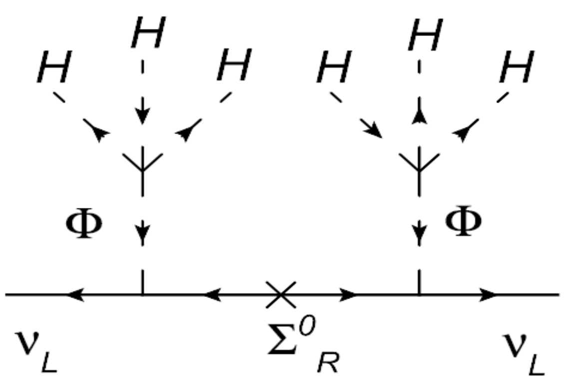


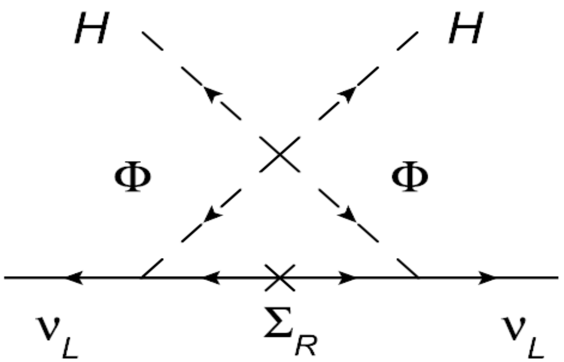
Tree w.r.t. Radiative dominance



Seesaw w.r.t. Radiative Masses: with higher Scalar and Fermion multiplets

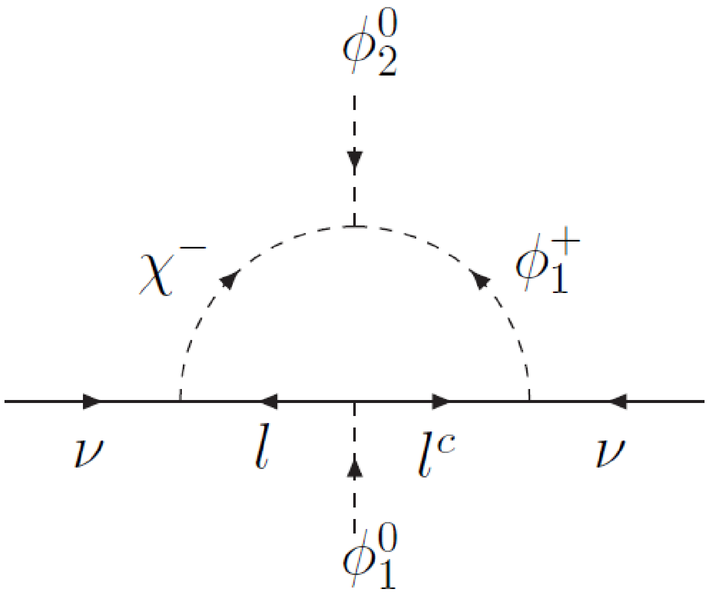
- Tree-level "type IV" (for 4-plet), "type V" (for 5-plet), ... Dim>5
- Loop-level generated Dim-5 operator (Radiative Mass if Tree-level forbidden)
- Scotogenic – with DM particle in loop





One-loop Radiative Models

Zee/Wolfenstein Model



the invariant combination of two (different) lepton doublets couples to a charged scalar singlet, i.e. $(\nu_i l_j - l_i \nu_j)\chi^+$

two different scalar doublets are also needed, i.e. $(\phi_1^+\phi_2^0 - \phi_1^0\phi_2^+)\chi^-$

Zee Model of Neutrino Mass

A. Zee (1980)

Simplest example of using $d = 7$ operator

Neutrino masses induced at one–loop

Loop and chiral suppression $\Rightarrow$ scale can be low

No right-handed neutrinos introduced, so the seesaw operator $\mathcal{O}_1 = L^i L^j H^k H^l \epsilon_{ik} \epsilon_{jl}$ absent at tree-level

Effective operator $\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$ induces neutrino mass

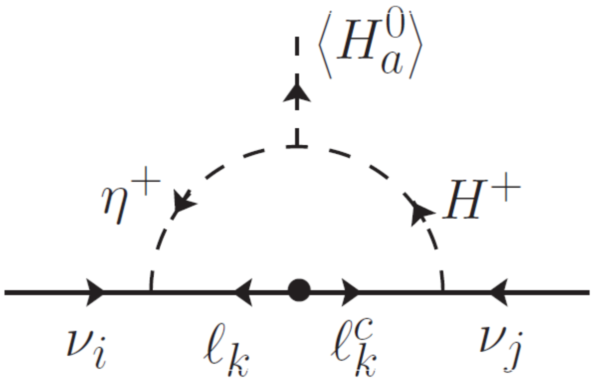
Introduces a second Higgs doublet and a charged singlet scalar η^+

Zee Model of Neutrino Mass (cont.)

$H_a(1,2,-\frac{1}{2}),\ \eta^+(1,1,1)$

$$\mathcal{L}_{\text{Yuk}} = f_{ij} L_i L_j \eta^+ + Y'_{ij} L_i e^c_j H_2 + h.c.$$

$$V = \mu H_1 H_2 \eta^+ + h.c. + \ldots$$



$f_{ij} = -f_{ji}$

$$M_\nu = \kappa \left(\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

$$\kappa = \frac{\sin 2\gamma}{16\pi^2} \log\left(\frac{M_1^2}{M_2^2}\right)$$

γ: $\eta^+ - H^+$ mixing angle, $M_{1,2}$: charged Higgs masses

In the Zee model, both Higgs doublets couple to leptons $\Rightarrow$ Flavor changing neutral currents at tree-level mediated by Higgs bosons

$$M_\nu = \kappa \left(\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

$\hat{Y}$ is arbitrary, so quantitative predictions difficult

Wolfenstein suggested a discrete Z_2 symmetry that allows only one Higgs doublet to couple to leptons

L. Wolfenstein (1980)

FCNC avoided, $\hat{Y} = \frac{M_\ell^{\text{diag}}}{v}$

Zee-Wolfenstein model very predictive for neutrinos

Smirnov, Tanimoto (1997)
Jarlskog, Matsuda, Skaldhauge, Tanimoto (1999)
Frampton, Glashow (1999)

Zee-Wolfenstein model:

$$M_\nu = \frac{\kappa}{v}\left[\hat{f}(M_\ell^{\text{diag}})^2 + (M_\ell^{\text{diag}})^2\hat{f}^T\right]$$

$$M_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

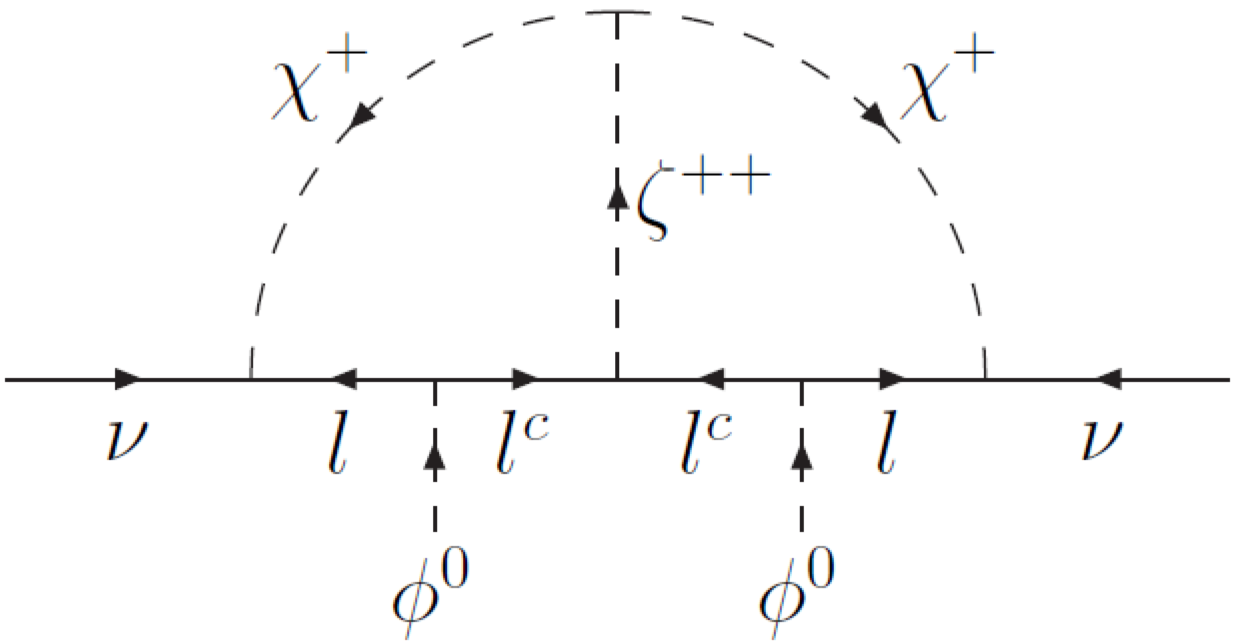
3 real parameters explain all neutrino oscillation data

Compatible with bimaximal neutrino mixing

KamLand and solar neutrino data excluded this possibility

Koide (2001)
X.G. He (2004)

Two-loop Zee/Babu



using the additional interactions $\zeta^{++}\chi^-\chi^-$ and $l_i^c l_j^c \zeta^{--}$

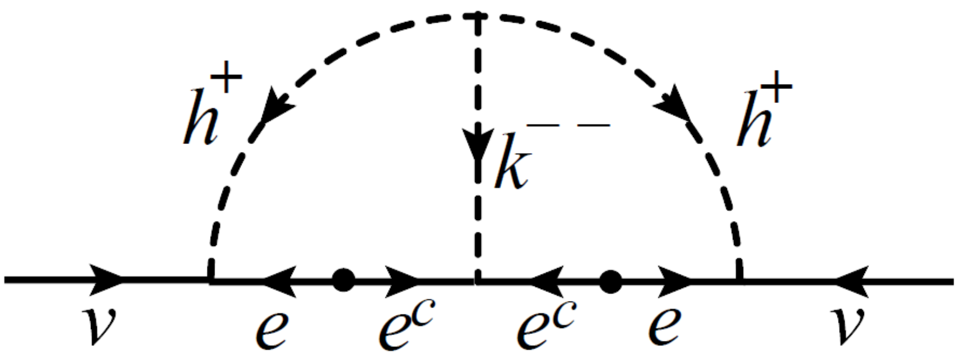
the second Higgs doublet is replaced by
a doubly charged singlet ζ^{++}

Two–loop neutrino mass generation

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + g_{ij} e_i^c e_j^c k^{--} + \mu h^+ h^+ k^{--} + \text{h.c.}$$

$$\Downarrow$$

$$\mathcal{O}_9 = L^i L^j L^k e^c L^l e^c \epsilon_{ij} \epsilon_{kl}$$



A. Zee, (1985)
Babu (1988)

Consistent with all neutrino oscillation data

Predicts doubly charged Higgs boson with TeV mass

One neutrino is nearly massless

CP violation in neutrino oscillation is expected

Two–loop neutrino mass model (cont.)

Fits to neutrino masses and mixing angles, consistent with perturbativity and boundedness of potential as well as FCNC limits sets constraints on h^+ and k^{++} masses:

$$\text{NH}: \quad 306 \text{ GeV} < m_{k^{++}} < 177 \text{ TeV}; \quad 779 \text{ GeV} < m_{h^+} < 63 \text{ TeV}$$

$$\text{IH}: \quad 997 \text{ GeV} < m_{k^{++}} < 25 \text{ TeV}; \quad 2.3 \text{ TeV} < m_{h^+} < 7.9 \text{ TeV}$$

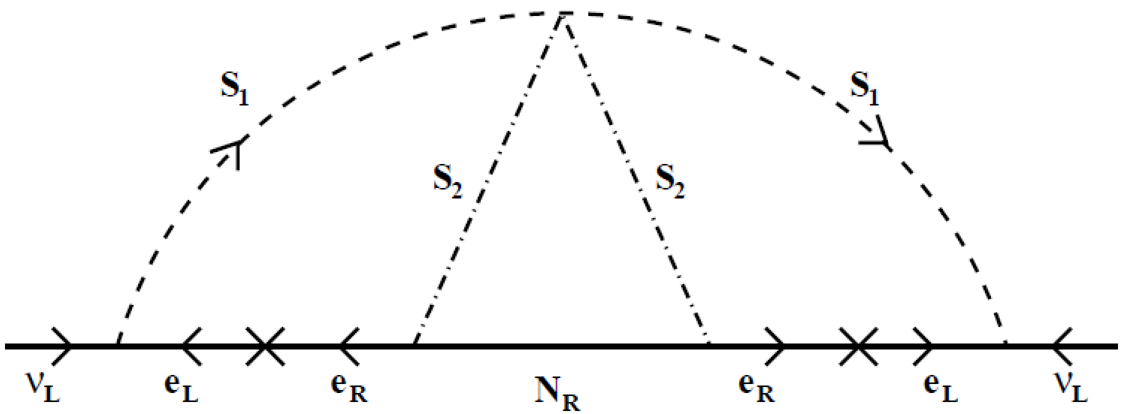
Since couplings are essentially fixed from neutrino masses, charged lepton flavor violation can be predicted

$\mu \to e\gamma$ and $\tau \to 3\mu$ limits used as input

Lower limits on branching ratios for $\mu \to 3e$, $\mu - e$ conversion in nuclei, as well as $\mu \to e\gamma$ and $\tau \to 3\mu$ follow

K.S. Babu, J. Julio (2013)
D. Schmitz, T.Schwetz, H. Zhang (2014)
J. Herrero-Garcia, M. Nebot, N. Rius, A. Santamaria (2014)
K.S. Babu, C. Macesanu (2005)
D. Sierra, M. Hirsch (2006)
M. Nebot, J. Oliver, D. Paolo, A. Santamaria (2008)

From Zee'80 to Krauss-Nasri-Trodden (KNT 2002)



$$\mathcal{L}_{\text{Zee}} = f_{\alpha\beta} L_\alpha^T C i\tau_2 L_\beta S^+ + \mu \Phi_1^T i\tau_2 \Phi_2 S^- + \text{h.c.}$$

Higgs doublets Φ_1 and Φ_2, and a charged field S

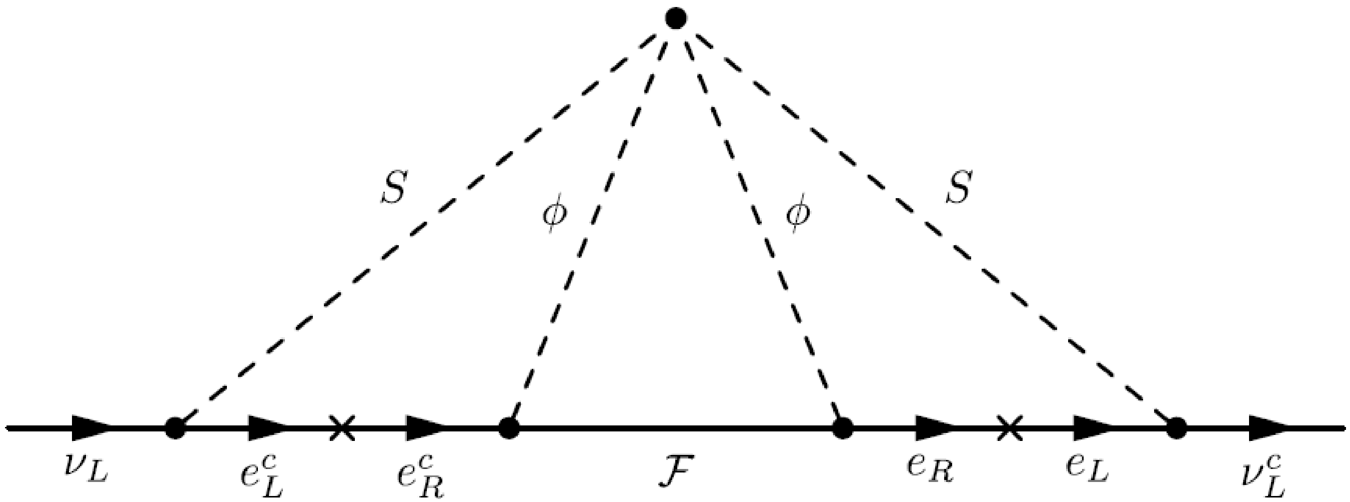
$$\mathcal{L}_{\text{KNT}} = f_{\alpha\beta} L_\alpha^T C i\tau_2 L_\beta S_1^+ + g_\alpha N_R S_2^+ l_{\alpha R} + M_R N_R^T C N_R + V(S_1, S_2) + \text{h.c.}$$

two charged singlet scalars S_1 and S_2 and one right handed neutrino N_R

S_2 and N_R transform as $Z_2 : \{S_2, N_R\} \longrightarrow \{-S_2, -N_R\}$

KNT Model

the exotic scalars $S \sim (1,1,2)$ and $\phi \sim (1,1,2)$
and the fermions $\mathcal{F}_{iR} \sim (1,1,0)$
such that ϕ and $\mathcal{F}$ are Z_2-odd, $\{\phi, \mathcal{F}\} \to \{-\phi, -\mathcal{F}\}$



A unified solution to the DM & nu-mass problem

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + \{f_{\alpha\beta}\,\overline{L^c_\alpha}\,L_\beta\,S^+ + g_{i\alpha}\,\overline{\mathcal{F}^c_i}\,\phi^+\,e_{\alpha R} + \text{H.c}\} - \frac{1}{2}\overline{\mathcal{F}^c_i}\,\mathcal{M}_{ij}\,\mathcal{F}_j - V(H,S,\phi)$$

$\alpha,\,\beta \in \{e,\,\mu,\,\tau\}$

i labels fermion generations

$$V(H,\,S,\,\phi) \supset \frac{\lambda_{\rm S}}{4}(S^*)^2\phi^2 + \text{H.c.}$$

$\mathcal{M} = \text{diag}(M_1,\,M_2,\,M_3)$

$M_1 < M_2 < M_3$

the loop diagram gives the mass matrix as

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\lambda_{\rm S}}{(4\pi^2)^3}\frac{m_\sigma m_\rho}{M_\phi}\,f_{\alpha\sigma}\,f_{\beta\rho}\,g^*_{\sigma i}\,g^*_{\rho i} \times F\left(\frac{M_i^2}{M_\phi^2},\frac{M_{\rm S}^2}{M_\phi^2}\right)$$

Z_2-odd fields with $M_1 < 225$ GeV and $M_\phi < 245$ GeV

3-plet & 5-plet Variants of KNT

- **Triplet Variant**

$SU(2)_L$ triplets, $\phi \sim (1,3,2)$ and $\mathcal{F} \sim (1,3,0)$

$\{\phi,\, \mathcal{F}\} \rightarrow \{-\phi,\, -\mathcal{F}\}$ The DM is the lightest $\mathcal{F}_1^0$,

$M_{\rm DM} \sim 2$ TeV

- **Larger Representations**
  - with or without DM?

the Z_2 symmetry need not be imposed

to preclude tree-level neutrino masses

Radiative Nu-Mass with or without DM

a charged scalar singlet, $S^+ \sim (1,1,2)$,

a complex scalar quintuplet, $\phi \sim (1,5,2)$,

and a real fermion quintuplet, $\mathcal{F} \sim (1,5,0)$

$$\mathcal{L} \supset \mathcal{L}_{\rm SM} + \{ f_{\alpha\beta}\, \overline{L^c_\alpha}\, L_\beta\, S^+ + g_{i\alpha}\, \overline{\mathcal{F}_i}\, \phi\, e_{\alpha R} + {\rm H.c} \} \\ - \frac{1}{2} \overline{\mathcal{F}^c_i}\, \mathcal{M}_{ij}\, \mathcal{F}_j - V(H,S,\phi)$$

$$V(H,S,\phi) \supset \frac{\lambda_S}{2}(S^-)^2 \{ \phi^{+++}\phi^- - \phi^{++}\phi^0 + \frac{1}{2}\phi^+\phi^+ \} + {\rm H.c.}$$

Softly broken accidental Z_2 symmetry

- **The 2nd term in the mixing potential**

$$V_m(H,\,S,\,\phi) = \frac{\lambda_S}{4}(S^-)^2\phi_{abcd}\phi_{efgh}\epsilon^{ae}\epsilon^{bf}\epsilon^{cg}\epsilon^{dh}$$

$$+\,\lambda S^-(\phi^*)^{abcd}\phi_{abef}\phi_{cdjl}\epsilon^{ej}\epsilon^{fl} + \text{H.c.}$$

as the sole source of Z_2 symmetry-breaking in the full theory.

the Z_2 symmetry $\{\phi,\,\mathcal{F}\} \to \{-\phi,\,-\mathcal{F}\}$, in the limit $\lambda \to 0$

making $\lambda \ll 1$ technically natural the DM width goes like $\Gamma_{\rm DM} \propto \lambda^2$

and one can make this sufficiently small

to obtain long-lived DM

2. Scotogenic Models with Scalar Doublets

$\Phi = (\phi^+, \phi^0)$ is the SM Higgs doublet

the Higgs boson $h = \sqrt{2}(Re\phi^0 - v)$

one extra scalar doublet (η^+, η^0)

3 singlet Majorana neutral fermions $N_{1,2,3}$

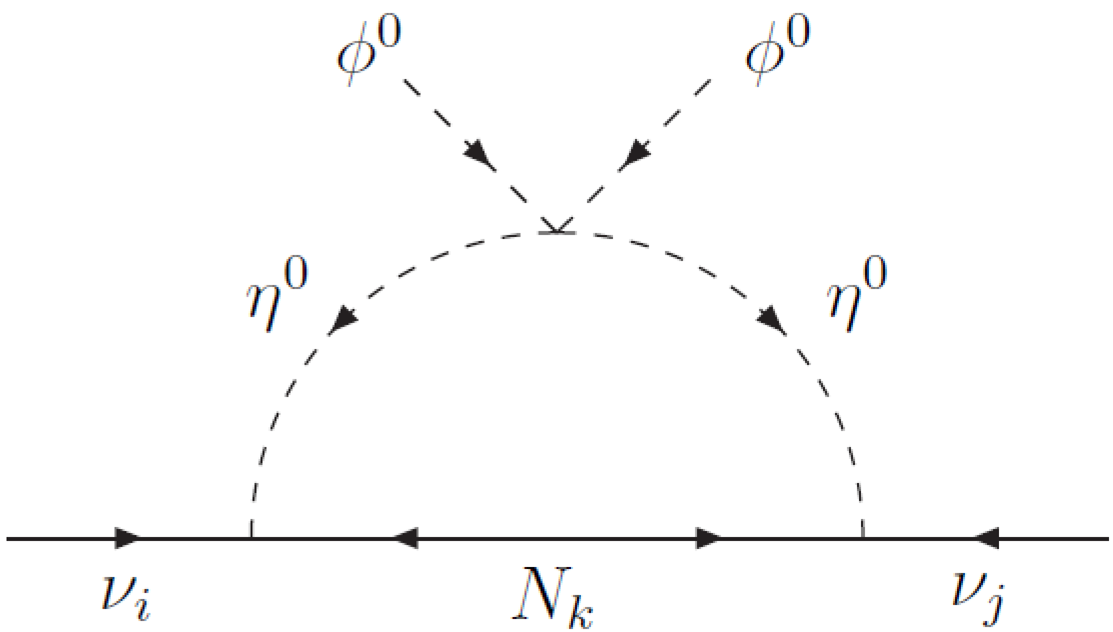
The Z_2 symmetry forbids νN coupling to ϕ^0

no Dirac mass linking ν to N

at the classical (tree) level

2.1 ORIGINAL SCOTOGENIC MODEL

E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, Phys. Rev. D 73 (2006) 077301, arXiv:hep-ph/0601225.



Radiative masses appear from electroweak symmetry breaking, i.e. $\langle \phi^0 \rangle = v$

- **Z2-odd 2nd SU(2) scalar doublet** $\eta = (\eta^+, \eta^0)$

a.k.a. inert Higgs doublet (IHD), which has inspired many studies

- **by Z2-allowed term** $(\lambda_5/2)(\Phi^\dagger \eta)^2 + H.c.$

η_R^0 and η_I^0 are split by $\langle \phi^0 \rangle = v$

One-loop diagram induces a Majorana mass for neutrino

$$\mathcal{M}_\nu = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}$$

$$(\mathcal{M}_\nu)_{ij} =$$

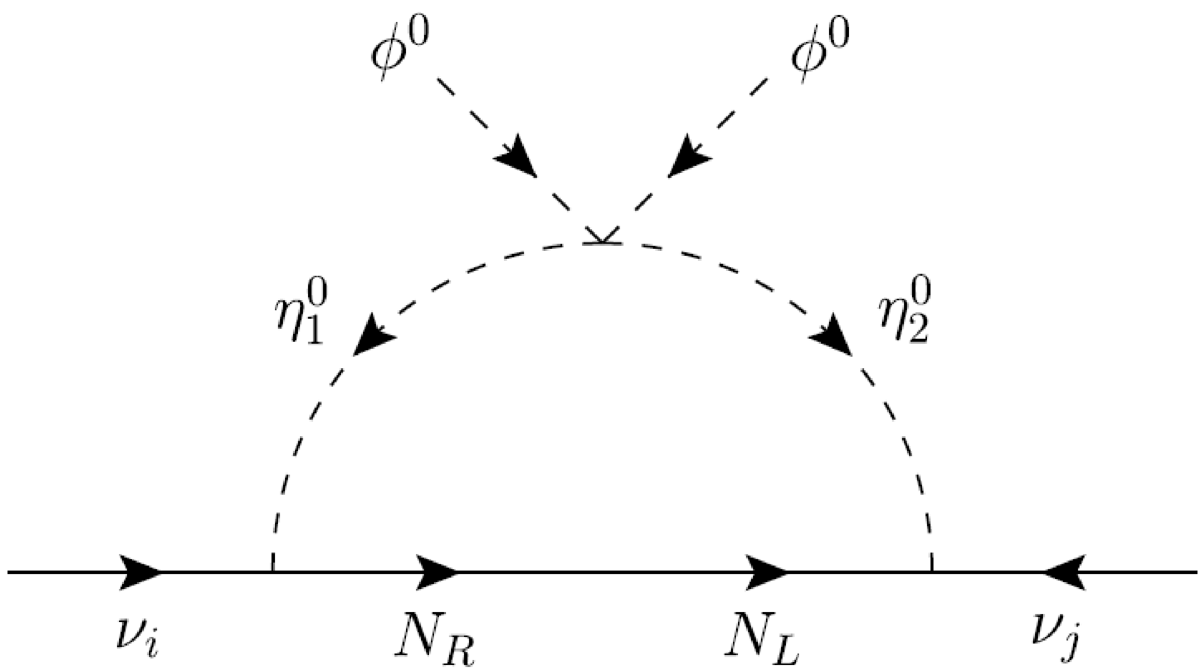
$$\sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2}\left[\frac{m_R^2}{m_R^2 - M_k^2}\ln\frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2}\ln\frac{m_I^2}{M_k^2}\right]$$

The lightest particle among η_R^0, η_I^0, $N_{1,2,3}$ is absolutely stable and is a good dark matter candidate.

2.2

2.2 U(1)D SCOTOGENIC MODEL

Z2 symmetry promoted to gauge U(1)D



E. Ma, I. Picek, B. Radovčić, New scotogenic model of neutrino mass with $U(1)_D$ gauge interaction, Phys. Lett. B 726 (2013) 744, arXiv:1308.5313 [hep-ph].

- **Two scalar doublets U(1) charged** $(\eta_1^+, \eta_1^0) \sim 1$ and $(\eta_2^+, \eta_2^0) \sim -1$

- **Three neutral singlet Dirac fermions, U(1) charged** $N_{1,2,3} \sim 1$

- **The allowed couplings completing the loop**

$$h_1 \bar{N}_R \nu_L \eta_1^0, \ h_2 N_L \nu_L \eta_2^0, \ \text{and} \ (\Phi^\dagger \eta_1)(\Phi^\dagger \eta_2)$$

which mixes η_1^0 and $\bar{\eta}_2^0$

$$\begin{pmatrix} \eta_1^0 \\ \bar{\eta}_2^0 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$\chi_{1,2}$ are mass eigenstates

The scotogenic neutrino mass

$$(\mathcal{M}_\nu)_{ij} = \sin\theta\cos\theta \sum_k \frac{[(h_1)_{ki}(h_2)_{kj} + (h_2)_{ki}(h_1)_{kj}]M_k}{8\pi^2} \times \left[\frac{m_1^2}{m_1^2 - M_k^2}\ln\frac{m_1^2}{M_k^2} - \frac{m_2^2}{m_2^2 - M_k^2}\ln\frac{m_2^2}{M_k^2}\right],$$

where $m_{1,2}$ are the masses of $\chi_{1,2}$ and M_k the mass of N_k.

- **(i)** If $U(1)_D$ is unbroken, only N_1 is a DM candidate
- **(ii)** If $U(1)_D$ is broken by $\zeta = (u + \rho + i\sigma)/\sqrt{2}$, $\zeta \sim 2$, there is a massive dark photon γ_D as well as a dark Higgs boson ρ

The lightest (ps)Dirac N as DM

$N\bar{N}$ would annihilate to $\gamma_D \gamma_D$ and $\zeta\zeta^*$

thermally averaged s-wave annihilation cross sections

$$\langle \sigma (N\bar{N} \to \gamma_D \gamma_D) v \rangle = \frac{\pi \alpha_D^2}{M_1^2},$$

$$\langle \sigma \left(N\bar{N} \to \zeta \zeta^*\right) v \rangle = \frac{(|y_L|^2 + |y_R|^2)^2 - (y_L y_R^* - y_L^* y_R)^2}{16\pi M_1^2}$$

- **Display on Fig. the values of DM couplings needed for the observed DM relic density**

$$\Omega_{DM} h^2 = 0.1187(17)$$

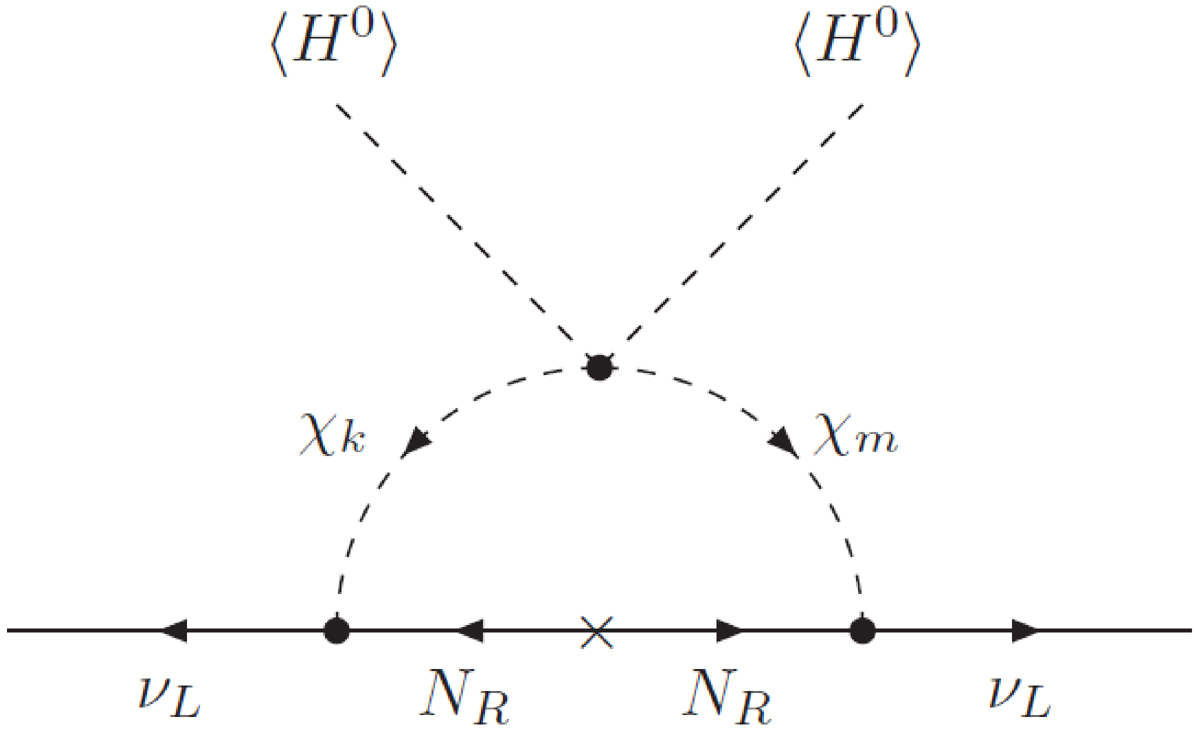
The standard collisionless CDM

agreement with many observational constraints, including CMB anisotropy [4–6], large-scale structure [7, 8] and the general properties of dark-matter-dominated halos [9–11], some crucial issues on small scales are subject to controversy (see Ref. [12] for a recent, brief review). First, hierarchical clustering in the standard CDM model overpredicts the number of substructures in a halo the size of the Local Group by an order of magnitude as compared with the number of satellite galaxies observed in the Local Group, a discrepancy referred to as

the "missing satellite problem" (see Refs. [13–16]).

N-body simulations show a universal profile with a central cusp ($\sim r^{-1}$ in the NFW profile [17]), while observations of low-surface brightness galaxies and dwarf galaxies mostly favor a flat central slope. This has been known as the "cuspy core problem" (see Refs. [18–21]).

3. Scotogenic Models with Minimal Dark Matter



$$N_R : (1,5,0)\,, \quad \chi : (1,6,-1/2)$$