HIGGSOV SEKTOR MSSM-a i SIMETRIJA PECCEI-QUINN

- PRIRODNOST I SUPERSIMETRIJA
- 2HD uz Peccei-Quinninu simetriju

Motivacija za dodatni HD

  - Nakon otkrića Higgsovog bozona možemo očekivati dodatne skalare

- U supersimetriji
- Uz Peccei-Quinnovu simetriju
- U modelima bariogeneze
- U modelima tamne tvari – od inertnog dubleta do skotogeničkih modela

PROBLEM HIJERARHIJE člana mase dim 2

SuSy & MSSM kao orijentiri

- Izjalovljena očekivanja MSSM-a (M.Mangano,ICTP 2015)
- Teorijska SuSy kao uspješna priča (M.Shifman/1211.0004) – M. Luty ICTP Lects on SuSy

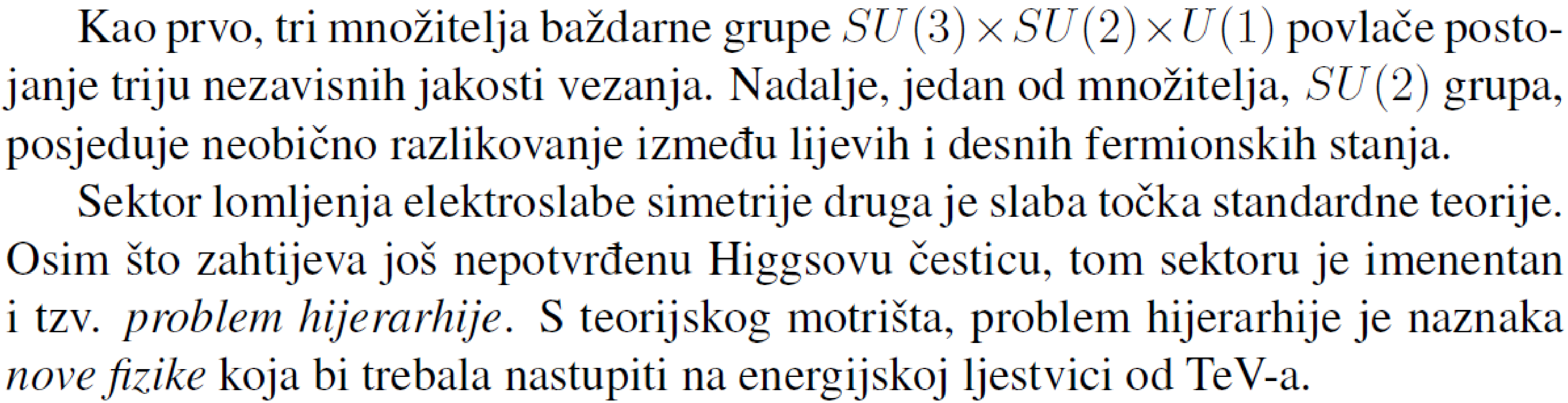
M.Mangano

- Until few yrs ago, we had a benchmark model, MSSM, expected to deliver the following:
  - low-mass Higgs h^0, no heavier than ~130 GeV
  - ~TeV scale squarks and gluinos, to be seen rapidly at the LHC
    - ⇒ solution to the naturalness problem
  - extra Higgses (A^0 /H^0 /$H^\pm$) observed at the LHC
  - candidate for DM, confirmed by direct detection
  - interesting flavour phenomenology
    - explanation of $(g-2)_\mu$
    - sizable deviations from SM in $B(B_S \to \mu^+ \mu^-)$
    - $\mu \to e\gamma$ observed at MEG, consistent with SUSY neutrino masses induced at the GUT scale
    - CPV in the Higgs or squark/gluino sector, to explain BAU
    - electric dipole moments (e, n) measured, consistent with previous point

- **None of the above happened.**
- Thus a radical change in attitude in BSM model building is taking place, focusing on schemes that address individual issues or anomalies, leaving for later the understanding of the "grand picture"
- The above scenario may still happen, with a few-year delay, perhaps stretching a bit the "naturalness".
- This expectation is still high, and well justified

- The observation of the Higgs where the SM predicted it would be, its SM-like properties, and the lack of BSM phenomena up to the TeV scale, make the ***naturalness issue more puzzling than ever***

FEČ 1997, pogl.7: Znakovi nepotpunosti SM-a

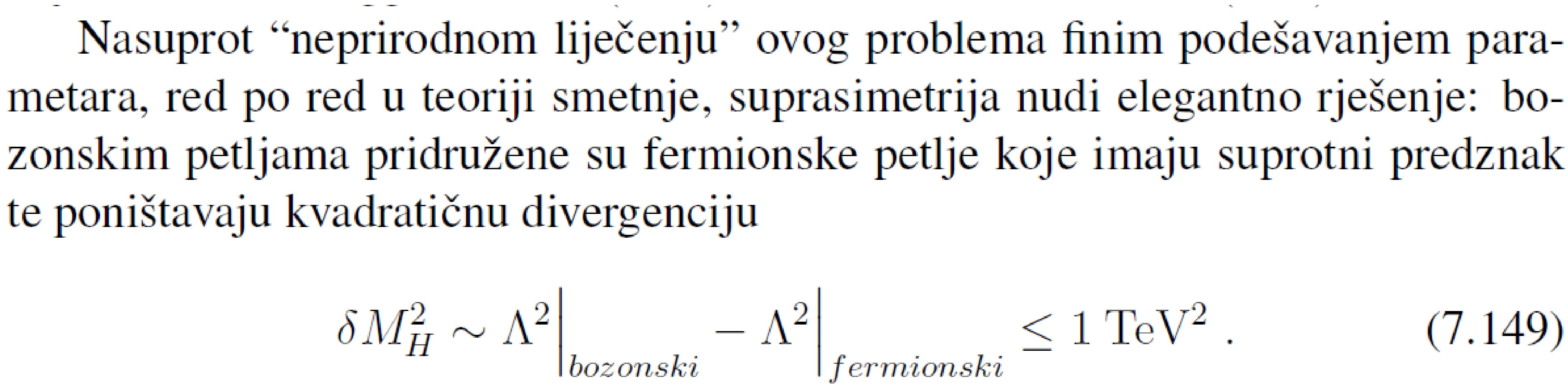


Kao prvo, tri množitelja baždarne grupe $SU(3) \times SU(2) \times U(1)$ povlače postojanje triju nezavisnih jakosti vezanja. Nadalje, jedan od množitelja, $SU(2)$ grupa, posjeduje neobično razlikovanje između lijevih i desnih fermionskih stanja.

Sektor lomljenja elektroslabe simetrije druga je slaba točka standardne teorije. Osim što zahtijeva još nepotvrđenu Higgsovu česticu, tom sektoru je imenentan i tzv. *problem hijerarhije*. S teorijskog motrišta, problem hijerarhije je naznaka *nove fizike* koja bi trebala nastupiti na energijskoj ljestvici od TeV-a.

KVADRATIČNO DIVERGENTNI DOPRINOSI MASI HIGGSOVOG BOZONA

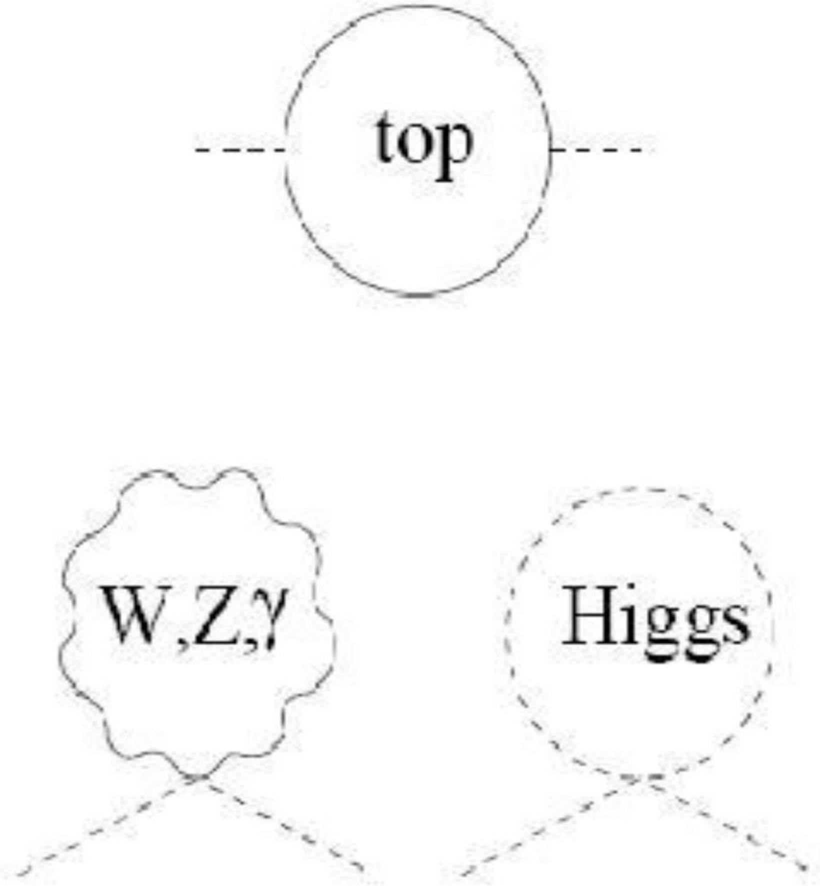
$$\delta M_H^2 \sim \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{k^2} \sim \Lambda^2$$



Nasuprot “neprirodnom liječenju” ovog problema finim podešavanjem parametara, red po red u teoriji smetnje, suprasimetrija nudi elegantno rješenje: bozonskim petljama pridružene su fermionske petlje koje imaju suprotni predznak te poništavaju kvadratičnu divergenciju

$$\delta M_H^2 \sim \Lambda^2\Big|_{bozonski} - \Lambda^2\Big|_{fermionski} \leq 1\ \mathrm{TeV}^2\ . \qquad (7.149)$$

Quant. Fluct. destabilise Higgs



$$-\frac{3}{8\pi^2}\lambda_t^2\Lambda^2 \ \sim\ -(2\,TeV)^2$$

from the top loop,

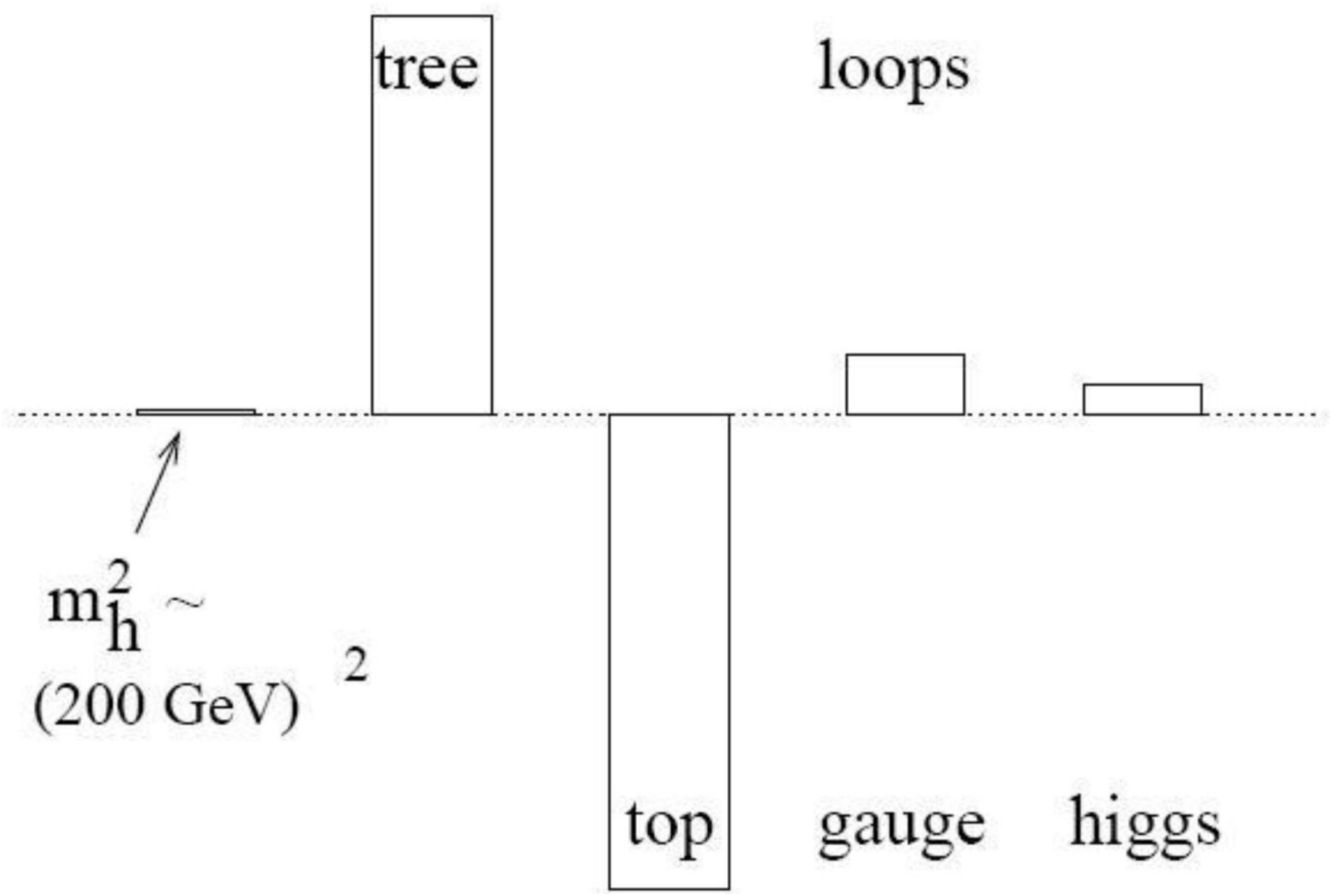
$$\frac{1}{16\pi^2}g^2\Lambda^2 \ \sim\ (700\,GeV)^2$$

from the gauge loop, and

$$\frac{1}{16\pi^2}\lambda^2\Lambda^2 \ \sim\ (500\,GeV)^2$$

from the Higgs loop.

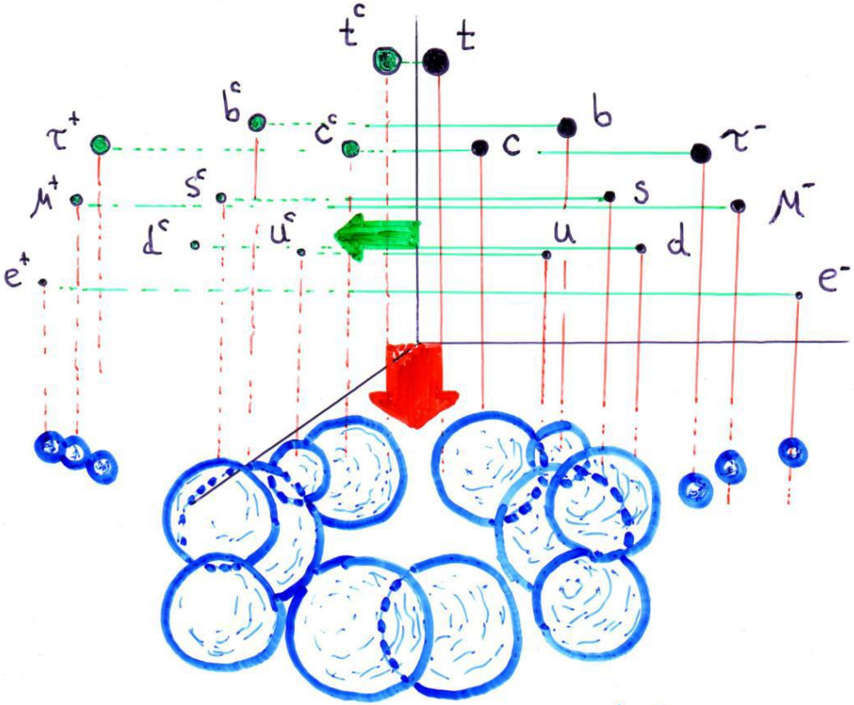
fine-tuning keeps the Higgs light



Supersymmetry

- ◇ CPT invariance of the Dirac eq. $(i\gamma^\mu\partial_\mu - m)\psi = 0$
  – the reason for the ANTIPARTICLES
- ◇ B ↔ F symmetry ⇒ 2nd doubling

antiparticles & particles



SuSy "shadow" particles
DARK MATTER

Čestica	suprasimetrični partner	spin partnera	ime
γ	$\tilde{\gamma}$	$1/2$	fotino
e_L	$\tilde{e}_L$	0	selektron
u_R	$\tilde{u}_R$	0	u skvark
g	$\tilde{g}$	1/2	gluino
ν_e	$\tilde{\nu}_e$	0	sneutrino
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Tablica 7.2: *Suprasimetrična stanja*

Kraćenje baždarne anomalije kao razlog za dodatni HD

- Jedan dodatni kiralni fermion dao bi nekompenzirani doprinos anomaliji

spin 1/2	spin 0	spin 1	spin 1/2
Q_L, u^c_L, d^c_L	$\tilde{Q}_L, \tilde{u}^c_L, \tilde{d}^c_L$	B	$\tilde{B}$
L_L, e^c_L	$\tilde{L}_L, \tilde{e}^c_L$	$W^\pm, W^0$	$\tilde{W}^\pm, \tilde{W}^0$
$\tilde{H}_1, \tilde{H}_2$	H_1, H_2	g	$\tilde{g}$

Multipleti MSSM modela: kiralni u prva dva stupca

Facets of SuSy

◊ SYMMETRY OF LAWS OF NATURE UNDER B ⟷ F

- as an ultimate symm. in FT
- as a spacetime symm. in an extended space – SUPERSPACE

◊ EFFECTIVE THEORY

- solving problems of the SM

  REASON FOR SCALARS

  STABILISING m_H against quant. corr's

  part of the HIERARCHY problem

  ↳ possibility of radiatively induced EW SB

- opening a window BSM

  GAUGE COUPLING UNIFICATION (SuSy GUT)

  A BRIDGE TO GRAVITY (SuGra)

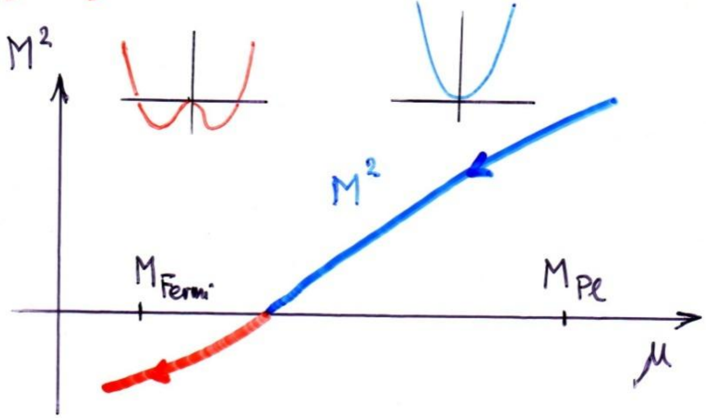
  part of "Nature's grand tapestry"

RADIATIVELY INDUCED — EW Symm. Breaking —

Positive M^2 at M_{Pl} can evolve to M^2 negative in the IR [Inoue et al '82, Ibanez & Ross '82]

- for $m_t > 160$ GeV ⟶
- once quadratic div. removed by SuSy

surprise!

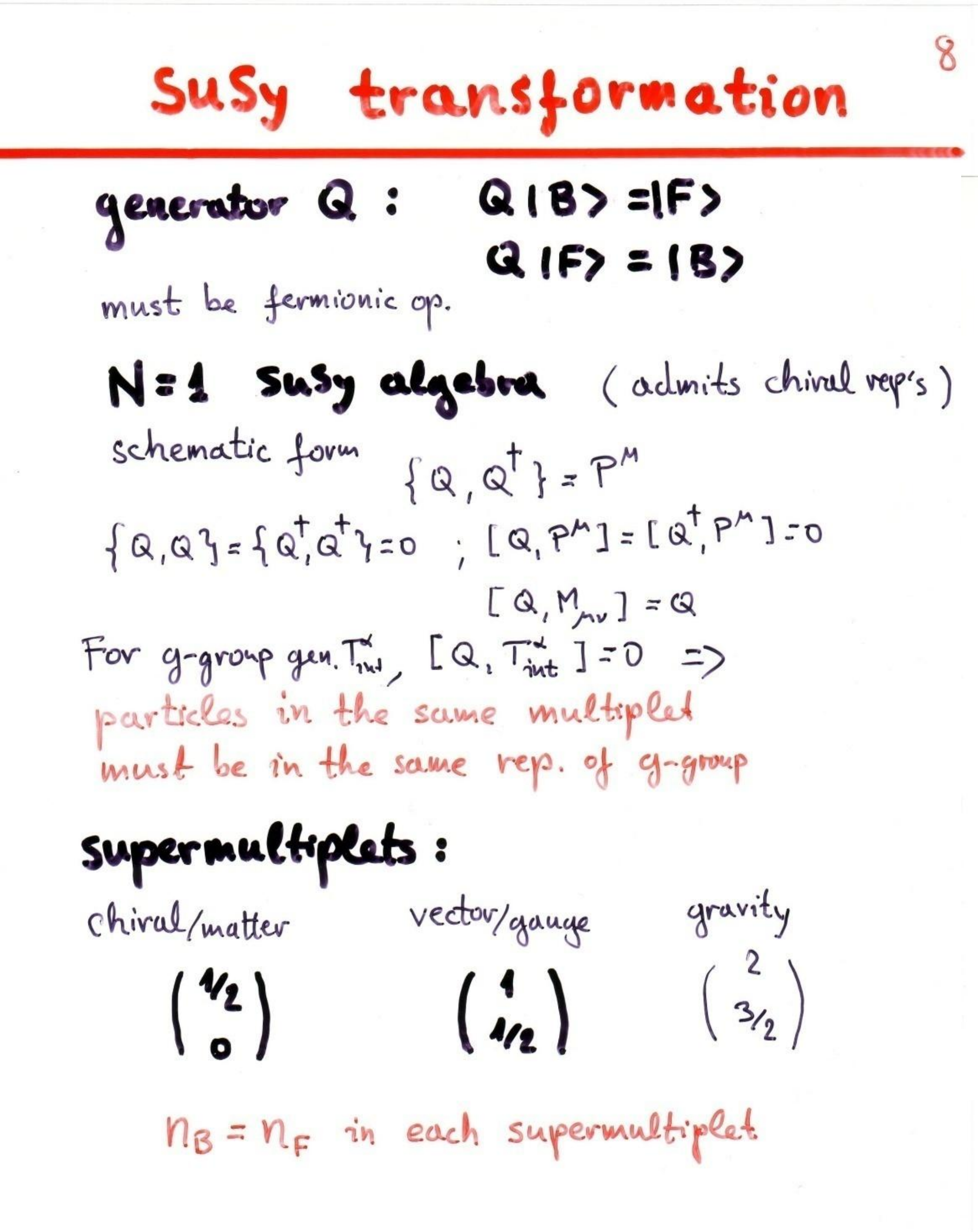


m_q, m_ℓ, $m_{W,Z}$ zero until M^2 becomes negative (during the Big Bang)

- for universal t-Higgs, W-Higgs coupling strength

SM postulates the scalar potential $V = M^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2$ with $M^2 < 0$ lowering the vacuum energy

⇒ Higgs boson, having v.e.v. $v = \sqrt{\frac{-M^2}{\lambda}}$



SuSy transformation

generator Q : $Q|B\rangle = |F\rangle$

$Q|F\rangle = |B\rangle$

must be fermionic op.

N=1 SuSy algebra (admits chiral rep's)

schematic form $\{Q, Q^\dagger\} = P^\mu$

$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$; $[Q, P^\mu] = [Q^\dagger, P^\mu] = 0$

$[Q, M_{\mu\nu}] = Q$

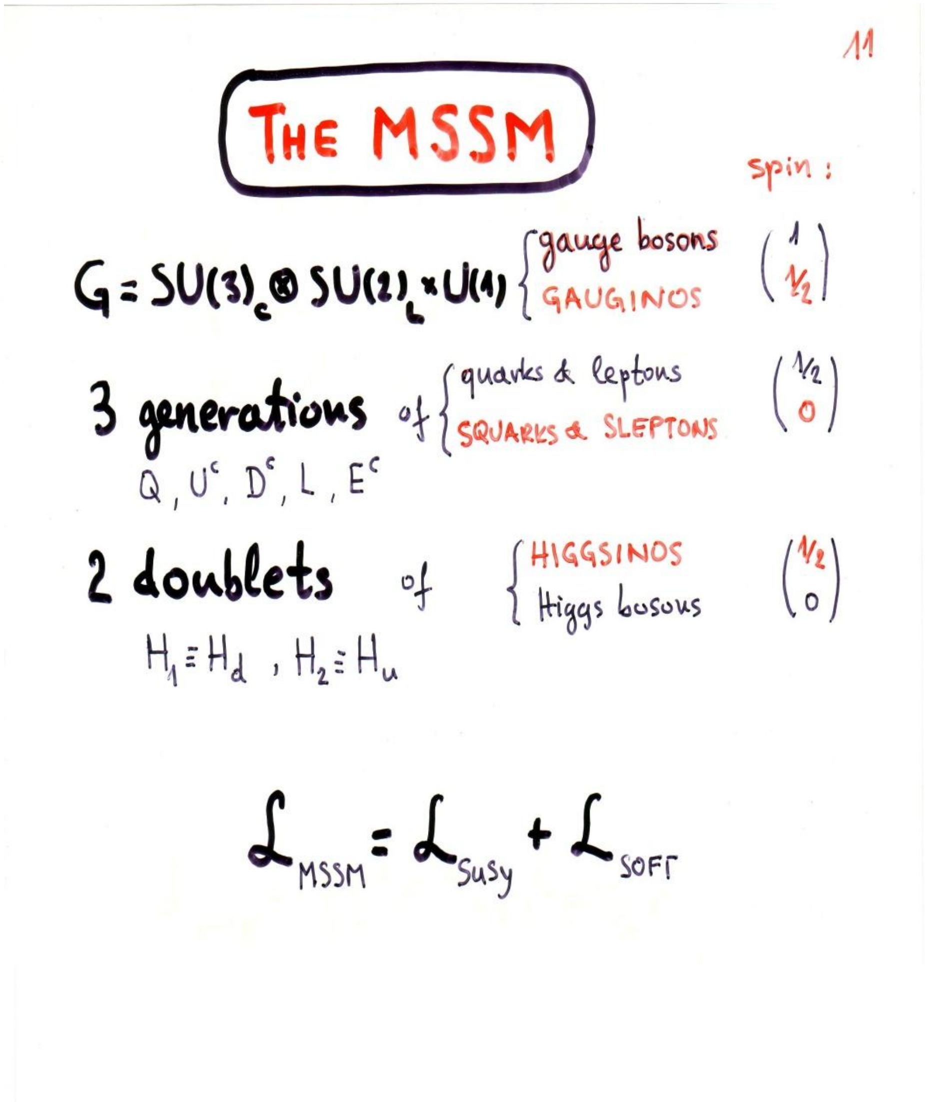
For g-group gen. T^α_{int}, $[Q, T^\alpha_{int}] = 0$ ⇒

particles in the same multiplet must be in the same rep. of g-group

supermultiplets :

chiral/matter	vector/gauge	gravity
$\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$	$\begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$

$n_B = n_F$ in each supermultiplet

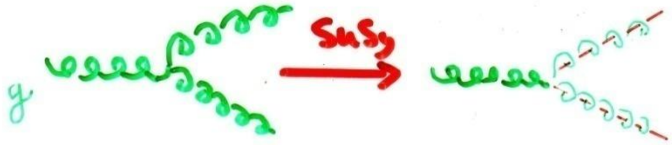


THE MSSM

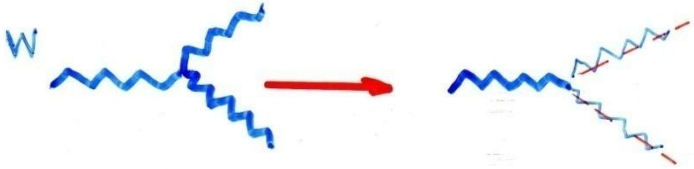
		spin :
$G = SU(3)_c \otimes SU(2)_L \times U(1)$	gauge bosons GAUGINOS	$\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$
3 generations of Q, U^c, D^c, L, E^c	quarks & leptons SQUARKS & SLEPTONS	$\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$
2 doublets of $H_1 \equiv H_d$, $H_2 \equiv H_u$	HIGGSINOS Higgs bosons	$\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$

$$\mathcal{L}_{MSSM} = \mathcal{L}_{SuSy} + \mathcal{L}_{SOFT}$$

Sector of forces



$SU(3)_c$ octet gluons (g) — octet fermions, gluinos ($\tilde{g}$)



$SU(2)_W$ I=1 bosons — I=1 fermions, winos

Triple boson gauge couplings → $g f_{abc} \bar{\lambda}^a \gamma^\mu \lambda^b A^c_\mu$

Names	spin 1/2	spin 1	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^\pm$ $\widetilde{W}^0$	$W^\pm$ W^0	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

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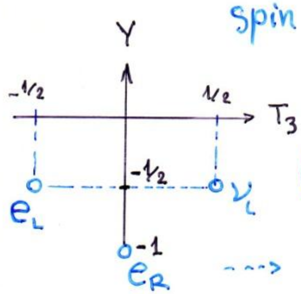
matter sector

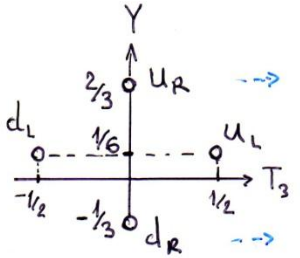
spin ½ — $SU(3) \otimes SU(2) \otimes U(1)_Y$ — spin 0



Sleptons

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $(1, 2, -\frac{1}{2})$ ↔ $\tilde{L}$

e_R ⇢ e^c_L $(1, 1, 1)$ ↔ $\tilde{E}^c$



squarks

u_R ⇢ u^c_L $(\bar{3}, 1, -\frac{2}{3})$ ↔ $\tilde{U}^c$

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ $(3, 2, \frac{1}{6})$ ↔ $\tilde{Q}$

d_R ⇢ d^c_L $(\bar{3}, 1, \frac{1}{3})$ ↔ $\tilde{D}^c$

Fermions satisfy anomaly cancellation condition

$$Tr(Y^3) = Tr(T_3^2 Y) = 0$$

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \ \widetilde{d}_L)$	$(u_L \ d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \ \widetilde{e}_L)$	$(\nu \ e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ *)
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	H_u	$(H^+_u \ H^0_u)$	$(\widetilde{H}^+_u \ \widetilde{H}^0_u)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_d	$(H^0_d \ H^-_d)$	$(\widetilde{H}^0_d \ \widetilde{H}^-_d)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ *)

*) reason for attempts to identify Higgs & sneutrino (doesn't work)

2 Higgs doublets (2HD)

$$\begin{array}{ccc} & S=0 \;|\; S=1/2 & \\ H_d = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} & \longleftrightarrow & \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix} \\ H_u = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} & & \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix} \end{array}$$

◆ **SuSy partner mix (after EW SB)**

NEUTRAL $\left\{ \begin{array}{l} \hat{H}_1^0, \hat{H}_2^0 \\ \hat{W}^3, \hat{B} \end{array} \right. \rightarrow$ **NEUTRALINOS** $\tilde{\chi}_i^0$, $i=1,2,3,4$

4 Majorana fermions

CHARGED $\left\{ \begin{array}{l} \tilde{H}_2^+, \hat{H}_1^- \\ \tilde{W}^+, \hat{W}^- \end{array} \right. \rightarrow$ **CHARGINOS** $\tilde{\chi}_i^\pm$; $i=1,2$

2 massive Dirac fermions

◆ **5 physical Higgs bosons**

3 (out of 8) d.o.f. eaten by $W^\pm, Z$

NEUTRAL $\left\{ \begin{array}{ll} CP(+) & h, H \\ CP(-) & A \end{array} \right.$ lighter & heavier

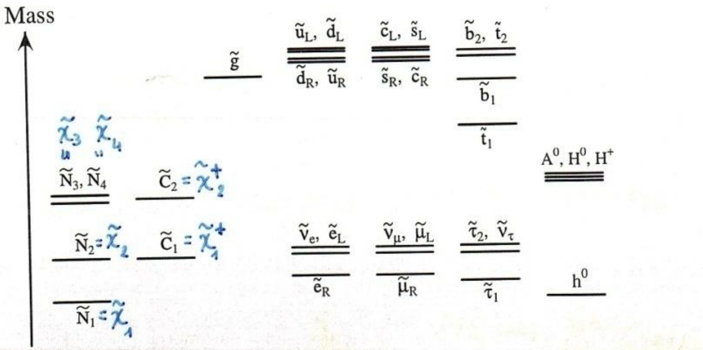
& CHARGED H^+, H^-

Undiscovered particles in the MSSM

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	+1	$h^0\ H^0\ A^0\ H^\pm$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$
squarks	0	−1	$\tilde{u}_L\ \tilde{u}_R\ \tilde{d}_L\ \tilde{d}_R$ $\tilde{s}_L\ \tilde{s}_R\ \tilde{c}_L\ \tilde{c}_R$ $\tilde{t}_1\ \tilde{t}_2\ \tilde{b}_1\ \tilde{b}_2$	" " " " $\tilde{t}_L\ \tilde{t}_R\ \tilde{b}_L\ \tilde{b}_R$
sleptons	0	−1	$\tilde{e}_L\ \tilde{e}_R\ \tilde{\nu}_e$ $\tilde{\mu}_L\ \tilde{\mu}_R\ \tilde{\nu}_\mu$ $\tilde{\tau}_1\ \tilde{\tau}_2\ \tilde{\nu}_\tau$	" " " " $\tilde{\tau}_L\ \tilde{\tau}_R\ \tilde{\nu}_\tau$
neutralinos	1/2	−1	$\tilde{N}_1\ \tilde{N}_2\ \tilde{N}_3\ \tilde{N}_4$	$\tilde{B}^0\ \tilde{W}^0\ \tilde{H}_u^0\ \tilde{H}_d^0$
charginos	1/2	−1	χ's $\tilde{C}_1^\pm\ \tilde{C}_2^\pm$	$\tilde{W}^\pm\ \tilde{H}_u^+\ \tilde{H}_d^-$
gluino	1/2	−1	$\tilde{g}$	" "
gravitino/ goldstino	3/2	−1	$\tilde{G}$	" "

& their schematic sample spectrum

(from S. P. Martin, hep-ph/9709356)



the squarks $\tilde{q}$ and the sleptons $\tilde{l}$, spin-0 partners

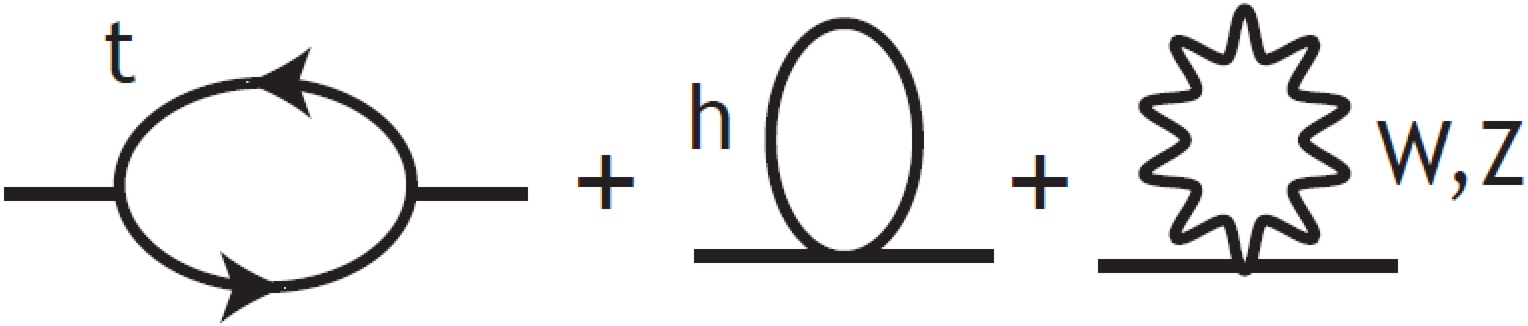
Similarly, Wino $\tilde{W}$, Bino $\tilde{B}$ and Higgsinos $\tilde{H}^{0,\pm}_{u,d}$ are the spin-1/2 superpartners
mix to give EWKinos decomposed in 4 neutralinos $\tilde{\chi}^{0}_{1,2,3,4}$ and 4 charginos $\tilde{\chi}^{\pm}_{1,2}$

colored gluinos $\tilde{g}$ and the gravitino $\tilde{G}$ are the partners of the gluon and graviton

Implikacije SuSy za Higgsovo polje

One-loop corrections to the Higgs field mass term in the Standard Model

$$\mu^2|\varphi|^2$$



depend quadratically on the ultraviolet cutoff,

$$\mu^2 = \mu^2_{bare} - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{\lambda}{8\pi}\Lambda^2 + \frac{9\alpha_w + 3\alpha'}{4\pi}\Lambda^2 + \cdots$$

Obećavajuća indikacija za moguća otkrića

a rough way to estimate the masses of new particles

to cancel the quadratic divergences in the diagrams

of mass less than 2 TeV to cancel the
top quark loop correction,
of mass less than 3 TeV to cancel the
Higgs loop correction,
of mass less than 5 TeV to cancel the
W and Z loop corrections.

Minimalno dva skalarna polja u SuSy-proširenju SM-a

a scalar field for each left- or right-handed fermion

two Higgs fields, H_u and H_d, are needed

The only allowed mass term is one involving the two Higgs fields H_u

$$\mu^2(|H_u|^2 + |H_d|^2)$$

Pokazuje se da samo jedno skalarno polje, Hu, poprima VEV

a potential energy function with a negative (mass)^2 for the H_u

U realističnom modelu – (spontano) slomljena SuSy

a model of EWSB along the following lines:

Spontaneous breaking of supersymmetry
gives mass to some new particles at very short distances,

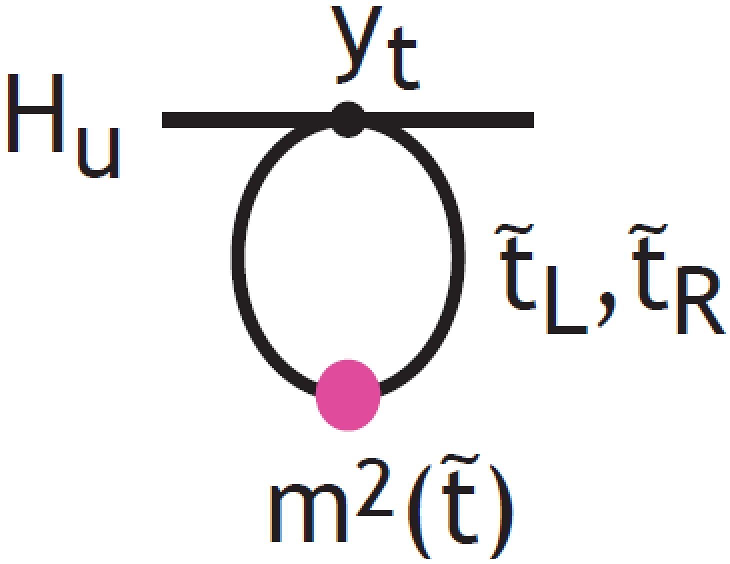
and this in turn gives mass to partners of SM particles.

- **Model s 3 skalarna polja u interakciji s Yukavinim vezanjem t-kvarka**

  $\widetilde{t}_L$ and $\widetilde{t}_R$, and the Higgs field H_u

Račun 1-petljenih korekcija masama skalara

A diagram by which the supersymmetry-breaking mass terms of $\widetilde{t}_L$, $\widetilde{t}_R$ renormalize the mass term of the Higgs field H_u



All three fields receive mass from supersymmetry

All three $(\text{mass})^2$ terms receive these negative contributions, but the correction to the Higgs mass is largest

Specifically, this can work as follows: Three scalar fields are coupled by the top quark Yukawa coupling—the scalar partners of the left- and right-handed top quarks, $\widetilde{t}_L$ and $\widetilde{t}_R$, and the Higgs field H_u. All three fields receive mass from supersymmetry breaking. Arrange that these (mass)^2 terms are all positive, approximatey equal, and of TeV size. Then compute the 1-loop corrections to these mass terms, which come from diagrams of the form of Fig. 5. This correction is negative by explicit calculation [50,51,52]. All three (mass)^2 terms receive these negative contributions, but the correction to the Higgs mass is largest, because of the factor of 3 from QCD color flowing around the loop. (The $\widetilde{t}_L$ and $\widetilde{t}_R$ mass terms also receive positive corrections from diagrams involving the supersymmetric partner of the gluon.) This calculation creates a potential energy function with a negative (mass)^2 for the H_u. It explains why this scalar field—and no other—obtains a vacuum expectation value.

Ultimately, this model of electroweak symmetry breaking is testable. The masses of the top quark partners and the Higgs boson spin-$\frac{1}{2}$ partners should not be too far above the 1 TeV mass scale. The Higgs partners, which are very difficult to discover at the LHC, could still be as light as 100 GeV [53,54]. If we could discover these particles and measure their masses and decay products, it will be possible to extract all of the parameters that enter the calculation of the Higgs potential [55]. If all of the pieces fit together, we could then claim to understand EWSB at the same level at which we understand the appearance of superconductivity in metals.

Anatomija MSSM-a

- Yukawa couplings $\Rightarrow$ Superpotential:

$$\mathcal{W} = \hat{\Phi}_1\hat{\Phi}_2\hat{\Phi}_3 \equiv \Phi_1\Phi_2\Phi_3 \quad \Longrightarrow \quad (\phi_1\phi_2)^2, \ldots, (\overline{\psi}_1\psi_2)\,\phi_3$$

- NB: part of scalar potential from gauge kinetic terms (light Higgs)
- MSSM superpotential

$$\mathcal{W}_{\text{MSSM}} = Y^u u^c Q H_u + Y^d d^c Q H_d + Y^e e^c L H_d + \mu H_u H_d$$

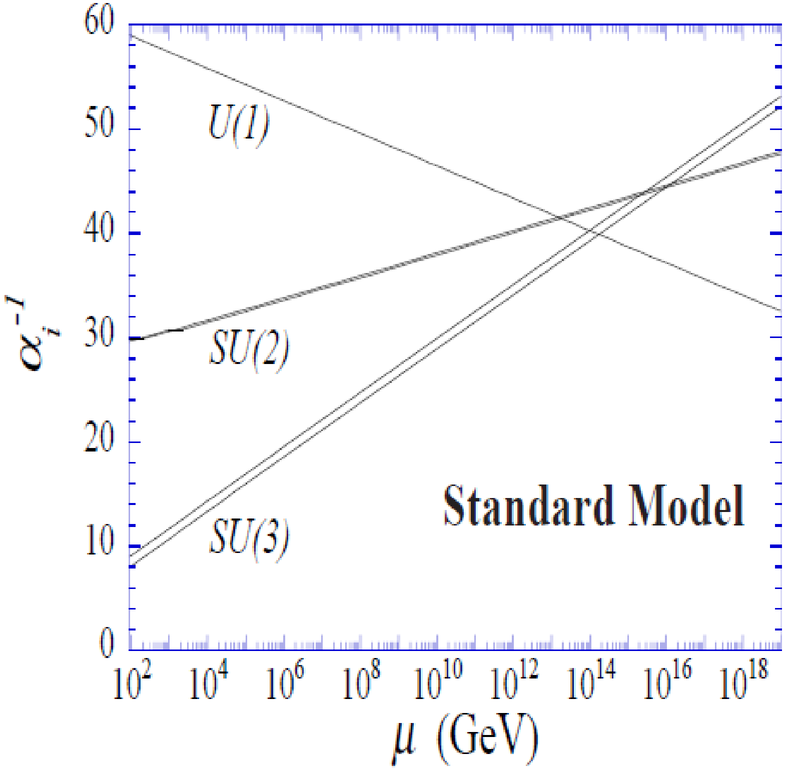
- Ignorance about SUSY breaking shows up as “soft-breaking terms”: Gaugino and sparticle masses, trilinear scalar potential terms
- μ problem: EWSB demands $\mu \sim \mathcal{O}(100\,\text{GeV} - 1\,\text{TeV})$
- Additional SUSY degrees of freedom modify vacuum polarization
  $\Rightarrow$ Unification of gauge couplings possible

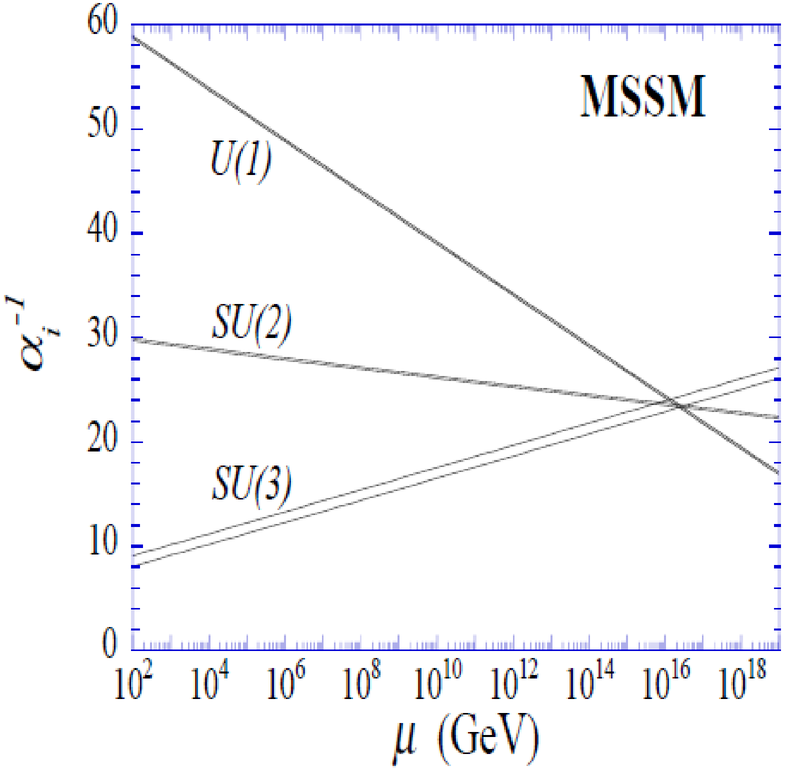
Klizna vezanja

$$\frac{dg_a}{d\log\mu} = \frac{g_a^3}{16\pi^2} B_a$$

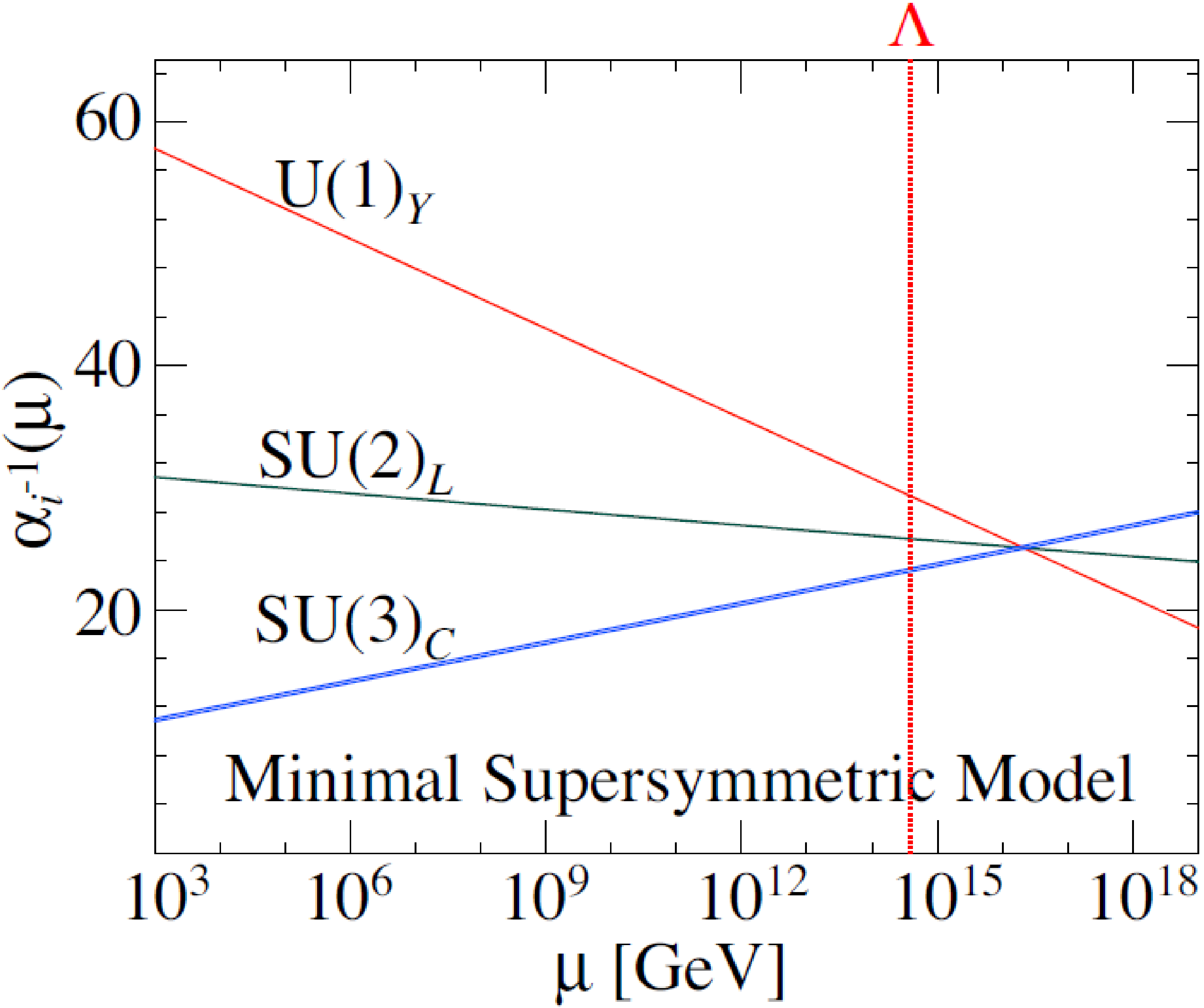
$$\text{SM} \quad B_a = \left(\tfrac{41}{10}, -\tfrac{19}{6}, -7\right)$$

$$\text{MSSM} \quad B_a = \left(\tfrac{33}{5}, 1, -3\right)$$





Skala baždarnog ujedinjenja u usporedbi s "njihalicom"



Peccei-Quinnina simetrija i aksioni

- Peccei-Quinnina simetrija
- FEČ str.390
- Aksioni – ICTP'15 Lects, S. Rajendran

θ-VAKUUMI, JAKO CP NARUŠENJE I AKSIONI

- ◇ $U(1)_A$ problem (problem velike mase η' čestice, s kraja odjeljka **??**) rješava se uvođenjem bogatije strukture **QCD** vakuuma, θ-vakuumom;
- ◇ Θ-parametar koji rješava $U(1)_A$ problem stvara problematično narušenje CP simetrije u jakom međudjelovanju;
- ◇ Činjenica da Θ-parametar mora biti fino podešen na vrlo malu vrijednost, upućuje na postojanje tzv. Peccei-Quinn (PQ) simetrije;
- ◇ Potreba da PQ simetrija bude spontano slomljena rezultira lakom pseudoskalarnom česticom *aksionom*, Goldstoneovim bozonom PQ simetrije.

θ-vakuumi

Pokazuje se da je vakuum u čistoj YM baždarnoj teoriji bez polja materije beskonačno degeneriran, s neiščezavajućim amplitudama prijelaza između baždarno rotiranih vakuuma [?],[?]. To znači da pravo vakuumsko stanje u Hilbertovom prostoru može biti napisano u obliku

$$|vac\rangle_\theta = \sum_{n=-\infty}^{\infty} e^{in\theta} \, |vac\rangle_n \;, \tag{7.88}$$

gdje n označava tzv. klasu homotopije. Taj je vakuum označen nekom vrijednošću θ, a koeficijenti $e^{in\theta}$ osiguravaju invarijantnost (do na fazni faktor) stanja $|vac\rangle_\theta$ na baždarne transformacije U_1. Naime, vakuumska stanja $|vac\rangle_n$ se na U_1 mijenjaju na način:

$$|vac\rangle_n \xrightarrow{U_1} |vac\rangle_{n+1} \;, \tag{7.89}$$

i zbog toga

$$
\begin{aligned}
|vac\rangle_\theta \quad \xrightarrow{U_1} \quad & \sum_{n=-\infty}^{\infty} e^{in\theta}|vac\rangle_{n+1} \\
= \quad & e^{-i\theta} \sum_{n=-\infty}^{\infty} e^{in\theta}|vac\rangle_n = e^{-i\theta}|vac\rangle_\theta \ .
\end{aligned}
\tag{7.90}
$$

Baždarne transformacije tipa $U_n = (U_1)^n$, koje mijenjaju klasu homotopije, ponekad se nazivaju *velike transformacije*. S druge strane, one transformacije koje su kontinuirano deformabilne u *identitet* i stoga *ne mijenjaju* klasu homotopije, nazivaju se *malim baždarnim transformacijama*. Vakuumi tipa (7.88) nazivaju se θ-*vakuumi*. S njima u vezi, u elektroslabom sektoru su *sfaleroni*, na koje ćemo se vratiti u odjeljku 7.3.2. Važnost θ-vakuuma pokazala se pri rješavanju $U(1)$ problema u **QCD**-u, spomenutog u odjeljku **??** [**?**]. No tada istovremeno iskrsava problem narušenja CP simetrije u jakom međudjelovanju.

Θ-član i jako CP narušenje

Unatoč tome što je vakuum **QCD**-a neinvarijantan na velike baždarne transformacije, pokazuje se da je moguće raditi s uobičajenim baždarno invarijantnim vakuumom. U pristupu integrala po putovima pokazuje se da se vakuumsko stanje može učiniti invarijatnim na sve baždarne transformacije ukoliko se u funkciju djelovanja doda član proporcionalan topološkom naboju, odnosno ako **QCD** lagrangianu (**??**) pribrojimo član proporcionalan gluonskoj anomaliji (**??**) :

$$\mathcal{L}(\Theta) = \Theta \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a = \Theta \mathcal{A} \,. \qquad (7.91)$$

Lako se uvjerimo da je **QCD** lagrangian za bezmasene kvarkove $\mathcal{L}_{QCD}(m = 0)$ kiralno invarijantan — simetričan na globalne kiralne rotacije $(\alpha \in \mathbf{R})$

$$\psi \to e^{i\alpha\gamma_5}\psi \; , \qquad \bar{\psi} \to \bar{\psi}e^{i\alpha\gamma_5} \; . \tag{7.92}$$

Za lijeva i desna polja $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$ te transformacije imaju oblik

$$\psi_L \to e^{-i\alpha}\psi_L \; , \qquad \psi_R \to e^{i\alpha}\psi_R \tag{7.93}$$

i pri tome bilježimo promjenu $\delta\mathcal{L} = -2\alpha\mathcal{A}$, dakle

$$\mathcal{L}_{QCD} \to \mathcal{L}_{QCD}(m = 0) + \mathcal{L}(\Theta - 2\alpha) \; . \tag{7.94}$$

Dakle, za $m = 0$ i jedan okus, Θ član se može odrotirati $U(1)$ transformacijom s $\alpha = \Theta/2$.

Ista se ideja može primijeniti i za realistični slučaj kvarkova s masama i N_F okusa. U tom slučaju vrši se nezavisna kiralna rotacija za svaki okus, što daje

$$\begin{aligned} \mathcal{L}_{QCD} \quad \rightarrow \quad & -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_j \bar{\psi}_j \left(i \gamma_\mu D^\mu \right) \psi_j \\ & - \sum_j \bar{\psi}_j m_j e^{2i\alpha\gamma_5} \psi_j + (\Theta - \sum_j 2\alpha_j) \mathcal{A} \; . \end{aligned} \tag{7.95}$$

Odavde možemo izdvojiti CP čuvajući dio $\mathcal{L}_{QCD}(\Theta = 0)$ od CP narušavajućeg $\delta\mathcal{L}_{CP}$

$$\mathcal{L} = \mathcal{L}_{QCD}(\Theta = 0, m_j \neq 0) + \delta\mathcal{L}_{CP} \; . \tag{7.96}$$

Pritom treba imati u vidu da matrice masa kvarkova dobivene spontanim lomljenjem simetrije nisu hermitske niti dijagonalne. Transformacija u bazu kvarkovskih okusa, gdje su matrice mase realne i dijagonalne, uključuje dodatnu kiralnu $U(1)_A$ rotaciju oblika (7.93) s

$$\alpha = \Theta_{EW}/2N_F \ , \qquad \Theta_{EW} = \arg\det M = \arg\left(\det M^U \det M^D\right) \ , \qquad (7.97)$$

gdje su M^U i M^D matrice mase gornjih i donjih kvarkova. Dakle, uključivanjem mase pribraja se originalnom **QCD** parametru Θ_{QCD} elektroslabi parametar Θ_{EW} :

$$\Theta \equiv \Theta_{QCD} \rightarrow \bar{\Theta} = \Theta_{QCD} + \Theta_{EW} \ . \qquad (7.98)$$

PQ simetrija i aksion

Malena veličina u fizici naznačuje prisutnost određene simetrije. Za bezmasene kvarkove smo vidjeli da se parametar Θ može ukloniti $U(1)$ kiralnom rotacijom. Za masivne kvarkove prepreka takvom odrotiranju dolazi od neinvarijantnosti člana $\bar{\psi}\psi$. Peccei i Quinn [**?**] su primjetili da se ideja kiralne rotacije može primijeniti i na takav član. U pristupu generiranja masa spontanim lomljenjem simetrije riječ je o Yukawinom članu $\bar{\psi}\psi\Phi$ koji se može učiniti kiralno invarijantnim ako odgovarajuća $U(1)_{PQ}$ simetrija istovremeno rotira i Higgsova polja! Pokazuje se da je za to potrebno imati *dva dubleta* (ϕ_1, ϕ_2) Higgsovih polja. Tada ukupni lagrangian posjeduje dodatnu simetriju $U(1)_{PQ}$ kojom se Θ može odrotirati!

Budući da Higgsov potencijal $V(\phi_1, \phi_2)$ mora biti $SU(2)_W \times U(1)_Y \times U(1)_{PQ}$ invarijantan, dodatna simetrija će također doživjeti spontano lomljenje na $U(1)_{em}$ putem vakuumskih očekivajućih vrijednosti

$$\begin{aligned} \phi_i &\rightarrow \phi_i + v_i \,, \qquad i = 1, 2 && (7.104) \\ \langle \phi_1 \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \,, && (7.105) \\ \langle \phi_2 \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \,. && (7.106) \end{aligned}$$

Stoga umjesto $U(1)_{PQ}$ očekujemo postojanje Goldstoneovog bozona pridruženog toj simetriji - *aksiona* [?], [?]. U najjednostavnijoj slici, gdje je skala PQ simetrije

$$\Lambda_{PQ} \simeq v = \sqrt{v_1^2 + v_2^2} \simeq 250 \text{ GeV} = \Lambda_{QFD}, \qquad (7.107)$$

očekuje se [?]

$$m_a \simeq \frac{m_\pi f_\pi}{v} \simeq 100 \text{ eV} \,. \qquad (7.108)$$